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TESTING FOR RATIONAL EXPECTATIONS IN FOREIGN EXCHANGE MARKETS

by

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Introduction

The rational expectations hypothesis implies that if investors are risk neutral (and if transactions costs are zero), the current price of foreign exchange for future delivery --the forward price --will be an unbiased predictor of the actual spot price at the time the forward contract matures. This proposition is conventionally tested by regressing the level of the current spot price on the level of the lagged forward price. This note proposes an alternative test, in which the change in the spot price, or rate of depreciation, is regressed on the forward discount rate. In general the two tests yield different results; it is further argued that the alternative test provides additional insight into the behavior of the forward exchange market. The two test equations are estimated for several exchange markets, and the alternative test is shown to reject rational expectations in a case where the conventional test does not.

Specification of a test of rational expectations

By definition of rational expectations (RE)

(1) $${}_t a_{t+1} = E_t(s_{t+1})$$

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where ${}_t a_{t+1}$ is the spot rate that investors at time t anticipate will hold at time t+1 and E_t denotes mathematical expectation, given the state of the world at time t. If the forward price of foreign exchange is in fact the anticipated future spot rate,[1/]

(2) $$f_t = {}_t a_{t+1}$$

then substitution gives

(3) $$f_t = E_t(s_{t+1})$$

The conventional test of (3) is to estimate the regression coefficients in

(4) $$s_{t+1} = b_0 + b_1 f_t + u_{t+1}$$ [2/]

The null hypothesis of rational expectations is that $b_0 = 0$, $b_1 = 1$, and that the error term u has no serial correlation.[3/]

Rational expectations implies in (4) that the forward price is an unbiased predictor of the level of the spot rate one period later,

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[1/] This requires two additional assumptions: that investors are risk neutral and that transactions costs are zero. Frankel (1978), Levich (1977), and Obstfeld (1978) discuss related problems of estimation and interpretation. As presented here the tests of RE actually test the joint hypothesis that expectations are rational and that the forward price is the anticipated price.

[2/] See, for example, Frenkel (1976), Frankel (1978), Obstfeld (1978). The equation is sometimes estimated in log form, although as Krugman (1977) shows, this introduces a specification error. Frankel (1978, p. 73) argues that the log form is preferred.

[3/] This null hypothesis is a sufficient but not a necessary condition for RE as defined in (3).
RE holds if $E(s_{t+1}) = f_t$, or $E(u_{t+1}) = (1-b_1) f_t - b_0$; $E(u_{t+1})$ may be interpested as ρu_t, where ρ is the autocorrelation coefficient and u_t is known at time t. This condition clearly holds under the given null hypothesis. However, it is also possible, if implausible, that the forward price, ceteris paribus, is a biased predictor ($b_1 \neq 1$) but that the bias is offset in the actual sample by other factors which are incorporated in the error term.

and also that the market is efficient. But the hypothesis further implies that the forward discount is an unbiased predictor of the change in the spot rate. Subtracting s_t from both sides of (3) we obtain

$$(5) \qquad (f_t - s_t) = E(s_{t+1}) - s_t$$

where $(f_t - s_t)$ is the forward discount on domestic currency and $E(s_{t+1}) - s_t$ is the expected depreciation of the domestic currency, given s_t. This condition can be tested using the regression coefficients in

$$(6) \qquad (s_{t+1} - s_t) = b_2 + b_3 (f_t - s_t) + v_{t+1}$$

where the null hypothesis is that $b_2 = 0$, $b_3 = 1$, and v_t is uncorrelated with its previous values.[1/]

Intuitively, it is clear that this is a more stringent test of rational expectations because it requires the forward market to predict without bias not only the level of the future spot rate but the change in that level. In many cases the forward discount is small in magnitude relative to the actual change in the spot rate, so that the market is essentially using the spot rate as a predictor of the future. The conventional test attributes to the forward price a predictive power which might equally well be assigned to the lagged spot price; the alternative test in effect asks whether the forward discount adds anything to the current spot rate as a predictor.[2/]

[1/] Again, this is a sufficient but not a necessary condition for RE. If $b_3 \neq 1$, equation (3) will hold if $E(v_{t+1}) = (1-b_3)(f_t-s_t) - b_2$.

[2/] Frankel (1978, p. 68) notes that for several modern currencies the mean squared prediction error is higher when the forward price is used as a predictor $(s_{t+1} - f_t)$ than when the spot price is used $(s_{t+1} - s_t)$.

Both the estimated coefficients and the test statistics will in general differ between the two test equations (4) and (6). This can in practice lead one test to reject RE while the other does not. Neither reject RE depends on the sample data and on the nature of the underlying mechanism which causes RE to fail. While there is no presumption that tests of RE using the two equations must necessarily be inconsistent, neither is it necessarily true that they will always agree with each other.

For example, suppose that in fact the forward discount is a biased predictor of actual depreciation. In this case $b_3 \neq 1$ in equation (6), and the null hypothesis of RE would be rejected, given sufficiently good data. Under these conditions, what would result from a test of RE using equation (4)? We can rewrite (6) to obtain

$$(7) \qquad s_{t+1} = b_2 + b_3 f_t + (1-b_3) s_t + v_{t+1}$$

If $b_3 = 1$ this equation reduces to (4). Otherwise, the conventional test equation is misspecified because the term $(1-b_3) s_t$ is omitted. The estimated coefficient on f_t will in general differ from b_3, depending on the correlation between f_t and s_t.

This can be seen by expressing s_t as a function of f_t in order to obtain a measure of their correlation:[1/]

$$(8) \qquad s_t = c_1 f_t + w_t$$

The error term w_t may well be autocorrelated, since it is possible that other variables than f_t explain s_t; it is also possible that $c_1 = 0$ and the two variables are perfectly uncorrelated. Substituting (8) in (7)

[1/] This is the "auxiliary regression" of Theil's specification analysis. Theil (1971), p. 549.

gives

$$(9) \qquad s_{t+1} = b_2 + \gamma f_t + [(1-b_3)\ w_t + v_{t+1}]$$
$$\gamma = b_3 + (1-b_3)\ c_1$$

which is the conventional test equation.

Rational expectations implies that the coefficient γ on f_t equals 1.0. This will be true if $b_3 = 1$ or if $c_1 = 1$. Otherwise γ will differ from 1.0 and from b_3, so that tests on the two coefficients may lead to different results.[1/] In particular, it is possible for the forward discount to be a highly biased predictor (e.g., $b_3 > 1$) while the spot and forward rates move so closely together (c_1 close to 1.0) that the coefficient γ is also close to 1.0. In this case it is conceivable that with a given body of data one could reject the hypothesis that $b_3 = 1$ but not that $\gamma = 1$.

Equation (9) also illustrates an ambiguity in the interpretation of the conventional test. An observed value of γ close to 1.0 may reflect the fact that the forward market does have some power to predict future changes in the spot rate, so that $b_3 = 1.0$. On the other hand, it may merely show that the spot and forward rates move closely together (perhaps because the forward discount is recursively determined by the interest parity condition), so that $c_1 = 1$. If there is no discernible trend in the spot rate this will yield $\gamma = 1$ even if

[1/] Another reason for this is that since the error term in (7) differs from that in (4) the test statistics in the two equations will in general differ.

the forward market has no ability at all to predict the future. This latter case is not inconsistent with the RE hypothesis; it means simply that we have not observed investors' expectations and the forward rate is not in fact equal to the anticipated spot rate.

Empirical Results

This section presents results for the two tests using monthly data for the French franc - pound sterling market in the 1920's.[1/] This case was selected because of its continuing historical interest (see, for example, Schuker, 1976) and because it has recently been cited in support of simple monetary models of the exchange rate (see Frenkel, 1978, and Krugman, 1978). Estimating the conventional test (equation 4) for France yields

Franc-sterling, Feb. 1921-Dec. 1926

$$s_t = \underset{(3.081)}{5.637} + \underset{(.0324)}{.941} f_{t-1} \tag{10}$$

$$R^2 = .951 \quad \text{rho} = .197\ (.117) \quad \text{SSR} = 4420.3 \quad n = 70$$

This equation and those that follow are estimated using ordinary least squares with the Cochrane-Orcutt correction for serial correlation. The numbers in parentheses are the standard errors.

The individual coefficients are significantly different at the 95% confidence level from their hypothesized values under rational expectations. However, a test of the joint hypothesis that $b_0 = 0$, $b_1 = 1$,

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[1/] The forward market data used are monthly averages of weekly data from Einzig (1937). The spot price of sterling is the monthly average Paris price given in Sauvy (1965). The period is from January 1921, when the forward market data start, to December 1926, when the franc was stabilized. The franc floated throughout; the float was "clean" except for French government interventions in March-April 1924 and July-December 1926.

rho = 0, yields the statistic $F(3,67) = 1.794$ (the SSR under the null hypothesis is 4775.3) and we cannot reject the null hypothesis at the 95% confidence level. Thus the data may be interpreted as being consistent with rational expectations.[1/]

The results for the alternative test, equation (6), are as follows

Franc-sterling, Feb. 1921-Dec. 1926

$$(11) \qquad (s_t - s_{t-1}) = \underset{(1.299)}{2.444} - \underset{(1.163)}{3.357}\,(f_{t-1} - s_{t-1})$$

$$R^2 = .122 \quad \text{rho} = .245\ (.116) \quad SSR = 3858.5 \quad n = 70$$

This is a startling result --the coefficient on the forward discount is significantly negative at the 99% level. A higher forward discount on domestic currency is associated with a more appreciated exchange rate in the next period. A test of the null hypothesis yields $F(3,67) = 5.307$, and we can reject RE at the 99% confidence level. Thus in this case the conventional test leads to a spurious acceptance of rational expectations.

The negative coefficient on the forward discount in (11) suggests at a minimum that there is more to this case than is revealed by the conventional test. While it is possible that the forward market did systematically guess wrong about the sign of future depreciation of the franc, it is also possible that the result is merely a statistical artifact. In fact, it is clear from the data that the

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[1/] This is the same conclusion reached by Frenkel (1978, p. 176) although the results differ slightly, since Frenkel estimated (4) in log form and over a shorter period (Feb. 1921 - May 1925). The conclusions obtained in this paper regarding the alternative test of RE can also be obtained using Frenkel's specification and time period.

coefficients in (11) are being dominated by the two periods of government in tervention referred to in footnote 1, page 6 during which the franc appreciated rapidly even though it had been at a forward discount. These observations also influence the conventional test.

If these interventions were truly exogenous, and rational investors did not anticipate them, then it is appropriate to test for RE by omitting these periods.[1/] Doing so, for the conventional test we obtain

Franc-sterling, 2/21-2/24,5/24-7/26

$$(12) \qquad s_t = \underset{(2.172)}{-6.190} + \underset{(.025)}{1.109}\, f_{t-1}$$

$$R^2 = .984 \quad \text{rho} = .299\ (.121) \quad \text{SSR} = 1036.8 \quad n = 62$$

and for the alternative test

Franc-sterling, 2/21-2/24, 5/24-7/26

$$(13) \qquad (s_t - s_{t-1}) = \underset{(.571)}{.520} + \underset{(1.135)}{9.737}\, (f_{t-1} - s_{t-1})$$

$$R^2 = .586 \quad \text{rho} = .114\ (.126) \quad \text{SSR} = 758.6 \quad n = 62$$

The coefficient on the forward discount now has the predicted sign, but is significantly greater than 1.0 at the 99% confidence level. The forward market consistently underestimates actual depreciation by a factor of ten. In this case rational expectations is rejected at the 99% level in both equations: (the F statistics are 18.13 and 31.99, respectively).

[1/] Tryon (1978, chapter 3) discusses the merits of this assumption. The residuals created by the interventions are very large and all of the same sign, so that in a small sample they bias the coefficients dramatically. If the sample were large enough they could be regarded simply as additional white noise, and their inclusion would make little difference.

This result might be interpreted in several ways. The apparent bias may reflect hedging by (risk neutral) investors against appreciation of the franc. This is the so-called "peso problem", analyzed by Krasker (1977). If so, events eventually proved investors right, although contemporary evidence goes against this interpretation. (See Tryon, 1978, ch. 3). It may also be that risk aversion caused investors to demand a premium for holding sterling (sterling consistently appreciated more than predicted), although this seems somewhat implausible given that sterling was returned to par in 1925.

Another explanation is, as Nurkse (1944, p. 118) and others since have argued, that speculation actually causes the exchange rate to depreciate more than initially anticipated. It is possible that all speculation occurs in the spot market, and that the forward discount is set by the interest differential, which for some reason is correlated with depreciation. Or, we could accept the result at face value, and conclude that investors were in fact wrong about the future path of the franc, but that the systematic error was just not worth competing away.

These possibilities cannot readily be tested, so we are not now in a position to say that rational expectations does fail in the French case. What we can say is that the alternative test presented here rejects the null hypothesis of rational expectations in a case where the conventional test does not, and that in so doing it raises a number of questions which would not otherwise be brought to the investigator's attention.

Finally, we estimate equations (4) and (6) for six dollar exchange rates for the current period of exchange rate floating. The currencies used are sterling, the French franc, the Deutschemark, the Italian lira, the Swiss franc, and the yen. The data used are 4-week averages of weekly observations of the spot and one-month forward dollar prices of foreign currency for the period March 1973 to December 1978, taken from the Harris Bank Weekly Review.

Table 1 presents the results. Columns 1 - 4 show the constant term, the coefficient on the lagged forward price or the lagged forward discount, and the autocorrelation coefficient, for each equation. The figures in parentheses are the standard errors. Column 5 shows the F - statistic testing the null hypothesis of rational expectations: its critical values at the 95% and 99% confidence levels are 2.73 and 4.07, respectively.

Rational expectations fails in four out of the six currencies --only for the DM and the Swiss franc is the null hypothesis not rejected. The two alternative tests of RE both give identical results except for the French franc and the yen, where the level of confidence with which RE is rejected differs between the two tests. For the DM and the French franc, the value of the test statistic is lower for the alternative test, indicating that in some cases at least the conventional test of RE is the more powerful of the two.

TABLE 1: ALTERNATIVE TESTS OF RATIONAL EXPECTATIONS
USING MONTHLY DOLLAR EXCHANGE RATES
MARCH 1978 - DEC. 1978

CURRENCY	Dependent Variable	Coefficients: Constant	f_{t-1}	$(f_{t-1}-S_{t-1})$	RHO	F(3,73)
Sterling	S_t	.089	.958		.474	7.09
		(.058)	(.028)		(.101)	
	S_t-S_{t-1}	-.015		-1.181	.484	7.43
		(.013)		(1.227)	(.100)	
French Franc	S_t	.035	.843		.319	4.69
		(.013)	(.060)		(.109)	
	S_t-S_{t-1}	-.0004		-.921	.252	3.43
		(.0011)		(1.14)	(.111)	
Deutschemark	S_t	.018	.960		.241	2.23
		(.016)	(.039)		(.111)	
	S_t-S_{t-1}	.0023		.467	.224	1.91
		(.0023)		(1.80)	(.112)	
Lira	S_t	.0085	.941		.536	11.78
		(.0038)	(.028)		(.097)	
	S_t-S_{t-1}	-.0013		-.489	.503	13.88
		(.0008)		(.492)	(.099)	
Swiss Franc	S_t	.0059	.992		.241	2.56
		(.0102)	(.025)		(.111)	
	S_t-S_{t-1}	.0041		-.041	.238	2.69
		(.0030)		(1.58)	(.111)	
Yen	S_t	.0064	.988		.297	3.66
		(.0102)	(.027)		(.110)	
	S_t-S	.0019		.384	.285	4.17
		(.0015)		(.496)	(.110)	

However, the two tests convey a very different impression of the forward exchange market. The conventional test gives coefficients on f_{t-1} which are close to one, suggesting that the forward rate is a good, if perhaps not a perfectly unbiased, predictor of the future spot rate. (None of the coefficients on f_{t-1} is significantly different from 1.0 at the 99% confidence level.) By contrast, the second test gives estimated coefficients on the forward discount, $(f_{t-1} - s_{t-1})$, which range from -1.18 to +.47, well away from the value of 1.0 implied by rational expectations.

The simple t-test of these coefficients is a direct test of whether the forward discount has any significant power to explain the change in the spot rate —this is equivalent to asking whether the forward market can improve upon the current spot rate as a predictor of the level of the future spot rate. In fact, none of the coefficients on $(f_{t-1} - s_{t-1})$ is significantly different from zero, and thus the forward discount has no significant explanatory power. This is true even in the two cases which are consistent with rational expectations. What these results show is that the conventional test of RE does not actually tell us very much about the forward exchange market: the alternative test proposed here helps to make clearer the predictive power of the forward market.

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