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THE ACCUMULATION OF HUMAN CAPITAL: ALTERNATIVE METHODS AND WHY THEY MATTER

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Abstract

We show how the ability to accumulate human capital through formal education and through a learning-by-doing process that occurs on the job affects the dynamic behavior of the human capital stock under a liquidity constrained and a non-constrained case. When there are alternatives to formal schooling in the accumulation of human capital, investing resources in increasing school enrollment rates in low-income countries may not be the most efficient means of increasing the human capital stock. In addition, removal of the liquidity constraints may not be sufficient to escape a development trap.

THE ACCUMULATION OF HUMAN CAPITAL. ALTERNATIVE METHODS AND WHY THEY MATTER

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1. Introduction

A survey of the recent growth literature would reveal at least one consistent policy prescription: invest in human capital. Although the importance of human capital has been established, the crucial question of how this policy should be implemented still remains to be answered. We take this question Up in this paper, considering how the ability to accumulate human capital through both formal training and through on-the-job experience affects the dynamic behavior of the human capital stock.

Empirical evidence from micro data has consistently shown that work experience is an important determinant of an individual human capital. The model we present In this paper acknowledges this evidence, broadening the definition of human capital to include on-the-job learning as well as general education. The insights gained through this approach allow us to draw several important conclusions. First, a policy that focuses exclusively on achieving large increases in enrollment rates may not be the most efficient policy for increasing an economy's stock of human capital. Alternative policies that encourage industries which produce jobs that allow for more on-the-job learning could achieve the same objective in a similar time frame with less resources. This conclusion is consistent with the recent experiences of several developing countries that have

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experienced large increases in enrollment rates but not corresponding increases in output.[2] In contrast, many of the East Asian economies specializing in manufacturing high-tech goods have experienced large increases m output with less dramatic rates of increase in school enrollment.

Recent work in the economics literature has provided mixed evidence in support of educational investment as a determinant of economic growth. On the one hand, Barro (991) and Mankiw, Romer and Weil (1992) demonstrate that human capital, even when defined only to include educational attainment, directly affects economic growth. 01] the other hand, Benhabib and Spiegel (1994) find empirical support for their claim that human capital is not a factor of production and is (rely relevant m determining the level of technology employed in an economy. Pritchett (1995) makes an even wronger point, showing that increases in educational capital due to improvements in the educational attainment of the labor force has a negative impact on the growth of output per worker.

In contrast to mixed evidence from aggregate data, micro data has consistently shown that earnings and productivity increase with an individuals education.[3] Our interpretation is consistent with both the micro and macro evidence; it allows individuals' productivity to increase with their level of education but recognizes that the extent of the productivity increase will depend on the existing level of human capital In this regard, our model is similar to some of the current work m the growth

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[2]Pritchett (1995) provides a summary of relevant stylized facts: In the yew-s between 1960 and 1985, the educational attainment of the labor force m the Sub-Saharan Africa grew at a faster rate than that in any other region. Yet growth of output was about half that of Latin America and about quarter that of more rapidly growing regions [See Easterly and Levine (1995) for a detailed account of the economic performance of countries in Sub-Saharan Africa]. Similarly, in other less developed countries as a whole, enrollment rates have increased significantly in the last thirty years while growth rates have been stagnating or even falling. In countries and regions that have recorded considerably high levels of educational attainment such as Sri Lanka, Costa Rica, Jamaica, the Philippines and parts of" India, output levels have been significantly low for educational levels.

[3]See, for example, Mincer (1993).

literature.[4] However, by acknowledging alternative methods of accumulating human capital, it differs significantly from the existing work in that it provides an explanation for why advocating increases in enrollment rates may not be the most efficient policy recommendation for economic development. In fact, in countries in which the existing level of human capital is very low, our model identifies the possibility that increases in enrollment rates could actually *decrease the* economy's stock of human capital in the short to intermediate term.[5]

The introduction of another method of accumulating human capital creates the possibility of multiple equilibria, even for preferences and technology that would otherwise not generate such a result. Importantly, this possibility is not erased when credit market imperfections are removed and there is no explicit cost to education. Another conclusion we are able to draw from our model is that the amount of education individuals desire may increase with the level of development. Thus, the causality between education and growth runs both ways. This conclusion is particularly strong when individuals cannot obtain loans to finance first period consumption.

Another conclusion of our analysis that is relevant to policy makers is that liquidity constraints, even when there are no out-of-pocket expenses for schooling, may limit the growth and level of per capita income. While the important role of liquidity constraints is not new to our paper, we add to this discussion the observation that although removal of liquidity constraints is a necessary

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[4]Some studies, such as Tamura (1991) and Galor and Tsiddon (1994) have focused on an education-type investment as the primary channel for human capital accumulation. Others, such as Stokey (1988) and Young (1991) have examined the growth effects of learning-by-doing. Our formulation for human capital accumulation, however, is perhaps most closely related to work that has been done by labor economists who have long recognized the importance of *both* work experience and education as a determinant of individual earnings [See, for example, Becker (1993), Mincer (1993) and Mincer (1995)].

[5]In a related paper, Fershtman, Murphy and Weiss (1996) also develop a model in which an increase in the number of educated workers can be associated with a decrease in the growth rate. In their model, a high social status of educated workers can result in an inefficient allocation of education as high-income, low-ability people "crowd (ret" low-income, high-ability people from growth-producing educated occupations."

condition for maximizing per capita income, it is not a sufficient condition. Economies with very low initial levels of human capital may not be able to escape a development trap through a policy that focuses exclusively on removing liquidity constraints.

Our conclusions are reached in the following four sections. Section 2 provides an overview of the model. Section 3 describes the basic elements of production and individual decision making. Section 4 considers the evolution of the human capital stock with and without liquidity constraints and Section 5 concludes.

2. Overview

The model below considers a small open economy in an overlapping generations framework in which both physical capital and a labor input are factors of production. In the first period of life, individuals divide their time between working and going to school. Both activities increase an individuals second period human capital and, thus, increase the amount of the labor input the individual can supply in the second period. A high parental level of human capital influences the effectiveness of time spent in training more than it influences the effectiveness of accumulating human capital through the learning-by-doing that occurs on the job. As a result, when the previous generation has a low level of human capital, individuals may prefer to accumulate human capital through work experience and not through training. As the level of parental human capital rises, however, schooling becomes a more effective means of accumulating human capital and individuals choose more education over work in the first period. When loans for first period consumption are not available, the amount of time spent in school in the first period gradually increases with the parental stock of human capital. When individuals can borrow against second period income to finance first period consumption, however, they either spend all or none of their time in school.

This setup generates the possibility of multiple equilibria. When credit markets are perfect, there can be two steady states: a low human capital steady state in which individuals accumulate human capital only through work experience and a high human capital steady state in which individuals spend all of the first period in school. Both of these steady states are stable. If there are credit market imperfections, there can be three steady states: the low human capital steady state analogous to the low human capital steady state mentioned above and two additional steady states in which individuals spend time both working and getting an education in the first period. In this case, however, only the upper and lower steady states are stable. The following section describes the model economy more completely.

3. The Model

3.1. Production

The output of the economy is a single homogeneous good produced by a CRS production function that uses capital and labor as inputs. The output produced at time t, Y_t, is given by

$$Y_t = F(K_t, L_t) = L_t f(k_t); \; k_t \equiv \frac{K_t}{L_t} \tag{1}$$

where K_t and L_t denote the quantities of physical capital and labor employed at time t. The production function $f: R_+ \to R_+$ satisfies the standard neoclassical assumptions. Namely, $f'(k_t) > 0$, $f''(k_t) < 0$, $\lim_{k \to 0} f'(k_t) = \infty$ and $\lim_{k \to \infty} f'(k_t) = 0$.

We assume that production is carried out in a perfectly competitive environment, and both factors earn their marginal products. Thus,

$$r_t = f'(k_t) \; and \; w_t = f(k_t) - f'(k_t)k_t \tag{2}$$

where r_t and w_t denote the interest rate on physical capital and the wage rate paid to labor at time t.

The economy under consideration is open and small, and the world interest rate is constant at $\bar{r}$. Since the open economy permits perfect capital mobility, its interest rate is constant at $\bar{r}$ as well. Therefore, the ratio of physical capital to labor, k_t, and the wage rate paid to labor, w_t, are also constant.

$$r_t = \bar{r} = f'(k_t) \;\; - \;\; k_t = \bar{k} = f'^{-1}(\bar{r}) \tag{3}$$

and,

$$w_t = f(\bar{k}) - f'(\bar{k})\bar{k} = \bar{w} \tag{4}$$

3.2. Individuals

Individuals live for two periods in overlapping generations. The size of the Population is normalized to one and there is no population growth. Individuals are endowed with one unit of time and with μ, $\mu > 0$, units of physical labor input in both periods. In the first period, they allocate their time and physical labor between work and education and they consume. In the second period, they supply all of their time and physical labor plus the human capital they accumulated in the first period to the labor market and consume. Thus, first period income is $x_t\mu w$, and second period income is $(\mu + h_{t+1})w$, where x_t is the fraction of time spent working in the first period and h_{t+1} is the human capital supplied in the second period. Because there are no out-of- pocket expenses associated with formal training, getting an education affects first period resources only by reducing the amount of labor income.

Both work experience and formal training in the first period increase the amount of human capital supplied in the second period. In addition, the parent's level of human capital enhances the child's human capital accumulation. Specifically, an individual's second period human capital is given by

$$h_{t+1} = v(h_t)x_t + z(h_t)e_t \tag{5}$$

where h_t is the parental level of human capital. x_t is the time spent working. e_t is the fraction of time spent in training. and v(h) and z(h) are mm-decreasing functions of the parental human capital such that

$$\begin{gathered} V'(h_t), z'(h_t) \geq 0, \quad v''(h_t), z''(h_t) \leq 0, \quad \forall h_t > 0 \\ v(h_t) \geq z(h_t) \ \forall \ h_t \leq \hat{h} \\ v(h_t) < z(h_t) \ \forall \ h_t > \hat{h} \\ \lim_{h \to \infty} z'(h_t) = 0 \end{gathered} \tag{6}$$

A key feature of the specification above is that at low levels of parental human capital, working m the first period will provide for higher levels of human capital m the second period; but at higher levels of parental human capital. formal training results in higher human capital accumulation.[6] We should also note that m the context of our model, education can be interpreted broadly to include not only traditional schooling but also other forms of formal training such as vocational training or apprenticeships, Thus, in our formulation. the human capital accumulated through work experience is exclusively a result of learning by doing.

The important role that a parent's human capital plays in increasing the efficiency with which a child accumulates human capital can be justified in light of Coleman et al. (1966). Hanushek (1986) and Fuchs and Reklis (1994) which show that parent's education is important in determining the child's level of education. More specifically. Coleman et al. (1966) investigate the relative importance of family backgrounds in educational attainment and conclude that differences m backgrounds and

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[6] An example of a function that satisfies the properties discussed above is:

$$h_{t+1} = ah_t^{\beta}x_t + bh_t^{\alpha}e_t \ ; \ 0 \leq \beta < \alpha < 1 \ ; \ a,b > 0$$

characteristics of peers in school play a more important role than quality differences among schools. Hanushek (1986), in a survey of the literature on educational studies, remarks that general conceptual models depict the achievement of a given student as a function of the inputs of family, peers and teachers interacting with innate personal abilities. Fuchs and Reklis (1994) provide evidence that family and child characteristics, but not schools, influence math achievement of eighth-grade students in the U.S.

In addition to this evidence on parental roles in children's educational achievement in a developed country, other studies have emphasized the importance of learning-by-doing and the transmission 01 this knowledge to other family members in developing countries. Rosenzweig and Wolpin (1985) document the positive influence that older, more experienced, family members have on farm profitability in India, finding that the presence of an older family memhcr increases txd-weather profits by 14% (compared to an 8.5% increase resulting from an additional year of schooling to the most educated individual in the household). Foster and Rosenzweig (1995) also show the importance of learning-by-doing in developing countries, finding that experience with high-yielding seed varieties is essential for generating profits with their use One of the results of' this study of Indian households is that experienced farmers were able to earn profits while reexperienced fanners, both educated and uneducated, could not. Losses to inexperienced educated farmers were smaller than those to inexperienced and uneducated farmers, indicating that education did add to a farmer's human capital, but to a lesser degree than previous experience.

Since, in the first period, individuals allocate their time between education and work, it follows that

$$e_t + x_t \leq 1 \quad (7)$$

Individuals receive utility from consumption in both periods, with the utility of an individual of generation t given by

$$U_t = U(c_t, c_{t+1}, \delta) \qquad (8)$$

where c_t and c_{t+1} denote the **consumption** of the individual **in the** first and second periods. δ, $0 \leq \delta \leq 1$, is the discount factor that the individual applies to second period consumption, and $U(c_t, c_{t+1}; \delta)$ satisfies the usual conditions that ensure an interior solution.

Individuals maximize their utility subject to (7) and the following budget constraint:

$$(1 + \bar{r})c_t + c_{t+1} + (1 + \bar{r})\bar{w}\mu e_t \leq (2 + \bar{r})\bar{w}\mu + \bar{w}h_{t+1} \qquad (9)$$

4. The Evolution of the Economy

The optimal amount of time allocated to education, e_t, and to work, x_t, by the individual in the first period (and, therefore the evolution of the economy as a whole), will depend on whether there exists binding liquidity constraints on individuals' first period consumption choices. In what follows, we first analyze the evolution of a non-liquidity constrained economy in which individuals can borrow to finance consumption. Then, we consider an economy in which individuals are liquidity constrained and cannot borrow to finance first period consumption.

The evolution of this economy, and, in particular, the evolution of the stock of human capital, $\{h_t\}_{t=0}^{\infty}$, is governed by an autonomous, first-order difference equation. The evolution of the human capital stock, $\{h_t\}_{t=0}^{\infty}$, in turn determines the evolutions of the time allocated to education, $\{e_t\}_{t=0}^{\infty}$, and to work, $\{x_t\}_{t=0}^{\infty}$, and, therefore, the evolution of per-capita income, $\{y_t\}_{t=0}^{\infty}$.

4.1. **The Case of No Liquidity Constraints**

Let h^* be the threshold level of parental human capital below which individuals do not choose any education in the first period. Then, given the assumptions outlined in (6), h^* is strictly greater than $\hat{h}$ since the opportunity cost of getting an education includes not only the foregone human capital accumulated through work experience but also the earnings associated with working. Since $\mu >$ 0. at $\hat{h}$ the opportunity cost of education is strictly greater than the opportunity cost of working. Thus, when there are no binding liquidity constrains on consumption in the first period. the optimal amount of time allocated to education by the individual. c,. is given by the following:

$$e_t = \phi(h_t) = \begin{cases} 0 & if\ h_t \leq h^* \\ 1 & if\ h_t > h^* \end{cases} \tag{lo}$$

where h" is such that v(h*) + u = /.(1?")

When there are not liquidity constraints. m the first period. individuals devote all of their time to either work or to education. with the choice depending on the relative marginal returns of each activity Specifically, individuals borrow to finance first period consumption and allocate all of their time in the first period to getting educated when their parental human capital stock is sufficiently high. Otherwise. at low levels of the parental human capital stock. individuals choose to spend all of their time at work in the first period.

Noting that e, + x, = 1, we can describe the time path of the human capital stock by combining equations (5) and (lo):

$$h_{t+1} = \psi(h_t) = v(h_t)[1-\phi(h_t)] + z(h_t)\phi(h_t) = \begin{cases} v(h_t)\ if\ h_t \leq h^* \\ z(h_t)\ if\ h_t > h" \end{cases} \tag{11}$$

where the initial stock of human capital. h,,. is historically given.

Along the dynamic path. h, evolves monotonically. Namely,

$$\lim_{h_t \to \infty} \psi'(h_t) = 0 \tag{14}$$

$$\frac{\partial h_{t+1}}{\partial h_t} = \psi'(h_t) = \begin{cases} v'(h_t) \geq 0 & if\ h_t < h^* \\ z'(h_t) > 0 & if\ h_t > h^* \end{cases} \quad (12)$$

$$\frac{\partial^2 h_{t+1}}{\partial h_t^2} = \psi''(h_t) = \begin{cases} v''(h_t) \leq 0 & if\ h_t < h^* \\ z''(h_t) \leq 0 & if\ h_t > h^* \end{cases} \quad (13)$$

We define a steady-state equilibrium as a stationary stock of human capital, $\bar{h}$, such that

$$\bar{h} = \psi(\bar{h}) \quad (15)$$

Once the human capital stock reaches its steady-state, $\bar{h}$, the amount of time that individuals allocate to education, time allocated to work, and per capita income reach their steady-state levels, respectively denoted by $\bar{e}$, $\bar{x}$, $\bar{y}$, as well. If $\psi(0) = v(0) > 0$ or if $\lim_{h \to 0} v'(h) > 1$, then we know a non-trivial steady state exists.[8] Figure 1 shows three possibilities for $\psi(h_t)$. In figure 1 A, there is a unique non-trivial steady state in which individuals choose no education, in 1 B there is a unique non-trivial steady state in which individuals do not choose to work in the first period, and in 1C, there are two steady states, one in which there is no work and one in which there is no education in the first period. In Section 4.4, we give an example in which, depending on parameter values, all three cases are possible.

4.2. The Case of Liquidity Constraints

When there are liquidity constraints, individuals cannot borrow to finance their consumption in the first period of their lives. As a result, individuals' first period consumption cannot exceed their first period income. Thus, to ensure positive consumption in both periods, they will always choose to allocate some portion of their time in work in the first period.

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[8]Even when only the trivial steady state, $h=0$, exists, $y > 0$ since individuals would still earn $w\mu>0$ in both periods. This event would occur if work experience and education provided relatively little human capital.

The optimal amount of time allocated to education by the individual. e_t, when there are binding liquidity constraints on consumption. is given by the following:

$$e_t = \phi(h_t) = \begin{cases} 0 & \text{if } h_t \leq h^* \\ e(h_t) > 0 & \text{if } h_t > h^* \end{cases} \tag{16}$$

Note that the amount of time allocated to education. e_t, is a non-decreasing function of the parental human capital stock. and the amount of time allocated to work in the first period. x_t, is a non-increasing function of the parental human capital stock. Thus. $\phi'(h_t) = 0$ when $h < h^*$, and $\phi'(h_t) > 0$ when $h_t > h^*$. We are able to describe the dynamic behavior of the human capital stock when there are liquidity constraints by combining equations (5) and (16):

$$h_{t+1} = \psi(h_t) = v(h_t)[1 - \phi(h_t)] + z(h_t)\phi(h_t) = \begin{cases} v(h_t) \text{ if } h_t \leq h^* \\ v(h_t)[1 - e(h_t)] + z(h_t)e(h_t) \text{ if } h_t > h^* \end{cases} \tag{17}$$

Moreover,.

$$\lim_{h_t \to \infty} e(h_t) < 1 \tag{18}$$

As in the case of no liquidity constraints examined above. h_t evolves monotonically along the dynamic path. **Specifically.**

$$\frac{\partial h_{t+1}}{\partial h_t} = \psi'(h_t) = \begin{cases} v'(h_t) \geq 0 & \text{if } h_t < h^* \\ z'(h_t)e(h_t) + v'(h_t)[1 - e(h_t)] + e'(h_t)[z(h_t) - v(h_t)] > 0 & \text{if } h_t > h^* \end{cases} \tag{19}$$

$$\frac{\partial^2 h_{t+1}}{\partial h_t^2} = \psi''(h_t) = \begin{cases} v''(h_t) \le 0 & \text{if } h_t < h^* \\ z''(h_t)e(h_t) + v''(h_t)[1-e(h_t)] + 2e'(h_t)[z'(h_t) - v'(h_t)] \\ \quad + e''(h_t)[z(h_t) - v(h_t)] & \text{if } h_t > h^* \end{cases} \tag{20}$$

where the sign of $\psi''(h_t)$ when $h_t > h^*$ is indeterminate.

As in the liquidity constrained case, if $\psi(0) = v(0) > 0$ or if $\lim_{h\to 0} v'(h_t) > 1$, a non-trivial steady state exists. When there are liquidity constraints, however, $\psi(h_t)$ is a continuous function of the human capital stock, h_t. Figure 2 shows three possibilities for the $\psi(h_t)$ function in the liquidity constrained case. Figure 2A generates a unique steady state with no education, 2B generates a unique steady state in which individuals both work and go to school and 2C shows the case in which multiple equilibria exist. At h_1 there is no education and at h_2, there is both work and education in the first period. (See the example in Section 4.4 that demonstrates the possibility of each of these three cases.)

4.3 Discussion of Policy Alternatives

In considering the effects of human capital accumulation on economic growth, the analysis above demonstrates that a broader definition of human capital that includes elements of formal training and on-the-job learning leads to significantly different policy implications than the ones suggested by models that define human capital more narrowly.

First, the presence of alternative means of human capital accumulation implies that increasing school enrollments will not necessarily increase an economy's stock of human capital. In fact, when the level of human capital in the economy is sufficiently low ($h_t < \hat{h}$) increasing school enrollments will actually decrease the economy's stock of human capital in the short run. This point is illustrated in Figure 3 which shows the time path of human capital for three different initial levels of human capital under the assumption that a policy devised so that the amount of time spent in school is set equal to the level that would be chosen in the high-income steady state. Thus, the presence of alternative methods of human capital accumulation can help explain the observations documented in Benhabib & Spiegel (1994) and Pritchett (1995) regarding the lack of increase in per capita income in low income countries that have recently experienced large increases in school enrollment rates.

In the absence of a scheme that commits future generations to subsidize the consumption of current generations, increasing school enrollment rates beyond the level chosen by utility-maximizing agents clearly trades off the welfare of current and future generations. Welfare along the transition to the steady state is lower in the presence of such a policy since, left to their own devices, individuals would have chosen less education. If the policy is successful in achieving the high-income steady state however future generations will be much better off than they would have been otherwise.~

While increasing school enrollment rates is a policy that will be successful in the long-run, as pointed out above, it may be costly in the short-run An additional policy that our approach identifies is that of attracting jobs to an economy that have a higher learning content, thus allowing individuals to increase their human capital without attending more school and foregoing first period wages. Figure 4 shows the time path of human capital for various levels of the marginal productivity of work experience and illustrates that fact that if the value of" work experience is either too high or [()() low the economy will not achieve the high-income steady state. When the value of work experience is too low, the economy cannot accumulate enough human capital to make education worthwhile: and when the value of work experience is too high, individuals choose not to become educated.

'The analysis above also shows that the existence 01 liquidity constraints affects the optimal amount of time allocated to work and to education. Specifically, as we show in the next section with

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'While the issue of education financing is not specifically addressed in our model, our analysis allows us to point out that, when the accumulation of human capital is positively influenced by the existing level of human capital, running a government budget deficit to finance the accumulation of education may be an appropriate out-of-steady-state policy since it allows future generations to make transfers to their ancestors. In the steady state, however, when the stock of human capital is no longer growing, this intergenerational transfer is no longer appropriate and the government budget deficit should tie approaching zero.

an example. the threshold level of parental human capital below which individuals devote no time to education. h*. does not depend 011 the discount factor that individuals apply to future consumption. when no liquidity constraints exists. whereas. it does depend on the discount factor when liquidity constraints exist The reason for this result is simple--when no liquidity constraints exist. individuals choose the optimal time devoted to education only co maximize their inter-temporal income: but when con.. traints exist. they take into account their consumption time preference as well. Put differently. the threshold level of parental education. h* will be greater when I iquidity constraints exist i! the discount factor is relatively low. An implication 01 this analysis is that policies that aim to remove liquidity con.. traints on individuals consumption in the earlier part of their lives will result in individuals devoting more time to education, provided that their parental human capital stock is sufficiently high. Therefore, the removal of liquidity constraints can help those less developed economies that have sufficiently high levels of parental human capital stock avoid a development trap (if it exists). Nevertheless. as we have shown above. multiple equilibria may exist even when there are no liquidity constraints. As a result. when an economy's stock of human capital is sufficiently low. the removal of liquidity constraints will not allow the economy to avoid the development trip.

Another observation that results from comparing the liquidity and non-liquidity constrained cases is that the level of human capital and per capita income in the high income steady state is higher when there are no liquidity constraints since the presence of' consumption loans allows individuals to spend all of the first period getting an education. Tires. wnen this observation is combined with the discussion m the previous paragraph. it is apparent that removing liquidity constraints is a necessary but not sufficient condition to achieve maximum per capita income.

Unfortunately. we cannot conclude thrs section with one overriding recommendation appropriate for all countries. Rather. we are able to use our model to identify the circumstances under which certain policies will he appropriate and to offer an explanation as to why increases in education do not always translate into increases in per capita income

4.4 An Example

In this section, we provide an example that illustrates the properties discussed above. Let

$$
\begin{aligned}
h_{t+1} &= a x_t + b h_t^{\alpha} e_t \\
U(c_t, c_{t+1}) &= \ln(c_t) + \delta \ln(c_{t+1}) \; ; \quad 0 \leq \delta \leq 1
\end{aligned}
\tag{21}
$$

Then, when there are no liquidity constraints

$$
e_t = \phi(h_t) = \begin{cases} 0 & \text{if } h_t \leq h^* = \left[\dfrac{a + (1+\bar{r})\mu}{b} \right]^{\frac{1}{\alpha}} \\ 1 & \text{if } h_t > h^* \end{cases}
\tag{22}
$$

$$
h_{t+1} = \psi(h_t) = \begin{cases} a & \text{if } h_t \leq h^* \\ b h_t^{\alpha} & \text{if } h_t > h^* \end{cases}
\tag{23}
$$

$$
\frac{\partial h_{t+1}}{\partial h_t} = \psi'(h_t) = \begin{cases} 0 & \text{if } h_t < h^* \\ \dfrac{\alpha b}{h_t^{1-\alpha}} > 0 & \text{if } h_t > h^* \end{cases}
\tag{24}
$$

$$
\frac{\partial^2 h_{t+1}}{\partial h_t^2} = \psi''(h_t) = \begin{cases} 0 & \text{if } h_t < h^* \\ -\dfrac{\alpha(1-\alpha) b}{h_t^{2-\alpha}} < 0 & \text{if } h_t > h^* \end{cases}
\tag{25}
$$

In this economy, there exists a steady-state equilibrium in which individuals allocate all of their time to work in the first period if $\lim_{h_t \to h^*} \psi(h_t) < h^*$. We find that this occurs when the physical labor endowment of individuals, μ, is sufficiently large and/or the effect of work experience on human capital accumulation, a, takes on an intermediate value. **Namely.**

$$\mu > \frac{a^{\alpha}[b - a^{1-\alpha}]}{1 + \bar{r}} \tag{26}$$

Equation (26) says that individuals will choose to work in the first period when 1) the opportunity cost of education is high (μ is large), or 2) if work experiences is particularly valuable in accumulating human capital (a is large). Note that very small values of a will also generate a steady state in which there is no education. While the choice not to get an education when work experience provides little human capital may appear irrational. it is not. A steady state with a low a is consistent only with a small amount of parental human capital. At this steady state, the low level of parental human capital results in a low marginal productivity for education in producing human capital. Thus. the positive opportunity cost of becoming educated causes individuals to chose work over education.

When there are no liquidity constraints. a steady state in which individuals choose not to work in the first period exists if the effect of education on human capital accumulation for a given level of the parental human capital stock. b, is sufficiently large relative to the effect of work experience on human capital accumulation. a. Specifically. $\lim_{h_t \to h^*} \psi(h_t) > h^*$ if and only if

$$b > [a + (1 + \bar{r})\mu]^{1-\alpha} \tag{27}$$

Interpretation of the condition in (27) is straightforward--individuals obtain an education when its marginal productivity in accumulating human capital is relatively high (b is large) and the opportunity cost of education is relatively low (a and μ are small). When parameter values are such that both equations (26) and (27) are satisfied. then without further restrictions. there exists multiple steady-slate equilibria in this economy.

When there are liquidity constraints, the optimal amount of time devoted to education and the evolution of the human capital stock can be characterized by

$$e_t = \phi(h_t) = \begin{cases} 0 & \text{if } h_t \leq h^* = \left[\dfrac{\mu + a(1-\delta)}{\delta b}\right]^{\frac{1}{\alpha}} \\ \dfrac{\delta(bh_t^{\alpha} - a) - (\mu - a)}{(1+\delta)(bh_t^{\alpha} - a)} > 0 & \text{if } h_t > h^* \end{cases} \tag{28}$$

$$h_{t+1} = \psi(h_t) = \begin{cases} a & \text{if } h_t \leq h^* \\ a[1 - \phi(h_t)] + bh_t^{\alpha}\phi(h_t) & \text{if } h_t > h^* \\ = \dfrac{\delta}{1-\delta}(bh_t^{\alpha} - \mu) & \end{cases} \tag{29}$$

As in the case of no liquidity constraints examined above, h_t evolves monotonically along the dynamic path. Specifically,

$$\frac{\partial h_{t+1}}{\partial h_t} = \psi'(h_t) = \begin{cases} 0 & \text{if } h_t < h^* \\ \dfrac{\delta}{1+\delta}\dfrac{\alpha b}{h_t^{1-\alpha}} > 0 & \text{if } h_t > h^* \end{cases} \tag{30}$$

$$\frac{\partial^2 h_{t+1}}{\partial h_t^2} = \psi''(h_t) = \begin{cases} 0 & \text{if } h_t < h^* \\ -\dfrac{\delta}{1-\delta}\dfrac{\alpha(1-\alpha)b}{h_t^{2-\alpha}} < 0 & \text{if } h_t > h^* \end{cases} \tag{31}$$

When mere are liquidity constraints, $\psi(h_t)$ is a continuous function of the human capital stock, h_t. Thus. there exists a s[cwty-stare equilibrium in this economy if $\psi(0) > 0$ and there exists h_t such that $\psi(h_t) < h_t$ for some $h_t > 0$. Since a > 0 and since individuals choose to allocate all of their time to work in the first period when the parental education stock is equal to zero, $\psi(0) > 0$. In the stew.iy

$$\lim_{h} -- \psi'(h_t) = 0 \tag{32}$$

state, it could be the case that individuals choose no education or choose some education. A steady state with no education exists if $\psi(h^*) < h$“. Equation (29) implies that $\psi(h^*) <$ h“ when

$$\mu > a^a [\delta b - (2'-''(1 + 8)] \tag{33}$$

Consequently, if equation (33) is satisfied. then without further parameter restrictions there exists a steady-state equilibrium in which individuals choose to allocate no time to education. denoted by h_1, where h_1 = u. Examination of (33) reveals that, as m the non-liquidity-constrained case, a **steady** state with no education is possible if the value of work experience in accumulating human capital. *a*. is either very high or very low. in contrast, if $\psi(h^*) > h^*$, then the shape of the $\psi(h_t)$ function characterized by equations (30) through (32) allow us to conclude that a unique steady state with education exists.

For a reasonable range of parameter values. it is possible that multiple equilibria exist in this economy. Multiple equilibria will exist when $\psi(h^*) < h'$. $\lim_{h\to\infty} \psi'(h_t) < 1$, and there exists an $h_t > h^*$ such that $\psi(h_t) > h_t$. As established above. $\psi(h^*) < h^*$ if equation (33) is satisfied, and as implied by equation (32). $\lim_{h\to\infty}$ W’(ii) < 1. In addition, a closer examination of (30) shows that the slope of $\psi(h_t)$ at h^* is increasing in *b* and δ and decreasing **m u. thus** ensuring for some range of parameter values that there exists an $h_t > h^*$ such that $\psi(h_t) > h^*$.[9] Therefore, it follows that for some range of parameter values, there exists two stable steady-state equilibria, denoted by $\bar{h}_1$ and $\bar{h}_2$, where $\bar{h}_2 > \bar{h}_1$. At $\bar{h}_1$, individuals obtain no education, and at $\bar{h}_2$, individuals choose to allocate some of their time to education. A third, unstable steady state, $\bar{h}_3$ such that $\bar{h}_1 < \bar{h}_3 < \bar{h}_2$, is also possible.[10] At this unstable steady state individuals would also choose to both work and go to school.

The parental stock of human capital required for individuals to choose to allocate a positive amount of time to education in the first period, h^*, will be greater when there are binding liquidity constraints and when individuals are sufficiently impatient (the discount factor, δ relative to the interest rate, r, is sufficiently low).[11] As we discussed in the preceding section, this condition allows the removal of liquidity constraints to be a viable policy alternative.

5. Conclusion

The theory we present above focuses on how the ability to accumulate human capital through both formal training and through on-the-Job experience affects the dynamic behavior of the human capital stock and of the economy, allowing us to draw several conclusions, some of which have important policy implications. First, a policy that focuses exclusively on achieving large increases in enrollment rates may not be the most efficient policy for generating economic development. Alternative policies that encourage industries which produce jobs that allow for more learning through on-the-job experience could achieve the same objective more efficiently. As some empirical studies have shown, this conclusion is consistent with the recent experiences of several developing countries

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[9]Evaluating (30) at h^* yields a value greater than 1 when $\alpha\delta b - a^{1-\alpha}(1+\beta) > 0$. This condition can be satisfied simultaneously with (33) for some parameter values.

[10]The third steady state is not present when $h^* = a$.

[11]h^* is greater under the liquidity constrained case when $(\mu+a)/\delta > (1+r)\mu$.

that have experienced large increases in enrollment rates without corresponding increases in output.

Another conclusion of the preceding analysis that is relevant to policy makers is that liquidity constraints, even when there are no out-of-pocket expenses for schooling, may limit the level and growth of per capita income. More importantly, the model demonstrates that although the removal of liquidity constraints is a necessary condition for increasing efficiency and for maximizing long-run per capita income, it is not a sufficient condition Economies with very low levels of initial human capital stock may not be able to escape a development trap through a policy that focuses exclusively on the removal of liquidity constraints.

Our paper also shows the conditions under which there will be multiple equilibria. Specifically, it shows that if the effect of on-the-job experience relative to that of training on human capital accumulation is sufficiently small, then there may exist a development trip. In that case, sufficiently low initial levels of human capital result in low levels of marginal productivity for education in producing human capital Thus, the positive opportunity" cost of becoming educated causes individuals to choose work over education

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Figure 1A

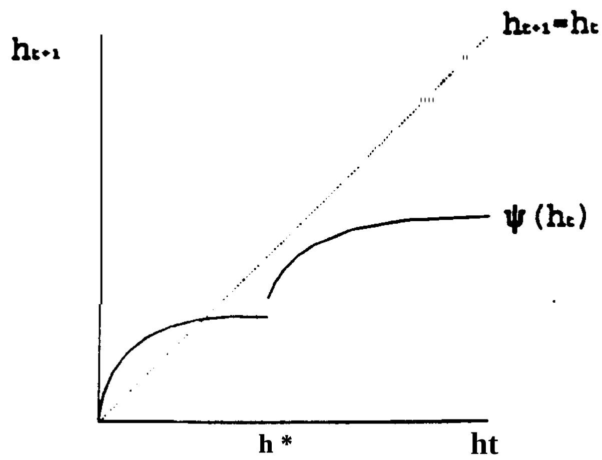


Figure 1B

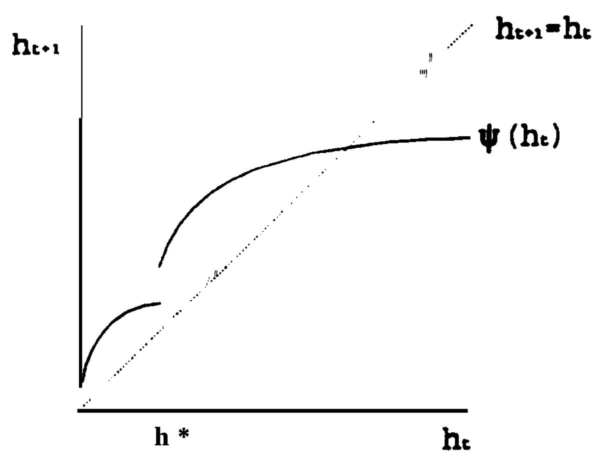


Figure IC

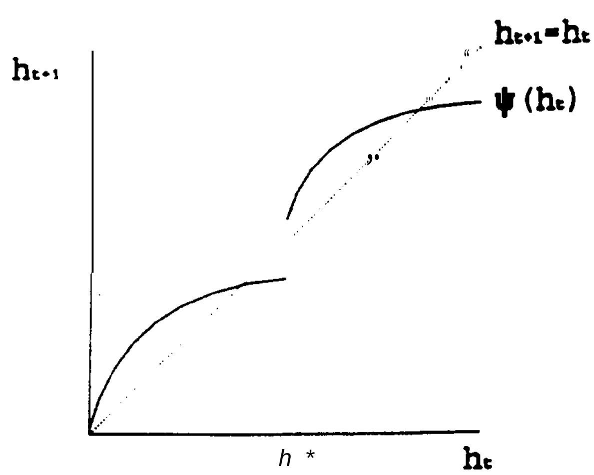
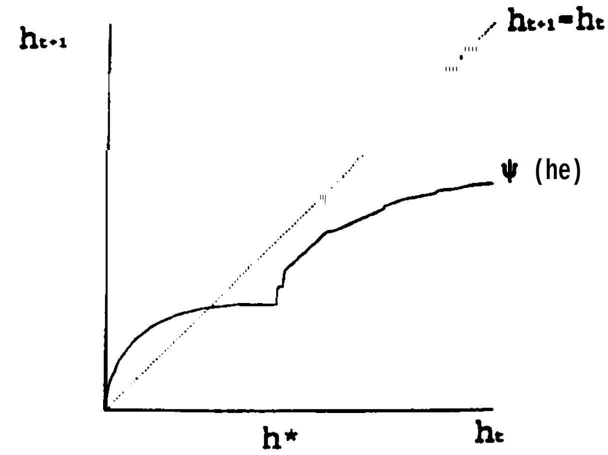


Figure 2A



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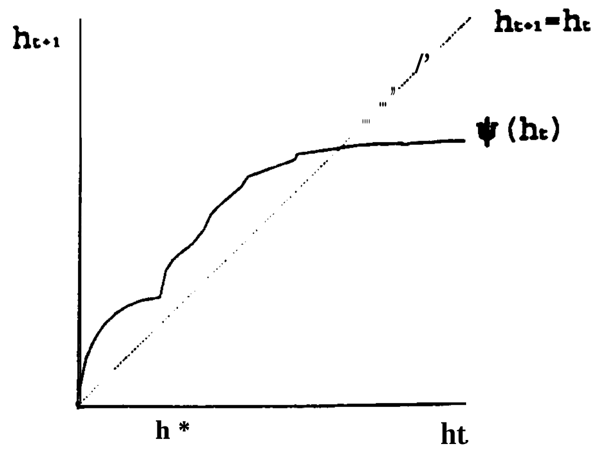

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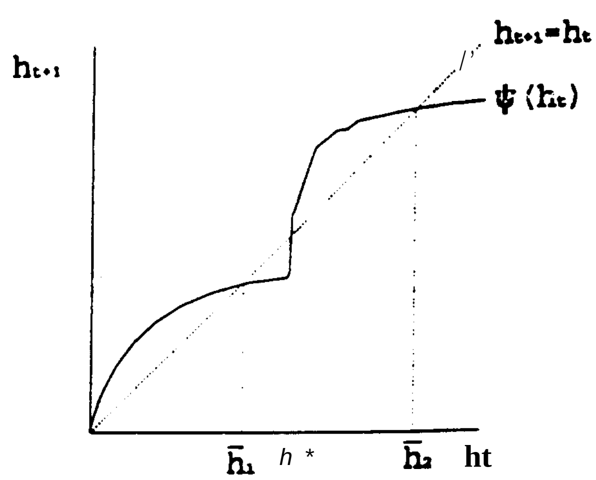


Figure 3

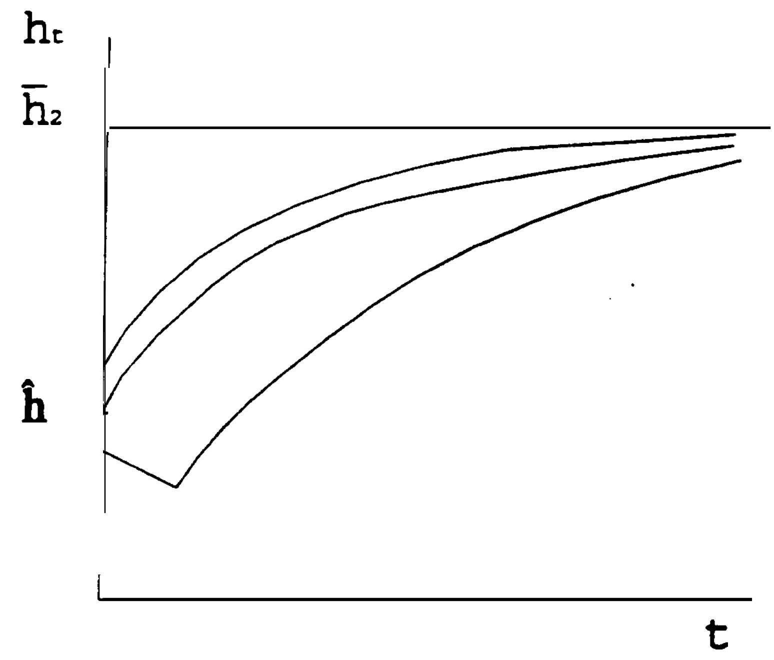
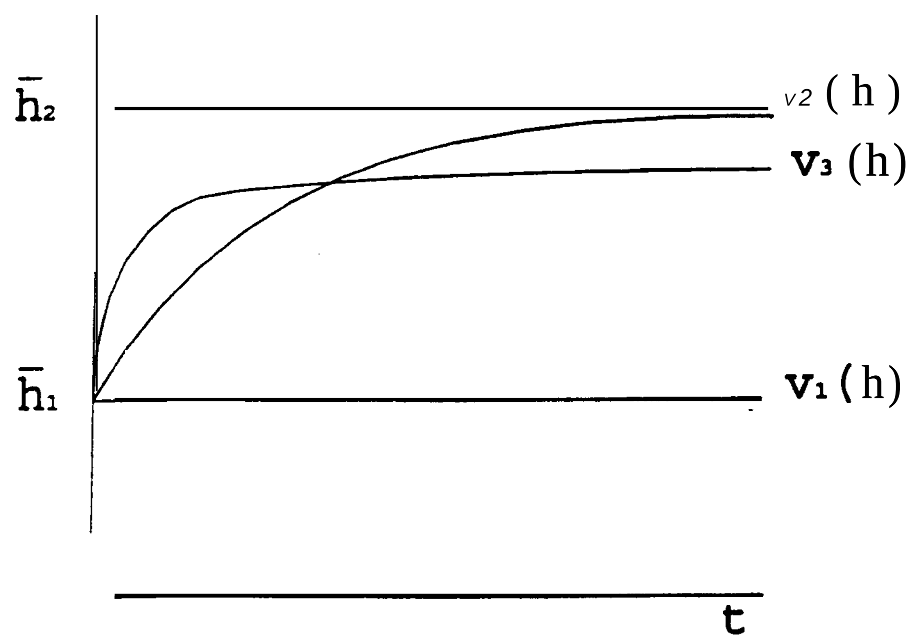


Figure 4



v_1 h) $< v_2$ (h) $< v_3$ (h) $\forall$ h

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