International Finance Discussion Papers

Number 220

April 1983

SPECULATION AND HEDGING USING OPTIONS ON FUTURES CONTRACTS

by

Laurence R. Jacobson

NOTE: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to International Finance Discussion Papers (other than an acknowledgment by a writer that he has had access to unpublished material) should be cleared with the author or authors.

Speculation and Hedging Using Options on Futures Contracts

by Laurence R. Jacobson*

1.) Background

Last year, a number of options contracts were initiated in several exchanges, and several others have been started in 1983. Trading began in the fourth quarter of 1982 for options to purchase Treasury bond futures, options for gold futures, and options for spot purchases of five major currencies. Options on stock index futures were initiated early this year. The rapid expansion of options contracts is probably related to the increased volatility (variance) of interest rates, exchange rates, stock prices, etc., as well as to the enlarged opportunity set for hedging and speculation. Some of the new option contracts may become moribund due to thinness of the market, although any prognosis at this time may be premature. As will be explained further below, the existing options on futures (gold, Treasury bonds) are only a subset of possible options on futures contracts, since the expiration date of available options always bear a fixed relationship to the delivery date of the futures contract. However, all other possible futures options prices would be determined by arbitrage possibilities.

The next section of this paper will explain the economic purpose of options on futures contracts in expanding the set of hedging opportunities available to entrepreneurs facing exchange rate risk (or interest rate risk, etc.). The relationship beweeen spot transactions, futures contracts, options on spot transactions, and options on future contracts will be described; in the case of currency markets, this yields a generalized interest rate parity condition.

*This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.

The third section gives a brief tutorial of different speculative opportunities provided by options, and describes how options can be used to partially hedge speculative positions. Since these strategies are described in detail elsewhere*, this section is intended mainly to serve as a tutorial for those unfamiliar with the market.

The final section of the paper is a description of some of the theory of options pricing which has been developed in the literature, and how the new options on financial instruments might provide useful information for economic analysis. Since the variance on the price of the underlying asset is a key variable affecting the price of an option, options may provide useful information as to the market's expected variance of exchange rates, interest rates, etc.

2.) Hedging with options on spot and forward transactions

Consider a U.S. importer who requires 1 million marks one year from now. Hedging of the exchange rate risk can take place by either of two methods if spot and futures markets exist: 1) The importer could purchase marks spot and invest the proceeds in mark denominated assets for one year, or 2) The importer could buy 1 million marks in a futures contract for delivery in one year. Of course, incomplete markets or institutional constraints (fixed dates for delivery, minimal size of transactions, etc.) will in general limit the ability of an importer to hedge exchange rate risk, and it would not generally be optimal to hedge all risk even if possible (see McKinnon, 1979). Moreover, covered interest arbitrage (under certain assumptions) assures that

---

* See the reports prepared by the exchanges, listed in the bibliography.

the opportunity cost of (1) must equal the cost of (2), so that interest rate differentials determine the forward discount or premium:

$$(1)\quad (1+r_d)/(1+r_f)\cdot S = F$$

where S = spot rate (\$/mark), F = forward rate, and r_d = interest rate on dollar denominated assets, and r_f = interest rate on mark assets. If $r_d > r_f$, the forward rate must have a discount (a higher dollar price of marks) equivalent to the interest rate differential.

Consider now an importer who is unsure whether or not he will need 1 million marks in one year. Suppose he will either require 1 million marks or no marks at all. Even with complete spot and futures markets (no discrete time problems, etc.) he will not be able to perfectly hedge his exchange rate risk. Under these conditions, the optimal hedge will be a function of the mean and variance of his probabilistic exchange needs and should be between zero and a million marks, but his actual need one year hence will result in an under-hedged or over-hedged position. Unlike the previous case, the importer has uncertainty about the magnitude of his foreign exchange needs as well as about the movement in exchange rates.

On the other hand, if an options market exists for spot foreign currency transactions, the importer could buy a call (an option to purchase) for 1 million marks which expires one year hence, and can let the option expire unused if the million marks are not required. At the cost of the option, the importer is now able to perfectly hedge his exchange rate risk. In fact, even if the importer does not require the million marks, the value of the option may well exceed zero at the termination date and the importer could sell or exercise the option and offset the initial option cost. The option price thus represents the maximum cost of hedging. Of course, an equivalent method of hedging in this case would be for the importer to purchase 1 million marks spot and buy a put (an option to sell) for 1 million marks one year hence. The costs of these alternative means of hedging must be equivalent or arbitrage will occur. If, for instance, the cost of a call option plus the cost if exercised exceeds the cost of the put option (for the same strike price) plus the interest opportunity cost of purchasing and holding mark denominated assets, arbitragers could make profits by writing calls (selling options to purchase) and purchasing puts, and would bid up the put price and bid down the call price until arbitrage opportunities disappeared.

That is, for a particular strike price (the price at which the option to purchase or sell the currency is set), the following relationship will hold, evaluated at the expiration date of the option:

$$(2)\quad (1+r_d)P^X_{p.G} + S\cdot X(1+r_d)/(1+r_f) = (1+r_d)P^X_{c.G} + G\cdot X$$

where $P^X_{p.G}$ = the price of a put option to sell X units of currency at price G and $P^X_{c.G}$ = the price of a call option to buy X units of currency at price G.

A third method of hedging in this case is to purchase 1 million marks forward rather than spot while buying a put. By substituting (1) into (2) this yields the following relationship between the put and call prices as a function of the forward rather than the spot exchange rate:

$$(3)\quad (1+r_d)P^X_{p.G} + F\cdot X = (1+r_d)P^X_{c.G} + G\cdot X$$

If the strike price equals the forward rate*, the price of the put and call options must be identical. If the call price exceeds the put price,

*In market jargon, an at-the-money option.

then an arbitrager could write calls and buy puts, while simultaneously buying the foreign currency spot (or forward). If the end-of-period spot rate exceeds the forward rate, the purchaser of the call will exercise the option, and the arbitrager will sell his foreign currency. If not, the call will expire and the arbitrager will exercise his put. In either case, the arbitrager earns the difference between the call option and put option prices and has no foreign currency position when the options expire. When the strike price is greater than the forward rate, the call option is priced less than the put option (and conversely when the strike price is less than the forward rate).

An option to purchase (sell) foreign currency spot may equivalently be thought of as a forward purchase (sale) with an option to back out of the agreement at any time up to the delivery date (that is, the expiration date of the option). The importer in effect is buying an insurance provision to back out of his forward contract at any date up to the expiration date of the option. However, this insurance protection might be more than ample (and thus more expensive) than what the importer actually needs. Suppose that the importer is uncertain whether he will need 1 million marks one year hence, but will receive information within the next six months that will enable him to determine with certainty his needs at the end of the year. In this case, the importer only needs an insurance provision to void a 1 year forward purchase of 1 million marks which expires after six months rather than 1 year. An option to purchase a futures contract (with delivery one year after the contract date) which expires in six months will satisfy this need. It is intuitively clear (and required by a simple 'dominance' theorem) that the price of an option which expires in six months to purchase currency at the end of a year will cost less than the option which expires at the end of a year.

However, while an option to purchase a futures contract might be better suited to the importer's needs than an option to purchase spot, note that the importer could also satisfy his needs by purchasing 1 million marks spot (or six months forward) and buying an option to sell 1 million marks spot which expires in six months. (If he discovers he will need the marks during the first six months, he will let his option expire, otherwise it will be executed.) Of course, the transactions cost may be greater for this operation (particularly, if the importer faces a 'borrowing constraint'), but it will always be true that an option on a futures contract is equivalent to an option for a spot transaction and a spot or forward transaction. In addition, the importer could also purchase an option to buy 1 million marks which expires in six months. In short, given the existence of a complete set of spot and futures markets, and spot options, prices of options to purchase futures contracts will be determined by arbitrage given spot and forward exchange rates, spot option prices, and interest rate differentials.

The existing options on gold futures and Treasury bond futures have expiration dates which are exactly one month prior to the delivery date on the futures contract (thus, only the delivery date is quoted in the market). The set of options prices generated by the one month ahead expiration clause is equally suited to the task of giving the importer the flexibility that would be allowed by having a set of options which expire on the delivery date (that is, spot options). In other words the arbitrage condition only requires that one vector of the matrix of expiration dates and delivery dates for call and put options on a commodity is necessary to determine the prices of all other options with alternate expiration and delivery dates, if complete markets exist for underlying futures (and spot) contracts. Increasing the variability of expiration dates relative to delivery dates will not add additional information to the market (although, might reduce transactions costs for some individuals). For a commodity which is costly to deliver (like gold), or when trading volume in near term futures contracts is high, it may be institutionally preferrable to have options on futures contracts (which expire at some set time prior to delivery) rather than options for spot transactions. The low volume of trading thus far for foreign currency options on the Philadelphia Exchange (which require spot delivery) may be indicative of a less than optimal institutional framework.

3.) Strategies for speculation using options

A wide range of speculation strategies exist for using options, sometimes in conjunction with futures contracts. Several possibilities are outlined in this section. Note that some popular strategies involve partially hedging speculative positions, and hence could be considered something between pure hedging and pure speculation.

a) An importer with a view on exchange rates desires a partial hedge.

Suppose an importer needs to acquire marks at some point in the future but expects the future spot rate to be less than or equal to the forward rate. Since he expects to be able to purchase the currency at a cheaper rate in the future, he would rather not acquire a forward contract. However, the importer may desire some protection against an adverse exchange rate movement (i.e., a mark appreciation). The importer may buy a call option with the strike price equal to the forward rate to limit his losses in case of a mark appreciation. If marks in fact are cheaper in the future as the importer expected, the option will not be exercised. The importer gains the difference between the actual future spot rate and the initial forward rate less the cost of the option as compared to the cost of complete hedging using the forward market. Compared to a completely unhedged position, the importer loses the cost of the option. If the future spot rate exceeds the forward rate, the option is exercised. In this case the importer loses the cost of the option relative to his position had he entered a forward contract, but is better off than a completely unhedged position if the actual additional cost of foreign exchange exceeds the option price.

If the importer purchased a call option with a strike price, say, 2 cents/mark higher than the forward rate, the option price would be much lower, but his loss would be higher in the case of an adverse rate movement. This would be analogous to an insurance policy with a lower premium but a higher deductible.

b) A speculator expects highly variable exchange rates, but has no view of the direction of movement.

A strategy for this speculator would be to purchase both call and put options for a particular strike price (and same expiration date). The speculator will have a gain on one option and will let the other option expire. If there is a big movement in price, the gain on one option will exceed the cost of the options.

This strategy could be modified by having different strike prices for the put and call options, if the speculator expects a greater likelihood of an exchange rate movement in a certain direction.

c) A speculator expects little movement in exchange rates.

This speculator could write a put and a call at the same strike price. He expects that the price he receives for selling the two options will exceed the loss which will occur when one or the other option is exercised.

d) A speculator expects a currency to depreciate (appreciate) more than the forward rate.

The speculator can sell (buy) futures and buy call (put) options. The option provides insurance as to the investor's maximum loss and also provides 'staying power'. With only a futures contract, an adverse rate movement can wipe out the initial margin requirement. A call option limits the maximum loss and can prevent a margin call if the speculator wants to maintain his position.

e) A speculator expects a currency to depreciate (appreciate) more than the current foward rate.

The speculator can write call (put) options. This strategy has unlimited loss potential (of course, the speculator can close out the position by buying a call (put) option at a subsequent time). The speculator will gain the amount of the option price if the exhange rate moves as expected, as the option will be allowed to expire. By writing calls (puts) with higher (lower) strike prices, the investor has less likelihood of loss, but will receive smaller fees from writing the options.

f) An institution holding foreign currency wishes to hedge against a decline in its value.

This might be considered pure hedging, but could also be considered hedging a speculative open position. The institution could purchase put options.

g) An institution holds several currencies for transactions purposes, and expects the future spot rate to equal the forward rate.

The institution could write call options, using its currency holdings to satisfy any margin requirements. The institution will gain if the currency does not appreciate enough to offset the premium received for writing its options. Losses are unlimited until the position is closed out. Of course, as with stock options, a common public misperception is that the institution cannot lose if it is holding the securities to back the call options it is writing. In fact, it may lose an unlimited amount on the option, although this will be offset by the gain on its security holdings.

4.) Some notes on option pricing.

A number of useful boundary conditions on option prices have been derived through dominance arguments. A dominant asset or portfolio is one in which the return on the asset is greater or equal to the return on another asset under all possible states of the world, and strictly greater for at least one state of the world. Such an asset cannot exist, since the relative price of that asset must be bid up until dominance disappears. Arbitrage must eliminate dominance. Interest rate parity and the correspondence between spot options prices and futures options prices discussed in Section 2 are dominance conditions. While dominance arguments place useful limitations on put and call prices, the level of options prices cannot be determined without a model of option price determination. In this section I will summarize some key dominance arguments described in detail by Merton, and will briefly describe the general equilibrium option pricing model developed by Black and Scholes (1973) and some extensions and applications made by other authors. A useful review of the literature through 1975 is found in Smith (1976).

Two types of options have been adopted in markets. American options allow the option to be exercised at any time between the contract date and the expiration date. European options can be exercised only on the expiration date. Clearly, an American option can be priced no lower than an otherwise identical European option. However, American call options will generally not be exercised before the expiration date if a secondary market exists. This occurs because the option price in the secondary market must be at least as great as the intrinsic value of the option if immediately exercised. Holders of American options will generally close out open positions by selling options rather than by exercising them. Certain problems (such as uncertain dividend payments in the case of equity options) might cause early exercise of American options.

American put options, however, have a positive probability of premature exercise even if technical difficulties concerning dividends, etc. are eliminated. This assymmetry is due to the fact that asset prices are bounded below. Suppose a holder of a gold futures put option at $400/oz. observes that the futures price falls to $1/oz. The option could immediately be exercised for a profit of $399; if not exercised, the price of the option could never exceed $400. If the (risk free) interest that could be earned on $399 prior to the expiration date exceeds the expected additional gain from holding the option (most likely), the option would be prematurely exercised.

Call options prices must be non-increasing functions of the exercise price, while put options must be non-decreasing functions of the exercise price. Both puts and calls on American options must be priced as non-decreasing functions of the expiration date. As noted in Section 2, if futures prices exist, then the call price will be greater/equal/less than the put price whenever the exercise price is less/equal/greater than the futures price. The table shows these relationships for actual trading in gold futures. This restriction also implies that the price of both call and put options less their intrinsic value will reach a maximum at the current futures price, as shown in the lower panel of the table.

Note, that arbitrage would also suggest that for any particular strike price, puts and calls should be priced identically once the intrinsic value is subtracted. If not, an arbitrager could write the higher priced call and fully cover himself by purchasing the lower priced call, earning the difference in the premiums. However, since the price paid for the option includes the intrinsic value, the interest cost of purchasing the higher priced option exceeds the interest received from selling the lower priced option. Thus, the price of an in-the-money call (a call with intrinsic value) less the intrinsic value should be lower than the put at the same exercise price, and conversely for in-the-money puts. This is generally true in the case shown in the table, although there are some slight discrepancies.

As can be observed in the table, the option price (both put and call) less the intrinsic value approaches zero as the strike price approaches zero or infinity. This results from the fact that such options will always or never be exercised. For example, I will gladly sell you an option to give you gold futures (exercise price of zero) at any margin above the futures price, since I could cover myself immediately and earn the margin. However, you obviously would be unwilling to pay a penny more than the futures price for this gift.

Black and Scholes use a general equilibrium asset model to derive options prices expressed as differential equations. The solution to the

Table
Prices of Gold Futures and Options on Gold Futures
(December 10 closing prices, WSJ)

Gold Future Prices

Spot	436.00
Feb.	441.50
April	448.30
August	462.50

Gold Future Options

Strike Price		Calls Feb.	Calls April	Calls August	Puts Feb.	Puts April	Puts August	
360			88.50			2.00		
380		62.00	71.00	88.00	1.50	4.50	9.50	
400		44.50	55.50	74.50	3.50	7.50	14.00	Increasing
420		27.50	41.50	62.00	7.50	12.00	21.50	
440	Decreasing	16.00	31.50	50.50	14.00	20.50	28.50	
460		9.00	21.00	42.50	26.50	31.00	37.50	
480		4.50	15.00	33.00		43.00	49.50	
500		3.00	10.00	27.00		57.00		
530			6.00	18.00			78.00	

Calls: Increasing →
Puts: Increasing →

Gold Future Options Less Intrinsic Value

Strike Price	Calls Feb.	Calls April	Calls August	Puts Feb.	Puts April	Puts August	
360		.20			2.00		Decreasing
380	.50	2.70	3.50	1.50	4.50	9.50	
400	3.00	7.20	12.00	3.50	7.50	14.00	
420	6.00	13.20	19.50	7.50	12.00	21.50	
440	15.50	23.20	28.00	14.00	20.50	28.50	(Peak at futures price)
460	9.00	21.00	40.00	8.00	19.30	37.50	
480	4.50	15.00	33.00		11.30	32.00	
500	3.00	10.00	27.00		5.30		
530		6.00	18.00			10.50	Decreasing

Calls: Increasing →
Puts: Increasing →

option pricing problem is a function of five variables: the asset price, the variance rate on the asset price, the strike price, the expiration date of the option, and the risk-free interest rate (which is assumed constant). In addition to yielding a solution which can be explicitly solved given the values of the five variables, several qualitative results are derived which conform to the dominance arguments already described. The price of calls and puts are also shown to be increasing functions of the variance of the asset price. The call price rises with an increase in the risk free interest rate, while put prices fall, since the present value of the exercise price falls. Note that options pricing models usually are concerned with European options (those which can be exercised only on the expiration date) or American call options. As a result of the possibility of premature exercise discussed earlier, no closed form solution exists for the valuation of American put options. However Brennan and Schwartz (1977) derive an algorithm for solving the put pricing problem under certain circumstances.

The Black-Scholes option pricing model assumes a continuous stochasic process for describing price movements. A solution using a jump stochastic process is derived in Cox and Ross (1977). Cox, Ross, and Rubinstein (1979) simplify the mathematical derivations by using a discrete binomial formula, which contains the Black-Scholes and jump process formulas as special limiting cases. The qualitative characteristics of options prices in relation to changes in the asset price, expiration date and other variables are not affected by the type of stochastic process assumed, although numerical results and forecasts may differ to some extent.

A number of empirical studies of options prices have been undertaken, using data for equity options which have been in existence for ten years. (See Beckers (1981), Brenner and Galai (1982), Latane and Rendleman (1976), and Schmalensee and Trippi (1978).) The Black-Scholes model, or variants thereof, can be used to derive the implied variance of the asset price given option prices, other known variables, and some assumption about the appropriate 'risk free' interest rate. The variance implied in options price can be empirically tested as a predictor of future volatility and compared with historical variance as a predictor. In general, actual option prices are reasonably predicted using the general equilibrium pricing model. Implied variances are most accurate for at-the-money options; for deep in-the-money or out-of-the-money options, the option price is minimally affected by the variance so that a small price differential implies a large difference in the implied variance.

Foreign currency options have recently been examined by Giddy (1982) and Garman and Kohlhagen (1982). Garman and Kohlhagen show formally that the currency spot call option price increases with the domestic interest rate and rises with the forward exchange rate. Although the option price also varies positively with the spot rate and negatively with the foreign interest rate, these are not independent conditions because of interest rate parity. Hedging using interest rate options (Treasury bonds, bills) has been discussed by Goodman (1982).

In summary, option pricing models may be useful empirically to derive estimates of the market's expected variance of interest rates, gold prices, exchange rates, etc., or to examine how quickly market participants react to actual changes in volatility of asset prices. In addition to the possible use of options prices in empirical work, direct observation of changes in option prices may provide additional insight into ongoing trends in financial markets.

BIBLIOGRAPHY

Beckers, Stan. Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Variability. Journal of Banking and Finance. 1981.

Black, F. and M. Scholes. The Pricing of Options and Corporate Liabilities. Journal of Political Economy. June 1973.

Brennan, Michael and Schwartz, Eduardo. The Valuation of American Put Options. Journal of Finance. May 1977.

Brenner, Menachem and Galai, Dan. A Note on Measuring the Risk of Common Stocks Implied by Options Prices. Working Paper #278. NYU Graduate School of Business Administration. 1982.

Chicago Board of Trade. Strategies for Buying and Writing Options on Treasury Bond Futures. 1982.

COMEX. Options on Comex Gold Futures. 1982.

Cox, John and Ross, Stephen. A Survey of Some New Results in Financial Option Pricing Theory. Journal of Finance. May 1976.

Cox, John and Ross, Stephen. The Valuation of Options for Alternative Stochastic Processes. Journal of Financial Economics. May 1977.

Cox, John, Ross, Stephen, and Rubinstein, Mark. Option Pricing: A Simplified Approach. Journal of Financial Economics. 1979.

Emanuel, David and MacBeth, James. Further Results on the Constant Elasticity of Variance Call Option Pricing Model. Journal of Financial and Quantitative Analysis. November 1982.

Feiger, George and Jacquillat, Bertrand. Currency Option Bonds, Puts, and Calls on Spot Exchange and the Hedging of Contingent Foreign Earnings. Journal of Finance. December 1979.

Garman, M. and Kohlhagen, Stephen. Foreign Currency Option Values. School of Business Administration, Berkeley. Working Paper. December 1982.

Giddy, Ian. Foreign Exchange Options. Working Paper Series #CSFM-47. Columbia Business School. December 1982.

Goodman, Laurie. Interest Rate Options and Financial Institutions. Draft paper. Federal Reserve Bank of New York. 1982.

Latane, Henry and Rendleman, Richard. Standard Deviations of Stock Price Rates Implied in Options Prices. Journal of Finance. May 1976.

McKinnon, R. Money in International Finance. 1979.

Merton, R. C. Theory of Rational Option Pricing. Bell Journal of Economics. Spring 1973.

Philadelphia Stock Exchange. Foreign Currency Options: The Third Dimension to Foreign Exchange. 1982

Schmalensee, Richard and Trippi, Robert. Common Stock Volatility Expectations Implied by Option Premia. Journal of Finance. March 1978.

Smith, Cifford, Jr. Option Pricing: A Review. Journal of Financial Economics. 1976

Wolf, Avner. Fundamentals of Commodity Options on Futures. Working Paper Series #CSFM-35. Columbia University. July 1982.