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WHEN DO LONG-RUN IDENTIFYING RESTRICTIONS GIVE RELIABLE RESULTS?

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Abstract

Many recent papers have tried to identify behavioral disturbances in vector autoregressions (VAR's) by imposing restrictions on the long-run effects of shocks. This paper argues that this approach will support reliable structural inferences only if the underlying economy satisfies strong restrictions. Absent restrictions linking long-run and short-run dynamics, every decomposition of a VAR is essentially equally consistent with any long-run restriction. Further, dynamic common factor restrictions must hold if the scheme is to work properly in small models estimated using time-aggregated data. The paper illustrates possible consequences of failure of these assumptions using bivariate models to identify aggregate supply and demand disturbances.

When do long-run identifying restrictions give reliable results?

Jon Faust and Eric M. Leeper[1]

Vector autoregressions have become an increasingly popular empirical tool since Sims [1980a] labelled the identifying assumptions of large structural econometric models as "incredible." He argued that many empirical questions could be answered with vector autoregressions (VAR's), which impose weaker restrictions on dynamic interactions among time series than are typically applied in traditional structural models. VAR's cannot sidestep the identification issue, however, and much recent work on VAR methodology focuses on finding credible restrictions that are minimally sufficient to identify economically interpretable shocks.

Sims [1980b] initially proposed identifying a VAR by assuming that contemporaneous interactions among variables are recursive. In this approach, one chooses an ordering for the variables and assumes that at each date, variables higher in the ordering are determined before any variable lower in the ordering. This approach rules out rich simultaneity in the determination of variables and is at odds with most plausible accounts of the macroeconomy.

Bernanke [1986], Blanchard and Watson [1986], and Sims [1986] retained the approach of imposing restrictions only on contemporaneous interactions, but broke away from the recursive structure. They impose only economically meaningful contemporaneous restrictions, allowing for more realistic simultaneity in the model. While this approach has proved quite useful, economic theory often does not provide enough contemporaneous restrictions to identify quantities of interest.

The search for additional identifying restrictions led Blanchard and Quah [1989], King *et al.* [1991], and Shapiro and Watson [1988] to base restrictions on long-run

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neutrality properties. For example, in some widely accepted models a shock to the level of the money supply has no long-run effect on output, while a shock to output may affect the long-run level of money. In a VAR, neutrality properties imply zero restrictions on the long-run effects of shocks. Because such restrictions are plausible *a priori* and easy to implement, use of long-run restrictions has grown very rapidly. Long-run restrictions have been used to study, for example, the sources of business cycles [Bayoumi and Eichengreen, 1992a,1992b; Moreno, 1992; Rogers and Wang, 1992], money supply and demand shocks [Lastrapes and Selgin, 1993], and the international transmission of shocks [Hutchison and Walsh, 1992; Hutchison, 1992; Ahmed *et al.*, 1993].

The long-run scheme rests on the view that if certain economically plausible long-run neutrality assumptions are imposed, then reliable inferences can be drawn about the short-run dynamics of behavioral disturbances in the economy. This paper argues that the explicit assumptions of the scheme will generally not be sufficient to draw reliable structural inferences.[2] A small VAR model estimated under the scheme must be viewed as an approximation to a part of a larger underlying structure. We show that the long-run scheme will reliably identify the economic quantities of interest only under three sets of strong restrictions on the underlying structure.

First, the long-run neutrality property must be tied to a restriction on finite-horizon dynamics. Without such a tie, structural conclusions derived under the long-run scheme are not robust *in any sample size* to changes in the parsimonious functional form used to fit the data.

A second set of restrictions must be satisfied for a VAR with a small number of variables to capture economically interpretable features of the larger underlying economy. The third set of restrictions is required for the identifying scheme to be appropriate for time-aggregated data. The identification problems that give rise to the need for these restrictions are quite familiar, stemming from problems in aggregating across variables and across time. The paper lays out sufficient restrictions for

[2] Hendry [1993] made this point in a more general context. This paper works out the stringent conditions under which the approach may be fruitful.

the long-run scheme to function properly given these generic aggregation problems and provides some informal means for assessing the plausibility of these restrictions.

The paper illustrates the potential importance of these theoretical results by comparing the results from identifying aggregate supply and demand shocks in three bivariate models. We find that the structural conclusions from these models are mutually inconsistent and that there is little *a priori* reason to expect that the required restrictions hold in one model and not the others. Thus, we conclude that the long-run scheme as applied in this exercise does not support reliable conclusions about the underlying structure of the economy.

1 Identifying VAR's using long-run restrictions

This section lays out the basic issues of identification in VAR's and describes the long-run identification scheme. If $X_t = (X_{1t}, \dots, X_{nt})'$ is covariance stationary, then ignoring deterministic components it will have a Wold representation,

$$X_t = F(L)u_t. \tag{1}$$

The disturbance term, u_t, has mean zero, is serially uncorrelated and has a fixed covariance matrix, $E[u_t u_t'] = \Sigma$ for all t. The term $F(L)$ is an $(n \times n)$ matrix whose typical element, $f_{ij}(L)$, is a polynomial in the lag operator, L, where $f_{ij}(L) = \sum_{k=0}^{\infty} f_{ijk}L^k$, and $L^k X_t = X_{t-k}$. The matrix polynomial will also be written $\sum_{k=0}^{\infty} F_k L^k$ where F_k is the $(n \times n)$ matrix with typical element f_{ijk}. In the Wold representation $F_0 = I$ by convention.

If $F(L)$ is invertible,[3] there will also be a VAR representation,

$$R(L)X_t = u_t, \tag{2}$$

where $R(L) = F(L)^{-1}$, and $R_0 = I$. Since this VAR expresses each variable at t in terms of predetermined variables and the current disturbances, it is a *reduced*

[3] There is a familiar case when $F(L)$ will not be invertible: if some X_{it} is the first difference of a stationary variable, or if X_{it} and X_{jt} are the first differences of cointegrated variables, then $F(L)$ will not be invertible. In both of these cases, the variables have in some sense been *over-differenced* to arrive at X_t. That is, some linear combination of the variables was stationary before differencing.

form. Throughout the paper, we limit ourselves to structures with invertible moving average representations; thus, none of our results stem from problems caused by nonfundamental representations that are the focus of Lippi and Reichlin [1993].

1.1 The identification problem

There are many observationally equivalent representations of the process for X_t. Taking any non-singular matrix A_0, such representations can be written

$$X_t = F(L)A_0A_0^{-1}u_t = A(L)\varepsilon_t, \tag{3}$$

where $A(L) = F(L)A_0$ and $\varepsilon_t = A_0^{-1}u_t$. Since $F_0 = I$, the lead matrix of $A(L)$ will be A_0. Each A_0 produces a *structure* consistent with the reduced form and the resulting disturbances, ε_t, are called *structural shocks.* Of course, each structure has a VAR representation that can be written

$$B(L)X_t = \varepsilon_t,$$

with $B(L) = A(L)^{-1}$.

Identifying a VAR can be seen as a matter of choosing a unique A_0. This choice determines the nature of contemporaneous interactions among the variables. For example, if A_0 is lower triangular, as in the recursive scheme proposed by Sims [1980a], the first variable affects all other variables contemporaneously, but not vice versa. The choice of A_0 can also be seen as choosing the covariance matrix of structural shocks. Since $\varepsilon_t = A_0^{-1}u_t$, we have

$$E[\varepsilon_t\varepsilon_t'] = A_0^{-1}\Sigma A_0^{-1\prime}.$$

Identification requires choosing n^2 elements of A_0. Almost all VAR approaches begin with $n(n+1)/2$ restrictions on the covariance matrix. The first n restrictions are normalizations that choose the units for the shocks; typically the standard deviations of the shocks are normalized to one. Another $n(n-1)/2$ restrictions come from the assumption that the structural shocks of interest are mutually uncorrelated. This assumption is consistent with the view that the structural shocks originate in behaviorally distinct sectors of the economy. Together, these restrictions on A_0 imply,

$$A_0^{-1}\Sigma A_0^{-1} = I,$$

where I is the identity matrix.

1.2 The long-run identification scheme

We need $n(n-1)/2$ more restrictions to identify the model. Blanchard and Quah, King *et al.*, and Shapiro and Watson suggested that some (or possibly all) of these restrictions could come from long-run neutrality properties suggested by economic theory. These long-run neutrality properties have implications for the long-run effects of structural shocks in a VAR. For example, suppose that one believes that a nominal shock should have no long-run effect on output. If output is the i^{th} variable in a VAR and the nominal shock is the j^{th} shock, then the restriction on A_0 can be written, $a_{ij}(1) = \sum_{k=0}^{\infty} a_{ijk} = 0$, or,

$$[F(1)A_0]_{i,j} = 0. \tag{4}$$

Some authors have used long-run restrictions for all of the required $n(n-1)/2$ restrictions [Blanchard and Quah, Ahmed *et al.*, Lastrapes and Selgin], while others have combined long-run restrictions with other restrictions to complete the identification [King *et al.*, Shapiro and Watson, Gali 1992].[4] If only long-run restrictions are used, the equations can be ordered such that the long-run effects matrix,

$$A(1) = F(1)A_0,$$

is lower triangular (with zeros above the main diagonal) so the structure is recursive in the long run.

The restrictions discussed so far pick out the magnitude of the elements of A_0, but not the signs of the elements on the main diagonal. Multiplying a shock and the corresponding polynomials by minus one leaves the empirical implications of the

[4] Typically the models are just identified. Shapiro and Watson's model, however, is overidentified by their assumption that oil prices are exogenous.

model unchanged. Generally, we have *a priori* views about the sign of the impact or the long-run effect of shocks to certain variables, and this information can be used to complete the identification.

1.3 Illustration: The Blanchard-Quah model

To illustrate the use of the long-run scheme, we consider the bivariate model of Blanchard and Quah, which includes the growth rate of output and the unemployment rate. Thus, $X_t = (Y_t, U_t)'$ is a vector of quarterly observations on the growth rate of GDP and the unemployment rate.[5] In a two-variable system, four identifying restrictions are needed; three come from normalizing and orthogonalizing the shocks. Blanchard and Quah identify real (aggregate supply) and nominal (aggregate demand) shocks by applying the assumption that nominal shocks have no long-run effect on the level of output. Taking the second shock as the nominal disturbance, this implies $a_{12}(1) = 0$.[6] Finally, the signs of the shocks are determined by assuming that positive demand and supply shocks increase output on impact.

The economic interpretation of Blanchard and Quah's identifying assumptions is straightforward. The restriction that identifies the demand shock implies that the long-run aggregate supply function is vertical. Neither disturbance has a long-run effect on unemployment, so the structure is consistent with there being a natural rate of unemployment. From an initial position of full employment, an aggregate demand shock moves the economy up the original short-run supply function. Over time, as behavior adjusts to the demand shock and the resulting increase in prices, the short-run supply function shifts back along the new demand curve. The economy

[5] Following Blanchard and Quah, output growth has had means extracted for the periods 1948:2 to 1973:4 and 1974:1 to 1992:4, and unemployment has been linearly detrended. The unemployment series is unemployment among males 20 years or older. The VAR analogous to equation (2) is estimated with eight lags and no constant term over the period 1950:2 to 1992:4, using 1948:2 to 1950:1 as initial conditions.

[6] Blanchard and Quah's work can be viewed simply as a decomposition of output into a permanent and transistory component. Of course, there are arbitrarily many ways to perform such a decomposition, and none is inherently more interesting than another. The problems arise only when one attempts to place an economic interpretation on the results. This paper focuses on the robustness of structural economic interpretations of VAR's.

reaches the long run and adjusts fully to the demand shock when the price level rises sufficiently that output has returned to the level consistent with the natural rate of unemployment.

The point estimates we obtained for the dynamic effects of supply and demand shocks are nearly identical to those reported by Blanchard and Quah (Figure 1).[7] Shocks that shift aggregate demand outward have a hump-shaped effect on the level of GDP that peaks after a few quarters and dies out in five years. The path of unemployment is the mirror image of output. The effect of supply shocks on output cumulates to reach a peak after two years before stabilizing at a permanently higher level. The unemployment rate, if anything, increases initially after a positive supply shock, but the response is not strongly positive for more than a few quarters.

The impulse response functions and implied forecast error variance decompositions suggest that demand shocks are the dominant source of output fluctuations in the short-run, and their effects are quite persistent (Table 1). By construction, supply shocks dominate output in the long run. At impact, demand and supply shocks appear to be equally important sources of unemployment fluctuations, while demand shocks dominate at longer horizons.

The validity of the inferences drawn about the relative importance of demand and supply shocks for output fluctuations rests on three aspects of the identification scheme: (1) whether the economic intuition underlying the long-run restriction actually restricts the data; (2) whether the identification correctly aggregates the many underlying behavioral disturbances into demand and supply shocks; (3) whether the identifying assumption that the behavioral disturbances are uncorrelated is valid when data are aggregated over time. The approach in the next three sections is the same: lay out a maintained model that is consistent with all assumptions of the

[7] The vertical axes measure the log of real GDP and the rate of unemployment, while the horizontal axes denote quarters following the shock. The point estimates of the dynamic responses lie inside 90 percent probability bands. We use the Bayesian Monte Carlo procedure described in the RATS manual, which takes random draws from the estimated asymptotic distributions of the VAR innovations and the VAR coefficients. This procedure is standard when the model is just identified. After 10,000 draws from the distributions, the impulse response functions are ordered and the 5^{th} and 95^{th} percentile responses are extracted.

long-run scheme and investigate what additional restrictions on this general model are needed for the long-run scheme to support reliable structural inferences.

2 Long-run restrictions and finite-horizon implications

Meaningful estimation of the infinite-horizon effect of shocks with a finite span of data requires smoothness assumptions linking aspects of the process observable in the given finite sample to the long-run properties of the process. Under the explicit smoothness assumptions generally used, the long-run restriction provides no reliable basis for selecting among the many structural forms consistent with any estimated reduced form.

2.1 Absent short-run restrictions, long-run restrictions are sterile

Begin with a maintained model that embodies all the restrictions of the long-run scheme. The model includes all covariance stationary n-dimensional vector time series,

$$X_t = A(L)\varepsilon_t,$$

satisfying $E[\varepsilon_t\varepsilon_t'] = I$, $A(1)$ finite, and satisfying $n(n-1)/2$ specified zero restrictions. Assume that the lag polynomial, $A(L)$, is invertible.

The scheme uses long-run restrictions to identify other, finite-horizon, aspects of the economic system. We assume that the features of interest are the sorts of things usually reported in VAR work. In particular, assume that the parameter of interest in the econometric exercise, θ, is a finite-dimensional vector containing any collection of VAR coefficients, impulse responses, autocorrelations, or cross correlations.

The estimation of θ begins with estimating a reduced form for the data and then the identifying restrictions pick out a unique parameter of interest, θ^*, consistent with the reduced form. Under the long-run scheme, one estimates a VAR reduced form for the data. Of course, there are arbitrarily many flexible models one might have chosen—for example, moving average models, Almon lag models, or flexible forms in the frequency domain. The choice of the VAR functional form is a pragmatic one based largely on ease of estimation. Ordinary least squares is typically used, which under the assumption of normal disturbances, is the maximum likelihood estimator conditional on the initial values of the lagged variables. The following proposition shows that the choice of θ^* over any other parameter of interest consistent with the estimated reduced form is almost strictly an artifact of the choice of the VAR functional form.

Proposition 1 *Take any sample X of size T. Suppose that the Gaussian maximum likelihood estimator selects an m-parameter VAR with likelihood L^* and that the identification scheme picks out a $(p \times 1)$ parameter of interest θ^*. Let Θ be the set of all θ's consistent with the reduced-form VAR. For every $\tilde{\theta} \in \Theta$ and every $\varepsilon_L, \varepsilon_\theta > 0$, there exists an m-parameter functional form containing a structure satisfying all restrictions of the maintained model with likelihood L' and parameter of interest θ' such that*

i) $|L' - L^*| < \varepsilon_L$, and

ii) $d(\theta', \tilde{\theta}) < \varepsilon_\theta$, where d is any continuous metric on R^p.

Proof: see the appendix.

Suppose that given the choice of an m-parameter VAR functional form, the OLS estimation procedure leads to a reduced form with likelihood L^*, and the long-run scheme picks out θ^* from among all parameters of interest consistent with the reduced form.[8] The Proposition shows that there is another m-parameter functional form including a structure with likelihood arbitrarily close to L^*, but for which the long-run scheme chooses a structure arbitrarily similar to $\tilde{\theta}$. Thus, the economic interpretation we put on the reduced form—the choice of θ^* over $\tilde{\theta}$—is almost strictly an artifact of the chosen functional form. This would not be a problem if economic theory provided us a reason to suppose that low-order VAR's were the appropriate model. It is difficult to imagine the *a priori* reasoning about macroeconomic systems that would justify such a VAR preference.

The intuition for the result is as follows. Suppose the reduced form is estimated using, say, 50 years of data. There is essentially no *direct* information in a 50-year sample about the infinite horizon. Any inference about the infinite horizon

[8] The Appendix proof holds for likelihood-based estimation procedures, and, more generally, for criterion functions with certain continuity properties. See the Appendix proof.

rests on extrapolating to the infinite horizon based on the given finite span of data. The particular nature of the extrapolation is imposed by the functional form. Two functional forms that fit the information in the sample equally well may extrapolate to the infinite horizon in very different ways, giving very different reduced form long-run effect matrices, $F(1)$. Thus, when the long-run scheme is imposed based on the estimated $F(1)$, as in (4), the two forms will suggest very different structural interpretations of the data.

The spirit of the constructive proof is easy to illustrate. Consider the m-parameter VAR structure $\tilde{B}(L)X_t = \varepsilon_t$, with corresponding moving average representation $X_t = \tilde{A}(L)\varepsilon_t$, where X_t is (2×1) and $E\varepsilon_t\varepsilon_t' = I$. Suppose that this structure is not consistent with the long-run scheme because $\tilde{a}_{12}(1)$ equals some arbitrary ξ, whereas the theory implies $\tilde{a}_{12}(1) = 0$. Proposition 1 says that there is an m-parameter model containing a particular structure that fits the data as well as that parameterized by $\tilde{A}(L)$ and that has nearly the same parameter of interest as that implied by $\tilde{A}(L)$, but which satisfies the long-run restriction. Form this new model by transforming each m-parameter VAR structure by subtracting ξ/k from the first k coefficients of the implied $(1,2)$ element in the moving average representation polynomial. Corresponding to $\tilde{B}(L)$ in the original VAR model there is a transformed structure, $\tilde{B}^{(k)}(L)$, with moving average representation $\tilde{A}^{(k)}(L)$, which satisfies the long-run restriction. For large k, $\tilde{A}^{(k)}(L)$ is the same as $\tilde{A}(L)$ except that k coefficients of $\tilde{a}_{12}(L)$ have been altered by a tiny amount. When k is large relative to the sample size, the reduced form of the structure based on $\tilde{A}^{(k)}(L)$ will be arbitrarily close to that of $\tilde{A}(L)$ in a likelihood sense, and the parameter of interest associated with $\tilde{A}^{(k)}(L)$ will be arbitrarily close to that for $\tilde{A}(L)$. This can be summarized as showing that we can alter the infinite-horizon properties of a structure without compromising the fit to the data and without altering the finite-horizon economic properties captured in the parameter of interest.

The claims of this section are unrelated to Blanchard and Quah's point that the long-run scheme should work equally well whether or not the true underlying structure *exactly* satisfies the long-run restriction. The analysis here assumes the ideal case where the restriction is satisfied exactly and shows that the scheme will not yield reliable results even in the ideal case. In short, unless there is some argument in favor of the particular way that the VAR model extrapolates from the sample information to the infinite horizon, there is little reason to prefer the economic interpretation suggested by θ^* over any other interpretation consistent with the reduced form.

2.2 Short-run implications of long-run neutrality

If we wish to exploit economic theory's assertions about long-run neutrality properties, we need more than *infinite* horizon implications. To understand more about the problem and its potential solution, return to the output-unemployment VAR. Suppose one asserts that the long-run scheme has simply mislabelled the two shocks: the estimated supply shock should have been labelled demand and vice versa. This new structural interpretation would seem to be inconsistent with the long-run scheme since the new *demand* shock has a permanent effect on output. A slight adjustment to the structure can reconcile it with the long-run restriction, however. Suppose that the *demand* shock's effect on output is the same as that labelled supply in Figure 1, but that the new *demand* shock's effect decays at some horizon beyond that shown in Figure 1. Such a structure can be constructed following the proof of Proposition 1 and cannot be ruled out based on any explicit assumption of the long-run scheme. Such structures also cannot be ruled out based on arguments about relative likelihood or based on an appeal to parsimony.

One can reject such structures, however, by claiming that the implied persistence of the demand shock is implausible. To rule out structures in which demand effects do not begin to die out for 10, 20, 30 years or more, one must impose restrictions on short-run—that is, finite horizon—dynamics. If economic theory or *a priori* reasoning provide a basis for such restrictions, then the problems of this section can be solved. Suppose one believed, say, that 90-percent of the real effects of any demand shock disappears in the first 10 years.[9] Any decomposition of the VAR satisfying this restriction would be equally plausible under such a restriction. The exercise of simply re-labelling the shocks in the output-unemployment exercise would no longer be consistent with the restrictions, however.

One might wonder whether the long-run scheme as generally implemented implicitly embeds a reasonable set of restrictions about short-run dynamics. For example, the set of structures satisfying a 90 percent decay rule might be adequately captured by considering only θ^* along with conventional standard errors. Unfortunately, the finite-order VAR embeds its own set of implicit short-run restrictions. There is no reason to believe that these implicit restrictions coincide with the short-run restrictions that might emerge from economic theory. Indeed, nothing guarantees that θ^* will satisfy any restriction on short-run dynamics. Hence, θ^*, and a confidence region around it need not bear any special relation to the class of structures satisfying any short-run restriction.

Overall, any preference for θ^* over other decompositions of the estimated reduced form rests on a strong implicit assumption—loosely speaking, a belief in the primacy of the VAR functional form. Without exploring the implications and plausibility of this assumption, there are no grounds for supposing that the long-run scheme provides a reliable basis for structural interpretations of data. One solution to this problem would be to add explicit assumptions about finite-horizon dynamics. This approach is straightforward to implement—specify restrictions and impose them—but it requires abandoning the view that no *explicit* restrictions should be imposed on finite-horizon dynamics.

3 The problem posed by multiple shocks

So far we have considered identifying n shocks in an n-shock model. Of course, the VAR methodology is usually applied in a low-dimensional model that must

[9] More generally, some probablistic restriction on the degree of decay at any horizon might be imposed. In practice, this would take the form of a Bayesian prior over the persistence of shocks.

be viewed as an approximation to a larger structure with many more sources of uncertainty. Thus, the n shocks identified must be viewed as aggregates of a larger number of underlying shocks. In the output-unemployment model, for example, the estimated supply shock must combine all real disturbances such as oil shocks, labor supply shocks, and productivity shocks. This section follows Blanchard and Quah [theorem, p.670] in considering the conditions under which an n-variable VAR system that is a subset of a larger underlying model driven by $m > n$ shocks will correctly aggregate and classify the multiple shocks.

3.1 Necessary and sufficient conditions for aggregation

Take an n-variable invertible structure driven by m shocks, $(m > n)$:

$$X_t = \tilde{A}(L)\tilde{\varepsilon}_t, \tag{5}$$

where X_t is an $(n \times 1)$ vector of data, $\tilde{A}(L)$ is an $(n \times m)$ matrix polynomial in the lag operator, and $\tilde{\varepsilon}_t$ is an $(m \times 1)$ vector of shocks, $E[\tilde{\varepsilon}_t \tilde{\varepsilon}_t'] = I$.

Suppose that there are n shock categories of interest and that each of the m underlying shocks falls into one of these categories. There are p_j shocks in category j, and the shocks are arranged by category. Partition $\tilde{A}(L)$ conformably with the shock categories, so that

$$\tilde{A}(L) = \begin{bmatrix} \tilde{A}_{11}(L) & \dots & \tilde{A}_{1n}(L) \\ \vdots & \ddots & \vdots \\ \tilde{A}_{n1}(L) & \dots & \tilde{A}_{nn}(L) \end{bmatrix},$$

where $\tilde{A}_{ij}(L)$ is a row vector of p_j scalar polynomials.

Assume that economic theory provides enough long-run restrictions on this model to make $\tilde{A}(1)$ block lower triangular. In the output-unemployment model, for example, this amounts to assuming that no demand shock in the underlying model has a long-run effect on output. If there were just one shock per category, this restriction would be enough to identify the model. The question of this section is whether under these conditions, one can apply the long-run scheme to estimate correctly one aggregate shock per category when there are multiple shocks.

Write down the n-shock model implied by (5) and apply the long-run scheme. The covariance stationary system in (5) has n-shock a Wold representation, say, $X_t = F(L)u_t$. Applying the long-run scheme by choosing $A(1)$ lower triangular gives,

$$X_t = A(L)\varepsilon_t, \tag{6}$$

with $E[\varepsilon_t \varepsilon_t'] = I$.

Even when $\tilde{A}(L)$ is block lower triangular in (5), each aggregate shock in the n-shock model, (6), generally will commingle the m underlying shocks at various points in time. Part (i) of the Proposition, which is a variant on and generalization of Blanchard and Quah's theorem [p.670], states when the shock categories but not the timing of shocks will be preserved. Part (ii) states when the shock categories and timing of shocks will be preserved.

Proposition 2 *Given the structure (5) and the n-shock representation (6),*

i) The shock ε_{jt} will be a linear function of the underlying category j shocks at time t and before if and only if

$$\tilde{A}(L) = \Gamma(L)D(L)$$

where $\Gamma(L)$ is $(n \times n)$ and $D(L)$ is $(n \times m)$ and block diagonal when partitioned conformably with the shock categories.

ii) The shock ε_{jt} will be a linear function of the underlying category j shocks at t if and only if part (i) holds and $D(L) = D$, a block diagonal matrix of scalars.

Proof: see the appendix.

The part (ii) conditions, which ensure both the categories and the timing of shocks will be preserved, are very strong: every category j shock must affect variable k in exactly the same way up to a scale factor. This restriction rules out any meaningful sense of multiple shocks in each category.

Part (i) gives a common factor restriction under which the shock categories will be correctly sorted out by the identification scheme, but the timing of the underlying shocks will be distorted in the aggregate shock. While the restriction is complicated to interpret, the conclusion of the proof is little more than a re-statement of the assumptions. To prove necessity of the restriction, equate the two representations (5) and (6):

$$X_t = \tilde{A}(L)\tilde{\varepsilon}_t = A(L)\varepsilon_t.$$

If each ε_{jt} is a linear combination of the underlying category j shocks at t and before, then there must be some $D(L)$ that is block diagonal when partitioned conformably with the shock categories such that $\varepsilon_t = D(L)\tilde{\varepsilon}_t$. Substituting, gives

$$X_t = \tilde{A}(L)\tilde{\varepsilon}_t = A(L)D(L)\tilde{\varepsilon}_t.$$

which is the essence of the required result.

3.2 Appraising the theoretical plausibility of the restrictions

Proposition 2 states that the usefulness of the long-run scheme rests implicitly on strong dynamic restrictions on the underlying model. To see how the common factor restriction in part (i) limits differences in the behavior of two shocks from the same category, return to the case of the output-unemployment model, in which the aggregate shocks are supply and demand. Consider shutting down all the shocks in the model except the k^{th} supply shock. If Proposition 2 holds, and assuming category 1 is supply, we can write,

$$\begin{aligned} Y_t &= \gamma_{11}(L)d_{1k}(L)\tilde{\varepsilon}_{kt} \\ U_t &= \gamma_{21}(L)d_{1k}(L)\tilde{\varepsilon}_{kt} \end{aligned}$$

or

$$U_t = \frac{\gamma_{21}(L)}{\gamma_{11}(L)}Y_t. \tag{7}$$

Since $d_{1k}(L)$ drops out of (7), the expression holds for *every* supply shock. Under the restriction, the response of U to every supply shock can be expressed as a single distributed lag on Y. The analogous result holds for the demand shock. In many cases this will be implausible.

Suppose the lead coefficient of the lag polynomial in (7) is negative. Equation (7) states that every supply shock that increases output growth on impact must also decrease the unemployment rate. Many models do not satisfy this restriction. For example, under standard assumptions, a positive productivity shock leads initially to an increase in output growth and a decrease in the unemployment rate. An exogenous increase in female participation in the labor force, however, might lead to an increase in the overall labor force, which initially increases employment, but not by as much as the increase in the labor force. At impact, output growth and the unemployment rate rise.

In general, Proposition 2 provides a basis for bringing theory to bear in assessing when a small VAR model identified under the long-run scheme will provide a reliable basis for inference. Of course, it would also be useful to have some way to evaluate the empirical importance of any violation of Proposition 2.

3.3 Appraising the empirical plausibility of the part (i) restrictions

The conditions in Proposition 2 require that the lag polynomial for the underlying model can be factored in a particular way. One way to test this would be to specify the larger model and use standard approaches to test the implied common factor restrictions [Hendry and Mizon, 1978]. Of course, this would require abandoning many of the practical advantages of using a small model.[10] We propose an alternative approach in the form of a simple robustness check.

Often there will be several different variables upon which a given analysis could be based. For example, while Blanchard and Quah used output and unemployment, output and prices might seem a more natural and directly interpretable choice since this quantity-price pair is the standard one used to analyze aggregate demand and supply. Alternatively, output and a nominal interest rate would fit into a traditional IS-LM framework. It is difficult on *a priori* grounds to choose a two-variable system that will best decompose output fluctuations into aggregate demand and supply

[10] Given a specification for the larger model and data for the added variables, one migth avoid some of the burden of estimating the full restricted model by using LM tests.

shocks.

The aggregation theorem may hold in none of these systems, it may hold for one system but not others, or it may hold for multiple systems. If the aggregation theorem holds for multiple systems, however, the supply shocks estimated in the models will be uncorrelated asymptotically with the demand shocks in the models.[11] If the supply shock from one model is correlated with the demand shocks from other models, there is clear evidence that one or all of the models have commingled the many underlying supply and demand shocks.

We explore the robustness of the results obtained from the output-unemployment (YU) model by estimating two other bivariate systems: output and inflation (YP) and output and a short-term nominal interest rate (YR).[12] The contemporaneous correlations among the estimated demand and supply shocks from the three models suggest the models fail to satisfy the shock aggregation theorem.

Demand shocks are moderately positively correlated across the models; supply shocks are weakly correlated (Table 2).[13] The YP and YR supply shocks, however, are more highly correlated with the YU demand shock (.56 and .54) than with the YU supply shock (.20 and .23). Supply shocks for the YP model are equally correlated with the supply and demand shocks from the model with interest rates. The pattern of large correlations between demand and supply shocks across models implies that each model aggregates the underlying shocks differently.

[11] Even if the aggregation assumptions hold for two different models, the models need not produce the same shocks: the two could alter the timing of the underlying shocks in a different manner.

[12] Bayoumi and Eichengreen also estimate YP systems for the United States and several European countries using annual data. The YR model is of independent interest because of the sizeable VAR literature that has explored the dynamic correlations between the two variables. In this literature, interest rate innovations have been variously interpreted as arising from technology (supply shocks) and from monetary policy (demand shocks). See, for example, Sims [1980b] and Novales [1990]. The YP model is estimated using quarterly growth rates in output and the GDP deflator; the YR model uses the quarterly growth rate in output and the level of the three-month Treasury bill rate. To be consistent with the specification of the YU model, each VAR is estimated over the same sample period with eight lags and no constant term. Means for the pre- and post-1974 periods were extracted from each data series. In YP and YR models we make the additional identifying assumptions that transitory disturbances that increase output at impact are positive demand shocks and permanent disturbances that increase output are positive supply shocks.

[13] The correlations are based on 171 observations, so ignoring the fact that the shocks are estimated, the typical formula implies a standard error of the correlations of about $1/\sqrt{171} = .08$.

The impulse response functions (Figures 2 and 3) and forecast error variance decompositions (Tables 3 and 4) for the YP and YR models reveal further differences in how the three systems aggregate underlying supply and demand shocks. In the YP and YR models, demand shocks produce a hump-shaped response in the level of output, but their influence dies out quickly. Output is determined largely by supply. Supply shocks increase output immediately and the long-run response of output is twice that estimated in the YU model. The YP and YR models attribute substantially less of output's forecast error variance to demand shocks at all horizons than does the YU model.

A closer look at the data and the models' estimated shocks underscores how the interpretation of historical episodes can vary with the bivariate system and helps to explain the differences across the three models. With the Korean War in 1950 and the price controls beginning in 1951 came wide swings in inflation, but no corresponding large changes in unemployment and the interest rate. The YP system decomposes the swings into a mixture of large positive and negative demand and supply shocks (Table 5). For example, in 1951:1, a modest drop in GDP coincided with a big spike in inflation, which the YP model attributes to a combination of a large outward shift in demand and an inward shift in supply. The other two models do not have to account for inflation fluctuations and report nothing extraordinary.

Different interpretations of the 1957–1958 recession also emerge from the three models. The YU model tells an exclusively demand-side story, with large negative demand shocks occurring immediately before the peak in 1957:3 and during the recession. The YP and YR models attribute the recession to both demand and supply shocks. The dramatic oil price increases in early 1974 and 1979 are identified as big negative supply shocks in the YP model and as a mixture of negative supply and demand shocks in the YR and YU models.[14]

The strong correlations between demand and supply shocks across the models imply that the three models have sorted the underlying behavioral disturbances into

[14] The supply shocks in the YP model occur about three quarters after the oil shocks hit. This lag is consistent with Hamilton's [1983] estimates of the effects of oil shocks on output.

aggregate supply and demand categories differently. We see no *a priori* grounds for selecting among the three models: the models are based on the same long-run restriction and none of the models identifies short-run behavior that might otherwise be used as a criterion for preferring a model. Likewise there seems to be no theoretical basis for supposing that the Proposition 2 restrictions are more likely to hold in one model than another. Thus, we are left to conclude that none of these models provides a basis for reliable structural inference.

4 Restrictions implied by assuming contemporaneously uncorrelated shocks

In VAR modelling, long-run restrictions typically are coupled with the assumption that structural shocks are orthogonal. The orthogonality assumption is often treated more as a normalization than as an essential part of the identification scheme.[15] In this section, we show that the reliability of inferences under the orthogonality assumption turns crucially on the frequency of observation of the data. Even if the orthogonality assumption is appropriate in sufficiently high frequency data, feedbacks among economic variables at higher frequency than that of the measured data can invalidate the orthogonality restriction. Following the form of the previous section, we derive the restrictions on an underlying high frequency model that would allow the long-run scheme to get the correct answer in data measured at lower frequency.

Identification in time-aggregated models is an issue that has been discussed before in the context of traditional models of supply and demand [Hendry, 1992] and in the more general time series literature. Telser [1967] showed that an underlying discrete-time AR structure is identified in lower frequency data when the order of the AR model is known. Phillips [1973] provided restrictions under which a continuous-time structure is identified in discrete data. Hansen and Sargent [1991] discussed

[15] Ahmed et al. [p. 336] say that their identification relies "exclusively on *long-run* restrictions" and Blanchard and Quah [p. 659] call the orthogonality restrictions a "nonissue."

when the cross-equation restrictions of rational expectations models would identify a continuous-time structure.

Most VAR modelling using the long-run scheme runs counter to the spirit in the general time series literature. In particular, the goal in the VAR literature is to specify a minimal set of assumptions that might be consistent with a broad range of theories and underlying structures. Thus, nothing is interpreted as a *deep* parameter, and no cross equation restrictions are imposed. This section examines when some of these general features remain identified in data sampled at some lower frequency than the underlying model.

4.1 Time aggregation and identification

Consider an n-variable structure driven by n shocks:

$$\tilde{X}_t = \tilde{A}(L)\tilde{\varepsilon}_t, \tag{8}$$

with $\tilde{A}(1)$ lower triangular and $E[\tilde{\varepsilon}_t\tilde{\varepsilon}_t'] = I$, $t = 1, \ldots, Tp$. Now consider a time-aggregated version of the model in which there is an observation only every p periods. If the original model involves quarterly data, $p = 4$ produces an annual model. The low frequency data might be end-of-period, or an average over the period. We suppose, generally, that the observed data are some linear function of the p observations making up the courser time period:[16]

$$X_t = M(L)\tilde{X}_t, \qquad t = p, 2p, \ldots, Tp,$$

where $M(L)$ is diagonal, of order $p-1$, and known. The model for X_t can be written

$$X_t = M(L)\tilde{A}(L)\tilde{\varepsilon}_t, \qquad t = p, 2p, \ldots, Tp.$$

This expresses X_t in terms of all the underlying shocks at the higher frequency; the lag operator operates on t, which is in the higher frequency units. There will also

[16] Linear time aggregation does not cover some interesting cases. If the quarterly data are logarithms of some underlying series and the annual data are logarithms of the annual average of the underlying series, then the time aggregation is nonlinear. The results of this section may approximately apply under a linear approximation to the aggregation scheme, however. In the examples here, the quarterly output numbers are the first difference of logs and the annual data are the fourth quarter over fourth quarter log growth rates, which do involve linear aggregation.

be a Wold representation for the X_t with only one shock per coarser period that can be written,

$$X_t = F(L^p)u_t, \quad t = p, 2p, \ldots, Tp.$$

Application of the long-run scheme leads to,

$$X_t = A(L^p)\varepsilon_t, \quad t = p, 2p, \ldots, Tp, \tag{9}$$

with $E[\varepsilon_t \varepsilon_t'] = I$ and $A(1)$ lower triangular.

When will ε_{jt} be an aggregate of the higher frequency $\tilde{\varepsilon}_j$'s in the p-periods underlying the coarser observation, or in all periods before t? The necessary and sufficient conditions for these outcomes can be stated in terms of a factorization of $\tilde{A}(L)$; the reasoning follows that in the previous section:

Proposition 3 *Given the structure (8) and the representation for the data sampled every p periods, (9), with time averaging $M(L)$,*

i) The shocks ε_{jt} will be a linear function only of $\tilde{\varepsilon}_{js}$, $s = t, t-1, \ldots$, for all j, if and only if,

$$M(L)\tilde{A}(L) = \Gamma(L^p)D(L),$$

where $D(L)$ is diagonal, and there exists a diagonal $\hat{D}(L^p)$ such that $\hat{D}(L^p)^{-1}D(L)\tilde{\varepsilon}_t$, $t = p, 2p, \ldots, Tp$, is white noise.

ii) The shocks ε_{jt} will be a linear function only of $\tilde{\varepsilon}_{js}$, $s = t, t-1, \ldots, t-p+1$ if and only if part (i) holds, and the diagonal elements of $D(L)$ are of order less than p.

Proof: see the appendix.

4.2 Appraising the time aggregation conditions theoretically

The simplest case when Proposition 3 is met is when $\tilde{A}(L)$ is block diagonal, implying that no variable Granger causes any other variable in the system. In this case, since $M(L)$ is diagonal, the common factor restriction holds trivially. In general equilibrium macroeconomic models, however, high frequency feedbacks may be of central importance. For example, when stock prices plummeted in October 1987, the Federal Reserve provided added reserves within a day of the initial shock. If the stock price movement were due to a supply shock, the high frequency feedback to a nominal variable such as the money supply could easily induce the type of correlations in quarterly or monthly data that would violate Proposition 3.

Proposition 3 does not rule out feedbacks at high frequency, but the types of feedback allowed are quite limited. Return to the quarterly YU model and consider when an annual model would properly sort out the supply and demand shocks. Suppose that the aggregation scheme for the two variables is the same ($M_{11}(L) = M_{22}(L)$). Following the argument of the multiple shock section, Proposition 3 implies that, for the supply (first) shock,

$$U_t = \frac{\gamma_{21}(L^4)}{\gamma_{11}(L^4)} Y_t \qquad t = 1, 2, \ldots, 4T.$$

The response of quarterly U_t to a supply shock must be expressible simply in terms $Y_t, Y_{t-4}, \ldots$. There is no strong reason to believe that this sort of periodic behavior is present in economic data. One might hope that feedbacks at frequencies higher than the measured data are sufficiently small that the restrictions approximately hold, however, so some way of assessing the likely magnitude of violations would be helpful.

4.3 Assessing the empirical relevance of Proposition 3

We estimate annual versions of the YU and YP models presented in the previous section, using annual average data.[17] The annual YP model is similar to the one used extensively by Bayoumi and Eichengreen to study supply and demand shocks in the OECD.[18] Many of the broad features of the quarterly models carry over to the annual model. In the YU system, much of the forecast error variance in output is attributed to demand, while the YP model gives much less precedence to demand.

One way to characterize how similarly the quarterly and annual models separate supply and demand is to assume that the estimated quarterly models are correct and to ask how the annual models aggregate the quarterly ones. Given a quarterly

[17] We also estimated the YR model in this case, but its inclusion added little.

[18] Annual averaging of the quarterly models implies that output and inflation were measured in fourth-quarter-over-fourth-quarter log growth rates. The unemployment rate is simply an average. Two lags were allowed in each model. The same means were taken out as in the quarterly models.

model based on $\tilde{A}(L)$ and a finite VAR approximation to an annual aggregation of the quarterly data, $A(L^p)$, there is a simple expression showing how the annual model aggregates the shocks from the quarterly model:

$$\varepsilon_t = C(L)\tilde{\varepsilon}_t,$$

$t = 4, 8, \ldots$, $C(L) = A(L^p)^{-1}M(L)\tilde{A}(L)$, or

$$\varepsilon_{it} = c_{i1}(L)\tilde{\varepsilon}_{1t} + c_{i2}(L)\tilde{\varepsilon}_{2t}.$$

The annual supply shock will generally be a distributed lag of the underlying supply and demand shocks, and similarly for the annual demand shock. Given the point estimates for the annual and quarterly models, we can compute the implied $C(L)$ and plot the lag polynomials illustrating how the two quarterly shocks are weighted in creating the annual shocks (Figure 4).

The annual YP model involves almost no commingling of the quarterly shocks. Thus, for the U.S. data, it appears that Bayoumi and Eichengreen's application of the long-run scheme in annual data did not lead to a different categorization of supply and demand shocks than the quarterly model. This result emerges because there are few feedbacks from Y to P (Y does not Granger cause P in the quarterly data) and the feedbacks from P to Y approximately satisfy the restriction of the proposition. In the YU model over one-quarter of the variance of the annual supply shock is accounted for by the quarterly demand shock (Table 6). The YU demand shock is somewhat less confounded, with about ten percent of the variance of the annual demand shock accounted for by quarterly supply shocks.

The annual models may also distort the timing of the underlying shocks. For three of the shocks, over 80 percent of the variance of the annual shock is accounted for by shocks in quarters constituting the annual observation. For the YU supply shock about half the variance comes from quarterly shocks in the previous year.

Overall, this exercise provides a basis for limited optimism regarding time aggregation problems in the cases examined. Given the ubiquitous feedbacks present in most theoretical models and the tight restrictions imposed by Proposition 3, it might seem likely that time aggregation would greatly muddle the results. The substantial absence of feedbacks in the quarterly YP model led to little commingling in the annual model. The quarterly YU model had larger feedbacks and showed more time aggregation bias.

5 Comments

The explicit assumptions of the long-run identifying scheme have been viewed as few in number and innocuous in content. Ahmed et al. [p. 342] note that they "cannot imagine an alternative set of restrictions that would be less restrictive" and still be sufficient for addressing the questions at hand. From this perspective, the long-run scheme may represent an extreme in the search for identifying assumptions that are protected from the *incredible* label stamped on earlier approaches.

This paper shows that structural inference under the long-run scheme will be reliable only if the underlying structure being approximated by the small VAR satisfies strong dynamic restrictions. In providing an explicit characterization of those restrictions, the paper provides a basis for evaluating whether heretofore implicit restrictions are consistent with the economic reasoning underlying the econometric exercise.

The results of this paper do not suggest that the long-run scheme should be abandoned. The results do not even provide a clear ranking of the scheme against other VAR approaches, or against structural modelling in the Cowles tradition, or against real business cycle modelling. We take the results as further evidence that identification in macroeconomics is a dirty business. Perhaps the most important issue in structural inference is assessing the robustness of inferences to changes in the unappealing aspects of the identification scheme. We hope this paper provides some insights to help in this assessment when using the long-run scheme.

Appendix

Proof of Proposition 1: Parameterize the n-variable VAR structures under consideration by $\beta = \{\alpha, \psi\}$ where $\alpha = \{A_i\}$ is the sequence of $(n \times n)$ coefficient matrices of the lag polynomial, $A(L)$, and ψ describes the parameters of the shock process.

Suppose that the VAR model is being estimated by maximizing a likelihood function $L(\beta|X^T)$ where β is $(m \times 1)$. Take the structure parameterized by $\beta^* = \{\alpha^*, \psi^*\}$ which is consistent with the maintained model and which has parameter of interest θ^*. Let $\tilde{\beta} = \{\tilde{\alpha}, \tilde{\psi}\}$ be the parameter of some VAR with the same reduced form as that parameterized by β^*. The parameter of interest for $\tilde{\beta}$ is $\tilde{\theta}$.

Let Q be the $(n \times n)$ matrix with $Q_{i,j} = 0$ if the i,j element of $\tilde{A}(1)$ is consistent with long-run restrictions of the maintained model, and $Q_{i,j} = [\tilde{A}(1)]_{i,j}$ else. Define a sequence of m-parameter models indexed by k: map each $\beta = \{\alpha, \psi\}$ consistent with the VAR model into a parameter $\beta^{(k)} = \{\alpha^{(k)}, \psi\}$ in the new model, where $\alpha^{(k)}$ is defined according to, $A_i^{(k)} = A_i - Q/k$ if $0 < i \leq k$ and $A_i^{(k)} = A_i$ else. Let $\tilde{\beta}^{(k)} = \{\tilde{\alpha}^{(k)}, \tilde{\psi}\}$ be the k^{th} model's structure corresponding to $\tilde{\beta}$ in the original model. Note that $\tilde{A}^{(k)}(1)$ satisfies the long-run restriction for all $k > 0$.

Part i. A sufficient condition for part (i) is that $L(\tilde{\beta}^{(k)}|X^T) \rightarrow_k L(\tilde{\beta}|X^T)$ for all X^T. This condition will be met if each linear combination of the random variables parameterized by $\tilde{\beta}^{(k)}$ converge in mean square to the corresponding linear combination of those parameterized by $\tilde{\beta}$.(Convergence in mean square implies convergence in distribution, which, under the assumption of continuous likelihoods, implies convergence of the likelihood.) Since X^T and $X^{(k),T}$ have the same shock process, we can write samples of size T for each structure as:

$$
\begin{aligned}
X_t &= \tilde{A}(L)\varepsilon_t \\
X_{kt} &= \tilde{A}^{(k)}(L)\varepsilon_t
\end{aligned}
$$

$t = 1, 2, \ldots, T$. Take the mean squared difference of an arbitrary linear combination of the random variables:

$$E(\sum_{t=1}^{T}\sum_{j=1}^{n} w_{tj}(X_{tj} - X_{tjk}))^2 \qquad (10)$$

Since each random variable has mean zero, the double sum is a variance. The double sum can be re-stated as a weighted sum of terms $Z_{ijtk} = (\tilde{a}_{ij}(L) - \tilde{a}_{ij}^{(k)}(L))\varepsilon_{jt}$ where the number of terms, call it M, is fixed by T and n and the weights are fixed by T, n, and the w_{tj}'s. The variance of such a term is bounded by M^2 times the maximum weight squared times the maximum variance of any Z term. Since M and the maximum weight are fixed independently of k, it is sufficient to show that the variance of all the Z terms goes to zero with k. The term $\text{var}(Z_{ijtk}) = \text{var}(\varepsilon_{jt})\|\tilde{\alpha}_{i,j} - \tilde{a}_{i,j}^{(k)}\|_2$. By construction this term goes to zero with k.

Note: The key feature of the sequence of models used in this proof is that $\tilde{\alpha}^{(k)}$ goes to $\tilde{\alpha}$ with k in the l^2 norm:

$$(\sum_{h=0}^{\infty}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{A}_{i,j,h} - \tilde{A}_{i,j,h}^{(k)})^2)^{1/2} \rightarrow_k 0.$$

The proof shows that the likelihood function is continuous in the α parameter in the l^2 sense. The proof could be repeated for any criterion function with this continuity property.

Part ii. A sufficent condition is that $\tilde{\theta}_i^{(k)} \rightarrow_k \tilde{\theta}_i$ for all i. The elements of θ are moving average or VAR coefficients, correlations, or forecast error variances. It is straightforward to verify that each of these is an l^2 continuous function of the α parameter in the sense discussed above. Thus, the required condition is satisfied. Q.E.D.

Proof of Proposition 2: Part i. Necessity can be proven following the discussion in the text. Consider sufficiency. Suppose $\tilde{A}(L) = \Gamma(L)D(L)$. Factor $D(L) = Q(L)\hat{D}(L)$ where $Q(L)$ is diagonal, $(n \times n)$, and $Q_{jj}(L)$ contains any factors common to all scalar polynomials in the j^{th} block diagonal element of $D(L)$. Form $\hat{\varepsilon}_t = \hat{D}(L)\tilde{\varepsilon}_t$. Each $\hat{\varepsilon}_{jt}$ is a function only of the shocks in category j, and no $\hat{\varepsilon}_{jt}$ has a unit root: each is a linear function of uncorrelated variables with no common unit roots. Thus, there is a $\bar{d}_j(L)$ such that $\bar{d}_j(L)^{-1}\hat{\varepsilon}_{jt}$ is white noise with standard deviation one. Form the diagonal matrix, $\bar{D}(L)$ with $\bar{d}_j(L)$ in the j^{th} diagonal element. We can now write the system,

$$X_t = \Gamma(L)Q(L)\bar{D}(L)\bar{D}(L)^{-1}\hat{D}(L)\tilde{\varepsilon}_t.$$

Defining $A(L) = \Gamma(L)Q(L)\bar{D}(L)$ and $\varepsilon_t = \bar{D}(L)^{-1}\hat{D}(L)\tilde{\varepsilon}_t$, we have $X_t = A(L)\varepsilon_t$, Since $\tilde{A}(1)$ is lower triangular, and $\bar{D}(1)^{-1}\hat{D}(1)$ has all elements non-zero by construction, the implied $A(1)$ is lower triangular.

Part *ii*. Repeat the proof of part *i* with $D(L) = D$. Q.E.D.

Proof of Proposition 3: Part *i*. Necessity: From the two representations of X_t and assuming the conclusion we have,

$$M(L)\tilde{A}(L)\tilde{\varepsilon}_t = A(L^p)\varepsilon_t = A(L^p)Z(L)\tilde{\varepsilon}_t$$

for some diagonal $Z(L)$. Thus, $M(L)\tilde{A}(L) = A(L^p)Z(L)$, proving the point.

Sufficiency. By assumption,

$$M(L)\tilde{A}(L)\tilde{\varepsilon}_t = \Gamma(L^p)D(L)\tilde{\varepsilon}_t$$

By assumption, there is a matrix polynomial $\hat{D}(L^p)$ such that $\hat{D}(L^p)^{-1}D(L)\tilde{\varepsilon}_t$, $t = p, 2p, \ldots$ is white noise. Write

$$M(L)\tilde{A}(L)\tilde{\varepsilon}_t = \Gamma(L^p)\hat{D}(L^p)\hat{D}(L^p)^{-1}D(L)\tilde{\varepsilon}_t$$

Take $A(L^p) = \Gamma(L^p)\hat{D}(L^p)$ and $\varepsilon_t = \hat{D}(L^p)^{-1}D(L)\tilde{\varepsilon}_t$. The matrix, $A(1)$ is lower triangular since $\hat{D}(1)^{-1}D(1)$ has all elements non-zero.

Part *ii*. Repeat the proof of part *i* with the order of $D(L)$ limited. Q.E.D.

References

Ahmed, Shaghil, Ickes, Barry W., Wang, Ping, and Byung Sam Yoo (1993), "International Business Cycles," *American Economic Review* 83, June, 335–59.

Bayoumi, Tamim and Barry Eichengreen (1992a), "Shocking Aspects of European Monetary Unification," Working Paper No. 3949, National Bureau of Economic Research, January.

________________ (1992b), "Macroeconomic Adjustment Under Bretton Woods and the Post-Bretton Woods Float: An Impulse-Response Analysis," Working Paper No. 4169, National Bureau of Economic Research, September.

Bernanke, Ben (1986), "Alternative Explorations of the Money-Income Correlation," in: Brunner, Karl and Allan H. Meltzer, eds., *Real Business Cycles, Real Exchange Rates, and Actual Policies*, Carnegie-Rochester Conference Series on Public Policy 25, 49–99.

Blanchard, Olivier J. and Mark W. Watson (1986), "Are Business Cycles All Alike?" in: Gordon, Robert J., ed., *The American Business Cycle: Continuity and Change* (Chicago: University of Chicago Press), 123–56.

Blanchard, Olivier J. and Danny Quah (1989), "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review* 79, September, 655–73.

Gali, Jordi (1992), "How Well Does the IS–LM Model Fit Postwar U.S. Data?" *Quarterly Journal of Economics* 107, May, 709–38.

Hansen, L.P., and T.J. Sargent (1991), "Identification of continuous time rational expectations models from discrete time data," in Hansen and Sargent, eds., *Rational Expectations Econometrics*, (Westview Press: Boulder), 219–238.

Hendry, David F. (1992), "An Econometric Analysis of TV Advertising Expenditure in the United Kingdom," *Journal of Policy Modeling*, 14, June, 281–312.

_______________ (1993), "The Roles of Economic Theory and Econometrics in Time Series Economics," invited lecture, European Econometric Society Meetings, Uppsala Sweden, August.

Hendry, David F. and Grayham E. Mizon (1978), "Serial Correlation as a Convenient Simplification not a Nuisance: A Comment on a Study of the Demand for Money by the Bank of England," *Economic Journal*, 88, Sept., 549–63.

Hutchison, Michael M. (1992), "Structural Change and the Macroeconomic Effects of Oil Shocks: Empirical Evidence from the United States and Japan," Working Paper No. PB92-06, Federal Reserve Bank of San Francisco, March.

Hutchison, Michael M. and Carl E. Walsh (1992), "Empirical Evidence on the Insulation Properties of Fixed and Flexible Exchange Rates," *Journal of International Economics* 32, May, 241–63.

King, Robert G., Plosser, Charles I., Stock, James H. and Mark W. Watson (1991), "Stochastic Trends and Economic Fluctuations," *American Economic Review* 81, September, 819–40.

Lastrapes, William D. and George Selgin (1993), "The Liquidity Effect: Identifying Short-Run Interest Rate Dynamics using Long-Run Restrictions," manuscript, University of Georgia, January.

Lippi, Marco and Lucrezia Reichlin (1993), "The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment," *American Economic Review* 83, June, 644–52.

Moreno, Ramon (1994) "Are the Forces Shaping International Business Cycles Alike? The Evidence from Japan," manuscript, Federal Reserve Bank of San Francisco.

Novales, Alfonso (1990), "Solving Nonlinear Rational Expectations Models: A Stochastic Equilibrium Model of Interest Rates," *Econometrica* 58, January, 93–112.

Phillips, P.C.B. (1973) "The problem of identification in finite parameter continuous-time models," *Journal of Econometrics*, 1, 351–362.

Rogers, John H. and Ping Wang (1992), "Sources of Fluctuations in Relative Prices: Evidence from High Inflation Countries," manuscript, Pennsylvania State University, December.

Shapiro, Matthew D. and Mark W. Watson (1988), "Sources of Business Cycle Fluctuations," in: Fischer, Stanley, ed., *NBER Macroeconomics Annual 1988* (MIT Press: Cambridge, MA), 111-48.

Sims, Christopher A. (1980a), "Macroeconomics and Reality," *Econometrica* 48, January, 1–48.

____________ (1980b), "Comparison of Interwar and Postwar Business Cycles: Monetarism Reconsidered," *Papers and Proceedings of the American Economic Association* 70, May, 250–57.

____________ (1986), "Are Forecasting Models Usable for Policy Analysis?" Federal Reserve Bank of Minneapolis *Quarterly Review*, Winter, 2–16.

Telser, L.G. (1967), "Discrete Samples and Moving Sums in Stationary Stochastic Processes," *Journal of the American Statistical Association* 62, 318, 484–499.

Table 1: Percent of forecast error variance due to demand in the YU system

quarter	lower	Y error point est.	upper	lower	U error point est.	upper
1	52.8	99.9	100.0	2.4	43.9	93.6
2	51.3	99.4	99.6	5.8	53.8	96.3
3	52.2	99.5	99.4	11.1	63.3	97.2
4	52.0	99.0	99.2	16.6	71.0	96.7
8	36.4	84.7	94.9	34.1	82.9	93.4
12	27.1	73.2	91.5	38.7	84.1	92.4
24	17.7	55.3	80.8	40.1	84.3	92.3
40	12.5	42.6	67.6	40.1	84.3	92.3

Bounds are the 5^{th} and 95^{th} percentile based on the simulations described in the text.

Table 2: Contemporaneous correlation among the shocks in the three models

	Demand Shocks			Supply Shocks		
	YU-D	YP-D	YR-D	YU-S	YP-S	YR-S
YU-D	1.00	0.65	0.67	0.00	0.56	0.54
YP-D		1.00	0.48	-0.13	0.00	0.41
YR-D			1.00	-0.13	0.45	0.00
YU-S				1.00	0.20	0.23
YP-S					1.00	0.43
YR-S						1.00

Table 3: Percent of forecast error variance due to demand in the YP system

	Y error			P error		
quarter	lower	point est.	upper	lower	point est.	upper
1	1.9	46.4	96.8	3.7	56.5	98.5
2	2.7	50.2	97.5	5.8	62.2	98.6
3	3.0	51.0	97.5	7.1	66.3	98.8
4	3.0	50.9	97.5	8.0	68.3	98.9
8	3.3	37.2	90.4	8.3	69.4	99.0
12	2.8	27.6	80.5	8.6	70.2	99.0
24	1.6	15.7	57.4	9.0	71.6	99.2
40	1.0	9.4	36.6	9.2	72.2	99.3

Bounds are the 5^{th} and 95^{th} percentile based on the simulations described in the text.

Table 4: Percent of forecast error variance due to demand in the YR system

	Y error			R error		
quarter	lower	point est.	upper	lower	point est.	upper
1	2.5	53.6	98.4	10.2	77.2	99.7
2	2.2	50.7	97.9	12.2	80.3	99.6
3	2.0	43.9	96.1	16.6	84.4	98.4
4	2.3	37.7	93.9	19.1	87.3	98.1
8	5.6	19.6	76.5	20.1	88.9	98.2
12	6.9	13.2	59.8	19.9	88.5	97.7
24	3.7	6.5	40.4	18.3	87.7	97.4
40	2.1	3.9	25.0	17.7	87.6	97.4

Bounds are the 5^{th} and 95^{th} percentile based on the simulations described in the text.

Table 5: Identified demand and supply shocks in three Models for selected periods

date	YU-D	YP-D	YR-D	YU-S	YP-S	YR-S
51:1	-0.0	3.0	-0.7	-0.9	-3.3	-0.0
51:2	1.5	-1.6	1.1	1.1	4.6	2.1
51:3	1.3	-1.4	0.3	3.6	3.3	1.0
51:4	-1.4	0.5	-1.3	0.8	-0.5	-1.0
57:2	-1.3	-1.7	-0.3	-0.7	-0.2	-1.2
57:3	1.3	0.3	0.5	0.2	0.6	0.5
57:4	-2.3	-2.5	-1.0	0.8	-0.7	-2.1
58:1	-2.6	-1.8	-3.0	0.7	-2.7	-1.2
58:2	0.8	1.0	0.5	2.2	0.9	1.1
74:1	-1.0	-1.8	-3.3	-0.3	0.2	2.8
74:2	0.7	0.6	1.0	-1.9	0.4	-0.6
74:3	-1.4	1.0	-2.4	-0.5	-2.4	-0.3
74:4	0.1	0.2	0.9	2.0	-0.4	-1.2
75:1	-2.0	-0.1	-2.6	2.7	-2.4	-1.1
75:2	2.3	-0.7	1.4	1.2	2.6	-0.5
79:1	-0.6	-0.5	0.7	0.0	-0.2	-1.1
79:2	-0.2	0.1	0.6	-1.5	-0.2	0.1
79:3	0.1	1.7	1.0	0.5	-0.1	0.4
79:4	-0.4	-0.4	1.6	0.3	-0.4	-2.2
80:1	-0.1	0.6	1.3	2.0	-0.3	-0.6
80:2	-3.0	-1.7	-4.0	1.7	-2.7	1.1

Table 6: Decomposition of annual shock variances for the YU and YP models

	Share of annual shock variance due to		
	quarterly	both quarterly shocks, horizon	
annual shock	supply shock	4 quarters	8 quarters
YU-S	0.73	0.51	0.96
YP-S	0.96	0.88	0.97
YU-D	0.09	0.86	0.97
YP-D	0.05	0.80	0.94

Figure 1

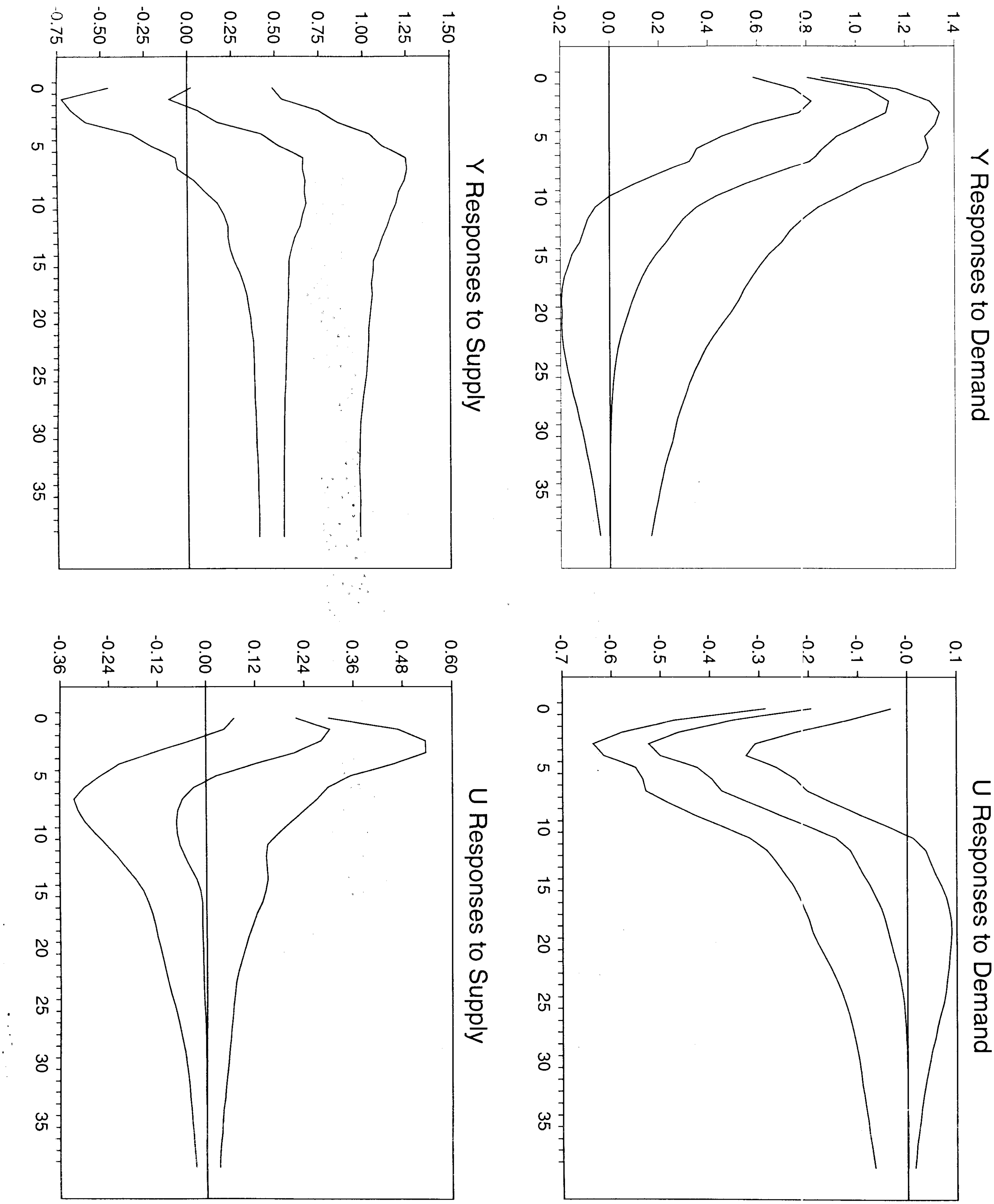

Y Responses to Demand
U Responses to Demand
Y Responses to Supply
U Responses to Supply

Figure 2

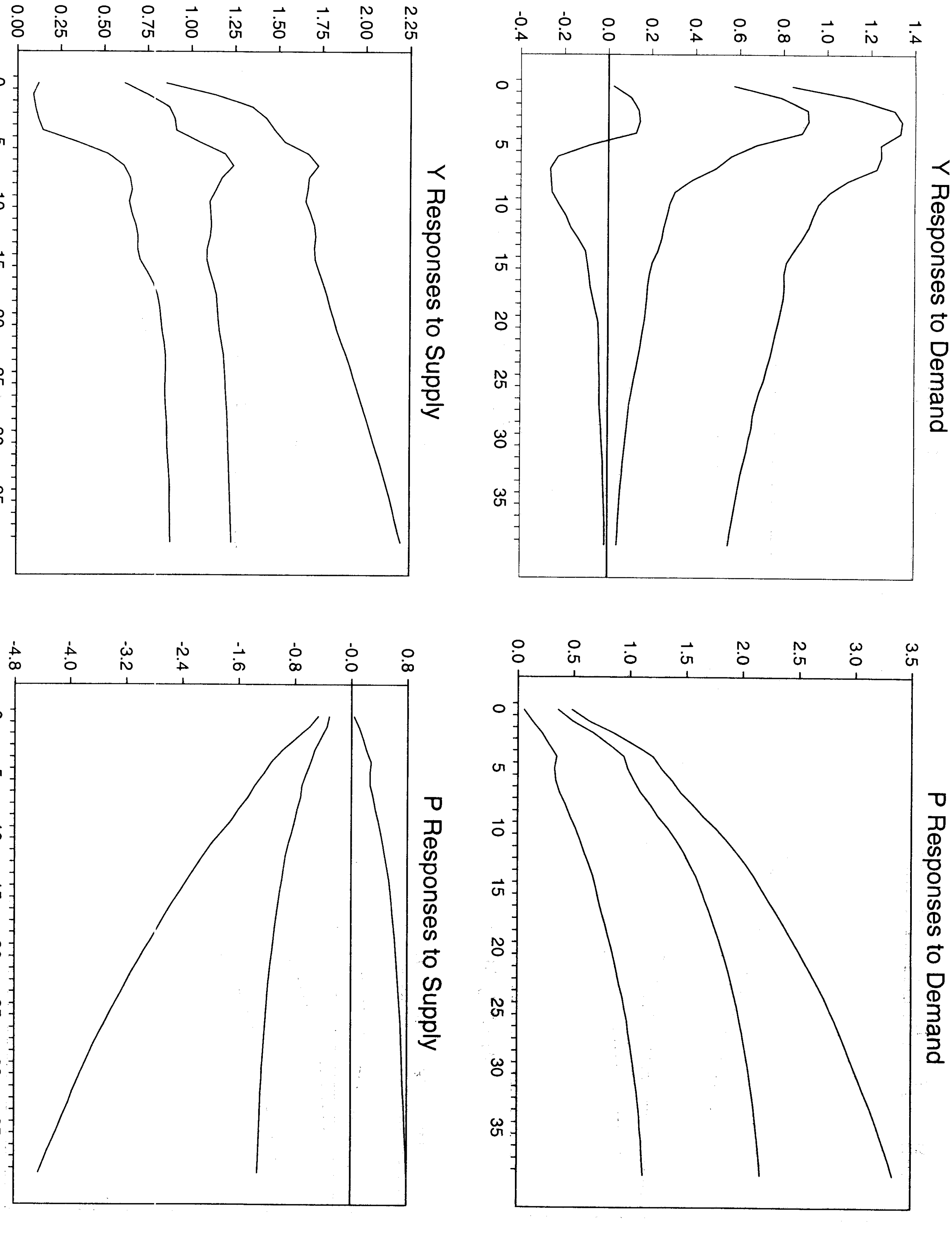

Y Responses to Demand
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
5
10
15
20
25
30
35
P Responses to Demand
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0
5
10
15
20
25
30
35
Y Responses to Supply
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
0
5
10
15
20
25
30
35
P Responses to Supply
-4.8
-4.0
-3.2
-2.4
-1.6
-0.8
-0.0
0.8
0
5
10
15
20
25
30
35

Figure 3

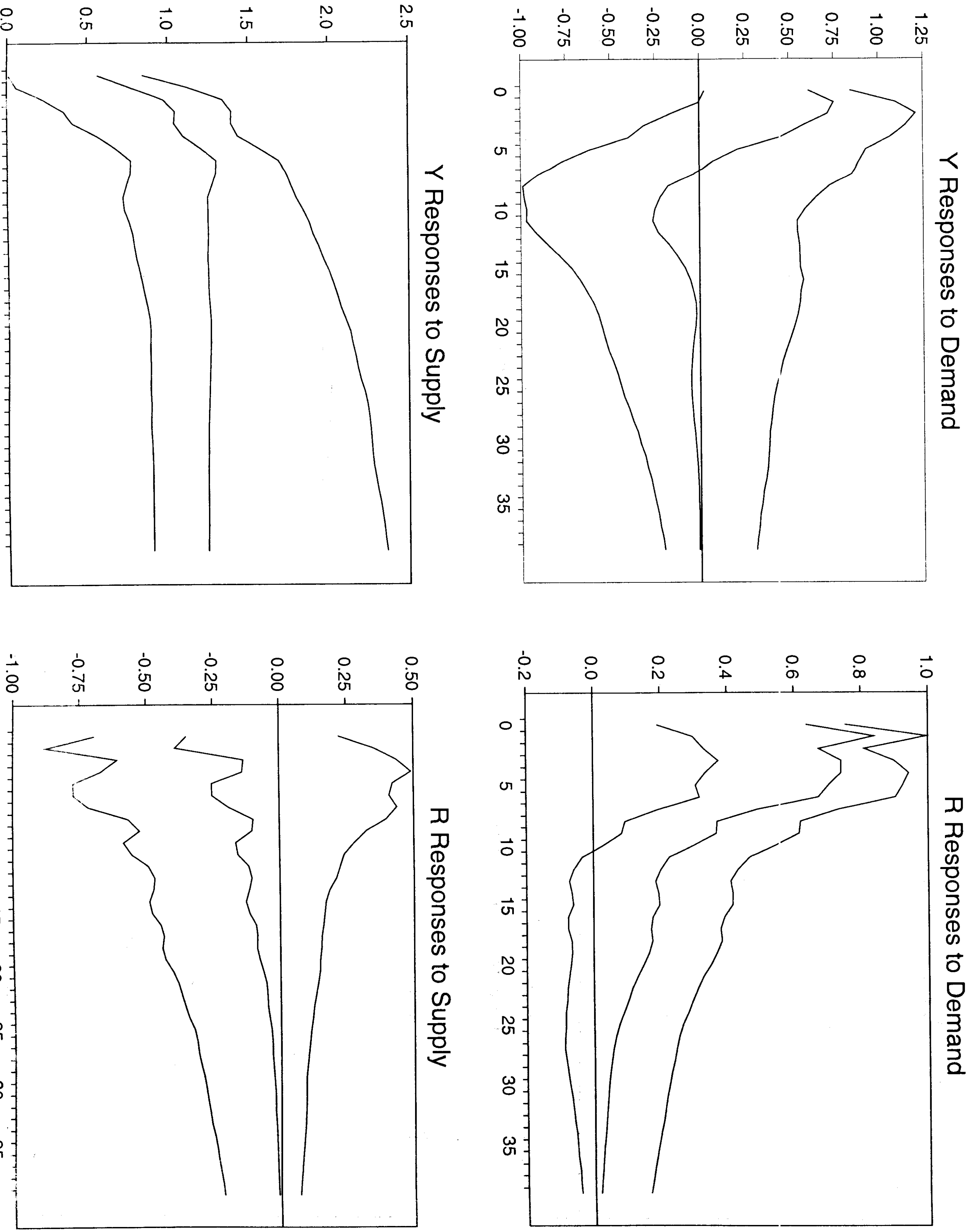
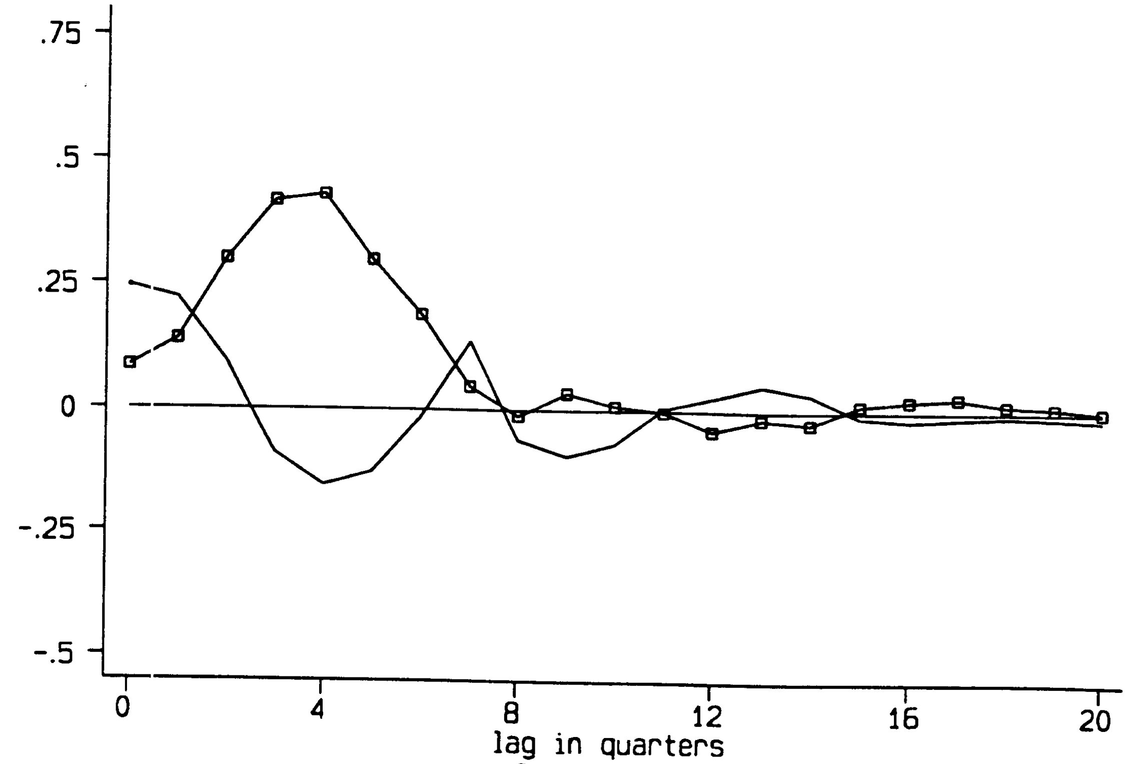

Y Responses to Demand
R Responses to Demand
Y Responses to Supply
R Responses to Supply

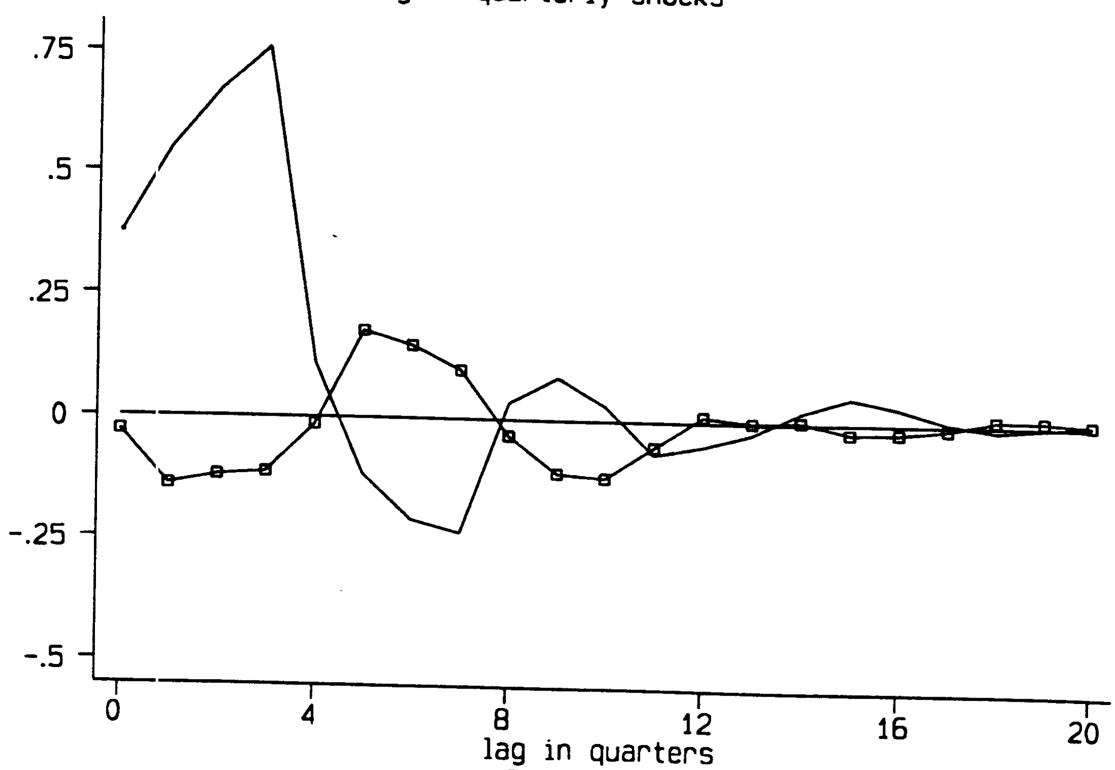
Figure 4

Annual YU model: supply shock
as a distributed lag of quarterly shocks



square=supply, dot=demand

Annual YU model: demand shock
as a distributed lag of quarterly shocks



square=supply, dot=demand

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