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A TECHNIQUE FOR EXTRACTING A MEASURE OF EXPECTED INFLATION FROM THE INTEREST RATE TERM STRUCTURE

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1. Introduction

Few macroeconomic variables are as crucial to so many current controversies, and yet as hard to measure in empirical work, as the expected inflation rate. Each of the proxies which are often used for the expected inflation rate has serious drawbacks. Public opinion survey data are not based on observed economic behavior. Lagged values of the actual inflation rate, or other relevant macroeconomic variables, cannot hope to capture all the information that enters into the formation of expectations (for example, the latest government announcements of money growth). Future values of the actual inflation rate differ from the expected inflation rate by large expectational errors (even though they have mean zero if expectations are rational).

One proxy sometimes used for the expected inflation rate is the nominal interest rate (minus a constant), under the hypothesis that the real rate of interest is constant. This proxy has the advantages that it is based on observed economic behavior and that it is capable of capturing the latest information available in financial markets. The disadvantage lies in the questionableness of the hypothesis that the real rate of interest is literally constant. Much has been written on both sides of this issue.[1]

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1. The controversy began when Fama (1975) accepted the joint hypothesis that the real rate of interest was constant and that expected inflation was an unbiased predictor of actual inflation. Fama's tests were subsequently attacked by Hess and Bicksler (1975), Joines (1977), Nelson and Schwert (1977) and Begg (1977).

While the present paper and Fama's work can both be interpreted as assuming how nominal interest rates are determined as a function of expected inflation, in order to obtain a measure of expected inflation, there is a much larger literature that assumes how expected inflation is determined, in order to estimate the effect of expected inflation on nominal interest rates. For example, Modigliani and Shiller (1973) assume that expected inflation is determined as a distributed lag of actual inflation in order to estimate the effect on nominal interest rates. Mishkin (1977) argues the limitations of using distributed lags to predict the effect of policy changes on interest rates. Pyle (1972) and Gibson (1972) assume that expected inflation is accurately reflected in survey data in order to estimate the effect on nominal interest rates; Gibson finds a greater effect for the one-year term than for the six-month term, supporting the approach of the present paper.

Most economists probably would not agree with the claim that the nominal interest rate fully reflects the expected inflation rate.[2] But a majority probably would agree that, in the absence of new disturbances (such as unexpected monetary expansions) the expected inflation rate will become increasingly incorporated into the nominal interest rate with the passage of time. Since long-term interest rates are known to reflect expected future short-term interest rates, an important implication is that long-term interest rates reflect the expected inflation rate more fully than do short-term interest rates. This principle is recognized in financial markets. For example, when President Carter announced a more restrictive monetary policy in late 1978, in order to reduce inflation and the depreciation of the dollar, the long-term interest rate fell below the short-term interest rate for the first time in many years. The Wall Street Journal characterized the reaction in bond markets as follows:

> Although analysts painted a bleak outlook for short-term interest rates, many said the dollar-defense program could help to lower longer-term bond yields. That's because investors might feel more convinced the government is determined to fight inflation. Indeed, prices on bonds jumped sharply yesterday, pushing yields down.[3]

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2. Appendix 1 reports some formal tests of the hypothesis that nominal interest rates fully reflect expected inflation (the Fisher hypothesis) jointly with the hypothesis that expectations are rational.

3. Wall Street Journal, Nov. 2, 1978, p.2.

This paper suggests a precise technique for extracting a measure of expected inflation from the term structure of interest rates. Briefly, the procedure is as follows. For a given term of maturity, the interest rate can be regarded as a weighted average of an instantaneously short-term interest rate which is sensitive to the current tightness of monetary policy and an infinitely long-term interest rate which reflects only the expected inflation rate. The weights in the average depend, first, on the speed with which the system converges to the steady-state inflation rate (in expectation), and, second, on the length of maturity of the bond in question. At any point in time, we can look at two maturities and extrapolate to infer the infinitely long-term interest rate which reflects only the expected inflation rate. The result is a time series which represents the market's expected inflation rate, up to a constant.

2. Theory

We begin with the assumption that at every point in time the public has an expected long-run inflation rate $\hat{\pi}$ (for example an expected long-run rate of monetary growth corrected for any trend in real income or velocity). It is not necessary for the public to expect that the inflation rate will be $\hat{\pi}$ in the near future; it is merely necessary that the public considers its expected value to converge to $\hat{\pi}$ in the long run. We also assume that the public expects the interest rate to converge in the long run to $\hat{\pi}$ plus a constant (the long-run real rate of interest). Specifically, the public expects the gap to be closed at a certain rate δ (in the absence of future disturbances):

$$(1) \qquad \frac{di}{dt} = -\delta(i - \hat{\pi} - \bar{r}),$$

where i is the (instantaneously) short-term interest rate and $\bar{r}$ is the long-run real rate of interest.[4]

Equation (1) implies that at time 0 the public expects the short-term interest rate at time t to be a weighted average of the long-run interest rate $(\hat{\pi}_o + \bar{r})$ and the current short-term interest rate (i_o):

$$(2) \qquad i_t = (1 - \exp(-\delta t))(\hat{\pi}_o + \bar{r}) + (\exp(-\delta t))i_o.$$

We need only one additional assumption: i_o^τ , defined as the interest rate on τ-maturity bonds (issued at time 0), is the average of the expected instantaneously short-term interest rates between time 0 and time τ, plus a possible constant liquidity premium term:[5]

$$(3) \qquad i_o^\tau = \frac{1}{\tau}\int_o^\tau i_t dt + k_\tau.$$

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4. The argument is that because prices are sticky, an unexpected monetary expansion in the short run raises the real money supply, lowering the real interest rate and stimulating demand. But as time passes, prices begin to rise more quickly in response to the high level of demand, the real money supply falls, raising the interest rate (and, incidentally, lowering the output level) back to their long-run equilibrium levels. Appendix 2 specifies such a model and demonstrates that rational expectations will take the form of (1). The crucial assumption is that the market expects prices to adjust exponentially in the short run and to increase at rate $\hat{\pi}$ in the long run. The rational value of δ, the speed of adjustment, turns out to depend positively on the responsiveness of the rate of price change to goods demand and negatively on the responsiveness of money demand to income and the interest rate.

5. In order for (3) to hold as an arithmetic average (rather than as a geometric average) and in order to enable us to work generally with linear equations, all rates are defined logarithmically:

$$i_o^\tau \equiv \log\ (1 + \tau\text{-maturity interest rate})$$

$$i_t \equiv \log\ (1 + \text{force of interest})$$

$$\pi_o \equiv \log\ (\text{future price level/current price level}).$$

For small rates, the log of one plus the rate is numerically close to the rate itself, which justifies the references in the text to i, etc., as "the interest rate."

The liquidity premium k_τ may or may not be a smoothly rising function of τ.[6]

By integrating (2), we find that the τ-maturity rate can be represented as another weighted average:

$$(4) \qquad i_o^\tau = (1 - \omega_\tau)(\hat{\pi}_o + \bar{r}) + (\omega_\tau)(i_o) + k_\tau,$$

where the weights are given by $\omega_\tau = \frac{1 - \exp(-\delta\tau)}{\delta\tau}$.

For any given point in time we can obtain the treasury bill rate for two (or more) maturities τ_1 and τ_2 (say 3 months and 12 months). Thus for any given point in time we have two (or more) equations like (4) which we could solve for the two unknowns $\hat{\pi}$ and i (up to constants),[7] if only we knew the parameter δ, and thus the weights ω_{τ_1} and ω_{τ_2}. The reduced form equations for $\hat{\pi}$ and i (up to constants) in terms of the observable long-term and short-term interest rates are:

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6. Hicks (1939) first argued that lenders require a liquidity premium for longer term maturities, to compensate for risk regarding future short-term rates. Pesando (1978) accepts the joint hypothesis that (1) the bond market is efficient and (2) the variation in long-term bond rates is due solely to expectations effects, suggesting that if there is a liquidity premium structure it must be time-invariant. However Sargent (1968), Roll (1970) and Cargill (1975) reject versions of this joint hypothesis.

McCulloch (1975) presents evidence that the liquidity premium curve is independent of the level of interest rates, and suggests a monotonically increasing functional form. Modigliani and Sutch (1966, 1967) developed a more general model in which lenders demanded a liquidity premium in order to commit themselves to a term different from their "preferred habitat;" a priori, the liquidity premium could have any shape, though it would presumably be smooth. Nelson (1972) argues that the liquidity premium could be zero and yet longer-term interest rates would appear to be higher on average because of a covariance term in the product of the shorter-term rates. (k would be zero in our model, because of the logarithmic specification).

7. Henceforth we drop the subscript on the time series variables $\hat{\pi}$, i, i^{τ_1} and i^{τ_2}.

$$(5)\quad \hat{\pi} = \frac{\omega_{\tau_1}(i^{\tau_2} - k_{\tau_2}) - \omega_{\tau_2}(i^{\tau_1} - k_{\tau_1})}{\omega_{\tau_1} - \omega_{\tau_2}} - \bar{r}$$

$$i = \frac{(\omega_{\tau_1} - 1)(i^{\tau_2} - k_{\tau_2}) - (\omega_{\tau_2} - 1)(i^{\tau_1} - k_{\tau_1})}{\omega_{\tau_1} - \omega_{\tau_2}} \; .$$

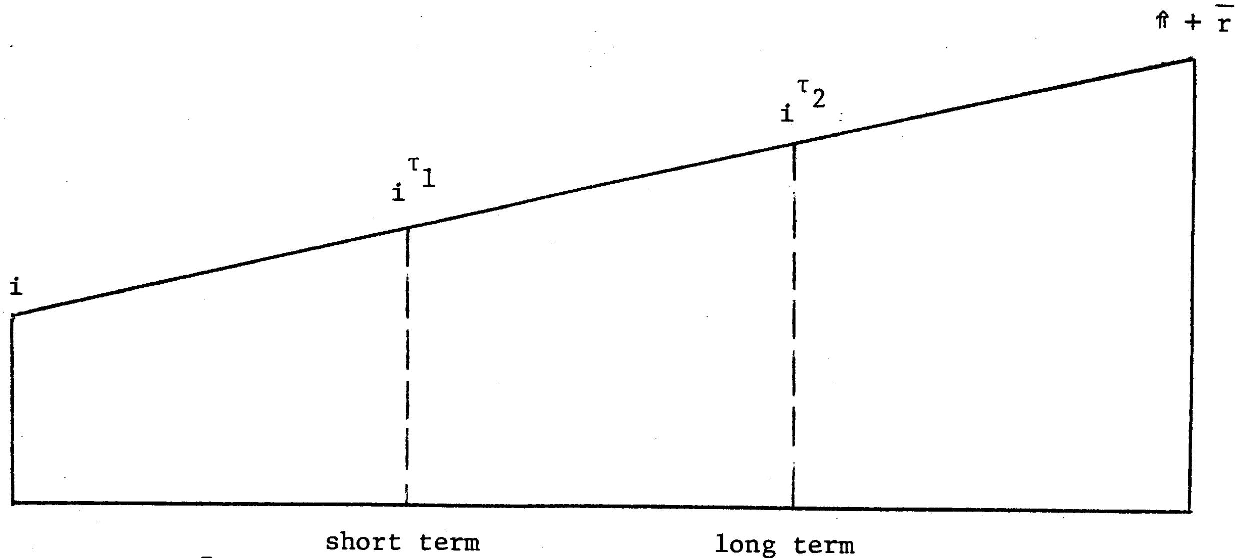
For example, if τ_2 were sufficiently large, ω_{τ_2} would be close to zero and $\hat{\pi}$ would be close to $i^{\tau_2} - k_{\tau_2} - \bar{r}$.

The relationship is illustrated schematically in Figure 1. At any point in time, the short-term and long-term nominal interest rates, i^{τ_1} and i^{τ_2}, are each weighted averages of i and $\hat{\pi} + \bar{r}$, with the long-term rate giving greater weight to $\hat{\pi} + \bar{r}$. We can extrapolate from the observed i^{τ_1} and i^{τ_2} to infer the value of $\hat{\pi} + \bar{r}$. (For simplicity, the figure omits the possible liquidity premium).

Thus if we knew the parameter δ we would know the weights and we could compute the series for the right-hand sides of (5) and thus the series for $\hat{\pi}$ and i. On the other hand, if we knew the series for $\hat{\pi}$ and i, and we were willing to assume that the public's estimate of δ is equal to the true value of δ (i.e., rational expectations), we could estimate the parameter δ as follows. We regress the real interest rate $i - \pi$ against its own lagged value to ascertain the speed with which the system tends to equilibrium. Equation (1) implies that the coefficient in such a regression, β, is equal to $\exp(-\delta/n)$,

Figure 1: Extrapolation from interest rates

to the expected inflation rate



$$i^{\tau_1} = \omega_{\tau_1} i + (1 - \omega_{\tau_1})(\hat{\pi} + \bar{r})$$

$$i^{\tau_2} = \omega_{\tau_2} i + (1 - \omega_{\tau_2})(\hat{\pi} + \bar{r})$$

where n is the number of observations per year (or 1/n is the length of the observation interval).

It might appear that we are trapped, unable to estimate the parameter δ without the series i and $\hat{\pi}$, and unable to estimate the series i and $\hat{\pi}$ without the parameter δ. But there is an easy solution. From (5), the reduced form for the real interest rate is

$$(7) \quad i - \hat{\pi} = \bar{r} + \frac{(i^{\tau_1} - k_{\tau_1}) - (i^{\tau_2} - k_{\tau_2})}{\omega_{\tau_1} - \omega_{\tau_2}} .$$

If we are only interested in regressing this expression against its own lagged value, we will get the same estimate for the coefficient regardless of the weights ω_{τ_1} and ω_{τ_2}. Thus we can estimate β simply by regressing the interest rate spread $i^{\tau 1} - i^{\tau 2}$ against its own lagged value.

3. Estimation

Table 1 reports the estimates of β from such regressions, along with the implied values for δ, ω_{τ_1} and ω_{τ_2}. Each regression represents a different pair of terms of maturity. If the theory held perfectly, the estimates of β would be the same.[8] The discussion will concentrate on the three-month and 10-year interest rates, as they constitute the greatest spread.

Table 1: Estimation of speed of adjustment

$$(i_t^{\tau_2} - i_t^{\tau_1}) = \alpha + \beta(i_{t-1}^{\tau_2} - i_{t-1}^{\tau_1}) + u_t$$

technique: OLS Sample: Aug. 1958 - Dec 1978, monthly

terms of maturity		regression results				implied parameter estimates		
τ_1	τ_2	$\hat{\alpha}$	$\hat{\beta}$	R^2	D.W.	$\hat{\delta}=-12\log\hat{\beta}$	$\hat{\omega}_{\tau_1} = -\frac{1-\beta^{\tau_1}}{\tau_1 \log \beta}$	$\hat{\omega}_{\tau_2} = -\frac{1-\beta^{\tau_2}}{\tau_2 \log \beta}$
3 mo	10 yr	.002 (.0002)	.9636 (.0186)	.92	1.78	.4444	.9465	.2224
3 mo	12 mo	.0008 (.0002)	.8452 (.0341)	.72	1.92	2.0178	.7853	.4297
1 yr	10 yr	.0001 (.0001)	.9628 (.1899)	.92	1.46	.4554	.8033	.2173

i^{τ}: log (1 + yield on U.S. Treasury securities of term τ months)

(Estimated standard errors are reported in parentheses.)

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8. Since the theory of course does not work perfectly, the question arises how to get an optimal estimator by using the entire term structure, rather than just two points. If we could ignore the existence of the liquidity terms, then one strategy would be to fit, at each point in time, a least - squares line through the points; the R^2s would then be tests of the validity of the theory. Given the existence of the liquidity terms, they would have to be estimated at the same time as the expected inflation time series, in a cross section/time-series data sample. This extension is left for future research.

The estimates of β and δ are interesting statistics in their own right. They can be interpreted as the speed (in discrete time and continuous time, respectively) with which the macroeconomic system adjusts to long-run equilibrium. For example an estimate of $\hat{\beta}$ = .964 indicates that when a monetary or other disturbance raises the real interest rate above its long run level, 96.4% of the effect remains one month later and 64.1% (which is $.964^{12}$) remains one year later.[9]

However the primary purpose for estimating β and δ was as a means of estimating ω_{τ_1} and ω_{τ_2} and computing a time series for $\hat{\pi}$. From equation (5), we have:

$$(5') \quad \hat{\pi} + \bar{r} + \frac{\omega_{\tau_1} k_{\tau_2} - \omega_{\tau_2} k_{\tau_1}}{\omega_{\tau_1} - \omega_{\tau_2}} = \frac{\omega_{\tau_1} i^{\tau_2} - \omega_{\tau_2} i^{\tau_1}}{\omega_{\tau_1} - \omega_{\tau_2}} .$$

Now computing $\hat{\pi}$ up to a constant is a simple matter of computing the appropriate linear combination of i^{τ_2} and i^{τ_1}. (Notice that the weights in this linear combination add up to unity, but that the weight on i^{τ_1} is negative, representing the fact that we are extrapolating beyond τ_2.) The resulting time series is the main end-product of this paper. It is printed out as Appendix 3. (The interest rates used are those on securities of maturity three months and ten years.)

As can be seen from (5') the constant by which this time series differs from $\hat{\pi}$ has two components: the long-run real rate of interest $\bar{r}$ and a term representing the liquidity premium spread (which may be very small). The

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9. Within the context of the macroeconomic model specified in Appendix 1, β is not only the speed with which the real interest rate adjusts to its long-run equilibrium value, but is also the speed with which the output level and price level adjust to their respective equilibrium values (though the equilibrium price level is a moving target, expected to increase at rate π). Frankel (1979) develops an international version of this same model. By estimating the effect of the real interest rate on the exchange rate, β is estimated to be .934 and δ to be .819, i.e. 44.1% of a disturbance is computed to remain after one year.

only obvious way to estimate this constant is to assume that public expectations of inflation were on average correct during this period, and to compute the constant as the average of the difference between the actual inflation rate and the value of the time series in Appendix 3. A drawback to this procedure is that the public may have underestimated the actual inflation rate during the latter part of this period.[10] The mean of the difference over the period was .0132. Since this number seems too low to be the sum of the long-run real interest rate and a liquidity premium term, it suggests that the market on average underpredicted the inflation rate. (An examination of the time series indicates that the underprediction was concentrated in the latter part of the period, particularly 1972-1974.) However the decision whether or not to subtract .0132 off to get our final estimate of the time series is not a crucial decision, because most purposes to which one would want to put the series require only that it be accurate up to a constant.

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[10]Some studies using price expectations survey data support this possibility. Carlson (1977) and Mullineaux (1978) use data from the Livingston price survey to test the hypothesis that the expected inflation rate has been an unbiased estimator of actual inflation, and obtain generally negative results.

4. Tests of rational expectations

One obvious and immediate application for the measure of expected inflation is to use it to test the hypothesis of rational expectations, jointly with the hypothesis that the model is valid (i.e. that the technique for extracting a measure of expected inflation from the term structure is an accurate one). The approach of this paper has been to take the validity of the model as given. But some readers may wish to take rational expectations as given and to consider these tests as tests of the validity of the model.

The conclusion that the public on average underpredicted the one-year inflation rate in the 1970s does not violate rational expectations unless the bias was statistically significant. The standard deviation on the difference between the actual and expected inflation rates is .0217. The implied confidence interval around .0132 is so large as to include any reasonable value for the real interest rate plus liquidity term.[11] Thus we cannot reject rational expectations on the basis of this (very weak) test.

11. In fact, the standard deviation is a downward-biased estimate of the true standard error for the reasons discussed in the next paragraphs, which implies an even larger confidence interval.

Regressing the actual one-year inflation rate against the expected inflation is a more powerful test of rational expectations.[12] In such a regression on monthly data (not reported), the null hypothesis of a unit coefficient would appear to be rejected easily, were it not for the presence of autocorrelation, biasing the reported standard errors downward.

Correcting for the autocorrelation is not a simple matter of correcting for a low-order autoregressive process. Nor does the existence of autocorrelation violate the requirement (under the null hypothesis of rational expectations) that the market's prediction errors should be serially uncorrelated. Both of these propositions are consequences of the fact that monthly observations of one-year predictions overlap. The information that becomes available between t and t + 12 is highly correlated with the information that becomes available between t + 1 and t + 13. The error process thus (under the null hypothesis) is an eleventh-order moving average. Correcting for a high-order moving

12. Technically, the π series measures the long-run expected inflation rate rather than the one-year expected inflation rate. The one-year inflation rate will be higher, for example, if monetary policy is currently expansionary so that the real interest rate is currently low. Within the context of the model in Appendix 2, it can be shown that the expected rate of price change τ^* years into the future is given by

$$(\hat{p}_{t+\tau^*} - p_t)/\tau^* = \hat{\pi}_t + \omega_{\tau^*}\rho\gamma\left(\frac{i_t^{\tau_1} - i_t^{\tau_2}}{\omega_{\tau_1} - \omega_{\tau_2}}\right),$$

where $\omega_{\tau}*$ goes to zero for large τ^*.

average process can be computationally difficult.[13]

The computationally-simplest strategy is to include in the sample only every twelfth observation. Since the prediction periods no longer overlap, the errors will be uncorrelated under the null hypothesis. The obvious drawback is that throwing away data reduces the power of the test. The solution adopted here is to perform separate tests on twelve sub-samples, each formed by taking every twelfth observation. Since the tests are not independent, there is no way of knowing how to combine optimally the results into one efficient estimator, but using all the data must add to our knowledge.

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13. This same problem has been encountered in the large (though mostly unpublished) literature which tests to see if the forward exchange rate is an unbiased estimator of the future spot exchange rate. Many such papers - for example, Frenkel (1977), Krugman (1977) and Frankel (1978) - adopt the simplest strategy of using samples that consist of every twelfth observation (or every fourth observation in the case of weekly observations of one-month forward rates). Others - such as Bilson and Levich (1977) and Obstfeld (1978) - use all observations, estimate the error process from the residuals of the first-stage OLS regression, and then attempt to transform the data appropriately, with the aim of attaining efficient estimators. However it has been recently pointed out - by Garber (1978), Hansen and Hodrick (1979), and Hakkio (1979) - that correlation between the transformed forward rate and the error term will render these two-step procedures inconsistent unless the forward rate is not only contemporaneously uncorrelated with the error term (as it is under the null hypothesis of rational expectations) but is also strictly exogenous (an unrealistic assumption). These authors suggest their own procedures to get around this problem.

Table 2 indicates that in each of the twelve sub-samples, the estimated coefficient $\hat{\beta}$ is insignificantly different from unity. It seems likely that rational expectations would not be rejected by the optimal test that combined all sub-samples.

Table 3 reports a test for serial correlation of the prediction errors; the difference between the actual inflation rate and the expected inflation rate is regressed against its own lagged value. This time the null hypothesis of a zero coefficient is rejected at the 95% level for all but the last of the twelve sub-samples. The rejection of rational expectations seems clear.

5. A comparison with survey data

Another proxy which is sometimes used for the expected inflation rate is survey data, such as the semi-annual Livingston survey of price expectations.[14] One possible way of evaluating different proxies is to compare their ability to predict actual inflation, though this procedure of course assumes a degree of rationality in expectations.

Table 4 reports the mean squared prediction error of several alternative proxies. Because the new term-structure measure is only meaningful up to a constant term, its mean squared prediction error is calculated as the variance of its prediction error; in other words it is normalized on the mean actual inflation rate. If we compare this number -- 4.716×10^{-4} for the 3-month to 10-year term spread -- to the mean squared error of the Livingston survey data -- 7.671×10^{-4} -- we find that the survey data do a much worse job of predicting the one-year inflation rate.[15] However one might argue that for

14. I am obligated to Stephen McNeese for access to the Livingston data.

15. The difference is even greater when the measures based on the 3-month to 12-month term spread or 1-year to 10-year term spread are used.

Table 2: Test for bias of prediction errors in expected inflation measure

$$\text{OLSQ } \pi = \alpha + \beta\hat{\pi} + u$$

	α	$s(\alpha)$	β	$s(\beta)$	R^2	D.W.
1	-.039	(.019)	1.415	(.317)	.54	1.36
2	-.039	(.021)	1.418	(.344)	.50	1.44
3	-.024	(.022)	1.479	(.358)	.50	1.42
4	-.041	(.023)	1.490	(.391)	.46	1.37
5	-.044	(.023)	1.547	(.394)	.48	1.37
6	-.042	(.022)	1.503	(.361)	.51	1.33
7	-.045	(.020)	1.563	(.332)	.57	1.42
8	-.046	(.018)	1.573	(.302)	.61	1.47
9	-.043	(.019)	1.515	(.302)	.60	.53
10	-.035	(.019)	1.340	(.308)	.54	1.29
11	-.038	(.021)	1.412	(.338)	.52	1.25
12	-.039	(.021)	1.435	(.346)	.52	1.37

π: inflation rate, calculated as $(CPI_{+12} - CPI)/CPI$

$\hat{\pi}$: measure of expected long-run inflation (up to a constant), equation calculated as in (5'), where $\tau_1 = 3$ mos. and $\tau_2 = 10$ yrs.

Sample: Aug. 1959 - Apr. 1979
19 monthly observations for subsamples 1-9
18 monthly observations for subsamples 10-12.

Table 3: Test for serial correlation of prediction errors in expected inflation measure

$$OLSQ\ (\pi - \hat{\pi}) = \alpha + \rho(\pi - \hat{\pi})_{-12} + u$$

	α	$s(\alpha)$	ρ	$s(\rho)$	R	D.W.
1	-.014	(.004)	.816	(.359)	.23	1.35
2	-.014	(.005)	.872	(.385)	.23	1.44
3	-.013	(.005)	.930	(.343)	.25	1.42
4	-.013	(.005)	1.001	(.406)	.26	1.39
5	-.012	(.005)	1.010	(.407)	.27	1.37
6	-.102	(.005)	.950	(.397)	.25	1.32
7	-.102	(.005)	.932	(.377)	.26	1.40
8	-.012	(.004)	.917	(.349)	.29	1.42
9	-.012	(.004)	.932	(.357)	.29	1.48
10	-.015	(.004)	.703	(.377)	.18	1.28
11	-.014	(.005)	.815	(.385)	.22	1.26
12	-.014	(.005)	.787	(.404)	.19	1.36

π: inflation rate, calculated as $(CPI_{+12} - CPI)/CPI$.

$\hat{\pi}$: measure of expected long-run inflation (up to a constant), calculated as in equation (5'), where $\tau_1 = 3$ mos. and $\tau_2 = 10$ yrs.

Sample: Aug. 1959 - April 1979
19 monthly observations for sub-samples 1-9
18 monthly observations for sub-samples 10-12

Table 4: Ability of Expected Inflation Proxies to Predict Inflation

Expected inflation proxy	One-year inflation rate		Five-year inflation rate	
	Mean Squared Error x 10^4	Number of Observations	Mean Squared Error x 10^4	Number of Observations
Measure extracted from term structure (normalized on mean inflation rate) τ_1 = 3 mo. τ_2 = 10 yr.	4.716	225	3.261	177
Livingston price expectations survey data				
normalized on mean inflation rate	5.009	37	2.813	29
not normalized	7.671		12.301	
3-month treasury bill yield (normalized on mean inflation rate)	3.634	225	3.399	177

purposes of in-sample prediction of inflation, it is not a fair comparison to give the new measure a degree of freedom without also giving the survey data a degree of freedom. This suggests comparing 4.716×10^{-4} with the <u>variance</u> of the survey data prediction errors -- 5.008×10^{-4}. The new measure still does a better job than the survey data, but only slightly.

Since the term-structure measure is constructed to reflect the long-term inflation rate, not the one-year inflation rate, Table 4 also reports mean squared errors using the five-year inflation rate, though the last five years of observations must be sacrificed. The difference between the mean squared error of the term-structure measure and that of the non-normalized survey data is even greater than before (as one might expect from the fact that the survey data refer to one-year expectations), but the term-structure measure does not do as well as the normalized survey data.

Table 4 also reports the mean squared error of the 3-month treasury bill yield, which does better at predicting one-year inflation than the term-structure measure, but does worse at predicting five-year inflation.

Figures A1 - A5 in Appendix 4 are graphs of the actual 12-month inflation rate, the measure of expected inflation, the Livingston price expectations survey data, and the yield on 3-month Treasury bills. One characteristic of the measure of expected inflation is that it has remained high ever since 1970, whereas the survey data show a dip in 1975-77, and the yield on Treasury bills shows pronounced dips in 1971-72 and 1975-77. The measure of expected inflation has risen only about one percentage point from 1977 to 1979, whereas the yield on Treasury bills has risen about five percentage points over the same period.

6. Summary

To summarize the procedure of the paper, a plausible model -- in which monetary policy can create short-run variations in the real interest rate due to sticky prices but in which the real interest rate tends to a constant in the long-run -- was shown to imply that the nominal interest rate for a given term of maturity can be expressed as a weighted average of the instantaneously short-term interest rate and the long-run expected inflation rate. The weights depend on a parameter representing the speed of adjustment of the system, which is estimated. Thus the interest rates for any two maturities can be extrapolated to infer the long-run expected inflation rate. The resulting time series for expected inflation is printed in Appendix 3. A comparison with the time series of actual inflation suggests that the market underestimated inflation in the 1970s. However the new measure of expected inflation does a slightly better job of predicting actual inflation, in terms of mean squared error, than survey data.

Appendix 1

Table Al reports tests of the joint hypothesis that (1) short-term nominal interest rates fully reflect expected inflation and (2) expectations of inflation are rational. The tests are performed for the one-month, three-month, six-month and twelve-month inflation and interest rates. In each case, the frequency of the observation is chosen to match the term. Otherwise - i.e. if the frequency of the observation is greater than the term - the expectation periods overlap, and the prediction errors will follow a moving average process, rather than being serially uncorrelated, even under the null hypothesis of a constant real rate and rational expectations.

When the actual inflation rate is regressed against the interest rate, the null hypothesis implies a unit coefficient. However the coefficient appears significantly greater than unity in two cases, and significantly less than unity in another.[16]

When the difference between the actual inflation rate and the interest rate is regressed against its lagged value, the null hypothesis implies a zero coefficient. But the coefficient is significantly greater than zero in three of the four cases.

The results point toward rejection of the joint hypothesis.

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16. The low Durbin-Watson statistics indicate the presence of autocorrelation, biasing downward the reported standard errors and invalidating the test. But if the prediction errors are indeed autocorrelated, as the last half of Table Al indicate they are, then the rational expectations hypothesis is violated anyway.

Table A1: Tests of the Fisher hypothesis

OLSQ $\pi^{\tau} = \alpha + \beta i^{\tau} + u$

Term τ	α	β	R^2	D.W.	nobs.	sample
1 mo.	-.033 (.007)	1.596 (.122)	.49	.49	183	6401-7904
3 mo.	.053 (.017)	-.130 (.244)	.00	.34	78	5908-7904
6 mo.	-.041 (.015)	1.258 (.209)	.49	.57	34	5908-7904
12 mo.	-.120 (.020)	1.886 (.231)	.80	1.61	19	5908-7904

OLSQ $\pi^{\tau} - i^{\tau} = \gamma + \rho(\pi^{\tau} - i^{\tau})_{-\tau} + u$

Term τ	γ	ρ	R^2	h^*	nobs	sample
1 mo.	.000 (.002)	.271 (.072)	.07	-3.24	182	6401-7904
3 mo.	-.003 (.002)	.492 (.101)	.24	-.42	77	5908-7904
6 mo.	-.004 (.003)	.387 (.155)	.15	-4.36	38	5908-7904
12 mo.	-.007 (.005)	.281 (.252)	.07		18	5908-7904

π^{τ} : inflation rate, calculated as $(CPI_{+\tau} - CPI)/CPI$
i^{τ} : yield on τ-month treasury bills

(Estimated standard errors reported in parentheses.)

*Durbin's h statistic tests for autocorrelation in the presence of a lagged endogenous variable. It breaks down in the 12-month regression due to the high standard error of the lagged endogenous variable.

APPENDIX 2

A SIMPLE MACROECONOMIC MODEL OF ADJUSTMENT IN THE PRICE LEVEL AND INTEREST RATE

Equation (1) asserts that the short-term nominal interest rate is expected to converge, at rate δ, to the long-run real interest rate plus the expected long-run inflation rate. This appendix demonstrates that this form of expectations is rational, within a simple macroeconomic model.

(A1) $y - \bar{y} = -\gamma(i - \hat{\pi} - \bar{r})$

(A2) $m - p = \phi y - \lambda i$

(A3) $dp/dt = \rho(y - \bar{y}) + \hat{\pi}$

where y is the log of output

$\bar{y}$ is the log of normal or potential output

i is the short-term nominal interest rate

$\hat{\pi}$ is the expected long-run inflation rate

$\bar{r}$ is the (constant) long-run real interest rate

m is the log of the money supply, and

p is the log of the price level .

(A1) is an IS relationship; it says that the output gap is related to the current real interest rate, through investment demand.* We note immediately that

---

*The description of $i - \pi$ as "the real interest rate" is not quite proper, since i is short-term and π is long-term. But the model would be unchanged if the interest and inflation rates were specified to be of the same term, for example, the short term:

$$y - \bar{y} = -\psi(i - dp/dt - \bar{r}).$$

We simply substitute (A3), and solve for $(y - \bar{y})$:

$$y - y = -\frac{\psi}{1 - \psi\rho}(i - \hat{\pi} - \bar{r}).$$

This equation is the same as (A1), with γ defined as $\frac{\psi}{1 - \psi\rho}$.

in the long run, when $y = \bar{y}$, we have $\bar{i} = \hat{\pi} + \bar{r}$.

(A2) is an LM relationship; it says that real money demand depends positively on income, with an elasticity of ϕ, and negatively on the interest rate, with a semi-elasticity of λ. In the long run, $dm/dt = dp/dt = \pi$.

(A3) is a supply relationship; it says that the rate of price change is given by the sum of an excess demand term and the expected steady-state inflation rate.

Differentiating (A2), we find

(A2′) $dm/dt - dp/dt = \phi\, dy/dt - \lambda\, di/dt$.

Differentiating (A1), we have

(A1′) $dy/dt = -\gamma\, di/dt$.

We substitute (A1′) and (A3) into (A2′):

$$\pi - [\rho(y - \bar{y}) + \hat{\pi}] = -(\phi\gamma + \lambda) di/dt.$$

Finally, we substitute in (A1), assume perfect foresight ($\pi = \hat{\pi}$), and solve for the expected rate of change of the interest rate:

(1) $di/dt = -\delta(i - \hat{\pi} - \bar{r})$ where $\delta \equiv \rho\delta/(\phi\gamma + \lambda)$.

This is equation (1) in the text.

The foregoing perfect foresight formulation can be transformed to a stochastic one by introducing future disturbances to the level (m) and trend (π) of the money supply. As long as these disturbances have expectation zero, (1) will describe the rationally-expected path of i. We could even allow for purely transitory disturbances in m.

Appendix 3

Measure of Expected long-run inflation rate (up to a constant), Computed as

$$\frac{\omega_{3mo} i^{3mo} - \omega_{10yr} i^{10yr}}{\omega_{3mo} - \omega_{10yr}.}$$

Aug. 1959 - Apr. 1979

OMEGAS = .94645
OMEGAL = .22238
XINFLC (A 237 COMPONENT ARRAY)

.046223	.047331	.045426	.045121	.046084	.046885	.045201
.044183	.044804	.045497	.045537	.042888	.041629	.041076
.042762	.04305	.042287	.042318	.041008	.040597	.041408
.040526	.042544	.043324	.044371	.043956	.04314	.042837
.043977	.043859	.043357	.041974	.040811	.041342	.041692
.042366	.042296	.042418	.041913	.04151	.040633	.040132
.041234	.041452	.041924	.04136	.0419	.041694	.041014
.041835	.041997	.041909	.042034	.042536	.042255	.043102
.043442	.043035	.042659	.042971	.042848	.042882	.042634
.041918	.041683	.041901	.041785	.041785	.041659	.041907
.042182	.041934	.042561	.042818	.043234	.044303	.045545
.04478	.047344	.048025	.046437	.046751	.047551	.049254
.051255	.049513	.04746	.049416	.046526	.04401	.045121
.044911	.046817	.050796	.053098	.052792	.0541	.053891
.055697	.058522	.057174	.054979	.055411	.057061	.055187
.057181	.055752	.053666	.053342	.053534	.054535	.055717
.058247	.057826	.059735	.06139	.059519	.061576	.063437
.063588	.063281	.068696	.068234	.068001	.072467	.074137
.069551	.068983	.073245	.078565	.078199	.074278	.07525
.074423	.074327	.070249	.065998	.065438	.066145	.062067
.062209	.068221	.067956	.068544	.068154	.063452	.061517
.060816	.062889	.065087	.067278	.065508	.067109	.066333
.065426	.065286	.066359	.068586	.067486	.064904	.064982
.06527	.066789	.066186	.065185	.067078	.065235	.06558
.066936	.064269	.063836	.061187	.06252	.064566	.066118
.066663	.069248	.070352	.070824	.075169	.073839	.076444
.076543	.073881	.07185	.075362	.076324	.080428	.086111
.085257	.082499	.082534	.085662	.086081	.083988	.084417
.083839	.082492	.083074	.081994	.080367	.083332	.082298
.082487	.081987	.080045	.078321	.077362	.07345	.076822
.078847	.079875	.078984	.078765	.076367	.076521	.076412
.074803	.075932	.076742	.078175	.080328	.081241	.081737
.083188	.085138	.085553	.086872	.083903	.081672	.083944
.08395	.085083	.085384	.085438	.085207	.086036	

Appendix 4

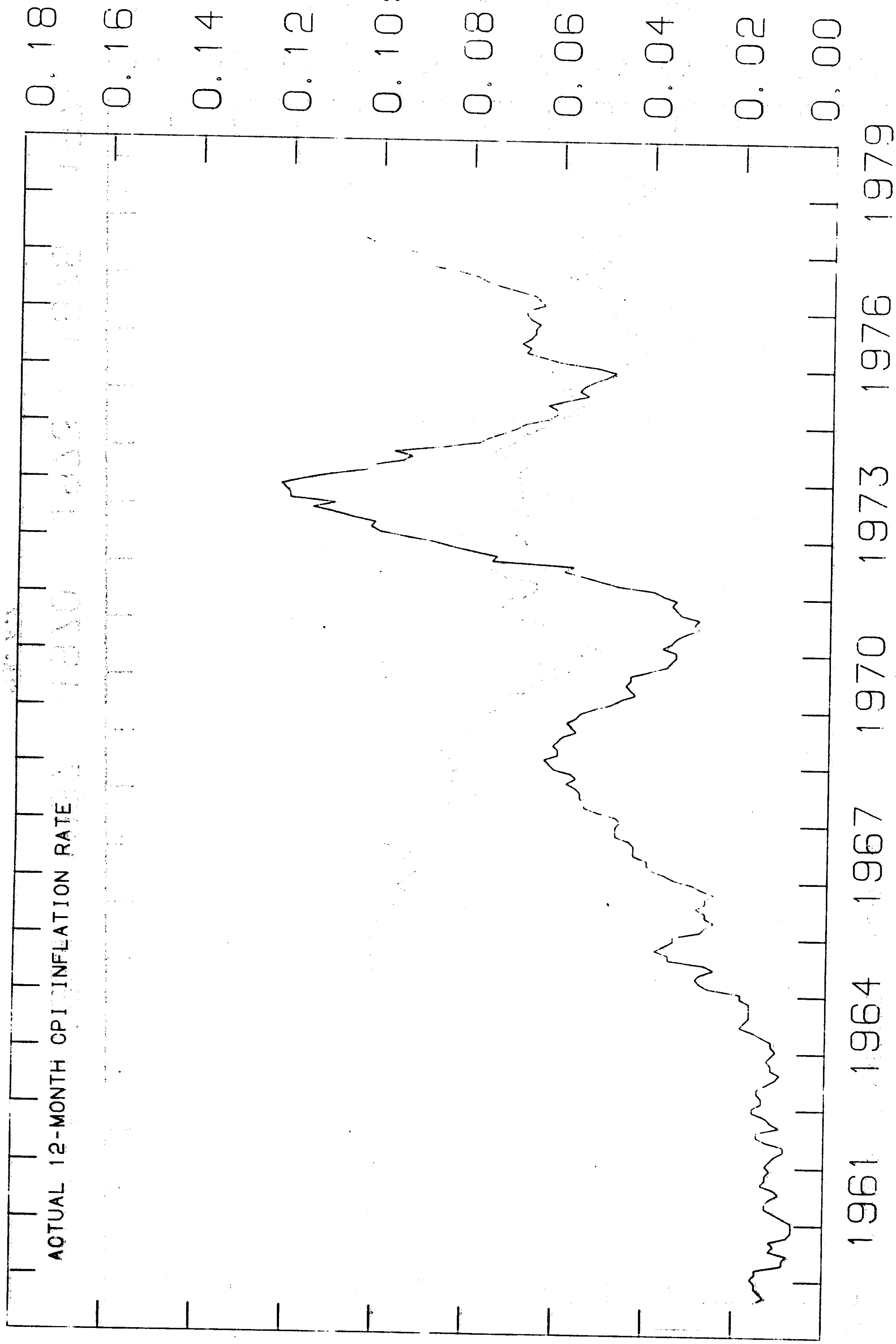

Fig. A.1

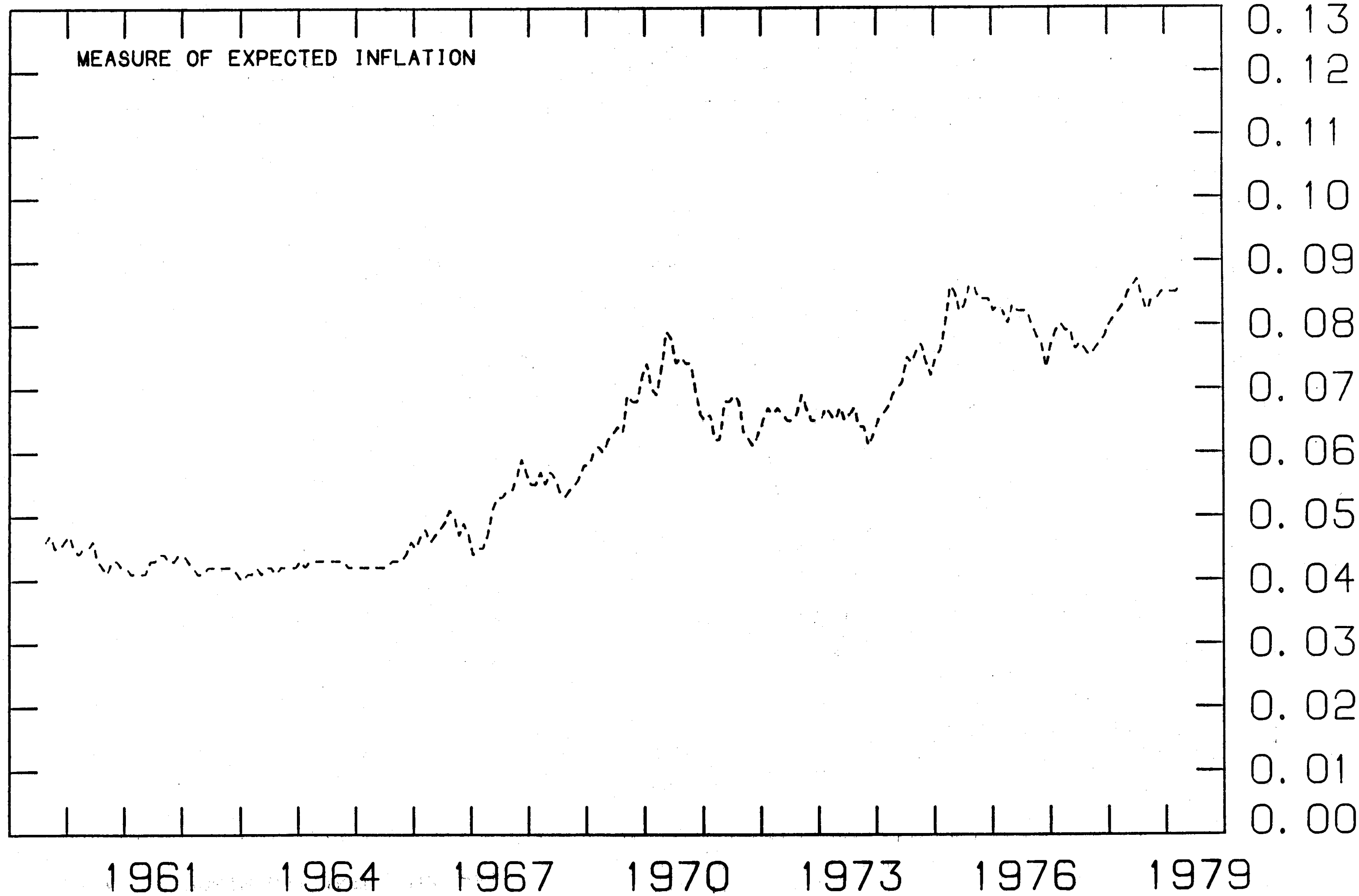
MEASURE OF EXPECTED INFLATION
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1961
1964
1967
1970
1973
1976
1979

Fig. A.2

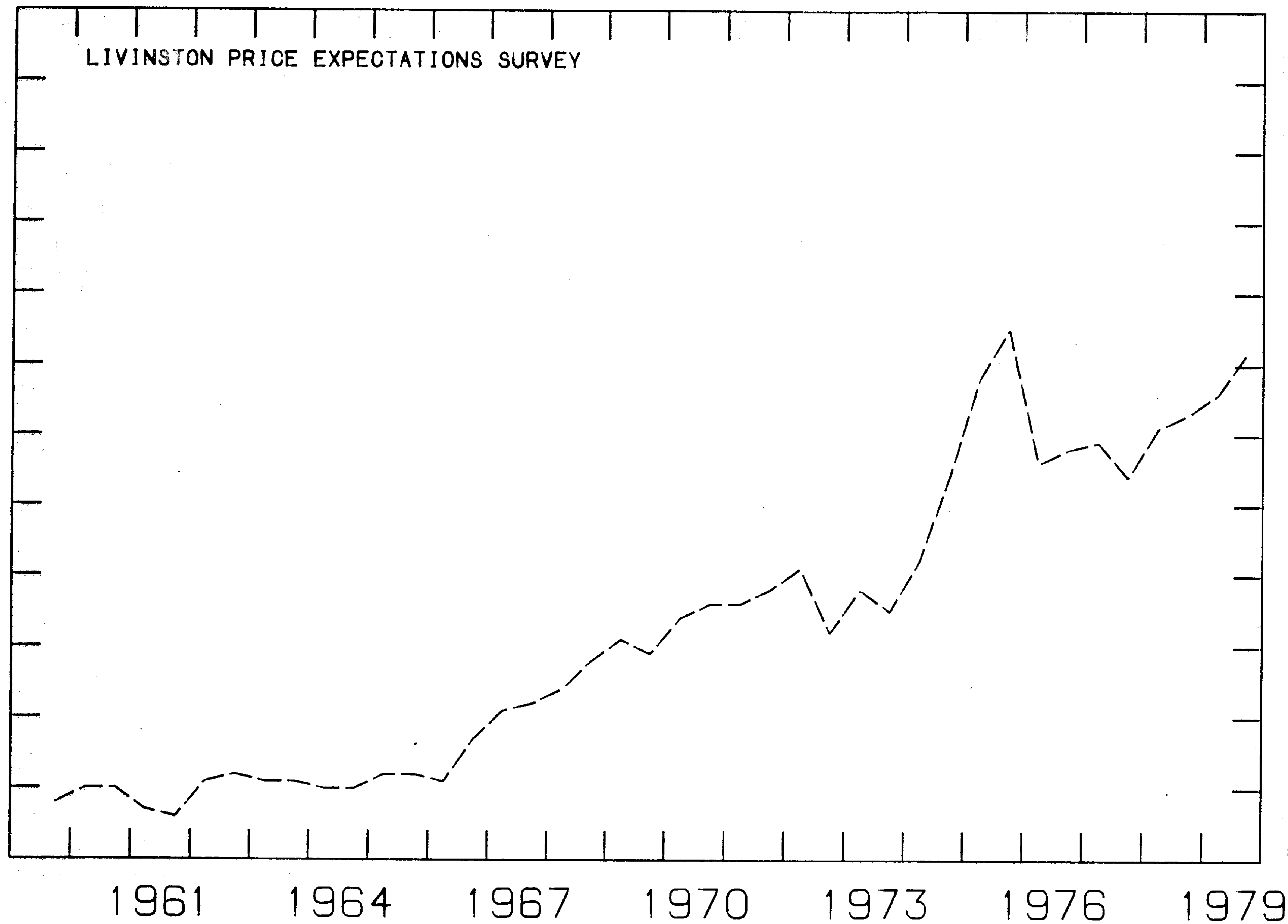
LIVINSTON PRICE EXPECTATIONS SURVEY
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1961
1964
1967
1970
1973
1976
1979

Fig. A.3

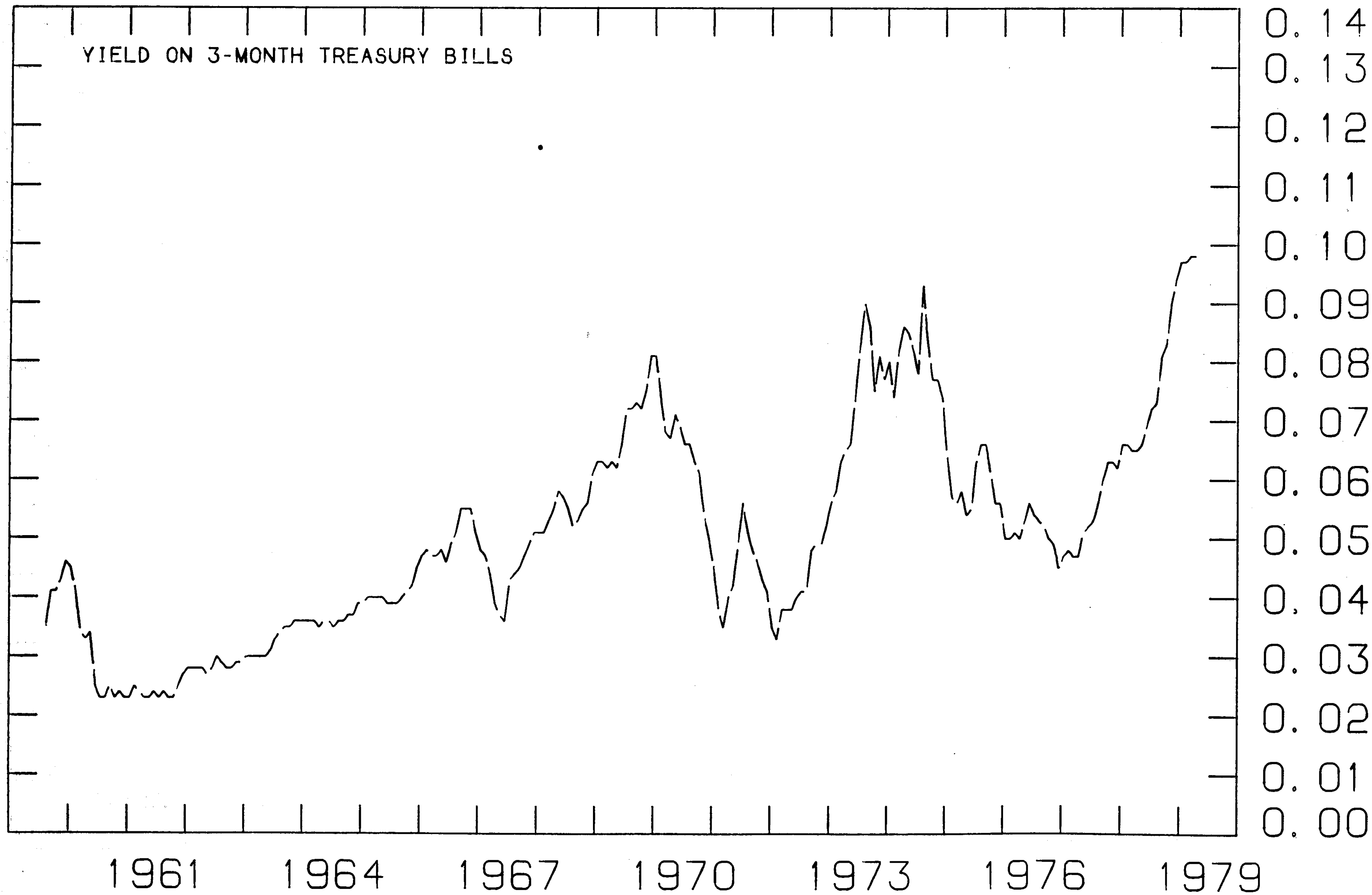
YIELD ON 3-MONTH TREASURY BILLS
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1961
1964
1967
1970
1973
1976
1979

Fig. A.4

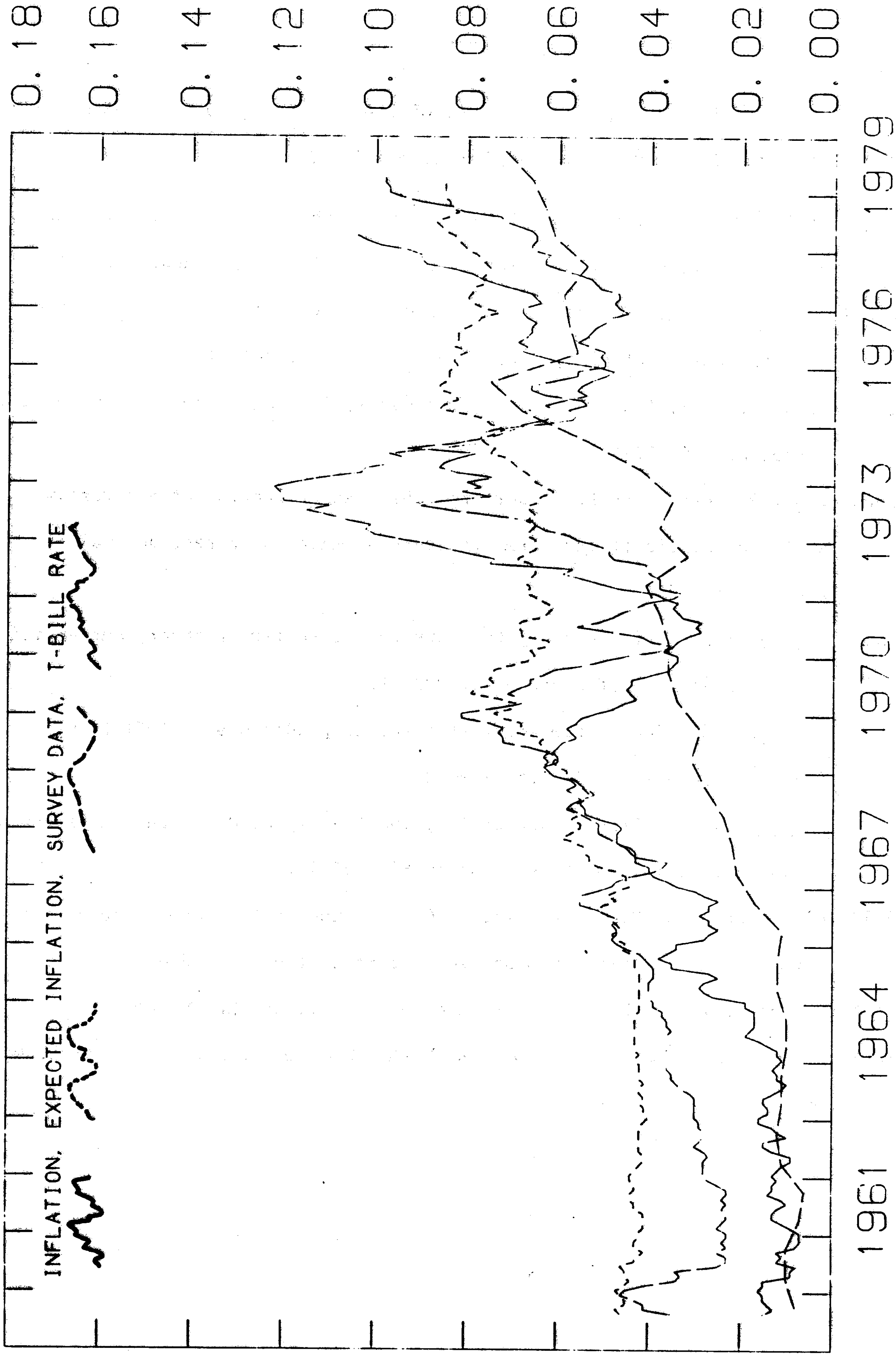
INFLATION, EXPECTED INFLATION, SURVEY DATA, T-BILL RATE
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1961
1964
1967
1970
1973
1976
1979

Fig. A.5

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