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OPTIMAL INTERNATIONAL BORROWING WITH DEFAULT

by

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One of the most conspicuous developments in international financial markets in the 1970's has been the rapid expansion of private lending to developing countries. Although private capital inflows to LDCs have undoubtedly been of great assistance in financing large current-account deficits, growing concern has been expressed over the ability of LDC borrowers to repay. In light of marked increases in the debt burden of many borrowing countries, observers in the international financial community have urged that banks exercise greater caution in their lending of this type. The prospect of more frequent default has also brought to public attention the urgent need for more complete information on private lending to LDC's and for improved analysis of the factors that may influence country default.1/

Most previous studies of default on international lending have followed an approach that parallels the treatment of default on domestic lending to individuals. Essentially this involves projecting a borrower's income stream and debt payments schedule to determine if, and when, he may be unable to meet his obligations. In international applications, this approach is reflected in the widespread use of measures such as the debt-service ratio as an indicator of a country's creditworthiness and default risk. In recent years, however, the

1/ A convenient summary of recent developments can be found in U.S. Senate (1977) and references therein. See also Solomon (1977) and Kapur (1977a).

This research was carried out in part while I was affiliated with the World Bank as a Brookings Institution Policy Fellow and while in my current position with the Federal Reserve Board of Governors. I have greatly benefited from conversations with members of the World Bank and Federal Reserve staffs, including Gershon Feder, Jo Saxe, Syamaprasad Gupta, Dale Henderson, Val Koromzay, Steve Salant and Yves Maroni. The views expressed here -- and any errors as well -- are, however, my own and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System or anyone else on its staff.

international banking community has become increasingly aware that in a macroeconomic context such a parallel approach may be of limited value -- and in some circumstances even misleading. Analytical problems arise not only because of the inherently greater number and uncertainty of elements to be forecast in projecting a country's income stream, but also because of the comparatively wide range of options available to policy makers in borrowing countries. More recent contributions have begun to take account of some of these aspects by introducing such factors as inflation, trade policy, exchange rate policy, and domestic fiscal and monetary policy into analyses of default.

This study extends this movement toward viewing the default decision within a broader policy framework. A basic premise is the notion that default is a legitimate option and, indeed, under some circumstances may even be part of a preferred strategy.2/ To illustrate this point, we construct a dynamic model of an open developing economy which allows for the option of default on international borrowing. Taking lenders' behavior as given, we focus on development strategies available to planners in the borrowing country and use the model to demonstrate conditions under which an optimal development program will include default. This choice and its timing are shown to be sensitive to various parameters of the system, including ones that are controlled primarily by lenders -- a property that suggests how lenders may be able to influence the default decision. Among other things, we show that the extension of additional foreign assistance, which has often been suggested as a remedy for potential default, may in fact have quite the opposite effect.

2/ It is recognized that in most cases default is closely followed by a rescheduling of payments. Unless otherwise indicated, the term "default", as used here, should be understood to mean default *cum* rescheduling.

The model also highlights the point that the default decision is a choice made over dynamic alternatives and, therefore, should not be viewed separately from its dynamic implications. Put another way, the decisions of how much to borrow, whether or not to default, and if so, when, are intimately bound to a country's growth objectives and savings decisions. Accordingly, information based only on the current state of the system (such as the current debt-service ratio or the ratio of debt to GNP, etc.), though important, may be insufficient to evaluate the liklihood of default. Information that bears on the future costs of default may be of equal significance, and its ommission from empirical analyses may constitute a serious specification error.

Empirical Studies of Default

An important reason for undertaking this study was to provide a sounder basis for empirical analysis than has been evident in work on default up to now. Understandably, a good deal of recent attention has been directed at the problem of measuring default risk and characterizing the conditions that precede repayments problems in order to establish standards for creditworthiness. In the last several years the state of the art in this area has moved rapidly from a relatively qualitative, subjective approach -- involving, for example, the use of "check lists" -- to considerably more advanced methods based on econometric techniques. In this regard two published studies stand out: that by Frank and Cline (1971) which uses discriminant analysis and that by Feder and Just (1977a) based on logit techniques. Although the details differ somewhat, both studies derive a weighted index of creditworthiness from pooled, time series and cross-sectional data on major economic aggregates that are commonly employed in analysis of debt.[3/] These indices are then used either (in the case of discriminant analysis) to determine whether or not a country is in the likely-to-default category or (in the case of logit analysis) to estimate the probability of default. In both cases statistical tests on the sample data are quite robust.

Despite the rather impressive results, the two studies share several important shortcomings. Because they are both based on pooled data, the resulting default prediction functions reflect the average properties of the sample. This is clearly unsatisfactory to an analyst interested in a particular country for which past performance and propensities may differ sharply from the group mean. Ideally, a country-specific default function would be used in this case, but this option is usually foreclosed by a lack of data points (i.e., actual defaults). Serious problems also arise when the default prediction functions are used to project beyond the original sample. In this regard, a recent study (Smith, 1977) has shown that the incidence of error increases dramatically when both the Feder-Just and Frank-Cline default prediction functions are used to forecast outcomes with data from more recent periods. It is, of course, not surprising that the functions perform

3/ The Frank & Cline study used observations on 26 LDC's during the 1960-1968 period, and included 13 cases of rescheduling. Statistical significance was established for the debt-service ratio, the ratio of amortization to debt, and the ratio of imports to reserves. The logit study by Feder & Just was applied to a sample of 41 countries over the seven years, 1965-1972. Significance was found for the same three variables, plus the growth rate of exports, the growth rate of per capita income, and the ratio of capital inflow to debt service. For related statistical studies, see Dhonte (1975), Kapur (1977b), Feder & Just (1977b), Sargen (1977), and Hanson & Hoban (1978).

less well outside the original sample, since conditions in world markets and in LDC's may well have changed. The decline in performance, however, does point to an underlying weakness of these methods -- namely, they are not based on any consistent structural model. The choice of explanatory variables in these studies, while generally sensible, is very much ad hoc in nature. Thus, when conditions are altered, it is difficult to determine how the weights in default functions should be adjusted or, indeed, in some cases even what the direction of the effect may be. It is to this point -- i.e., the development of a consistent theoretical framework for analysis of default -- that the model below is directed.

The Default Model

To analyze the interaction of growth, borrowing, and default we shall use an extended version of the familiar neo-classical, single-sector growth model. We assume that the preferences of planners in the borrowing country are described by a discounted community welfare function dependent only on per capita consumption, i.e.,

$$(1) \quad u(c(t))\, e^{-\gamma t}, \qquad 0 < \gamma < 1,$$

where u is an index of community welfare or utility, c is consumption per capita, γ is the discount rate, and t is instantaneous time measured from the start of the analysis ($t_0 = 0$).4/ Primarily for

4/ We ignore the role of the government sector here. Strictly speaking, c should be interpreted as per capita absorption, rather than the narrower concept of consumption.

convenience in computation, we also assume that the utility function is of the following specific form:

$$(2) \quad u(c) = c^{\beta}, \qquad 0 < \beta < 1,$$

where β is the (constant) elasticity of utility with respect to per capita consumption.

Output is produced with inputs of capital and labor according to a neo-classical production function,

$$(3) \quad y = f(k),$$

where y is output per capita and k is the ratio of capital employed in domestic production to labor.[5/] Since capital is internationally mobile in this analysis, it is important to distinguish the ratio of employed factors, k, from the ratio of domestically owned factors-- i.e., per capita domestically owned capital (equity)-- for which we use the symbol, b. The stock of net debt per capita, d, is related to b and k by

$$(4) \quad d = k - b,$$

and will be assumed to be always positive for an LDC. In addition, the labor force (population) is assumed to grow at the exogenously given rate, n.

5/ This production relationship is assumed to exhibit constant returns to scale and to satisfy the usual Inada conditions.

Loans to the country in question are available in the international capital market at a given fixed interest rate, r_N^*. It will be convenient to assume that after each period the current stock of debt is repaid with interest and that new loans are taken out -- in effect, that outstanding balances can be continuously rolled over at the fixed market rate.[6/] To a certain extent this approach is at variance with the observed facts of international borrowing, since many LDC loans are contracted for relatively long periods. It does, however, coincide more closely with practices in the Eurodollar market where shorter-term loans with frequent rollovers are common. The fact that it is loans of this type that have grown most rapidly and over which the greatest concern has been expressed lends support to this view of the borrowing and repayment process.

The system spelled out so far is quite standard; similar formulations have been used in other analyses of optimal borrowing and growth, such as those by Bardhan (1967) and Hanson (1974). The main innovation in this study is the specification of the consequences of default. We start by recognizing that default typically brings both benefits and costs to the debtor. To model the benefit side, we assume that the debt relief associated with default gives rise to a (virtually) immediate windfall gain. Accordingly, we model this aspect by specifying that in the event of default the debtor's total equity, b, will be augmented by an amount, b_D, that measures the debt from which a defaulting country can expect relief. The value of b_D is assumed to be known

6/ r^* is such that the country remains a net borrower over the planning period; we shall also assume that r^* is always above the modified golden-rule rate of return or so-called "natural rate", equal to $(\eta+\gamma)$. The subscript N is used to indicate that this borrowing rate applies when a country has not defaulted; a subscript D will be used for the borrowing rate after a default, as described below.

to planners in advance and to be independent of other variables of the system. On the cost side, we shall require that after default a debtor country will face higher interest charges on subsequent borrowing as lenders react to demonstrated greater risk as well as the desire to recoup losses. The size of this conditional shift in r* is also known to planners in advance.[7]

It will be evident from the analysis below that results depend closely on the specification of these cost and benefit rules. In particular, the assymmetry between short-run benefits versus penalty costs applied continuously over a longer time horizon is an important feature of the decision framework. It should be recognized that this particular specification captures only the broadest features of the consequences of default, and is but one member of a wide class of possible cost and benefit rules. This approach does illustrate quite well, however, a common property of most plausible specifications -- namely, that default amounts, in effect, to an option whereby the debtor can restructure the time profile of borrowing costs in favor of immediate or near-term gains in return for higher future costs.

Even with this highly simplified structure, the planner in a borrowing country faces a complex decision. Starting from given initial conditions, including a fixed set of resources (equity per

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7/ As indicated earlier, in this analysis default is always presumed to be accompanied by a rescheduling of terms. Thus, an outright repudiation of the obligation to repay represents the extreme limiting case; more typically default *cum* rescheduling will translate into a net windfall gain to the defaulting country that is much smaller than the size of the total debt outstanding. We should also point out that the cost and benefit parameters may not always lie in the range indicated above. For example, after a default creditors may sometimes relax borrowing terms, at least in the short-run; likewise, in some circumstances it is possible for the defaulting country to experience a windfall *loss*. We shall exclude these possibilities by assumption, since their implications for optimal strategy are clear.

capita), he has to select among paths for both consumption and external borrowing so as to meet a specified objective at the end of the planning period (in this case, a target terminal level of per capita equity) in such a way as to maximize the cumulated value of community value. His options include both "normal" borrowing programs in which debt is repaid according to the originally scheduled terms and programs that include default, at a time of his choosing, subject to the rules laid out above. In general, the default option brings greater immediate resources, which can be translated through capital accumulation into increased income and consumption in later periods. On the other hand, default also brings penalties that make later borrowing more expensive, thereby reducing subsequent welfare. Although this basic tradeoff can be described easily enough, the answers to the questions of whether or not default should be chosen, and, if so, when -- questions to which we shall turn directly -- are not intuitively obvious.

This problem can be formalized by casting the planner's choice as a problem in constrained dynamic optimization. In effect, he seeks to maximize the functional,

$$(5) \quad \int_0^T u(c(t))e^{-\gamma t}dt,$$

over $c(t)$ and $d(t)$, subject to

$$(5a) \quad b(0) \leq b_0,$$

$$(5b) \quad b(T) \geq b_T, \text{ and}$$

$$(5c) \quad \dot{b} = f(k) - c - \pi b - r_N^* d,$$

where T designates the end of the relevant planning period, and b_0 and b_T

are initial and terminal conditions on the level of domestically owned resources, respectively.[8/] Condition (5c) divides per capita income into per capita investment ($\dot{b} + \pi b$) and consumption (c). Imbedded in equation (5c) is also an assumption that any current-account deficit (in this case, net imports plus interest payments) is balanced by new capital inflow. This continuous external equilibrium is assumed to be accomplished by an appropriate exchange rate adjustment or its equivalent.

The formulation of the problem in (5) applies to "normal", or non-default paths, with d being determined by equation (4). The possibility of default, however, introduces additional side conditions. If we designate the time of default by τ, then condition (5c) applies only in the pre-default segment of the program when $0 \leq t < \tau$. In the post-default segment, the higher cost of borrowing implies instead

$$(5c') \quad \dot{b} = f(k) - c - \pi b - r_D^* d, \qquad \tau \leq t \leq T ,$$

where r_D^* is the higher, post-default borrowing rate. In addition, to reflect any penalties that may apply to periods beyond T, we require that in the event of default planners must meet a terminal target that is higher by an amount b_P.[9/] Accordingly, under default, (5b) is modified to

$$(5b') \quad b(T) \geq b_T + b_P .$$

Finally, the windfall associated with default is given by

$$(5d) \quad b^+ = b^- + b_D,$$

8/ The variables c, b, and d are all time-dependent. When the interpretation is clear, the time argument has been ommitted.

9/ This feature is added in part to rule out "last-moment" defaults -- i.e., default at time T, by which all the gains of default would be realized without cost. The value of b_P is assumed to be set so that the planner would be neutral to events after T in the sense that optimal programs from T onward under default and non-default produce equal discounted utility. To be effective, clearly b_P must be at least as large as b_D.

where b^- and b^+ are the values of equity immediately before and immediately after default, respectively. (In general, superscripts - and + will be used to refer to variables in the pre- and post-default portions of default programs, evaluated immediately before and after τ.)

Solutions to the alternate versions of the problem in (5) are found by application of optimal control techniques. From an analytic point of view, the main difference between the default version and the more standard non-default version lies in the discontinuity of the state variable at the time of default, shown in (5d), and the shift at τ in the system's equation of motion, implied by (5c'). Accordingly, to solve for the optimal default path, conditions (5c') and (5d) should be adjoined to the maximand, (5), and differentiation should then be performed with respect to c, d, τ, b^-, and the adjoint variables.[10/]

The resulting necessary conditions for a maximum produce the following results:

On the optimal non-default path

$$\text{(6a)} \quad r_N^* = f'(k), \qquad 0 \leq t \leq T,$$

and

$$\text{(6b)} \quad \dot{c}/c = \frac{dc/dt}{c} = [f'(k) - (\eta+\gamma)]/(1-\beta), \qquad 0 \leq t \leq T.$$

On the optimal default path, equation (6a) applies over the pre-default segment ($0 \leq t < \tau$), but in the post-default segment it must be modified to

$$\text{(6a')} \quad r_D^* = f'(k), \qquad \tau \leq t \leq T.$$

10/ More detail on the solution technique for this type of problem can be found in most standard references on control theory, such as Bryson & Ho (1969).

Furthermore, in the default case maximization with respect to b^- and τ implies

$$\text{(6c)} \quad u'(c^-) = u'(c^+),$$

and

$$\text{(6d)} \quad u(c^-) + \dot{b}^- \; u'(c^-) = u(c^+) + \dot{b}^+ \; u'(c^+),$$

respectively.

Equations (6a) and (6a') are the familiar conditions, derived by Bardhan and others, that on the optimal path the marginal cost of borrowing should be set equal to the domestic rate of return. Equation (6b), also a standard result, shows how the rate of growth of consumption depends on the relationship between the domestic rate of return and the natural rate, $n + \gamma$. Inasmuch as we have assumed that r_N^* and r_D^* exceed the natural rate, it is apparent from these equations that on the optimal path consumption will increase at a constant positive rate of growth. Furthermore, this growth rate will take a larger value on the post-default segment.

Equations (6c) and (6d), which are unique to the default case, indicate that on the optimal path changes in either the state variable at the moment of default, b^-, or the timing of default, τ, must have an equal impact at the margin on the value of the pre- and post-default segments. Since the same utility function is used throughout, equations (6c) and (6d) can be simplified to yield the following useful results:

$$\text{(7)} \quad c^- = c^+,$$

and

$$\text{(8)} \quad \dot{b}^- = \dot{b}^+.$$

Evidently, if default at an "interior" point ($0<\tau<T$) is optimal, there will be no gap in consumption at that moment, although the rate of change of consumption will show a discrete jump. Conversely, the level of equity will display an instantaneous jump at the moment of default (due to the default windfall), but the rate of change of equity across the default threshold will be unchanged.

The character of the optimal solutions and the nature of the choice between default and non-default programs can perhaps be better understood by considering how the value of the optimal program under default is affected by its timing. Consider for the moment the function

$$V_D(\tau) = \max_{c,d,b^-} \int_0^T u(c(t))\,dt \quad ,$$

subject to (5a), (5b), (5b'), (5c), (5c'), and (5d). When expressed in this form, $V_D(\tau)$ represents the value of an optimal default program for a specific choice of τ after maximization across c, d, and b^-. Several possible configurations of $V_D(\tau)$ are shown in Figure 1. In each panel the value of τ that maximizes the value function, $V_D(\tau)$, is indicated by τ^*; for reference we also show the value of the optimal non-default program by the horizontal line at V_N.[11/]

When V_D is aligned as it is in the left-hand panel, default at τ^* will clearly be preferred to default at any other time <u>and</u> to the non-default option. In the case illustrated, default at an "interior" (i.e., future) point in time would be selected. Under other conditions, however, τ^* might coincide with the left-hand boundary -- i.e., a case in which the optimal policy would be an immediate default. The middle panel shows a configuration in

11/ Notice that the curves in Figure 1 are drawn on the assumption that $V_D(\tau)$ is concave. Some additional comments on concavity are found in the appendix.

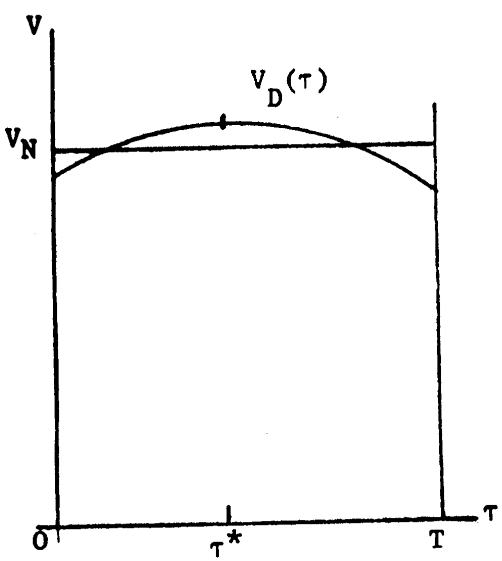

FIGURE 1a

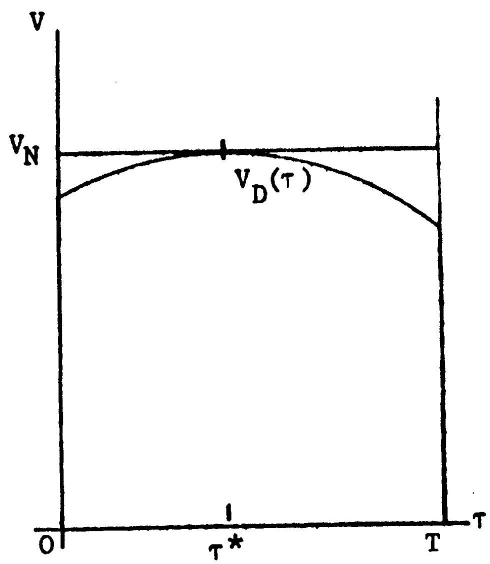

FIGURE 1b

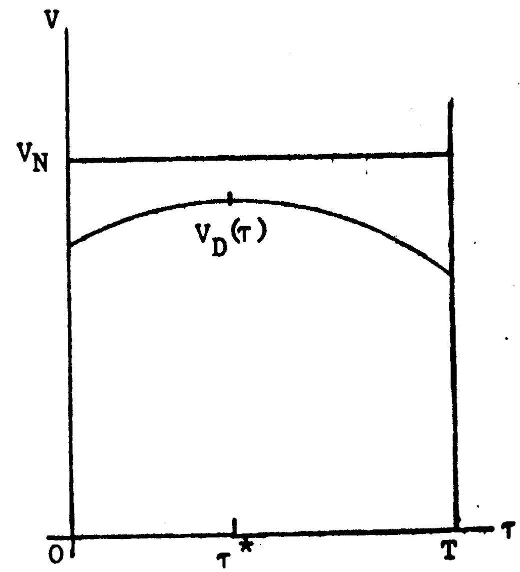

FIGURE 1c

which a planner would be just indifferent between a program of normal borrowing and repayment and a program that includes default at time τ^*. Finally, in the right-hand panel the non-default program dominates even the best default strategy. Obviously, whether or not default is chosen and its best timing are controlled by the shape of the function $V_D(\tau)$ and its position relative to V_N, characteristics which are determined, in turn, by the structural equations and parameters of the model. These relationships will be explored further below.

Additional detail on how key variables move over time is provided by Figures 2a and 2b. Each phase diagram is divided into four quadrants by the two steady-state equations,

$$\text{(9a)} \quad \dot{c} = 0 = [(r_N^* - (\pi+\gamma))/(1-\beta)]\, c,$$

$$\text{(9b)} \quad \dot{b} = 0 = [f(k) - r_N^* k] + (r_N^* - \pi)\, b - c,$$

and the direction of motion of the system in each quadrant is indicated by corresponding vectors. Figure 2a shows the trajectory for $(c(t), b(t))$ on the optimal non-default path. Starting from the required initial level of b_0, both b and c increase monotonically until the required terminal stock of equity, b_T, is attained at time T.[12/] Since b increases throughout, this means that the level of debt and interest payments will decrease monotonically over the course of the program.

Analysis of the default case shown in Figure 2b is somewhat more complex. At the moment of default the stock of equity is augmented by b_D, resulting in a discontinuous jump to the right in the optimal

12/ In line with our intention of describing a representative LDC, we assume here that the time horizon is long enough to ensure that the optimal path for b(t) is monotonic. Similarly, in the default case discussed below, we assume that the windfall gain from default is not so large as to violate this assumption.

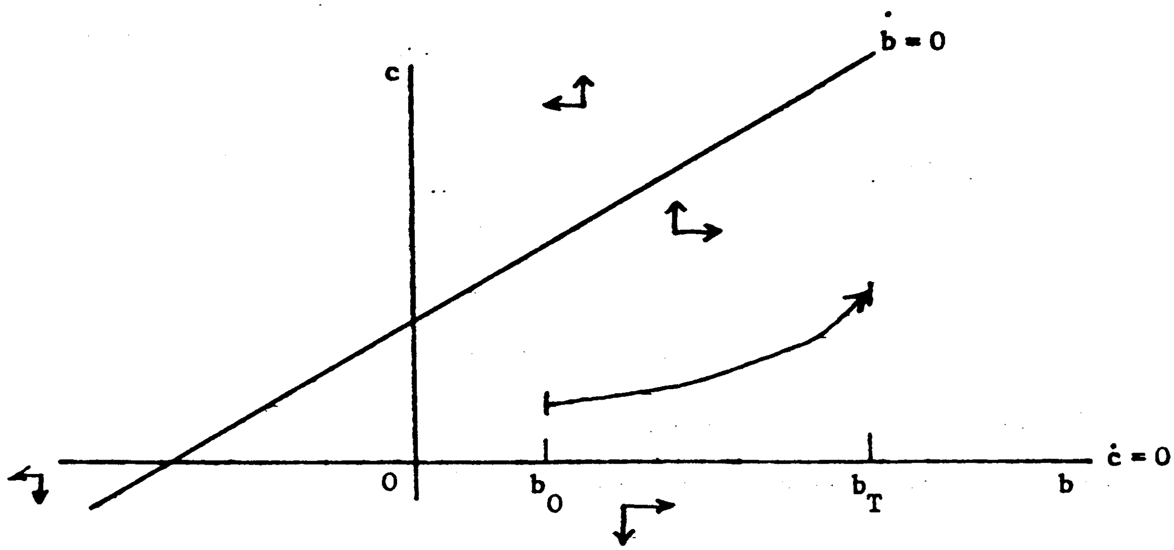

FIGURE 2a

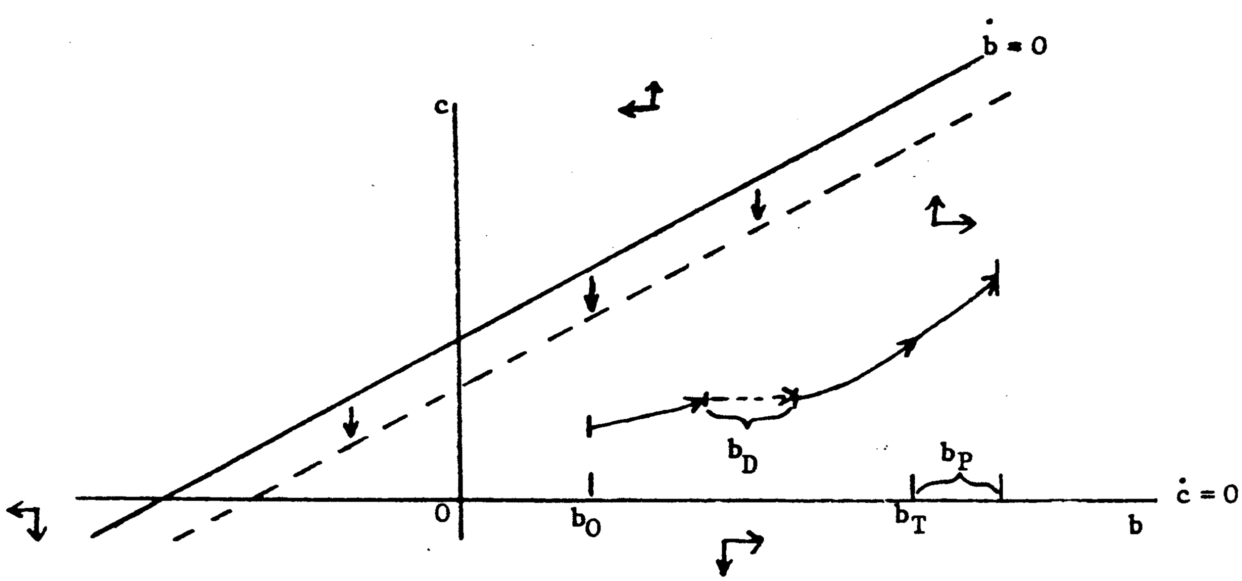

FIGURE 2b

trajectory, as illustrated in the diagram. At the same time the steady-state frontier for $\dot{b}$ (and all other $\dot{b}$ isoquants) is shifted downward by the change in the cost of borrowing. The offsetting effect of the two shifts maintains $\dot{b}(\tau)$ at the same level momentarily, as required by equation (8). Aside from these changes, the general character of the optimal default path is similar to the previous case; both consumption and equity increase monotonically over the full program.

With this information on how the system behaves on alternative optimal trajectories, we can now turn to the issue of what determines the choice between the best default and non-default path. It should be clear that from the decision maker's point of view, the available options are fully described by the initial and terminal conditions, the objective function, and other constraints of the system. Thus, assuming for the moment that the last two elements are invariant, the planner's choice can be regarded as depending only on the level of equity that he inherits from the previous period and the target level. Consider for the moment a simplified version of the problem in which the only alternative to normal borrowing is immediate default. (I.e., we temporarily exclude "interior" future defaults from consideration.) It can be shown that the value functions for the two options have the following form:

$$(10) \quad V_i = (-\mu_{0_i}(G_i) + \bar{\mu}_i)^{\beta}/\mu_{0_i}, \qquad i = N,D, \quad ,$$

where μ_{0_i} and $\bar{\mu}_i$ are positive constants (that differ across regimes), and G is defined by

$$G_i \equiv b_T e^{-(r_i^* - \eta)T} - b_0 , \qquad i = N ,$$

$$G_i \equiv (b_T + b_P)_e - {}^{(r_i^* - \eta)T} - (b_0 + b_D) , \qquad i = D.$$

The composite parameter, G_N (or G_D), may be interpreted as a measure of intended growth, since it is the difference between target equity, discounted by $(r_N^*-\eta)$, and initial equity. It is relatively easy to show that, when $r_D^* > r_N^*$, then $\mu_{0_D} < \mu_{0_N}$; thus, at any point where $V_D = V_N$,

$$\frac{dV_D/db_0}{dV_N/db_0} > 1 ,$$

$$\frac{dV_D/db_T}{dV_N/db_T} > 1 .$$

Furthermore, the same property of relative slopes can be shown to apply to the value function $V_D(\tau)$, for any τ $/0<\tau<T$, again at points where $V_D(\tau) = V_N$. Accordingly, if both V_N and the upper envelope (across all values of τ) of $V_D(\tau)$ curves are regarded now as functions of either b_0 or b_T, then relative slopes at a point of intersection of these two functions will be in the relationship indicated above. This property is illustrated in Figure 3 where intended real growth, G, is measured on the horizontal axis and V_D is now meant to represent the upper envelope of all V_D curves.[13/] In drawing Figure 3, we have assumed that default is the preferred option for at least some range of G; the point at which the value of the two options is equal is indicated by $\tilde{G}$.

Figure 3 can be used to illustrate several important points that may be of value in statistical analyses of default. Evidently, the relative attractiveness of default is enhanced by lower levels of G --

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13/ More detail on the derivation of the value functions for this case is given in the appendix. In interpreting Figure 3, it is convenient to assume that variations in G come from variations in either b_0 or b_T, but not both at once; this allows us to depict V_D and V_N unambiguously as functions of G (i.e., either G_N or G_D) without specifying which version of intended growth is used. Use of either growth measure produces the same qualitative results.

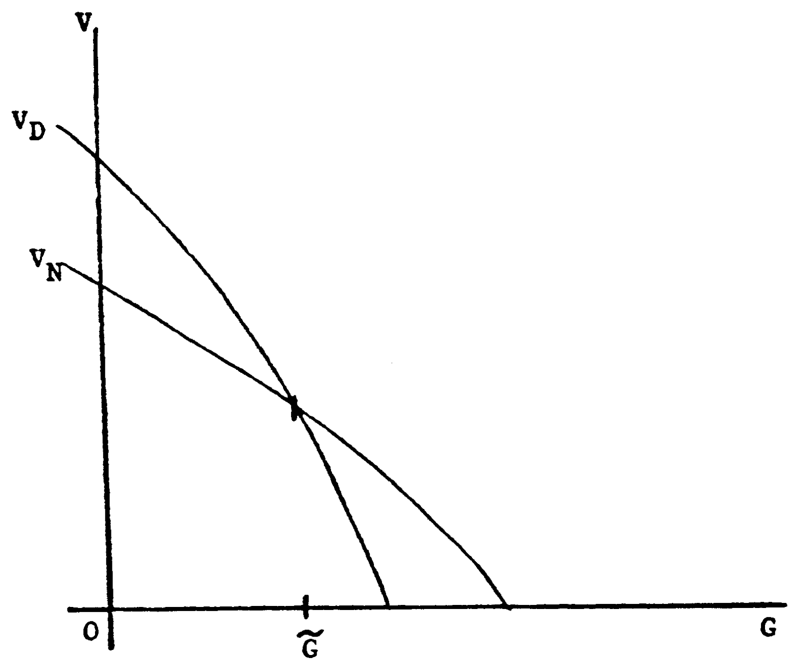

FIGURE 3

i.e., by <u>high</u> values of b_0 or <u>low</u> levels of b_T. The reason for this is that default brings with it higher subsequent borrowing costs. As a result, following a default greater near-term investment and lower near-term consumption will be favored in order to reduce the associated debt-service payments over the remaining course of the program. Countries with higher levels of initial equity need not sacrifice as much near-term consumption to accomplish this, and, hence, on balance will tend to prefer the default option.

The reader may find it quite surprising that our results suggest that, <u>ceteris paribus</u>, default will be <u>more</u> likely to occur among countries with <u>high</u> levels of per capita equity, inasmuch as most statistical studies have come to an opposite conclusion.[14/] These findings, however, are not necesarily inconsistent with the model. Returning to equation (10) and Figure 3, it can be seen that the relative return from default depends on <u>both</u> initial and terminal levels of equity. If countries with low levels of equity (or high levels of debt, etc.) also tend to be countries with low targets to the degree that growth aspirations, as measured by G, are low as well, then the model indicates that a <u>greater</u> incidence of default could be expected from this group, rather than the reverse. Put another way, the current level of equity -- or other variables closely correlated with equity, such as current debt, debt service, debt/GNP ratios, etc. -- may be operating as effective proxies for intended growth in default prediction functions. This suggests, of course, that variables related more directly to G would serve as superior explanatory variables. As a practical matter, devising measures of growth

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<u>14/</u> Similar findings have been reported for the empirical counterparts of variables such as per capita debt (d), per capita debt service payments ($r_N^* d$), their respective ratios to per capita output or income, and the aggregate debt-equity ratio (d/b). Variables that measure the debt burden tend to be positively associated with default, whereas our model suggests a contrary result.

aspirations might be difficult, but variables such as publicly announced growth targets and past growth performance of equity per capita suggest themselves immediately.15/

For similar reasons one should be careful about drawing strong inferences regarding possible default from observations on current variables alone. For example, a country which has recently suffered setbacks that have lowered its net stock of equity and increased its debt would not necessarily be a candidate for default unless growth aspirations had been reduced as well. Conversely, a country which appears to be maintaining a stable level of debt in an acceptable range might be shifted into the potential defaulter category by a downward revision in growth aspirations -- a change that would be likely to go undetected in default prediction functions of the usual sort. Also, it should be obvious that the performance of countries that tend to be outliers in default prediction functions -- such as Mexico and Brazil -- is consistent with this view of default. Although both countries are very heavy borrowers in international capital markets, they are evidently high growth countries as well. By the criterion developed above, therefore, they are not likely to find default to be an attractive option.

Despite the model's message, it is an indisputable fact that default does occur even among borrowers that could be fairly described

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15/ The model can also be used to establish the following point. Consider for the moment the sub-population of borrowers which should prefer default at some future point in time. A comparison of the pre-default path of b for a country in this group with the path of b for the same country if the default option were not available reveals that on the pre-default portion of the path leading to default consumption is lower and the level of equity is higher. This suggests that, if within the group of potential defaulters some borrowers are constrained from defaulting for some exogenous reason (e.g. ethical considerations) and some countries are not, consumption or equity levels could be used as screening variables to distinguish the two groups.

as high-growth countries. In many cases these defaults are brought on by special factors not accounted for in our model, such as a temporary inability to generate sufficient foreign exchange or "bunching" of the repayments stream. Even though the analysis above does not deal with these issues directly, it does indicate that in cases where a non-default program is optimal planners should seek to break such temporary constraints rather than expose themselves to the penalties associated with default -- even if doing so implies a short-term net loss of equity. A measure of the maximum net equity loss that would be acceptable to maintain creditworthiness is given by the distance between points on V_D and V_N connected by a horizontal line in that part of Figure 3 that lies to the right of $\tilde{G}$.

Finally, before concluding this section an additional word of warning is in order. Our remarks above on the effects on default of systematic differences in the terminal condition should alert the reader to the dangers latent in the <u>ceteris paribus</u> assumption. It may not be fair to assume that in fact all borrowers are identical or nearly alike in all other important respects. For example, the parameters that characterize borrowing costs, the benefits and penalties from default, the production function, and utility function could vary over a rather wide range. If any of these variables are also systematically related to the relative return to default and to our variable that measures intended growth then the conclusions of this section would have to be modified. Determining the nature of these possible interactions, however, is beyond the scope of this paper and has been left for later research.

Lender Control of Default

A natural question to raise at this point is how the incidence of default may be influenced by changes in exogenous conditions -- in particular, by conditions under the control of lenders. In analyzing this general issue, there are two separate effects that should be distinguished. Changes in external conditions typically will affect both the choice of whether or not to default and the timing of default. The first effect amounts to a change in the stock of defaulters, whereas the other may be viewed as a short-run change in the flow of defaults. Although from a lender's point of view both an increased number of defaulters and an accelerated pace of defaults would manifest themselves as an increase in the incidence of default, the two effects may respond in different ways to changes in conditions. Accordingly, in the analysis below we shall treat these two aspects separately.

A. Effects on the Default Choice

To determine the effect of a change in some policy parameter, π, on the default decision, refer again to Figure 3. A shift in π will usually give rise to shifts in both V_N and V_D. (Again, V_D is the upper envelope of $V_D(\tau)$ curves, defined across all values of τ.) After these shifts have occurred, a new value of the break-even growth level, $\tilde{G}$, will be found; its location will depend on the size and direction of the shifts in the two schedules and on their slopes in the neighborhood of $\tilde{G}$. More specifically it is easily shown that

$$(11)\quad \frac{d\tilde{G}}{d\pi} = -\frac{(\partial V_D/\partial\pi)-(\partial V_N/\partial\pi)}{(\partial V_D/\partial G)-(\partial V_N/\partial G)} .$$

But, since we have established that the denominator in the expression is negative, the direction of movement of the break-even point is determined by the relative impact on the two value functions of a change in π.

As a simple application of these propositions, consider the effect on $\tilde{G}$ of a change in the "rules of the game" -- for example, an increase in the expected windfall from default. It is easily confirmed that a rise in b_D will bring about an upward shift of V_D and have no effect on V_N. As a result, the value of $\tilde{G}$ will be increased, and some borrowers that had previously been on the non-default (right-hand) side of $\tilde{G}$ will be shifted into the zone where default is favored. It should be obvious from this example that any other policy change that is contingent on default is susceptible to a similar analysis.

An issue of considerable interest in connection with international borrowing by LDC's is the question: To what degree will direct transfers tend to mitigate or accentuate the likelihood of default? Since foreign aid and borrowing are regarded as substitutes in many contexts, one might conclude that increases in foreign aid will tend to reduce borrowing and diminish the incentive to default. With a few modifications of our model, we can determine whether or not this is indeed the case. Foreign aid flows can be added to the model by altering equation (5c) and (5c') to

$$\text{(5c.a)} \quad \dot{b} = f(k) - c - \eta b - r_N^* d + a, \qquad 0 \leq t < \tau,$$

$$\text{(5c.a')} \quad \dot{b} = f(k) - c - \eta b - r_D^* d + a, \qquad \tau \leq t \leq T,$$

where the parameter, a, measures the flow of per capita aid in each period. If we consider the effects of a unit change in aid flow on V_N and V_D in this modified system, it can be shown that both V_D and V_N will shift upward but that the shift in V_D will be larger.[16/]

16/ More precisely, the $V_D(\tau)$ curve for every τ shifts upward more than V_N shifts; hence, the upper envelope of these curves, V_D, shifts by a greater amount as well.

Hence, the numerator in (11) is positive, $\tilde{G}$ moves to the right, and a greater number of borrowers become defaulters. The reason for this outcome is that, at the margin, an increase in foreign aid flow provides greater relief in the post-default period and, therefore, makes default relatively more attractive. It is important to keep in mind, however, that this result assumes that there is no aid-related upward revision in the terminal equity target. From our earlier discussion it should be clear that the conclusion in this case would have to be modified or even reversed by a sufficiently large change in the target. Without such a revision, however, the analysis suggests that increases in aid will tend to increase the incidence of default, rather than the reverse.17/

Although we have assumed so far that planners have certain knowledge of the elements of the model, in fact, many, if not most of the key parameters could be estimated only with a considerable degree of inaccuracy at best. In this regard parameters such as $\bar{b}_D$ and r_D^* stand out; since their estimation requires that planners guess in advance the reaction of lenders to their future default. It may be of interest, therefore, to consider the implications for the default decision of changes in the degree of uncertainty about the value of these parameters.

Let us assume that planners have a subjective p.d.f. for b_D, $g(b_D)$, with mean, $\bar{b}_D$, and variance $\sigma^2_{b_D}$. The expected value of V_D for a given value of G and τ can be written as

$$(12)\quad E(V_D(G,\tau)) = \int g(b_D) \cdot V_D(G,\tau;b_D)db_D.$$

17/ An initial, once-for-all aid transfer will have a similar effect. This can be seen by noticing that such an increase in aid is equivalent to an increase in b_0 and will affect the relative attractiveness of default in the same way.

Inspection of the expanded expression for $V_D(G,\tau)$ in the appendix will show that $V_D(G,\tau)$ is concave in b_D; accordingly, an increase in the variance of b_D will reduce the expected value of $V_D(G,\tau)$. Since V_N is not affected by b_D, it is apparent that increased variance (greater uncertainty) will tend to reduce the number of potential defaulters. In broad terms, the reason is that realizations of b_D on the high side of $\bar{b}_D$ are outweighed by losses from outcomes on the low side -- primarily because of decreasing returns in the utility function. Because of this property of the model, lenders may find that a clear announcement of the size of the default windfall (or penalty), by virtue of its effect on reducing uncertainty, will tend to increase the incidence of default. From the point of view of reducing the likelihood of default, the terms of default are better left vague.

B. <u>Effects on the Timing of Default</u>

To illustrate how changes in conditions can affect the timing of default, let $V(b^-,\tau)$ represent the value of the default program at b^- and τ, after maximizing over $c(t)$ and $d(t)$. Since we are considering, by assumption, effects in the neighborhood of an interior maximum point, the impact of a small change in a policy parameter, π, on τ is given by

$$(13)\quad \frac{d\tau}{d\pi} = \frac{V_{b^-\tau}V_{b^-\pi} - V_{b^-b^-}V_{\tau\pi}}{V_{b^-b^-}V_{\tau\tau} - (V_{b^-\tau})^2} .$$

Since at a maximum point the denominator of (13) is positive, the sign of (13) is the same as that of the numerator (which we shall designate N hereafter). The expressions for the second-order partials, $V_{b^-b^-}$ and $V_{b^-\tau}$, are

$$(14a)\quad V_{b^-b^-} = -u''(c)\left[\frac{\partial c^-}{\partial b^-} - \frac{\partial c^+}{\partial b^+}\right] ,$$

$$\text{(14b)}\quad V_{b^-\tau} = (u'(c)+\dot{b}u''(c))\left[\frac{\partial c^-}{\partial b^-}-\frac{\partial c^+}{\partial b^+}\right],$$

where all variables are evaluated at τ. Expressions for $V_{b^-\pi}$ and $V_{\tau\pi}$ will vary according to our interpretation of π.

For example, consider the effect on timing of a change in the default windfall, b_D. For this choice $V_{b^-\pi}$ and $V_{\tau\pi}$ are given by

$$\text{(15a)}\quad V_{b^-\pi} = u''(c)\,\frac{\partial c^+}{\partial b^+} < 0,$$

$$\text{(15b)}\quad V_{\tau\pi} = u''(c)\dot{b}^+\frac{\partial c^+}{\partial b^+} - u'(c)\ (r^*_D - \eta)\ ,$$

and, therefore,

$$N = \left[(r^*_N - \eta)\,\frac{\partial c^+}{\partial b^+} - (r^*_D - \eta)\,\frac{\partial c^-}{\partial b^-}\right] u''(c)u'(c).$$

But, since $\partial c^-/\partial b^- < 0$ and $\partial c^+/\partial b^+ > 0$, then $N < 0$. Thus, increases in b_D tend to accelerate the timing of default. The larger windfall makes the penalties of the post-default period less burdensome and, at the margin, allows the planner to shift the timing of default forward in order to apply the windfall over a longer period.

If, instead, we let π stand for foreign aid flows, then $V_{b^-\pi}$ and $V_{\tau\pi}$ are given by

$$\text{(16a)}\quad V_{b^-\pi} = u''(c)\left[\frac{\partial c^-}{\partial a}-\frac{\partial c^+}{\partial a}\right] ,$$

$$(16b)\quad V_{\tau\pi} = -u(c)\dot{b}\left[\frac{\partial c^-}{\partial a} - \frac{\partial c^+}{\partial a}\right],$$

and

$$N = -u''(c)u'(c)\dot{b}\ (r^*_N - r^*_D)\left[\frac{\partial c^-}{\partial a} - \frac{\partial c^+}{\partial a}\right].$$

In this case the sign of $\left[\frac{\partial c^-}{\partial a} - \frac{\partial c^+}{\partial a}\right]$ is not obvious, but we have shown elsewhere by applying dynamic sensitivity analysis to the optimal path that this expression must be positive.[18/] Hence, $N<0$ and additional aid flows also tend to accelerate the timing of default. The reason is not unlike that of the previous example. If there is no aid-induced offset in the terminal target, more aid provides greater relief in the post-default period, and encourages the planner to default at an earlier date in order to balance returns in the two segments of the program.

Finally, let us consider the effect of increased variance in b_D on the timing of default. Since for any given b_D

$$V_D(b_D) = \int_0^T u(c(t;\ b_D))dt,$$

then $$E \equiv E(V_D(b_D)) = \int g(b_D)V_D(b_D)db_D.$$

By analogy with equation (13),

$$(17)\quad \frac{d\tau}{d\pi} = \frac{E_{b^-\tau}E_{b^-\pi} - E_{b^-b^-}E_{\tau\pi}}{E_{b^-b^-}E_{\tau\tau} - (E_{b^-\tau})^2},$$

for $\pi = \sigma_{b_D}$, and the sign of $d\tau/d\pi$ is determined by the sign of the numerator of (17). Since $V_{b^-} > 0$ and $V_{b^-b^-} < 0$ for any b_D, $E_{b^-\tau} > 0$ and $E_{b^-b^-} < 0$.

18/ This technique requires differentiation of the system's equations and boundary conditions and evaluation of the resultant first-order system. For more detail, see, Freeman (1978) and Oniki (1973). A similar conclusion can be demonstrated for lump-sum aid transfers given at any time in the program.

Furthermore, since it can be shown that ordinarily[19/] $V_{\bar{b}-b_D b_D} > 0$ and $V_{\tau b_D b_D} > 0$, this implies that

$$E_{\bar{b}-\pi} = \frac{\partial E_{\bar{b}-}}{\partial \sigma^2_{b_D}} > 0,$$

$$E_{\tau\pi} = \frac{\partial E_{\pi}}{\partial \sigma^2_{b_D}} > 0,$$

and

$$\frac{d\tau}{d\pi} > 0.$$

Evidently, greater uncertainty about the value of b_D means that borrowers will tend to delay the onset of default. Hence, in all three cases that we have considered the effect of a parameter shift on timing tends to complement its impact on the critical value of growth, $\tilde{G}$. Higher, more certain windfall or reduced foreign aid will tend to produce both more numerous and earlier defaults.

Concluding Remarks

The main purpose of this study has been to establish a framework for analysis of default on international lending which may be helpful both in forecasting and in devising policy to influence the default decision. In this regard, the model's implications for the default-enhancing effect of

19/ When the post-default segment of the optimal path is not concave across τ (i.e., when $V_{\tau\tau} \geq 0$), we cannot exclude the possibility that $V_{\tau b_D b_D} < 0$ and $\frac{d\tau}{d\pi} < 0$. This outcome appears to be highly unlikely, however, and not of great empirical significance.

low growth aspirations and increases in foreign assistance deserve special emphasis, since they seem not to have been recognized in current thinking about the problem.

It is important to bear in mind, however, that these findings are relevant only to the degree that economic conditions and objectives in a borrowing country conform to these imbedded in the model. Moreover, our conclusions depend to a degree on certain technical features of the model's structure -- i.e., the particular form of its structural equations and other special assumptions. Accordingly, an agenda for future research would include systematic investigation of how our findings might be affected by variations in the model's parameters and by introducing additional structural detail. With regard to the latter issue, it is worth pointing out that in the current, stripped-down version of the model in most periods the national income identity provides the only constraint on borrowing and consumption. Given the prominence of foreign exchange constraints in the literature on default, it would be particularly interesting to see how the default decision would be influenced by the introduction of additional assumptions and side constraints to reflect foreign exchange limitations. Similarly, the characterizations in our model of the international capital market and the "rules of the game" for default are highly idealized. In a more elaborated version one might consider how global or domestic inflation, varying interest rates, or an entirely different set of rules could affect the default choice.

Finally, throughout this analysis the objectives of lenders have been demoted to the background. One can well imagine, however, that a similar, symmetrical analysis could be developed to describe their best strategies. In such a framework one might ask (and answer) whether or not, under changed circumstances, there could be strategies for restructuring debt payments whereby both borrower and lender are made better off. These refinements, however, must await further research.

APPENDIX

I. Solution for the default value function.

To derive the value function shown in equation (10), consider first the equations of motion for the optimal non-default path,

(A.1) $\dot{c} = \emptyset c,$

(A.2) $\dot{b} = \theta_o + \theta b - c,$

where,

$$\emptyset = \frac{r_N^* - (\eta + \gamma)}{1 - \beta} > 0,$$

$$\theta = r_N^* - \gamma > 0,$$

$$\theta_o = f(k) - r_N^* k > 0,$$

Differentiating (A.2) with respect to time, and substituting we have

(A.3) $\ddot{b} = (\theta + \emptyset)\, \dot{b} - \theta\emptyset b - \theta_o\emptyset.$

When $b(0) = b_0$, and $b(T) = b_T$, the particular solution to this equation is

(A.4) $b(t) = z_1 e^{x_1 t} + z_2 e^{x_2 t} - \frac{\theta_o}{\theta},$

where,

$$z_1 = \frac{b_T - e^{\emptyset T} b_0 + (1 - e^{\emptyset T})(\theta_o/\theta)}{e^{\theta T} - e^{\emptyset T}},$$

$$z_2 = -\frac{b_T - e^{\theta T} b_0 + (1 - e^{\theta T})(\theta_o/\theta)}{e^{\theta T} - e^{\theta T}}$$

From (A.2)

(A.5) $c(0) = \theta_o + \theta b(0) - \dot{b}(0),$

and from (A.4)

$$\dot{b}(t) = x_1 z_1 e^{x_1 t} + x_2 z_2^{x_2 t},$$

(A.6) $\dot{b}(0) = x_1 z_1 + x_2 z_2.$

Combining (A.5) and (A.6), we get

$$\text{(A.7)} \qquad c(0) = \mu_{0_N} b_0 + \mu_{T_N} b_T + \bar{\mu}_N ,$$

where

$$\mu_{0_N} = \frac{\phi - \theta}{e^{(\phi-\theta)T} - 1} > 0,$$

$$\mu_{T_N} = -\mu_{0_N} e^{-\theta T} < 0,$$

$$\bar{\mu}_N = \mu_{0_N} (\theta_o/\theta)(1-e^{-\theta T}) > 0.$$

The value of the optimum program is given by

$$\text{(A.8)} \qquad V_N = \int_0^T u(c(t)) e^{-\gamma t} dt.$$

But

$$c(t) = c(0) e^{\phi t},$$

and

$$\begin{aligned} u(c(t)) e^{-\gamma t} &= (c(t)^{\beta}) e^{-\gamma t}, \\ &= c(0)^{\beta} e^{(\beta\phi - \gamma)t}, \\ &= c(0)^{\beta} e^{(\phi-\theta)t}. \end{aligned}$$

Hence,

$$\begin{aligned} V_N &= \int_0^T c(0)^{\beta} e^{(\phi-\theta)t} dt, \\ &= c(0)^{\beta} \int_0^T e^{(\phi-\theta)t} dt, \end{aligned}$$

$$\text{(A.9)} \qquad V_N = \frac{c(0)^{\beta}}{\mu_{0_N}} .$$

Substituting (A.7) in (A.9) yields

$$V_N = (\mu_{0_N} b_0 + \mu_{T_N} b_T + \bar{\mu}_N)^{\beta} / \mu_{0_N} ,$$

$$\text{(A.10)} \qquad = [-\mu_{0_N}(b_T e^{-\theta T} - b_0) + \bar{\mu}_N]^{\beta} / \mu_{0_N} .$$

A similar expression can be derived for the case of immediate default. Furthermore, it is easily shown that

$$d\mu_0/dr^* < 0,$$

and, thus, the two curves in Figure 3 are configured as shown.

The derivation of the value function, $V_D(\tau)$, for a given time of default, τ, is similar except that, according to equation (7), expressions for pre- and post- default values of $c(\tau)$ must be obtained for each segment and set equal. The final result is

$$V_D(\tau) = \lambda c(0)^\beta,$$

$$V_D(\tau) = \lambda[(1+\Psi e^{\phi_N \tau})(\mu_{0_N} b_0 + \bar{\mu}_N) - \Psi(\mu_{\tau_D} b_D + \mu_{T_D} b_T + \bar{\mu}_D)]^\beta, \qquad \text{(A.11)}$$

where

$$\lambda = \overline{(1/\mu_{0_N})} + ((e^{(\theta_D - \phi_D + \beta\phi_N)\tau})/\mu_{\tau_D}) > 0,$$

$$\Psi = \mu_{\tau_N}/(\mu_{\tau_N} e^{\phi_N \tau} - \mu_{\tau_D}) > 0,$$

$$\mu_{0_N} = \frac{(\phi_N - \theta_N)}{e^{(\phi_N - \theta_N)\tau} - 1} > 0,$$

$$\mu_{\tau_N} = -\mu_{0_N} e^{-\theta_N \tau} < 0,$$

$$\mu_{\tau_D} = \frac{(\phi_D - \theta_D)}{e^{(\phi_D - \theta_D)(T-\tau)} - 1} > 0,$$

$$\bar{\mu}_N = (\theta_{o_N}/\theta_N)\,\mu_{0_N}(1 - e^{-\theta_N \tau}) > 0,$$

$$\bar{\mu}_D = (\theta_{o_D}/\theta_D)\mu_{\tau_D}(1 - e^{-\theta_D(T-\tau)}) > 0,$$

$$\mu_{T_D} = -\mu_{\tau_D} e^{-\theta_D(T-\tau)} < 0.$$

II. Concavity of the default program.

To establish conditions for local concavity across b^- and τ for an interior critical point, consider the value function for default at b^- and τ, $V(b^-,\tau)$, after maximizing over $c(t)$ and $d(t)$. The second-order condition for a maximum at an interior point is

$$D = V_{\tau\tau} V_{b^-b^-} - (V_{b^-\tau})^2 > 0.$$

These three partial derivatives can be written as,

$$V_{b^-b^-} = -u''(c) \left(\frac{\partial c^-}{\partial b^-} - \frac{\partial c^+}{\partial b^+}\right) < 0,$$

$$V_{b^-\tau} = u'(c) \left(r_N^* - r_D^*\right) + \dot{b}u'(c) \left(\frac{\partial c^-}{\partial b^-} - \frac{\partial c^+}{\partial b^+}\right),$$

$$V_{\tau\tau} = -\dot{b}\, V_{b^-\tau},$$

where all terms are evaluated at τ. Hence, after combining and simplifying,

$$D = V_{b^-\tau}[-u'(c) \left(r_N^* - r_D^*\right)].$$

Since the term in brackets is always positive, D will be positive if $V_{b^-\tau} > 0$, or equivalently, if

$$\frac{-u''(c)}{u'(c)} \dot{b} \left(\frac{\partial c^+}{\partial b^+} - \frac{\partial c^-}{\partial b^-}\right) > r_D^* - r_N^*.$$

Since $\left(\frac{\partial c^+}{\partial b^+} - \frac{\partial c^-}{\partial b^-}\right)$ can be made very large in the vicinity of the end points of the interval [0,T], this suggests that optimal default programs with an interior maximum can be devised by an appropriate selection of parameters. Notice also that the above condition establishes only a local maximum, not a global maximum.

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