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ON THE DYNAMIC PROPERTIES OF ASYMMETRIC MODELS OF REAL GNP

Allan D. Brunner

ABSTRACT

There is now a substantial body of evidence that suggests business cycles are asymmetric. However, the evidence has been accumulated using a wide array of statistical techniques and, consequently, is based on various definitions of asymmetry. This paper examines several parametric models that have been used to study asymmetries in real GNP. Although these models capture asymmetries in very different ways, their dynamic properties are remarkably similar.

On The Dynamic Properties of Asymmetric Models of GNP

Allan D. Brunner[1]

I. Introduction

Linear time-series models have been used widely and quite successfully by economists for several decades. These models are based on the classical framework set forth by Box and Jenkins (1976), which assumed a linear model and a Gaussian, homogeneous error distribution. These assumptions, however, place strong restrictions on the time-series behavior of economic variables. Most importantly, they imply several types of symmetric behavior. For example, positive and negative shocks of equal magnitude have symmetric effects on the dependent variable using such a model.

Although there is now a substantial body of evidence that suggests that business cycles are not symmetric, that evidence is based on a variety of statistical models and, implicitly, on a variety of definitions for asymmetry. Initially, evidence of asymmetry was based on nonparametric tests. In a seminal article, Neftci (1984) proposed a nonparametric test for "steepness" in economic time-series. He concluded that contractions are steeper than expansions for postwar unemployment data, and Rothman (1991) confirmed those results. DeLong and Summers (1986), using an alternative test for steepness, found similar results. Sichel (1993) proposed a test for "deepness" and found evidence in unemployment variables that contractions are deeper than expansions.

More recently, the evidence of asymmetries has been based on various parametric models. While these models have properties that overlap somewhat, they can be categorized roughly by the way they relax the classical assumptions. One category has focused on the nonlinear behavior of the

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conditional mean. For example, Terasvirta and Anderson (1991), Potter (1991b) and Beaudry and Koop (1993) have used the threshold autoregressive model to study cyclical asymmetry. Their results generally show that contractions are less persistent than expansions.

More recently, economists have turned their attention to the time-varying properties of higher moments and, in particular, to conditional heteroskedasticity. Again, there are a number of such models, including the autoregressive conditional heteroskedasticity (ARCH) model, the generalized ARCH (GARCH) model, and the exponential GARCH (EGARCH) model. Conditional heteroskedasticity has been found in many economic variables, including employment data, GNP, consumption, investment, inventories, durable and nondurable goods, asset prices, and producer and consumer prices. For examples, see Engle (1982), Bollerslev (1986), Nelson (1991), French and Sichel (1993), and Brunner and Hess (1993).

A final category of models relaxes the Gaussian error distribution assumption. The most general is Gallant and Tauchen's (1990) seminonparametric (SNP) model, which accommodates arbitrary departures from Gaussianity and conditional heterogeneity. Brunner (1992, 1994) and Hussey (1992) have used SNP models to assess the properties of conditional distributions of several economic variables. Each study found strong evidence that the shape and time-varying characteristics of distributions during contractions are quite different from distributions during expansions.

This paper compares several asymmetric models of real GNP growth. Although these models differ dramatically in the way they model asymmetries, the dynamic properties of the models are remarkably similar. In particular, the most prominent feature of each model is conditional heteroskedasticity: The conditional variance of output increases dramatically during contractionary episodes. In addition, there is some evidence of nonlinear behavior in the conditional mean. Overall, this behavior is analogous to the notions of steepness and deepness that is suggested by nonparametric definitions of asymmetry.

II. Modelling Conditional Asymmetries

This section briefly outlines a framework for modeling conditional asymmetries, allowing for departures from linearity, Gaussianity, and homogeneity. This framework nests several models that have been used to study asymmetries, including the SETAR model, the SNP model, the ARCH family of models, and the EGARCH model. These models will be used to investigate conditional asymmetries in next section.

Let Δy_t denote the growth rate of real GNP, and let x_{t-1} denote a vector containing the history of Δy_t. Consider the following framework for modelling y_t:

$$\begin{aligned} \Delta y_t &= f(x_{t-1}) + \sigma_t \cdot z_t \\ \sigma_t &= h(x_{t-1}) \\ z_t &\sim g(x_{t-1}) \end{aligned} \tag{1}$$

In equation (1), the conditional mean, the conditional variance, and the error distribution are state-dependent functions of x_{t-1}. In this framework, $f(\cdot)$ allows for possible nonlinearities in the conditional mean, $h(\cdot)$ permits conditional heteroskdasticity, and $g(\cdot)$ permits more general forms of non-Gaussianity and conditional heterogeneity. By contrast, if $f(\cdot)$ is a linear function of x_{t-1}, $h(\cdot)$ is time-invariant, and $g(\cdot)$ is Gaussian and time-invariant, the model in equation (1) is a standard auto-regressive time-series model.

This framework nests several models that permit departures from linearity, homogeneity, and Gaussianity. The remainder of this section outlines a few of these models that will be used in the next section to study the asymmetric properties of real GNP.

<u>SETAR models</u>. Although many nonlinear models have been developed, the threshold

autoregressive model has been used quite successfully to study business cycle asymmetries.[2] Potter (1991b), for example, has introduced the self-exciting threshold autoregressive (SETAR) model. The SETAR model relaxes both the linearity and homoskedasticity assumptions by allowing the parameters of the autoregressive model to switch between various states. The switches are driven by the value of the current state vector (x_{t-1}). The SETAR(k,d,p) model has the following general form:

$$\Delta y_t = \sum_{j=1}^{p} \theta_{i,j} \cdot \Delta y_{t-j} + \sigma_i \cdot z_t \qquad \text{if } \Delta y_{t-d} \in A_i \quad (i=1,...k) \qquad (2)$$

$$z_t \sim N(0,1)$$

where p denotes the number of autoregressive lags; σ_i is a constant; k denotes the number of possible states; d denotes the specific lag, Δy_{t-d}, that drives the regime shifts; and A_i denotes the range of values for Δy_{t-d} that are associated with regime i, i=1,..k.

ARCH-type Models. In a seminal article, Engle (1982) introduced the ARCH model, which models the conditional variance as a function of lagged, squared forecast errors. Bolerslev (1986) extended the ARCH model -- called the generalized ARCH (GARCH) model -- to permit the effects of an increase in conditional variance to decay slowly over time. One drawback to the ARCH and GARCH models is that both positive and negative forecast errors lead to an increase in conditional variance. Brunner and Hess (1993), using state-dependent models of conditional variance (SDM-V), relaxed this symmetry condition and allowed the conditional variance to be an asymmetric function of either lagged levels of the dependent variable or lagged forecast errors.[3] The AR(p) model with SDM-V(k,l,m) errors can be written as follows:

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[2] For a complete treatment of nonlinear models, see Priestley (1988, 1989). Hamilton's (1989) switching regime model has also been used extensively to study business cycle asymmetries. That model does not fit into the framework in this paper, however, since the model depends on an unobservable, exogenous variable, rather than the history of the dependent variable.

[3] Nelson's (1991) EGARCH model allows for asymmetric effects of forecast errors on the conditional variance but does not nest ARCH and GARCH models.

$$\Delta y_t = \sum_{j=1}^{p} \theta_j \cdot \Delta y_{t-j} + \sigma_t \cdot z_t$$

$$\sigma_t^2 = \sigma_0^2 + \sum_{j=1}^{k} \beta_{1j}^2 \cdot (\Delta y_{t-j} - \gamma_{1j})^2 + \sum_{j=1}^{l} \beta_{2j}^2 \cdot (z_{t-j} - \gamma_{1j})^2 + \sum_{j=1}^{m} \beta_{3j}^2 \cdot \sigma_{t-j}^2 \tag{3}$$

$$z_t \sim N(0,1)$$

SNP Models. Gallant and Tauchen's (1990) seminonparametric (SNP) models are able to accommodate arbitrary departures from both Gaussianity and homogeneity. The SNPRX(p,K_Z,K_X) model can be written as follows:

$$\Delta y_t = \sum_{j=1}^{p} \theta_j \cdot \Delta y_{t-j} + \sigma_t \cdot z_t$$

$$\sigma_t^2 = [\sigma_0 + \sum_{j=1}^{p} \beta_j \cdot \Delta y_{t-j}]^2 \tag{4}$$

$$z_t \sim g(x_{t-1})$$

Gallant and Tauchen model g(·) as a Hermite polynomial expansion of a Gaussian density, which can approximate any general departures from normality. The degree of the polynomial is K_Z. In addition, the parameters of the polynomial are allowed to be K_X-degree polynomials in x_{t-1} in order to capture more general forms of heterogeneity. See Gallant and Tauchen (1990) for more technical details.[4]

III. Asymmetric Models of Real GNP

This section of the paper describes the results of estimating three asymmetric models of real GNP -- the SETAR model, the SNPRX model, the SDM-V model. The data are 175 quarterly observations of U.S. real GNP growth in \$1982 from 1947 to 1990. An "optimal" model in each

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[4] Gallant and Tauchen (1990) used the absolute value of Δy_{t-j} in their specification of h(·), which imposes symmetry with respect to Δy_{t-j}. Brunner (1992) found the asymmetric relationship in equation (4) to be important in stabilizing the conditional heterogeneity found in real GNP.

category was chosen on the basis of several model selection criteria.[5] The optimal specifications are the SETAR(2,2,2) model, the SNPRX(2,2,2) model, and an AR(2) model with SDM-V(0,1,1) errors. A simple AR(2) model will serve as the benchmark for the analysis.

Table 1 presents the results of several diagnostic tests that were performed on the residuals from each selected model. These tests are designed to detect model misspecification related to linearity, Gaussianity, and homogeneity. The first two tests are standard tests for detecting evidence of non-Gaussianity -- see Greene (1990). The remaining tests are Lagrange multiplier (LM) tests of the type suggested by Engle (1982), Breusch and Pagan (1978), White (1982), Newey (1985) and Tauchen (1985). The LM tests can be divided into misspecification tests for the conditional mean and the conditional variance. For mean tests, the residuals from each model were regressed on lagged values of the residuals (serial correlation), and on lagged values and lagged squared-values of real GNP growth rates (level effects). Likewise, for variance tests, squared residuals were regressed on lagged values of squared residuals (ARCH effects), and on lagged values and lagged squared-values of real GNP growth rates (level effects).

The results of the diagnostic tests for the simple AR(2) model suggest that real GNP growth is not well captured by a linear model with Gaussian, homogeneous errors. There is statistically significant evidence of kurtosis, as well as marginally-significant evidence of nonlinearities in the conditional mean and of time-varying conditional variance. The SETAR model, which models heteroskedasticity and nonlinearities in the conditional mean, provides somewhat mixed results. Although the model removes any evidence of nonlinearity in the conditional mean, there is still significant evidence of non-Gaussianity and marginal evidence of heterogeneity.

The SNPRX model, which models both non-Gaussianity and heterogeneity, appears to pass all

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[5] The model selection process is described in a supplemental appendix available from the author upon request. An EGARCH model was also estimated and provided nearly identical to those for the SDM-V model.

of the diagnostic tests. There is a very marginal amount of ARCH left in the residuals, however, as evidenced by the p-value at the fourth lag. That could be the result of the way the SNP framework models the time-varying conditional heteroskedasticity, as a function of lagged output rather than lagged forecast errors. The AR(2) model with SDM-V errors also appears to perform well, even though it relaxes only the heteroskedasticity assumption.

Table 2 presents other criteria for evaluating the asymmetric models of real GNP growth. The number of estimated parameters and the value of the negative log-likelihood function are listed in the first and second rows of the table, respectively. Since each model nests the AR(2) model, likelihood ratio tests for this restriction are reported in the third row of the table. Conditional symmetry can be rejected at the 1% significance level in all cases. The final rows of the table present two standard criteria for comparing non-nested models. Minimizing the Akaike information criteria (AIC) results in selecting the SNPRX model. The Schwarz criterion, which is known to be more conservative than the AIC in small samples, selects the SETAR model (the smallest of the asymmetric models).

The results presented so far are somewhat inconclusive. While there is no doubt that real GNP has asymmetric properties, it is unclear what the optimal specification for asymmetry should be. The SNPRX model is the only model that is both suggested by a model selection criterion (the AIC) and passes the battery of diagnostic tests. The SDM-V model passes the diagnostic tests, but finishes second-to-last using either model selection criteria. Although the SETAR model is chosen by the Schwarz criteria, it does not pass all of the diagnostic tests. Rather than choose an optimal model at this stage, the next section takes a closer look at the asymmetric properties of these models using analytical tools discussed in Potter (1991a), Brunner (1992), and Gallant, Rossi, and Tauchen (1993).

IV. Asymmetric Properties of Real GNP

Nonlinearities. The SETAR(2,2,2) specification entails an AR(2) model, with the parameters of the model switching between two sets of values based on whether output growth two periods in the past was negative or positive. The parameter estimates (and their standard errors) are:

$$
\begin{aligned}
\Delta y_t &= \underset{(2.8)}{1.57} + \underset{(3.9)}{.31}\,\Delta y_{t-1} + \underset{(1.6)}{.20}\,\Delta y_{t-2} + \underset{(19.7)}{3.50}\,z_t \qquad \textit{if } \Delta y_{t-2} > 0 \\
\Delta y_t &= \underset{(-1.3)}{-1.61} + \underset{(2.0)}{.44}\,\Delta y_{t-1} - \underset{(-1.9)}{.79}\,\Delta y_{t-2} + \underset{(10.8)}{4.82}\,z_t \qquad \textit{if } \Delta y_{t-2} < 0
\end{aligned}
\tag{5}
$$

and indicate both a nonlinear conditional mean and conditional heteroskedasticity.

In order to examine the nonlinear properties that are implied by the SETAR model, Figures 1a through 1c plot responses of both y_{t+i} and σ_{t+i} to impulses of various magnitudes -- $z_t = +2, +1, -1$, and -2.[6] The impulse response functions in Figure 1a have been conditioned on output growth having been at its unconditional mean for the two preceding quarters ($\Delta y_{t-1} = \Delta y_{t-2} = 3.2$ percent). Although the responses of the level of GNP to each shock are fairly similar, there is some evidence of nonlinearities: Note that the responses to positive shocks are fairly gradual, while the effects of a negative shock accumulate somewhat more quickly. By contrast, the responses of the conditional standard deviation are radically different. Negative shocks lead to strong increases in uncertainty about future values of output, since these shocks put GNP growth near zero. Positive shocks have little impact on the conditional variance, however, because the model has been conditioned on output growth being greater than zero.

Figure 1b shows the responses when output growth has been about 1 standard deviation below its unconditional mean (-1.1 percent) in two preceding periods. In this case, there appears to be more evidence of nonlinearities. With respect to the level of GNP, even bad news leads quickly to expansionary growth after a couple of quarters, as a result of the negative second autoregressive coefficient in equation (5). In addition, similar to the results by Beaudry and Koop (1993), positive

[6] The impulse response functions were simulated in RATS 4.0, using 10,000 replications. The simulations are based only on estimated model parameters and ignore any additional structure implied by the diagnostic tests presented in Table 1.

shocks are much more persistent than negative shocks. Indeed, the cumulative effect of a negative shock is not much greater than the initial impulse.

Figure 1c shows the responses when the SETAR model has been conditioned on a very good state-of-the-world -- GNP growth has been one standard deviation above its unconditional mean (about 7.4 percent growth). The responses of GNP are nearly identical to those shown in Figure 1a. In addition, since output growth is almost always positive after any shock in this state, uncertainty about future values of output growth is not affected by new information.

Non-Gaussianity. The SNPRX(2,2,2) model was chosen as the optimal SNP specification, which indicates statistically significant departures from both Gaussianity and heterogeneity. Figure 2 shows two possible densities from the SNPRX model. The density to the right is conditioned on 7.4 percent growth in two previous quarters, the density to the left is conditioned on minus 1.1 percent in the two previous quarters. While the density on the right -- corresponding to a contraction in GNP -- has a larger conditional variance, both densities appear to be fairly Gaussian. Table 3 presents a more comprehensive analysis of possible departures from Gaussianity. Each row corresponds to a different set of conditioning information (previous values), ranging from 13.9 percent growth to minus 7.5 percent in two previous quarters. The values in columns 2 through 9 are cumulative probabilities found in the tails of the densities relative to various critical values. For reference, the last number in each column (at the bottom of the table) corresponds to the probability value associated with a standardized Gaussian distribution.[7] These results are consistent with the features of the SNPRX model discussed earlier (Figure 2): After correcting for heteroskedasticity, the SNPRX model is fairly Gaussian when growth has been close to its unconditional mean. Departures from Gaussianity appear only when growth has been very strong or very weak.

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[7] The numbers in the table are analogous to the cumulative values used in Kolmogrov-Smirnov tests; see Bradley (1968, pg. 296).

Heterogeneity. The SNPRX(2,2,2) specification also indicate a departure from homogeneity. The results presented in Table 3, however, suggest that the model is primarily capturing conditional heteroskedasticity and not higher-order forms of heterogeneity. Further evidence is presented in Figure 3, which plots the model's impulse response functions using the same conditions as in Figure 1. As with the SETAR model, there is some evidence of nonlinearity in the conditional mean and a sharp increase in the conditional standard deviation during contractionary episodes.

Since heteroskedasticity appears to be the most important feature of the SETAR and SNPRX models, it could be more efficient to use a model with this distinct feature. Recall that an AR(2) model with SDM-V(0,1,1) errors was chosen as the optimal specification within the SDM-V class of models. The parameter estimates for this model are:

$$
\begin{aligned}
\Delta y_t &= \underset{(3.5)}{1.58} + \underset{(3.0)}{.28}\,\Delta y_{t-1} + \underset{(1.9)}{.17}\,\Delta y_{t-2} + \sigma_t \cdot z_t \\
\sigma_t^2 &= \underset{(1.4)}{1.23^2} + \underset{(2.4)}{.31^2}\,(z_{t-1} - \underset{(-1.7)}{.95})^2 + \underset{(10.8)}{.83}\,\sigma_{t-1}^2
\end{aligned}
\qquad (6)
$$

Note that with this specification of the conditional variance, uncertainty decreases for some positive shocks and increases sharply for all negative shocks. As before, uncertainty is fairly autocorrelated, and the effects of "news" take several periods to die out.

Figure 4 shows the impulse response functions based on an AR(2) model with SDM-V(0,1,1) errors. Since Δy_t is a linear function of x_{t-1} and σ_t is not a function of x_{t-1} in this model, the impulse response functions are impervious to all sets of conditioning information. Moreover, responses to the level are symmetric with respect to the magnitude of the impulse; that is, a +2 shock has the exact opposite response as a -2 shock. Finally, as suggested by the previous results, the impulse response functions for the conditional variance are remarkably similar to impulse responses using the SETAR and the SNPRX models. Still, although negative shocks increase uncertainty about future values of output growth, positive shocks have little impact on the conditional variance. The small impact of positive "news" is similar to the behavior in Figures 1a and 3a, however, and presumably reflects the fact that the SDM-V model averages across all values of x_{t-1}.

V. Conclusion

This paper compares several asymmetric models of real GNP growth. While it is not clear which model is best for capturing asymmetries in real GNP, each asymmetric model studied in this paper exhibits the same dominant feature: During contractionary phases of the business cycle, the conditional variance of forecasts increases dramatically. In addition, there is some evidence of nonlinearities in the conditional mean, as evidenced by the SETAR model.

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Table 1. Diagnostic Tests on Standardized Residuals
From Various Models of Real GNP Growth
(all values are significance levels)

		Model Specification			
Diagnostic	Lag Length	AR(2)	SETAR(2,2,2)	SNPRX(2,2,2)	SDM-V(0,1,1)
Skewness	-	.78	.07	.30	.33
Kurtosis	-	.02	<.01	.52	.47
Serial Corr. in Mean	1	.78	.84	.61	.70
	2	.68	.88	.87	.83
	3	.40	.65	.63	.63
	4	.71	.97	.81	.90
Level Effects in Mean	1	.42	.81	.49	.54
	2	.18	.94	.61	.41
	3	.11	.35	.40	.28
	4	.33	.59	.74	.64
ARCH Errors	1	.30	.63	.21	.48
	2	.25	.89	.45	.78
	3	.31	.45	.14	.50
	4	.63	.95	.70	.92
Level Effects in Variance	1	.16	.12	.83	.96
	2	.09	.26	.83	.67
	3	.16	.18	.88	.90
	4	.11	.85	.71	.49

Table 2. Summary Information for Various Models of Real GNP Growth

	Model Specification			
	AR(2)	SETAR(2,2,2)	SNPRX(2,2,2)	SDM-V(0,1,1)
Number of Parameters	4	6	23	7
-L.L.F. Value	478.2	469.3	446.1	472.2
Symmetry Test	-	<.001	<.001	.01
AIC	482.2	475.3	469.1	479.2
Schwarz	488.5	484.7	505.2	490.2

Note: The test for symmetry is a likelihood ratio test, comparing each asymmetric model to the simple AR(2) model.

Table 3. Cumulative Probability Under
Standardized SNPRX(2,2,2) Densities

Previous Values	Area Relative to Various Critical Values							
	< -2.33	< -1.65	< -1.28	< -0.84	> 0.84	> 1.28	> 1.65	> 2.33
13.9	.00	.00	.02	.11	.15	.11	.09	.07
11.8	.00	.02	.06	.15	.10	.04	.03	.03
9.6	.01	.04	.09	.18	.17	.07	.03	.01
7.4	.01	.05	.10	.20	.20	.09	.04	.01
5.3	.01	.06	.10	.20	.21	.10	.04	.00
3.2	.01	.05	.10	.20	.20	.10	.04	.01
1.0	.01	.05	.10	.20	.20	.09	.04	.01
-1.1	.01	.05	.10	.19	.19	.09	.04	.01
-3.2	.01	.05	.10	.19	.19	.08	.03	.01
-5.4	.01	.05	.09	.19	.18	.08	.03	.01
-7.5	.01	.04	.09	.19	.17	.07	.03	.01
Gaussian Density	.01	.05	.10	.20	.20	.10	.05	.01

Note: Each row denotes a density that is conditioned on two lags of GNP growth with values shown in column one. Critical values are standard deviations relative to the conditional mean, based on the conditional mean and variance of the corresponding density.

Figure 1. Impulse Response Functions Using the SETAR Model

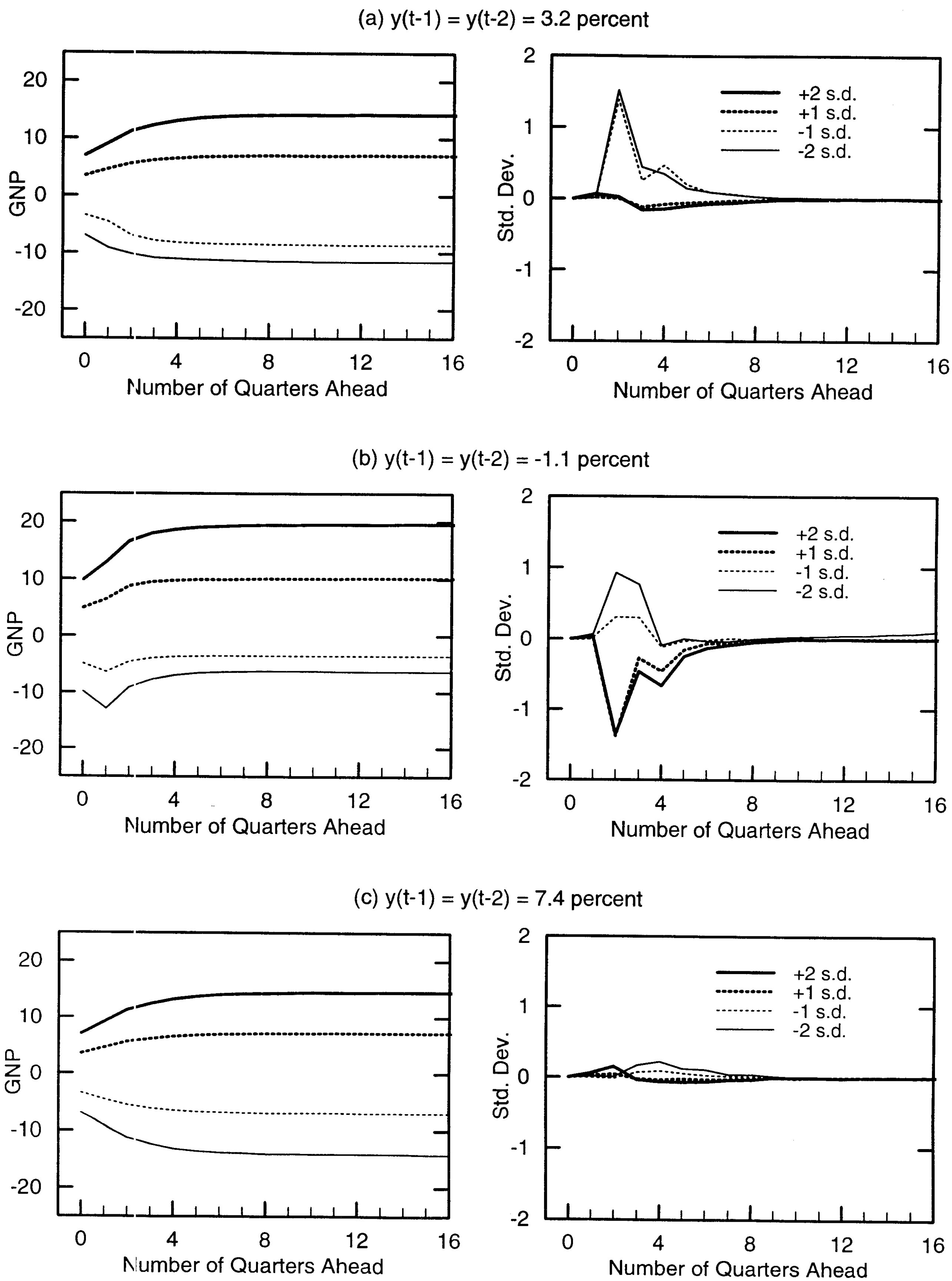


Figure 2. Conditional Densities Using the SNPRX Model

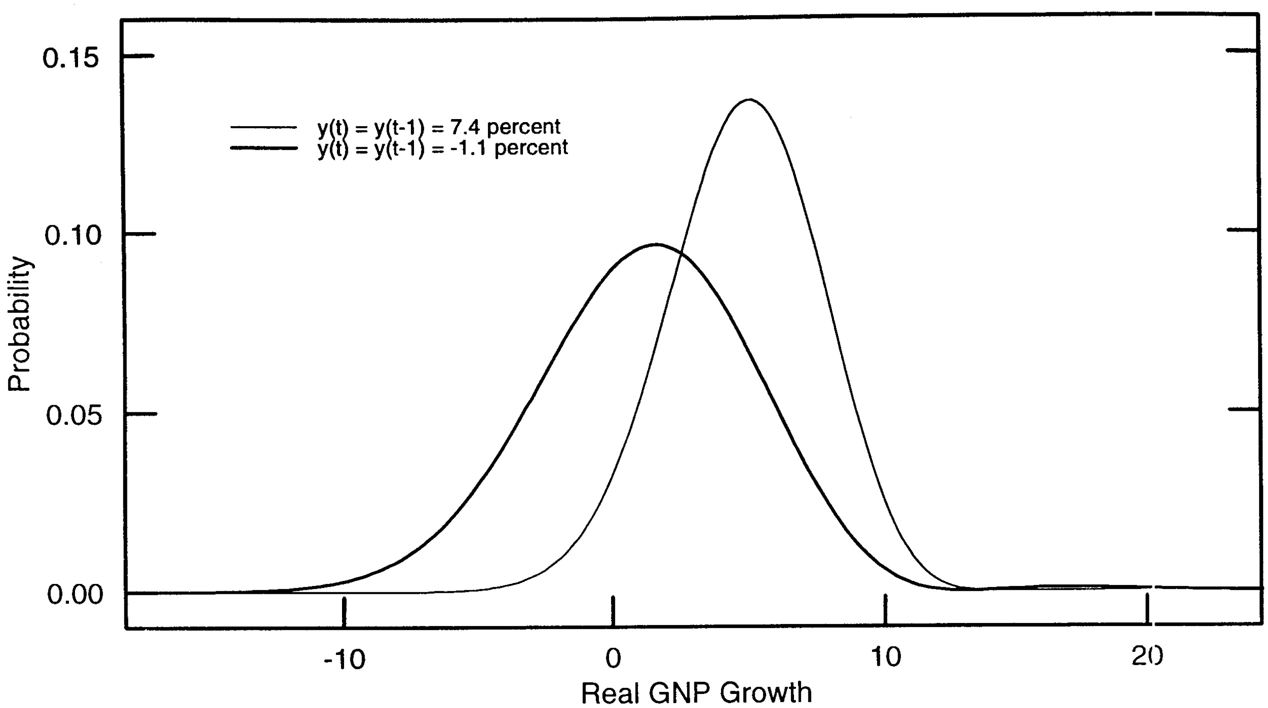
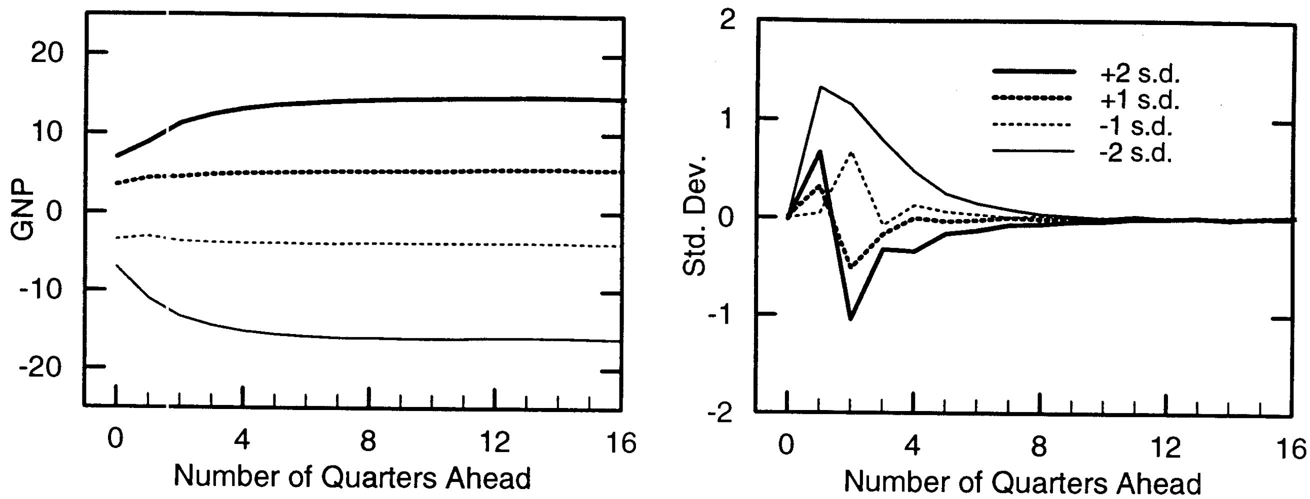
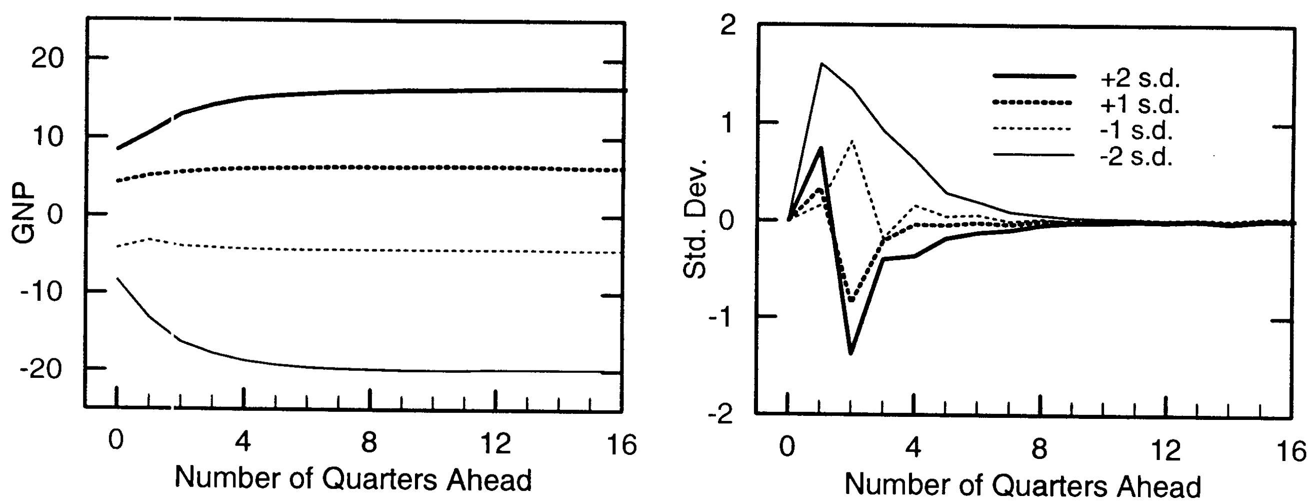


Figure 3. Impulse Response Functions Using the SNPRX Model

(a) y(t-1) = y(t-2) = 3.2 percent



(b) y(t-1) = y(t-2) = -1.1 percent



(c) y(t-1) = y(t-2) = 7.4 percent

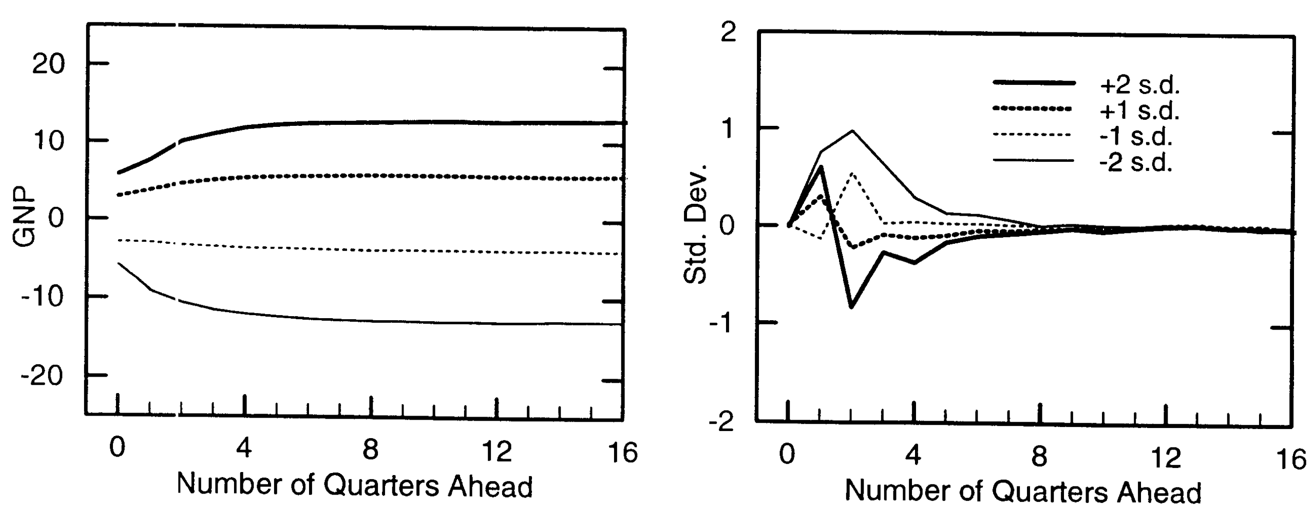
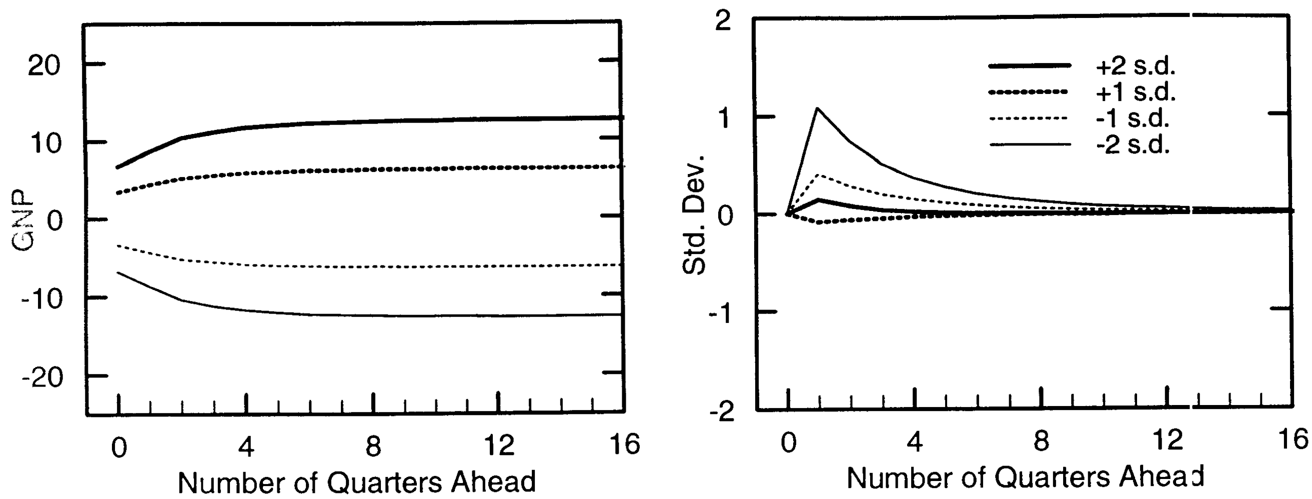


Figure 4. Impulse Response Functions Using the SDM-V Model



Supplemental Appendix

This appendix describes how specifications of the SNPRX, the SDM, and the EGARCH models were obtained for real GNP growth rates. The SETAR specification used in the paper is nearly identical to the specification discussed in Potter (1991b). Tables A1-A3 present detailed explorations of the likelihood surface for each of the models. To illustrate the selection process, consider the results shown in Figure A1. Each row of the table corresponds to a different SNPRX(p,K_z,K_x) model. The first three columns of the table describe characteristics of the model. The total number of estimated parameters for the model is listed in column four. The fifth column contains the value of the average objective function, evaluated at the optimum.

The p-values contained in columns 6 through 8 correspond to the p-values of a chi-square statistic for a test that compares the model in that row to its SNP <u>successor</u> for that column. For example, the p successor of an SNPRX(1,2,1) model is an SNPRX(2,2,1) model. The chi-square statistic for comparing the two specifications is $(2)\cdot(171)\cdot(1.29797 - 1.22769) = 24.04$ with $(14 - 9) = 5$ degrees of freedom. Since the corresponding p-value is less than 0.01, the SNPRX(1,2,1) model is easily rejected in favor of the SNPRX(2,2,1) model. Likewise, the p-value for comparing an SNPRX(2,2,1) to an SNPRX(2,2,2) model -- the K_X successor -- is 0.01, which provides considerable evidence towards rejecting the SNPRX(2,2,1) in favor of an SNP(2,2,2).

Based on p-values, the results in Table A1 can be summarized as follows. First, the appropriate lag length is 2. Second, there is some evidence of departures from Gaussianity ($K_Z>0$). When p=2, a quadratic polynomial is preferred to no polynomial at the 1% level, but cannot be rejected in favor of a cubic polynomial. Third, there is also strong evidence of conditional heterogeneity ($K_X>0$) -- at the 1% level, the SNPRX(2,2,0) model is easily rejected in favor of an SNPRX(2,2,1) model, and an SNPRX(2,2,1) models is rejected in favor of an SNPRX(2,2,2) model.

Based on p-values, the optimal model is an SNPRX(2,2,2) model.[8]

The final two columns of Table A1 present two alternative methods of model selection which place more weight on selecting a parsimonious model than does selecting according to p-values. The first criterion is the Akaike information criterion (AIC). The AIC adds a "penalty" of p_θ/N to the objective function. Minimizing the AIC also results in selecting the SNPRX(2,2,2) specification. The second criterion is the Schwarz criterion which adds a penalty of $p_\theta \cdot \log(N)/2N$ to the objective function. The Schwarz criterion, which is known to be more conservative than the AIC in small samples, selects the SNPRX(2,2,0) model. Since the SNPRX(2,2,0) and SNPRX(2,2,1) models did not pass the diagnostic tests described in the text and the SNPRX(2,2,2) did, the optimal specification for real GNP appears to be the SNPRX(2,2,2) model.

The SDM-V and the EGARCH models, shown in Tables A2 and A3, were chosen in a similar fashion. As before, the general rule was to select the most parsimonious model that was suggested by a model selection criterion and that passed the diagnostic tests.

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[8] Although larger models could have been fit, it did not seem prudent to do so as the SNPRX(2,2,2) model already involves a saturation ratio of about 7 observations per parameter

Table A1. SNPRX Models of Real GNP, 1947:II - 1990:IV
(171 effective observations)

					p-values			Model Selection	
P	K_Z	K_X	p_θ	$s_N(\theta)$	P	K_Z	K_X	AIC	Schwarz
1	0	0	4	1.34634	.02	.11		1.36973	1.40648
1	2	0	6	1.33353	<.01	.01	.01	1.36862	1.42374
1	2	1	9	1.29797	<.01		.03	1.35060	1.43328
1	2	2	12	1.27122	<.01			1.34140	1.45163
1	3	0	7	1.31556	<.01			1.35650	1.42080
2	0	0	6	1.32105	.11	<.01		1.35614	1.41126
2	2	0	8	1.27891	.67	.85	.01	1.32569	**1.39918**
2	2	1	14	1.22769	.10		.01	1.30956	1.43817
2	2	2	23	1.15670	.05			**1.29120**	1.50248
2	3	0	9	1.27881	.67			1.33144	1.41412
3	0	0	8	1.30818		.01		1.35496	1.42845
3	2	0	10	1.27657		.85	.01	1.33505	1.42691
3	2	1	19	1.20089			.01	1.31200	1.48654
3	2	2	37	1.09593				1.31230	1.65219
3	3	0	11	1.27647				1.34080	1.44185

Note: P indicates number of autoregressive lags in the conditional mean and the conditional variance. $K_Z>0$ indicates departure from Gaussianity and $K_X>0$ indicates departure from homogeneity. p_θ indicates the number of estimated parameters. The log-likelihood function values -- $s_N(\theta)$ -- are not directly comparable to the values for the SDM or EGARCH models, since the objective functions are slightly different.

Table A2. SDM-V Models of Real GNP, 1947:II - 1990:IV
(171 effective observations)

P	K	L	M	p_θ	$s_N(\theta)$	p-values P	p-values K	p-values L	p-values M	Model Selection AIC	Model Selection Schwarz
1	0	0	0	3	2.80571	.08	1.00	.40		2.82325	**2.85081**
1	0	1	0	5	2.80042	.07	1.00	<.01	<.01	2.82966	2.87559
1	0	2	0	7	2.76349	.08	.20		.23	2.80443	2.86873
1	1	0	0	5	2.80571	.08		.40		2.83495	2.88088
1	1	1	0	7	2.80042	.08		<.01		2.81136	2.90566
1	1	2	0	9	2.75395	.10				2.80658	2.88926
1	0	1	1	6	2.77189	.05		.12		2.80698	2.86210
1	0	1	1	8	2.75933	.10				2.80611	2.87960
2	0	0	0	4	2.79671	.05	1.00	.39		2.82010	2.85685
2	0	1	0	6	2.79114	.03	1.00	<.01	<.01	2.82623	2.88134
2	0	2	0	8	2.75434	.02	.94		.31	2.80112	2.87461
2	1	0	0	6	2.79671	.01		.39		2.83180	2.88692
2	1	1	0	8	2.79114	.03		<.01		2.83792	2.91141
2	1	2	0	10	2.75395	.02				2.81282	2.90468
2	0	1	1	7	2.76112	.07		.12		2.80206	2.86636
2	0	2	1	9	2.75137	.03				2.80400	2.88668
3	0	0	0	5	2.78588		.28	.25		2.81512	2.86105
3	0	1	0	7	2.77787		1.00	<.01	<.01	2.81881	2.88311
3	0	2	0	9	2.73744		1.00		1.00	**2.79007**	2.87275
3	1	0	0	7	2.77835			.92		2.81929	2.88359
3	1	1	0	9	2.77787			<.01		2.83050	2.91318
3	1	2	0	11	2.73744					2.80177	2.90282
3	0	1	1	8	2.75175			.09		2.79853	2.87203
3	0	2	1	10	2.73744					2.79592	2.88778

Note: P indicates the number of autoregressive parameters in the conditional mean. K and L denote the number of lags of output growth and residuals, respectively, in the conditional variance. M indicates whether the conditional variance was allowed to be autoregressive. p_θ indicates the number of estimated parameters. The log-likelihood function values -- $s_N(\theta)$ -- are not directly comparable to the values for the SNPRX model, since the objective functions are slightly different.

Table A3. EGARCH Models of Real GNP, 1947:II - 1990:IV
(171 effective observations)

P	L	M	p_θ	$s_N(\theta)$	p-values P	p-values L	p-values M	Model Selection AIC	Model Selection Schwarz
1	0	0	3	2.80571	.08	.35		2.82325	**2.85081**
1	1	0	5	2.79964	.09	.03	<.01	2.82888	2.87481
1	2	0	7	2.77818	.04		.01	2.81912	2.88342
1	1	1	6	2.77125	.07	.12		2.80634	2.86146
1	2	1	8	2.75861	.05			2.80539	2.87888
2	0	0	4	2.79671	.05	.39		2.82010	2.85685
2	1	0	6	2.79120	.03	.01	<.01	2.82629	2.88141
2	2	0	8	2.76563	.02		.03	2.81241	2.88590
2	1	1	7	2.76164	.08	.09		2.80258	2.86688
2	2	1	9	2.74778	.09			2.80041	2.88309
3	0	0	5	2.78588		.34		2.82146	2.86739
3	1	0	7	2.78586		.01	<.01	2.82680	2.89110
3	2	0	9	2.75577			.03	2.80840	2.89108
3	1	1	8	2.74988		.25		**2.79666**	2.87015
3	2	1	10	2.74187				2.80035	2.89221

Note: P indicates the number of autoregressive parameters in the conditional mean. L denotes the number of lags of residuals in the conditional variance. M indicates whether the conditional variance was allowed to be autoregressive. p_θ indicates the number of estimated parameters. The log-likelihood function values -- $s_N(\theta)$ -- are not directly comparable to the values for the SNPRX model, since the objective functions are slightly different.

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