"Forecasting the Forecasts of Others:" Expectational Heterogeneity and Aggregate Dynamics

Antulio N. Bomfim*
Division of Research and Statistics, Mail Stop 61
Board of Governors of the Federal Reserve System
Washington, DC, 20551
abomfim@frb.gov

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Abstract

I construct a dynamic general equilibrium model where agents differ in the way they form expectations. Sophisticated agents form model-consistent expectations. Rule-of-thumb agents' expectations are based on an intuitive forecasting rule. All agents solve standard dynamic optimization problems and face strategic complementarity in production. Extending the work of Haltiwanger and Waldman (1989), I show that even a minority of rule-of-thumb forecasters can have a significant effect on the aggregate properties of the economy. For instance, as agents try to forecast each others' behavior they effectively strengthen the internal propagation mechanism of the economy. I solve the model by assuming a hierarchical information structure similar to the one in Townsend's (1983) model of informationally dispersed markets. The quantitative results are obtained by calibrating the model and running a battery of sensitivity tests on key parameters. The analysis highlights the role of strategic complementarity in the heterogeneous expectations literature and precisely *quantifies* many qualitative claims about the aggregate implications of expectational heterogeneity.

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1 Introduction

The assumption that agents form rational (model-consistent) expectations is a standard fixture of most work in modern macroeconomics. Though widespread, such reliance on the rational expectations hypothesis has not gone unquestioned. For example, Frydman and Phelps (1983), Board (1994), and Arthur (1994) argue that human rationality is *bounded* and thus the rational expectations assumption imposes extreme informational and computational requirements on agents (see also Sargent, 1993). In addition, other researchers base their reservations on concerns about the observable characteristics of rational expectations equilibria. For instance, De Long, Schleifer, Summers, and Waldman (1990) construct a model in which behavior based on irrational noise trading helps explain a number of observed phenomena in financial markets, such as the excess volatility of asset prices and the equity premium puzzle. Also in the finance literature, Roll (1996) mentions incomplete (bounded) rationality as a possible explanation for the observation of large trade volumes in debt markets. Whereas these reservations have led some economists to discard the rational expectations hypothesis altogether, others have sought to reconsider it in the context of environments with less demanding informational assumptions and more plausible observable implications. Lucas (1975) and Townsend (1983) were among the first to take up this line of inquiry. Townsend, in particular, analyzed a model where agents form *heterogeneous expectations* because their forecasts are conditioned on different subsets of the relevant data. He showed that, as agents attempt to "forecast the forecast of others," the economy converges to a rational expectations equilibrium.

More recently, the work of Haltiwanger and Waldman (1985, 1989) brought a new perspective to the analysis of forecast heterogeneity. Rather than working with *dynamic* classical models with no externalities (the Lucas-Townsend approach), Haltiwanger and Waldman analyzed environments that allowed for strategic complementarity.[1] In addition, instead of focusing on agents with different access to the *data*, they built models where forecast heterogeneity arises because some agents may use more sophisticated forecasting methods than others. Accordingly, while some may form model-consistent expectations, others rely less on the *structure* of the model and form expectations based on a simple *rule-of-thumb*. Unlike the Lucas-Townsend tradition, the class of simple, *static* models analyzed by Haltiwanger and Waldman gave rise to environments where forecast heterogeneity did cast some doubt on the aggregate implications of the rational expectations hypothesis. They showed that, with strategic complementarity, less sophisticated forecasters may have a sizable effect on the evolution of aggregate output, effectively driving the economy away from its pure rational expectations equilibrium.

[1] Cooper and John (1988) define strategic complementarity and discuss its implications for macroeconomics. For the purpose of my paper, strategic complementarity involves a situation where an individual's output decision is increasing on the level of aggregate output.

In this paper I introduce and solve a model that brings together important issues stemming from both the Lucas-Townsend and Haltiwanger-Waldman approaches to analyzing heterogeneous-expectations models. Capturing the key insights of Haltiwanger and Waldman, my analysis allows for strategic complementarity and heterogeneous forecasting rules. However, to bring the discussion more into the current stage of macroeconomic thought—which emphasizes dynamic rather than static frameworks—I extend the Haltiwanger-Waldman analysis to a richer dynamic general equilibrium model, which I solve with a methodology that is very close in spirit to Townsend's.

In choosing a specific dynamic modeling framework, I opted for the class of models in the real business cycle tradition. The advantages of this choice are two-fold. First, the virtues and limitations of the RBC framework are well understood by the profession. Thus, the results I obtain under forecast heterogeneity and strategic complementarity can be *directly* and *quantitatively* compared to those generated by standard RBC models with homogeneous, rational expectations. Second, by introducing forecast heterogeneity into the RBC framework, I am able to address a recurring theme in the literature: the weak internal propagation mechanism that underlies many equilibrium business cycle models (King and Plosser, 1988).

The paper's main results and methodology can be summarized as follows. With a sufficiently strong degree of strategic complementarity, I show that even if only a small subset of agents forecasts according to a simple but reasonable rule of thumb, aggregate output exhibits more persistence than warranted by either (i) the degree of serial correlation in the productivity shock process, or (ii) the share of rule-of-thumb forecasters in the total population. More to the point, because agents engage in forecasting each others' forecasts, rule-of-thumb forecasters have a quantitatively important impact on the serial correlation properties of the business cycle. The results are obtained by calibrating all standard RBC parameters and then running a battery of sensitivity tests on the forecast-heterogeneity and strategic-complementarity parameters. The sensitivity tests highlight the role of strategic complementarity in the forecast-heterogeneity debate and precisely *quantify* the qualitative claims made by Haltiwanger and Waldman and others.

2 The Model

Apart from the issues of heterogeneity, the model described here is very close to that of Baxter and King (1991).

2.1 Strategic Complementarity and the Production Function

Individual output is a function not only of inputs and a productivity shock, but also of an index of per capita aggregate output. This index is a geometric average of the per capita output decisions of two types of agents: [2]

$$Y_t = Y_{S,t}^{\theta_s} Y_{R,t}^{(1-\theta_s)} \tag{1}$$

where θ_s is the proportion of total population represented by sophisticated forecasters (type S agents), and $Y_{S,t}$ is the per capita output decision of these agents. The R subscript denotes variables pertaining to the rule-of-thumb forecasters.

An agent of type i faces the production function,

$$y_{i,t} = \exp(A_t) F(k_{i,t-1}, n_{i,t}) Y_t^{\phi} \qquad i = R, S \tag{2}$$

where $y_{i,t}$ denotes individual output, and $k_{i,t-1}$ and $n_{i,t}$ correspond to capital and labor inputs. The shock A_t is assumed to capture stochastic shifts in total factor productivity. It follows a first-order autoregression:

$$A_t = \rho A_{t-1} + a_t \tag{3}$$

where $|\rho| \leq 1$, and $\{a_t\}$ is a zero-mean, normally distributed white noise process with variance σ_a^2.

$F(.)$ is a Cobb-Douglas production function:

$$F(k_{i,t-1}, n_{i,t}) \equiv k_{i,t-1}^{\theta_k} n_{i,t}^{\theta_n}$$

which is homogeneous of degree 1 ($\theta_k + \theta_n = 1$).[3] The ϕ parameter in equation (2) embodies the complementarity assumption; it determines the extent to which individual output, $y_{i,t}$, depends on aggregate output, Y_t. ($0 \leq \phi < 1$)

[2] Lucas (1972, 1973) uses this index instead of the conventional definition of aggregate output. This, as noted in Blanchard and Fischer (1989), amounts to "defining aggregate output as the product of individual outputs, rather than their sum" (p. 358). (See also Sargent 1987, p. 442). The use of such index simplifies tremendously the algebra, both here and in the referenced works.

[3] Including labor-augmenting technical change in the production function to allow for economic growth would have required "detrending" the model before proceeding to the solution algorithm; the results would have been unchanged. As in King, Plosser, and Rebelo (1988), detrending would involve nothing more than dividing all growing variables by the labor augmenting factor. For the sake of brevity, we have chosen to start with a stationary model from the beginning.

2.2 Evolution of the Capital Stock

Output not consumed constitutes gross investment, $i_{i,t}$. With $k_{i,t}$ representing the capital stock at the end of period t and assuming that this stock depreciates at the rate δ, $0 \leq \delta < 1$,

$$k_{i,t} = (1-\delta)k_{i,t-1} + i_{i,t} \tag{4}$$

2.3 Preferences

Preferences are homogeneous throughout the economy. The momentary utility function of a representative agent is:

$$u(c_{i,t}, l_{i,t}) = \log(c_{i,t}) + \theta_l \log(l_{i,t}) \tag{5}$$

where $c_{i,t}$ denotes consumption, and $l_{i,t}$ is leisure—expressed as a proportion of the unit time endowment.

3 Individual Behavior

The economy is populated by infinitely-lived, forward-looking agents who discount the future at the rate β and maximize expected utility over an infinite horizon. As in Townsend (1983), we can think of individual behavior as the outcome of two separate problems: dynamic optimization and inference.[4] The solution to the first problem yields the perfect-foresight equilibrium laws of motion of all choice variables, which express the optimal values of these variables as a function of past, current, and future states of the economy. By subsequently solving their inference problems, the agents convert these equilibrium laws of motion into decision rules that express all choice variables as functions only of the observed states of the economy.

My assumption of heterogeneous expectations amounts to saying that agents rely on the same mechanism to solve their dynamic optimization problem, but not their inference problem. In the next section I will discuss how different agents tackle their inference problem, but first I will focus on the aspects of individual behavior that are common to both types of agents.

[4] Invoking the separation or certainty-equivalence principle is common-place in standard real business cycle models with homogeneous, rational expectations (see, e.g., King et al., 1988).

3.1 The Dynamic Optimization Problem

Grouping all agents according to the way they form expectations, I assume that each individual takes as given both the group- and economy-wide levels of all relevant variables.[5] In addition to equations (1) through (5), the solution to the agents' dynamic optimization problem must satisfy the usual time and goods constraints,

$$l_{i,t} + n_{i,t} \leq 1 \tag{6}$$

$$c_{i,t} + i_{i,t} \leq y_{i,t} \tag{7}$$

and a symmetry condition that says that individuals who rely on the same forecasting mechanism must behave identically. This last condition implies that the equilibrium quantities for a given type i agent are equal to the respective per capita quantities for all agents of that type ($y_{S,t} = Y_{S,t}$, $n_{R,t} = N_{R,t}$, etc). When applied to the equilibrium levels of capital, labor, and output, the symmetry condition implies

$$Y_{i,t} = \Upsilon_i(A_t, K_{i,t-1}, N_{i,t}, Y_{jt}) \equiv [\exp(A_t)F(K_{i,t-1}, N_{i,t})]^{\eta_i} Y_{jt}^{s_j} \tag{8}$$

where, for $i = S, R$, $j \neq i$, $\eta_i \equiv 1/[1-\phi(1-\theta_j)]$ and $s_j \equiv \phi\theta_j\eta_i$

Euler Equations. Given equations (1) through (7), the agent's dynamic optimization problem reduces to solving

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ u(c_{i,t}, 1-n_{i,t}) + \lambda_t[\exp(A_t)F(.)Y_t^{\phi} - c_{i,t} - k_{i,t} + (1-\delta)k_{i,t-1}] \right\} \tag{9}$$

subject to

$k_{i,-1}$ and $\{A_t, Y_t\}_{t=0}^{\infty}$ given; and $\lim_{t\to\infty} \beta^t \lambda_t k_{i,t} = 0$. [6]

The derivation of the system of Euler equations that corresponds to (9) is straightforward. After imposing the symmetry conditions on this system we obtain: [7]

$$u_C(C_{i,t}, 1-N_{i,t}) - \Lambda_{i,t} = 0 \tag{10}$$

[5] Note that given that the solution to the dynamic optimization problem requires no forecasting, both agents behave identically in so far as this problem is concerned. For completeness, I will retain, however, the i index throughout this subsection.

[6] The last equation is the transversality condition. Its finite horizon analog says that individuals would place no value in holding capital after the end of the last period of the planning horizon. λ_t is the discounted Lagrange multiplier relevant for time t ($\lambda_t^* \equiv \beta^t \lambda_t$ is the undiscounted one)

[7] $u_i(.)$ $[F_i(.)]$ corresponds to the first derivative of the utility [production] function with respect to i. Note, e.g., that the private marginal product of labor can be written as:

$$\exp(A_t)F_N(k_{i,t-1}, n_{i,t})Y_t^{\phi} = (1-\theta_k)y_{i,t}/n_{i,t}$$

$$u_L(C_{i,t}, 1 - N_{i,t}) - (1-\theta_k)\Lambda_{i,t}\Upsilon_i(A_t, K_{i,t-1}, N_{i,t}, Y_{jt})/N_{i,t} = 0 \quad (11)$$

$$\beta\Lambda_{i,t+1}[\theta_k\Upsilon_i(A_{t+1}, K_{i,t}, N_{i,t+1}, Y_{jt+1})/K_{i,t} + (1-\delta)] - \Lambda_{i,t} = 0 \quad (12)$$

$$\Upsilon_i(A_t, K_{i,t-1}, N_{i,t}, Y_{jt}) - C_{i,t} - K_{i,t} + (1-\delta)K_{i,t-1} = 0 \quad (13)$$

which are the familiar optimality conditions also found in other equilibrium business cycle models (see, e.g., Baxter and King, 1991).

Equilibrium Laws of Motion. The perfect-foresight equilibrium paths of consumption, investment, and labor effort are given by the solution to the system formed by equations (10) through (13). It is well known, however, that there is no closed-form solution to this system so I will focus instead on an approximate solution, obtainable by log-linearizing the system around its steady state.[8] The resulting (approximate) equilibrium laws of motion take the form

$$x_{i,t} = \Pi_{i,k}\hat{K}_{i,t-1} + \Pi_{i,\lambda}\tilde{\lambda}_{i,t} + \Pi_{i,e}e_{i,t}, \qquad i,j = S,R,\ j \neq i \quad (14)$$

where $x_{i,t} \equiv [\hat{N}_{i,t}, \hat{K}_{i,t}, \hat{C}_{i,t}]'$, $e_{i,t} \equiv [A_t, \hat{Y}_{j,t}]'$, and

$$\tilde{\lambda}_{i,t} \equiv \sum_{h=0}^{\infty} \mu_i^{-h}(F_{i,1}e_{i,t+h+1} + F_{i,2}e_{i,t+h}) \quad (15)$$

A "caret" over a symbol denotes that the variable is expressed in percentage deviations from the steady state (e.g., $\hat{Y}_{i,t} \equiv \log(Y_{i,t}/\bar{Y}_i)$). The matrices Π_i and F_i, as well as the μ_i parameter, are all functions of various steady-state properties of the model, such as the economy's capital-output ratio and the steady-state labor's share of total income.[9]

Equation (14) corresponds to the solution to the agents' dynamic optimization problem, which abstracts from stochastic considerations. To fully characterize individual behavior, I still need to explicitly address the issues of uncertainty and expectational formation.

3.2 The Inference Problem

By solving their inference problem, agents convert the perfect-foresight equilibrium laws of motion just derived into the decision rules that make up their behavior. In this subsection I describe those aspects of the information structure that are common to both sophisticated and rule-of-thumb forecasters.

[8] The log-linear approximation method used here is described in detail in King et al. (1990).

[9] The derivation of (14) follows King et al. (1990) very closely.

For each period t, I assume that all agents follow a two-stage decision process.[10] At the beginning of the period, the first stage takes place: agents make their labor supply and capital accumulation decisions before being able to observe the current value of the productivity shifter, A_t, or the current output decision of the other agents in the economy. The factor-allocation decision rules take the form:

$$z_{i,t} = \pi_{i,k}\hat{K}_{i,t-1} + \pi_{i,\lambda}E^{(i)}\left[\tilde{\lambda}_{i,t} \mid \Omega_{t-1}\right] + \pi_{i,e}E^{(i)}\left[e_{i,t} \mid \Omega_{t-1}\right] \tag{16}$$

where $z_{i,t} \equiv [\hat{N}_{i,t}, \hat{K}_{i,t}]'$, and the π_i parameters correspond to the appropriate elements of the Π_i matrices from equation (14). Ω_{t-1} is the information set available at the beginning of period t; it contains the whole history of the economy up to period $t-1$. $E^{(i)}[.|\Omega_{t-1}]$ denotes the expectation of a type i agent conditioned on Ω_{t-1}.

Once the factor-allocation decisions are made, production takes place and the agents move to the second and last stage of their decision making process. Assuming that both sophisticated and rule-of-thumb forecasters observe each others' output as soon as production takes place, each agent can use its knowledge of the production function to deduce the current value of the productivity shifter (A_t). Therefore, the consumption decision is based on a larger information set, $\Omega_{0,t} \equiv \{\Omega_{t-1}, A_t, Y_{S,t}, Y_{R,t}\}$.

Equation (16) makes explicit two points advanced earlier in this paper. First, agents of different types are informationally linked; to generate their own decision rules they must forecast the behavior of the other agents in the economy—recall that $\hat{Y}_{j,t}$ is an element of $e_{i,t}$. Second, equation (16) highlights the channel through which agents' expectations affect their behavior. Accordingly, the different expectational rules embedded in $E^{(S)}$ and $E^{(R)}$ can lead to potentially different responses to the same fundamental shocks.

4 Expectational Heterogeneity

As stated before, type S agents are sophisticated forecasters; their forecasts are fully consistent with the rational expectations hypothesis. Accordingly, they have full knowledge of the structure of the model, including the expectational behavior of type R agents. For any given variable ψ_t, their expectations can be formally defined as

$$E^{(S)}[\psi_t|\Omega_{t-1}] = E[\psi_t|\Omega_{t-1}] \tag{17}$$

where $E[\psi_t|\Omega_{t-1}]$ is the mathematical expectation of ψ_t conditioned on the true structure of the *entire* model (Muth, 1961).

[10] Kydland and Prescott (1982) assume a similar information structure.

4.1 Rule-of-Thumb Forecasting

Type R agents are called rule-of-thumb forecasters because their expectations are based on a simple expectational rule. In allowing for the existence of these agents, I am motivated by a number of considerations. First, many researchers, especially those in the noise trading literature, have implicitly assumed frameworks where agents are endowed with different expectational formation capabilities. For instance, De Long et al. (1990) assume that some agents' misperceptions lead them to form incorrect expectations about the price of risky assets, while others form model-consistent expectations. Second, a series of recent papers in the macroeconomics literature has argued that even agents who are equally endowed with respect to their expectation formation capabilities could optimally adopt different expectational behaviors. According to this view, if the implementation of rational expectations is costly, otherwise identical agents who face different constraints and opportunities may choose different levels of sophistication when generating their forecasts. For instance, Evans and Ramey (1992) and Sethi and Franke (1995) developed theoretical models that include explicit costs of forming rational expectations and show that equilibria with a mix of rational and rule-of-thumb expectations are possible.[11]

Though partly motivated by both the noise-trading and costly-computation literatures, my analysis of the business-cycle implications of forecast heterogeneity is also driven by a third question: whether or not the general topic of bounded rationality matters for quantitative equilibrium business cycle analysis. Along these lines, one can think of this paper as an attempt to quantitatively assess the robustness of the RBC framework to a partial relaxation of the rational expectations hypothesis. Thus, while implicitly taking as given the existence of agents who, in a statistical sense, make less-than-efficient forecasts, my goal is to use the tools of the RBC literature to assess their potential impact on aggregate dynamics and to determine what features of the real world, if any, are missed by exclusively considering models with homogeneous, rational expectations.[12]

4.2 A Forecasting Rule for Type R Agents

Three criteria guided the specification of an illustrative forecasting rule for type R agents. First, to capture the concerns of the costly-implementation literature, the rule must be simple to implement. Second, the forecasting rule should generate “reasonable” forecasts, i.e., it should be consistent with simple, well-known characteristics of the economy. Finally, the expectational scheme assumed for type R agents must not be at odds with their ability to solve their dynamic optimization problem. In other words, when solving their

[11]For tractability, however, Evans and Ramey and Sethi and Franke had to assume a higher level of abstraction than the one assumed here, making their models not as suitable for exercises in quantitative analysis.

[12]Kydland and Prescott (1996) and King (1995) discuss the use of RBC models in quantitative analysis.

dynamic optimization and inference problems, type R agents must rely on a single, consistent pool of information about the behavior of the economy.

We saw in equation (16) that a type i agent must forecast current and future movements in total factor productivity (A_t) and the per capita output of the other agents in the economy ($\hat{Y}_{j,t}$). For illustrative purposes, suppose that type R agents assume an autoregressive forecasting model for these variables:

$$E^{(R)}[A_{t+h}|\Omega_{t-1}] = \alpha^{h+1}A_{t-1} \quad (18)$$

$$E^{(R)}[\hat{Y}_{S,t+h}|\Omega_{t-1}] = \alpha^{h+1}\hat{Y}_{S,t-1} \quad (19)$$

To see how the above forecasting structure fares with the three criteria listed above, note the following. First, especially for α close or equal to ρ, the forecasting model in equation (18) is not only "simple, reasonable, and consistent with their ability to solve their dynamic optimization problem," but also, for $\alpha = \rho$, perfectly rational. Second, turning to equation (19), it is obvious that it corresponds to a less-than-perfect approximation to the true process governing the evolution of $\hat{Y}_{S,t}$. However, despite its simplicity, it captures a well-known feature of traditional RBC models: the fact that output persistence is tightly linked to the degree of serial correlation in the productivity shock series (King and Plosser, 1988). Thus, rather than taking the time and resources to compute a fully model-consistent forecast for $\hat{Y}_{S,t}$, a practice that would considerably complicate the solution to the model, type R agents use the simple rule given by (19). Finally, note that the ability of type R agents to solve their dynamic optimization problem with the same of level of sophistication as type S agents is not inconsistent with the relatively unsophisticated methods they use in solving their inference problem. The solution to the dynamic optimization problem requires structural information only about one's *own* constraints and opportunities; by assumption, both types of agents use this information. However, to solve their inference problem in a way consistent with the rational expectations hypothesis, individuals also need complete structural information on the constraints and opportunities facing the *other* agents in the economy; by assumption, type S agents have this information, type R agents do not. Thus, agents solve their dynamic optimization and inference problems in a way that is fully consistent with the information assumed to be in their information sets.

4.3 Hierarchical Forecasting Structure

In several aspects, the expectational assumptions made so far are similar to Townsend's (1983) description of a hierarchical informational structure. Sophisticated forecasters are placed higher in the hierarchy; in addition to the information that enables them to solve their dynamic optimization problem, they also know the precise nature of the dynamic optimization *and* inference problems being solved by the rule-of-thumb agents. Thus, their forecasts incorporate *structural* information about the *whole* economy. In contrast, the structural information embodied in the forecasting behavior of the rule-of-thumb forecasters is *self-contained*; they use information about their *own* constraints and opportunity sets, but not those of type S agents.[13]

5 Decision Rules under Heterogeneous Expectations

Given the hierarchical information structure, the model can be solved sequentially in two steps. Starting at the bottom of the forecast hierarchy, I first derive the decision rules of type R agents and then use the results to compute the much more complicated decision rules of the sophisticated forecasters.[14]

5.1 Rule-of-Thumb Agents

The factor-allocation decision rules of type R agents, can be written as

$$z_{R,t} = \pi_{R,k}\hat{K}_{R,t-1} + \pi_{R,A}A_{t-1} + \pi_{R,Y}\hat{Y}_{S,t-1} \tag{20}$$

which is obtained by replacing the expectations formulae—equations (18) and (19)—into the perfect-foresight equilibrium law of motion of $z_{R,t}$—equation (16).[15] Given the factor-allocation decisions, production and consumption take place.

[13] In the illustrative case analyzed by Townsend, all agents use the same information about the structure of the economy. However, they have different access to the data; certain agents only had observations on their own local markets, while others observed a larger dataset. The difference between Townsend's assumptions and mine is relatively straightforward. While Townsend focused on agents whose forecasts were conditioned on different subsets of the data, I examine agents whose expectations are more or less structurally based depending on whether or not they form sophisticated forecasts.

[14] Note that this sequential scheme is valid for any hierarchical forecasting structure, regardless of the specific forecasting rules used by type R agents. Also note that a hierarchical forecasting structure eliminates concerns about the "infinite regress problem" (Townsend, 1983).

[15] The coefficients of equation (20) are defined as follows:

$$\pi_{R,A} \equiv \pi_{R,x}/(1-\mu_b^{-1}\alpha)(F_{R,1}^{(1)}\alpha + F_{R,2}^{(1)})\alpha + \pi_{R,e}^{(1)}\alpha$$
$$\pi_{R,Y} \equiv \pi_{R,x}/(1-\mu_b^{-1}\alpha)(F_{R,1}^{(2)}\alpha + F_{R,2}^{(2)})\alpha + \pi_{R,e}^{(2)}\alpha$$

where $F_{R,1}^{(m)}$ denotes the m^{th} element of $F_{R,1}$, and all other parameters come directly from (16).

5.2 Sophisticated Agents

Like the rule-of-thumb forecasters, type S agents make their labor, investment, and consumption decisions in two states and subject to the same information sets, Ω_{t-1} and $\Omega_{0,t}$. Thus, their factor-allocation decisions are made before they can observe either the output decision of the rule-of-thumb agents or the current state of productivity. However, to form expectations about $Y_{R,t}$, type S agents look not only at their own dynamic optimization problem, but also at the forecasting and decision rules adopted by type R agents. This information is subsumed in equations (14), (15), (18) through (20), and (3), which can be grouped together to form a two-sided matrix difference equation,

$$H_{-1}X_{t+1} + H_0 X_t + H_1 X_{t-1} = \epsilon_t \tag{21}$$

where the H_i are square coefficient matrices, $X_t \equiv \left[\begin{array}{cccccc} x'_{S,t} & \tilde{\lambda}_{S,t} & x'_{R,t} & \hat{Y}_{S,t} & \hat{Y}_{R,t} & A_t \end{array} \right]'$ and ϵ_t is a vector of zeros everywhere, except for the row corresponding to A_t, which contains a_t.

The first-stage inference problem of type S agents entails solving (21) for X_t and computing expectations subject to Ω_{t-1}. To solve this problem I use the generalized saddle-path algorithm described in Anderson and Moore (1985). The algorithm maps equation (21) into its stable VAR representation:[16]

$$X_t = \Gamma\ X_{t-1} + S\ \epsilon_t \tag{23}$$

where Γ and S are functions of the H_i matrices.

Given (23), the j-step ahead forecast of $Y_{R,t}$, made at time t based on time $t-1$ information, is

$$E^{(S)}[\hat{Y}_{R,t+j}|\Omega_{t-1}] = \imath^* \Gamma^{j+1} X_{t-1} \tag{24}$$

where $\imath^*$ is the vector that selects the row of $\Gamma^{j+1}X_{t-1}$ that corresponds to $\hat{Y}_{R,t}$. By plugging the above expression—along with the corresponding prediction formula for the productivity shock ($\rho^{j+1}A_{t-1}$)—into (16), I obtain the labor supply and capital accumulation decision rules of type S agents. Given these decisions, production takes place; $\hat{Y}_{S,t}$, $\hat{Y}_{R,t}$, and A_t become observable; and the consumption decision is implemented.

[16]The first step in the Anderson-Moore algorithm is to find a transformation of (21) such that a companion matrix representation exists:

$$V_{t+1} = G\ V_t \tag{22}$$

with V_t defined as $[X'_{t-1},\ X'_t]'$ The stability conditions of the transformed system are combined with the original "untransformed" system to generate equation (23).

5.3 Aggregation

The previous subsections described how different types of agents go about solving their respective utility maximization problems. Our ultimate interest, however, lies on the analysis of the dynamics of the economy as a whole. As it turns out, the evolution of aggregate variables can be easily derived from the individual decision rules computed above—see equation (1). Moreover, it can also be shown that movements in these variables obey a state-space form that has the following general representation:

$$P_t = A\,P_{t-1} + Bv_t \tag{25}$$

$$Z_t = Q\,P_t \tag{26}$$

where P_t and Z_t are vectors of individual and aggregate variables, respectively, and A, B, and Q are appropriately defined matrices.[17]

6 Quantitative Business-Cycle Analysis

The main question asked in this paper is whether the introduction of bounded rationality affects the dynamic properties of a real-business-cycle economy. In particular, given the forecasting rule of type R agents, my goal is to gauge how the θ_R and α parameters affect the cyclical properties of Z_t. To answer this question I run what Kydland and Prescott (1996) call a *computational experiment*—see also King (1995). The previous sections implemented the initial steps in the experiment: (i) posing the question that the experiment will address; (ii) constructing the theoretical model where the analysis will be carried out, and (iii) solving the model to compute its equilibrium properties. The final steps involve calibrating the model to allow for meaningful quantitative analysis and running the experiment itself.

6.1 Model Calibration

With the exception of the forecast-heterogeneity and complementarity parameters, $[\theta_R, \alpha, \phi]$, all model parameters are calibrated as in King et al. (1988). The first panel of table 1 shows this basic parameterization. The parameters shown in the second panel are discussed below.

Strategic complementarity parameter. The calibration of the strategic complementarity parameter (ϕ) is

[17] Equation (25) contains the expressions describing the decision rules of both types of agents. For instance, the rows of A and B that correspond to $z_{R,t}$ are set according to equation (20). Equation (26) maps individual variables into aggregate ones using expressions similar to equation (1), expressed in percentage deviations from steady-state values.

guided by the empirical work of Baxter and King (1991), Caballero and Lyons (1992), and Cooper and Haltiwanger (1993).

Baxter and King (1991) estimated ϕ using aggregate data by running instrumental-variable regressions of output growth on input growth. In principle, their estimation approach is very straightforward: choose an appropriate instrument set and run the usual first- and second-stage regressions. In practice, however, Baxter and King expressed concern about the lack of precision of their estimation results: of the three instrument sets they experimented with, neither generated a first-stage R^2 higher than 0.08, and the resulting estimated values of ϕ ranged from 0.1 to 0.45. For their final simulation results, however, Baxter and King set ϕ at 0.23, about the mid-point of their range of estimates and in line with the early estimates obtained by Caballero and Lyons (1989). However, based on the more recent work of Caballero and Lyons (1992) and Cooper and Haltiwanger (1993), there is some reason to believe that this number might be higher, perhaps even above the upper end of the Baxter-King estimates.

Caballero and Lyons (1992) found evidence of a strong reduced-form relationship between disaggregated (two-digit) productivity and aggregate activity. They offered two possible explanations for this finding. First, they posited that the estimated external effects are simply the result of true externalities, an assumption that is consistent with the model discussed by Baxter and King and in this paper (equation (2)). Based on the true-externalities specification, Caballero and Lyons estimated ϕ to be in the 0.32-to-0.49 range, depending on the set of instruments and on how energy-price effects are modeled. Their second explanation for the estimated external effects allows for the possibility that unobservable variations in effort, and not just true externalities, might also be behind the measured "external effects." If confirmed, this second explanation would imply that the estimates obtained under the true-externalities specification are potentially biased upwards. Here their results are mixed: though they did find some evidence for unobserved effort variation, the coefficient on one of their effort proxies, while statistically significant, actually came in with the wrong sign across all of their estimated equations.

Using industry and aggregate manufacturing data, Cooper and Haltiwanger (1993) reported even larger values for their own estimates of ϕ. Moreover, their estimates remained large even after accounting for potential biases associated with measurement error. In fact, the Cooper-Haltiwanger estimates for certain manufacturing sectors were so large that they would violate the saddle-path stability conditions of the model presented in this paper. Therefore, rather than using any of their estimates, I take the Cooper-Haltiwanger results as evidence favoring the higher range of estimates obtained by Caballero and Lyons (1992).

An obvious conclusion of this brief overview of the empirical strategic complementarity literature is that it is hard to pin down with confidence what the true value of ϕ actually is. For the purposes of this paper, rather than defending any particular parameterization of ϕ, I run my experiments over the full of estimates reported above. Thus, my range of values will include Baxter and King's (1991) Caballero and Lyons's (1992) lower bonds—0.10 and 0.32, respectively—as well as their estimated upper bounds, 0.45 and 0.49. My goal is to trace the consequences of these different estimated values of ϕ for the aggregate implications of expectational heterogeneity.

Serial correlation and volatility of technology shocks. Two parameters that do not affect the steady-state properties of the model, but play a crucial role in aggregate fluctuations, are the innovation variance and the autoregressive coefficient of $\hat{A}_t$, (σ_a^2 and ρ). To calibrate σ_a^2, I simply set it to a value that makes the model's output variance equal to its empirical counterpart—this value is shown in the second panel of table 1.[18] Nevertheless, it is important to note that, as long as my focus rests on the propagation mechanism, my results are invariant to the particular parameterization of σ_a^2.

The parameterization of ρ is designed to highlight a recurring weakness of the standard RBC model with homogeneous expectations: the lack of a quantitatively important internal propagation mechanism. As is well known, in order to be able to mimic the degree of serial correlation in the data, most RBC models require near-unit root processes for A_t, effectively implying that persistence is exogenously imposed on the system, rather than explained by it. To isolate the role of expectational heterogeneity in the persistent generation process, I start by setting ρ at 0.5, about half the usual parameterization adopted in other RBC models—later in this paper I will experiment of other values of ρ.[19]

Expectational parameters. If there were some precision concerns surrounding the available estimates of the strategic complementarity parameter, we are hard pressed to find *any* estimates, however imprecise, for the expectational parameters of the model (θ_R and α).[20] Thus, rather than trying to defend any particular values for these parameters, I will treat θ_R and α as semi-free parameters and experiment with a wide range of values for each of them.[21] Therefore, the focus of my analysis is not to determine just how much and what type of rule-of-thumb forecasting is out there. My emphasis is on assessing the quantitative implications of different degrees of rule-of-thumb forecasting. Accordingly, I view my results as a mapping from the magnitudes of the ϕ, θ_R and α parameters to the properties of the artificial time series generated by the

[18] It is customary to assess the validity of RBC models according to its ability to capture the observed amplitude of aggregate fluctuations (King, 1995 Kydland and Prescott, 1991). Obviously, this is not the approach taken in this paper since, by construction, I obtain a ratio of model and empirical output variances that is equal to one.

[19] Prescott (1986) and Plosser (1989) have argued that the near-unit root assumption on the technological shock process is justified by estimated autocorrelations of Solow residuals. However, the assumption that Solow residuals actually capture shifts in technology is not universally accepted (see, e.g., Hall, 1987).

[20] A natural strategy to parameterize α is to set it equal to ρ, an approach I take in several of the cases analyzed below.

[21] Kydland and Prescott (1982) adopted a similar approach to deal with the difficulty in obtaining estimates of the time non-separability parameter of the utility function (King, 1995).

model. The rest of this section provides a sensitivity analysis that reflects this mapping.[22]

6.2 Expectational Heterogeneity and the Propagation Mechanism

In analyzing the aggregate implications of heterogeneous expectations, I primarily focus on how the existence of less sophisticated forecasters affects the propagation mechanism of the economy. Towards the end of this section I discuss the implications of forecast heterogeneity for other selected moments of the data.

Model without external returns. I start by examining an economy without strategic complementarity ($\phi = 0$). Figure 1 shows the autocorrelation function of aggregate output under alternative values of the expectational parameters. In particular, the figure summarizes the results of a computational experiment that explores different degrees of type R agents' misperception about the persistence of technological shocks—for $\rho = 0.5$, α is allowed to vary from 0.01 to 0.95—and for different shares of type R agents in the total population—θ_R varies from 0 to 0.9.

Two main results are evident in figure 1. First, except for the case where the beliefs of type R agents are wildly at odds with reality ($\alpha = .01$), their impact on the autocorrelation function of output is very small, even if we allow these agents to make up the vast majority of the population (θ_R = 0.9). Second, though quantitatively small, the particular expectational model used by type R agents has a noteworthy property: these agents' misperceptions about the persistence of the technological shock are reflected in the actual serial correlation pattern of output. Whenever they expect the shocks to be more [less] persistent than warranted by the data generating process, aggregate output ends up slightly more [less] persistent than otherwise.

On the whole, however, the inclusion of unsophisticated forecasters in the RBC model without complementarity produced quantitatively insignificant results. With ϕ set to zero, small deviations from the rational expectations assumption produce only small deviations from the standard RBC results, shown as the dotted lines in figure 1. Moreover, for the case where type R agents know the correct value of ρ—$\alpha = \rho$, not shown in figure 1—the behavior of aggregate output is virtually unaffected by the presence of type R agents.

Expectational Heterogeneity under Strategic Complementarity. Figure 2 summarizes the results of a computational experiment identical to the one described above, except that now I set the strategic complementarity parameter at 0.49, the upper end of the range of values discussed in the previous section. The plots in this figure stand in stark contrast to the ones in figure 1. According to figure 2, even if only a minority of agents forecasts according to the rule of thumb, the serial correlation properties of aggregate output may be affected in a quantitatively important way. For instance, as shown in the lower-left panel of figure 2, if the

[22] I am grateful to an anonymous referee for suggesting this course of inquiry.

rule-of-thumb agents over-estimate the persistence of the technological shock by only two decimal points (α = 0.70, ρ = 0.50), aggregate output becomes more persistent than in the case of purely rational expectations. Moreover, the impact of forecast-heterogeneity remains sizable even when the unsophisticated forecasters represent as little as 30 percent of the total population. Thus, unlike the case of no strategic complementarity, when external returns are high even small deviations from the rational expectations hypothesis can lead to sizable deviations from standard RBC results.

Some might question the different parameterizations of α in figure 2. In particular, given the simple data generating process for A_t, one might want to see the effects of eliminating type R agents' misperception about ρ. The results of this experiment are shown in figure 3, where I allow the rule-of-thumb forecasters to correctly infer the degree of serial correlation in the technology shock ($\alpha = \rho$). As shown, even without any misperceptions about the nature of the technological shock process, type R agents still have a significant effect on the serial correlation of aggregate output; deviations of output from its trend become more persistent than in the case with no rule-of-thumb forecasting (shown as the dotted line in figure 3).

Figures 2 and 3 highlight an important feature of the model. Note that, for given α and ϕ, the effect of rule-of-thumb forecasting on the persistence of aggregate output does *not* generally monotonically increases with θ_R. For instance, as shown in the two lower panels of figure 2, output actually becomes *less* persistent as the share of rule-of-thumb forecasters in the population *increases* from 0.60 to 0.90. This finding has important implications for the study of the macroeconomic effects of expectational heterogeneity. What it says is that persistence is not simply being exogenously generated as a result of the introduction of type R agents. Undoubtedly, even in isolation, these agents do affect the serial correlation properties of output (see figure 1); however, in addition to this exogenous factor, there is also an endogenous component of the type of expectations-induced propagation mechanism featured in this paper. As we saw before, strategic complementarity strengthens the informational links between the two types of agents, which in turn leads the sophisticated forecasters to respond to the perceived behavior of the rule-of-thumb forecasters. Accordingly, output persistence is affected not just by the ad hoc introduction of type R agents, but by the *interactions* among agents operating under different expectational assumptions. Now, as we allow the share of rule-of-thumb forecasters to increase further, we dampen the extent of interactions between the two types of agents—obviously, there less of the sophisticated forecasters to interact with. This explains why the relative effect of rule-of-thumb forecasting actually decreases at higher values of θ_R.

So far I have reported on the effects of rule-of-thumb forecasting under what might be called two polar assumptions about the magnitude of the strategic complementarity parameter: the zero-lower bound featured in most RBC models and 0.49, at the high end of the estimates discussed in the previous section. A question of interest is what happens at intermediate values of ϕ. Accordingly, figure 4 shows the autocorrelation function of aggregate output under four different parameterizations of strategic complementarity. These alternative values of ϕ, [0.10, 0.32, 0.45, 0.49], correspond to the range of estimates obtained by Baxter and King (1991) and Caballero and Lyons (1992), but do not include the higher estimates reported by Cooper and Haltiwanger (1993). As shown in this figure, the impact of the type of expectational heterogeneity examined in this paper is still quite sizable for $\phi = 0.45$, but the results are clearly not as dramatic for the two lower values of the external returns parameter $\phi = 0.10$ and $\phi = .32$. Figure 4 highlights the fact that more precise estimates of the actual degree of strategic complementarity are crucial for a more definitive assessment of the aggregate (quantitative) implications of forecast heterogeneity.

6.3 Model Evaluation

It is customary in the RBC literature to use the data to calibrate all parameters of the model and then compare its time series properties with selected moments of the data. This approach cannot be fully implemented in the model presented here because of the two semi-free parameters discussed above (θ_R and α) and the uncertainty surrounding the magnitude of the strategic complementarity parameter (ϕ).[23] Nevertheless, it would be useful to verify whether some plausible parameterization of the heterogeneous-expectation RBC model with strategic complementarity can make it roughly consistent with the data.

Table 2-A, extracted from King et al. (1988), summarizes the selected moments of the U.S. data that the model will try to match. The corresponding model moments are shown in table 2-B. The results reported in this table are obtained by assuming a relatively high degree of complementarity (ϕ = 0.49), while potentially allowing for only a limited role for bounded rationality (θ_R = 0.30). To highlight the internal propagation mechanism coming from heterogeneous expectations under strategic complementarity, I arbitrarily set the persistence parameter (ρ) at 0.70, lower than what a standard RBC model would require to capture the serial correlation observed in the data. Assuming no misperceptions from the part of type R agents ($\alpha = \rho$), the model replicates well the serial correlation of the data, especially for consumption and output. The observed relative volatilities of output, consumption, and investment are also largely captured by the model, though consumption and investment are a bit too volatile and hours do not vary as much as in the data.[24]

To compare the internal propagation of the model with the standard RBC framework, table 2-C shows the same selected moments shown in table 2-B, but now the complementarity and expectational parameters are all set to zero. As expected, without strong serial correlation in the shocks, the standard RBC model fails

[23] Though one could argue that α could be set to ρ, we are still left with very imprecise measures of ϕ and no measures at all for θ_R.

[24] Again, these results are only suggestive since the semi-free parameters can not be calibrated with an acceptable degree of confidence.

to capture the serial correlation that characterizes observed business cycles.

7 Interactions Between Sophisticated and Rule-of-Thumb Agents

In a statistical (mean-squared-error) sense, the forecasts formed by rule-of-thumb forecasters are less efficient than the model-consistent expectations of the sophisticated forecasters. When motivating this difference, I appealed to, among other things, the issues raised in the computation literature, whereby some agents might face a trade-off between forecast efficiency and computational costs. The goal of this section is to informally check whether this reliance on statistically suboptimal forecasts translates into significant behavioral differences between sophisticated and rule-of-thumb forecasters. If these differences are large, either the (unspecified) computational costs are large or the rule-of-thumb forecasters are inherently irrational. On the other hand, if the behavior of type R agents resembles that of type S agents, then even small computational costs could implicitly justify the existence of "rational" rule-of-thumb forecasting. In other words, small differences would suggest that the rule-of-thumb forecasters behave like near-rational agents (Akerlof and Yellen, 1985a, 1985b). The analysis performed in this section is *suggestive*, however, and only looks at the resulting behavior of each type of agent; neither the costs of becoming a sophisticated forecaster nor the potential utility loss from forecasting with rules-of-thumb are modeled explicitly.

The left-hand-side panel of figure 5 plots the simulated output paths for representative sophisticated and rule-of-thumb agents.[25] As shown in this panel, despite significant differences in the way they form expectations, sophisticated and rule-of-thumb forecasts behave in an almost identical manner. In particular, as sophisticated agents anticipate and react to the imperfections embedded in the forecasting schemes of type R agents, they effectively end up mimicking their actions. In the presence of complementarities, it pays to produce more [less] whenever aggregate output is higher [lower], even if the rise [decline] in output is largely due to the suboptimal forecasts of type R agents (and not warranted by true fundamentals). The left-hand-side panel of figure 5 shows that even in their investment decisions, which correspond to much more volatile series, the actions of sophisticated and rule-of-thumb agents are remarkably similar.

Taken together, the plots shown in figure 5 suggest that the potential utility losses from being a rule-of-thumb forecaster are likely small, implying that this course of action might well constitute a near-rational strategy. Moreover, this figure reinforces a notion introduced earlier in this paper. The aggregate effects of rule-of-thumb forecasters cannot be solely explained by simply looking at their actions in isolation; a much more interesting and important factor lies in the endogenous response that their actions elicit from the sophisticated forecasters. As noted in the previous section, it was primarily this endogenous response that

[25] The parameter settings are the same used in table 2-B ($\alpha = \rho = 0.70$, $\phi = 0.50$, $\theta_R = 0.30$).

accounted for the stronger persistence found in the heterogeneous expectations model.

8 Summary and Concluding Remarks

The paper introduced expectational heterogeneity in a dynamic general equilibrium model of strategic complementarity. I found strong quantitative effects at the aggregate level from allowing even a minority of agents to form expectations according to a sensible rule of thumb. Namely, when combined with strategic complementarity, forecast heterogeneity can strengthen the internal propagation mechanism of the model.[26] As sophisticated agents try to forecast the forecasts (and actions) of others, they effectively end up reinforcing the perceptions of less sophisticated forecasters, *even if these are not entirely consistent with the structure of the economy*. By definition, strategic complementarity raises the individual reward for producing more whenever aggregate output is higher. Intuitively, the representative sophisticated agent ultimately cares about aggregate state of the economy; this agent will produce more whenever it foresees gains in aggregate output, regardless of whether or not these gains are coming from the unsophisticated forecasts of the rule-of-thumb agents.

The above findings are quantitatively relevant only if the degree of strategic complementarity is sufficiently high (in the upper half of the range of estimates obtained in the empirical strategic complementarity literature).[27] Nevertheless, the sensitivity of the results to the strategic complementarity parameterization should not be a basis for concluding that expectational heterogeneity does not matter for macroeconomic analysis. First, there is still much uncertainty surrounding the empirical measures of the degree of complementarity: for instance, many of the (sectoral) estimates obtained by Cooper and Haltiwanger (1993) would place ϕ well above the range of values analyzed in this paper. Second, this paper can be interpreted as a relatively conservative approach to the issue of bounded rationality: after all, the rule-of-thumb forecasters depicted herein are still highly sophisticated individuals. In particular, they are smart enough to solve their dynamic optimization problem optimally and sufficiently well informed to know the exact nature of the stochastic process governing the evolution of the technology shocks and observe the contemporaneous actions of all the agents in the economy.[28] Given how much the type R agents are allowed to know, one might even be surprised as to the extent to which such a small limitation in their behavior mattered as much

[26] This confirms the qualitative findings of Haltiwanger and Waldman (1989) and Oh and Waldman (1994).

[27] For low degrees of strategic complementarity, forecast heterogeneity mattered only when the beliefs of rule-of-thumb forecasters are wildly at odds with the data generating process of the exogenous productivity shock, a case most economists would find less interesting.

[28] Krusell and Smith (1996) analyze an artificial economy where agents are allowed to adopt simple savings rules of thumb instead of basing their savings behavior on the solution of a standard dynamic optimization problems. They show that the aggregate time series properties of the rule-of-thumb economy are substantially different from those of a traditional artificial economy.

as it did for *any* degree of strategic complementarity.

In addition to analyzing the effects of heterogeneous expectations in the RBC framework, the model developed in this paper was designed to encompass aspects of two different approaches to analyzing macroeconomic models of expectational heterogeneity. The Lucas-Townsend approach is centered on dynamic, classical models of incomplete information: agents have only a limited access to the data and form rational expectations accordingly (Lucas, 1975; Townsend, 1983). The result is a rational expectations equilibrium with heterogeneous expectations. The Haltiwanger and Waldman (1989) approach was developed in the context of simple, static models of strategic complementarity: although potentially sharing an equal access to the data, agents differ in the extent to which their expectations are based on the structure of the model (model-consistent vs. rule-of-thumb). The result is a heterogeneous-expectations economy whose equilibrium characteristics are, at least in a qualitative sense, fundamentally different from a pure rational expectations economy. By bringing the issues raised by Haltiwanger and Waldman to bear onto a methodology and modeling environment that are closer in spirit to the work of Lucas and Townsend, I found that the differences in the results arrived by the two lines of inquiry are more a matter of degree than of substance. To be precise, whether or not the properties of the heterogeneous-expectations economy are consistent with those of a pure rational expectations economy is largely a function of the degree of strategic complementarity displayed by the economy. With strong enough complementarity, the findings of Haltiwanger and Waldman prevail; without it, they lack quantitative relevancy. In the end, the debate is likely to be settled empirically as we develop a better understanding of the nature and extent of strategic complementarity in the macroeconomy. For now, the available range of estimates of the strategic complementarity parameter is still too wide to allow us to quantify with precision the aggregate effects of expectational heterogeneity.

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Table 1 — Parameter Values and Definitions

Parameter	Definition
	A. Standard RBC Parameters[a]
$\theta_n = 0.58$	long-run share of labor income
$\delta = 0.025$	quarterly rate of depreciation
$\bar{N} = 0.20$	steady-state hours (proportion of time spent working)
$\beta = 0.988$	utility discount rate
ρ	AR(1) coefficient of technology shock[b]
σ_a^2	variance of technology innovation[c] (a_t)
	B. Complementarity and Expectational Parameters
ϕ	strategic complementarity parameter[d]
α	perceived value of ρ (type R agents)[e]
θ_R	proportion of type R (rule-of-thumb) agents in total population[e]

[a] Source: King, Plosser, and Rebelo (1988), unless otherwise noted.

[b] Set according to the experiment being run. See text and figures.

[c] Calibrated so that the model mimics the observed variance of output

[d] Calibrated according to the range of values estimated by Baxter and King (1991), Caballero and Lyons (1992), and Cooper and Haltiwanger (1993). (see text, tables, and figures)

[e] Semi-free parameter. A sensitivity analysis was run over a wide range of parameter values. (see text, tables, and figures)

Table 2 — Comparing Selected Moments[a]

Series	Std Dev	Rat. SD	auto(1)	auto(2)	auto(3)
A. U.S. Postwar Quarterly Data[b]					
Output	5.62	1.00	.96	.91	.85
Consumption	3.86	0.69	.98	.95	.93
Investment	7.61	1.35	.93	.78	.62
Hours	2.97	0.52	.94	.85	.74
B. Model with S.C. and Heterogeneous Expectations[c]					
Output	5.62	1.00	.97	.91	.86
Consumption	4.73	0.84	.96	.95	.95
Investment	10.96	1.95	.85	.68	.54
Hours	1.89	0.34	.81	.59	.42
Prdvty Shock	0.53	0.09	.70	.49	.34
C. Standard RBC Model[d]					
Output	5.62	1.00	.85	.63	.47
Consumption	4.46	0.79	.16	.21	.25
Investment	16.24	2.89	.67	.44	.29
Hours	3.19	0.57	.66	.43	.26
Prdvty Shock	3.85	0.69	.70	.49	.34

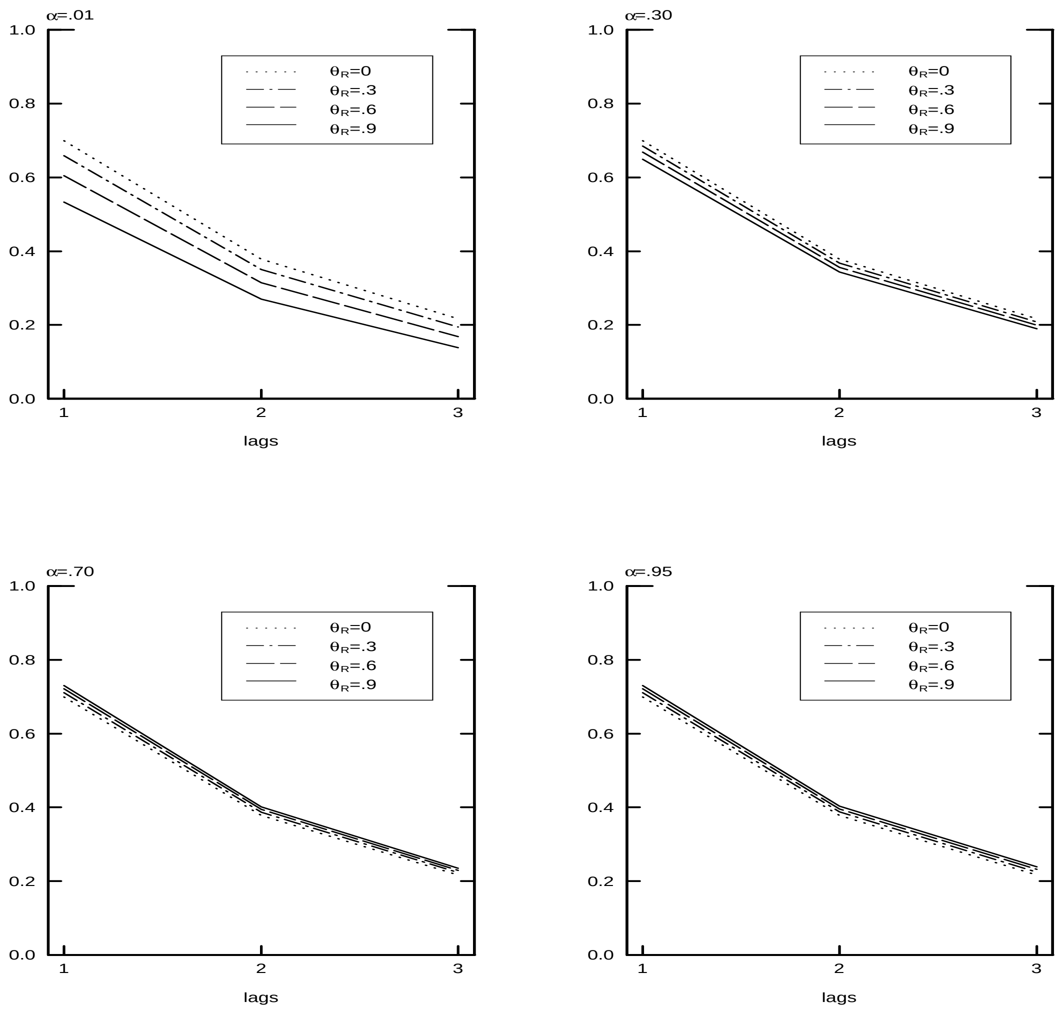
[a]The first column of numbers shows the standard deviation of each series; the second column shows ratios of standard deviations of each series with output. Columns 3 through 4 show first, second, and third autocorrelation coefficients.

[b]Source: King, Plosser, and Rebelo (1988).

[c]$\theta_R = 0.30$, $\phi = 0.50$, $\alpha = \rho = 0.70$. All other parameters calibrated as shown in Table 1, panel A.

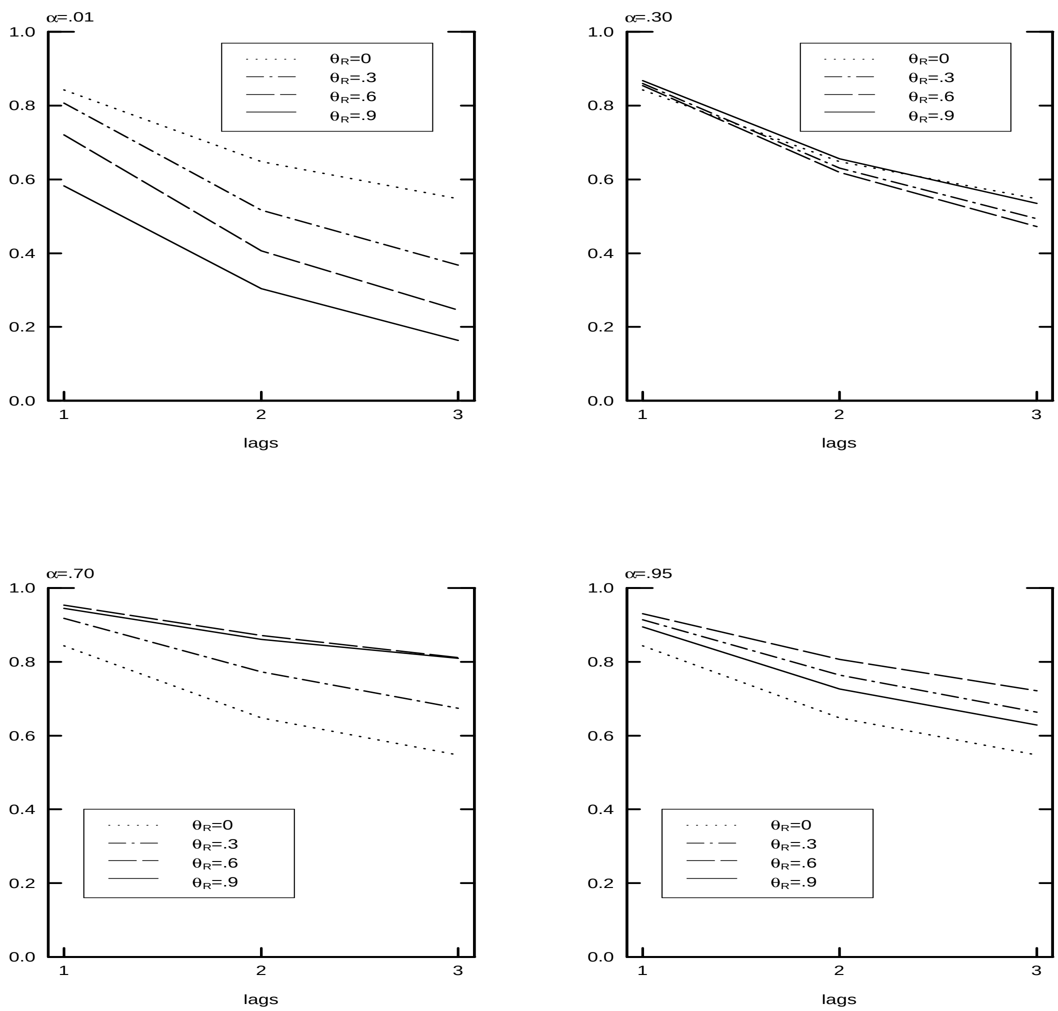
[d]$\theta_R = 0.00$, $\phi = 0.00$, $\rho = 0.70$. All other parameters calibrated as shown in Table 1, panel A.

Figure 1
Model without Strategic Complementarity ($\phi = 0$)
-Effect of alternative expectational assumptions on the autocorrelation of aggregate output*
($\rho = .50$)



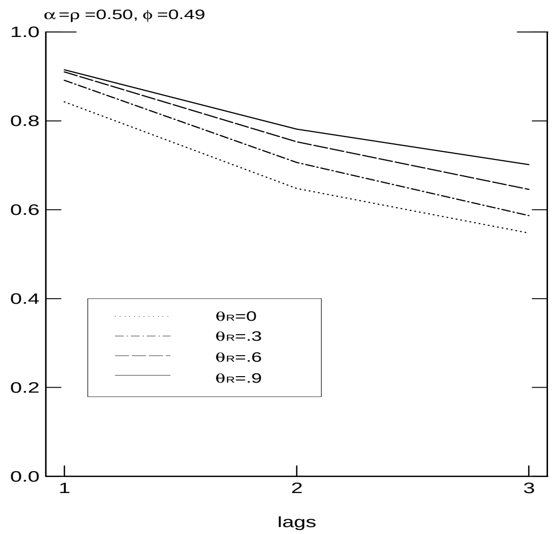
*** For different values of α and θ_R, the figures show the autocorrelation function of aggregate output. Each expectational assumption is defined by a [α,θ_R] pair. α is the perceived value of ρ (the AR(1) coefficient of the technology shock). θ_R is the proportion of rule-of-thumb forecasters in the population. All other parameters are set as in table 1.**

Figure 2
Model with Strategic Complementarity ($\phi = 0.49$)
-Effect of alternative expectational assumptions on the autocorrelation of aggregate output*
($\rho = 0.50$)



*** For different values of α and θ_R, the figures show the autocorrelation function of aggregate output. Each expectational assumption is defined by a [α,θ_R] pair. α is the perceived value of ρ (the AR(1) coefficient of the technology shock). θ_R is the proportion of rule-of-thumb forecasters in the population. All other parameters are set as in table 1.**

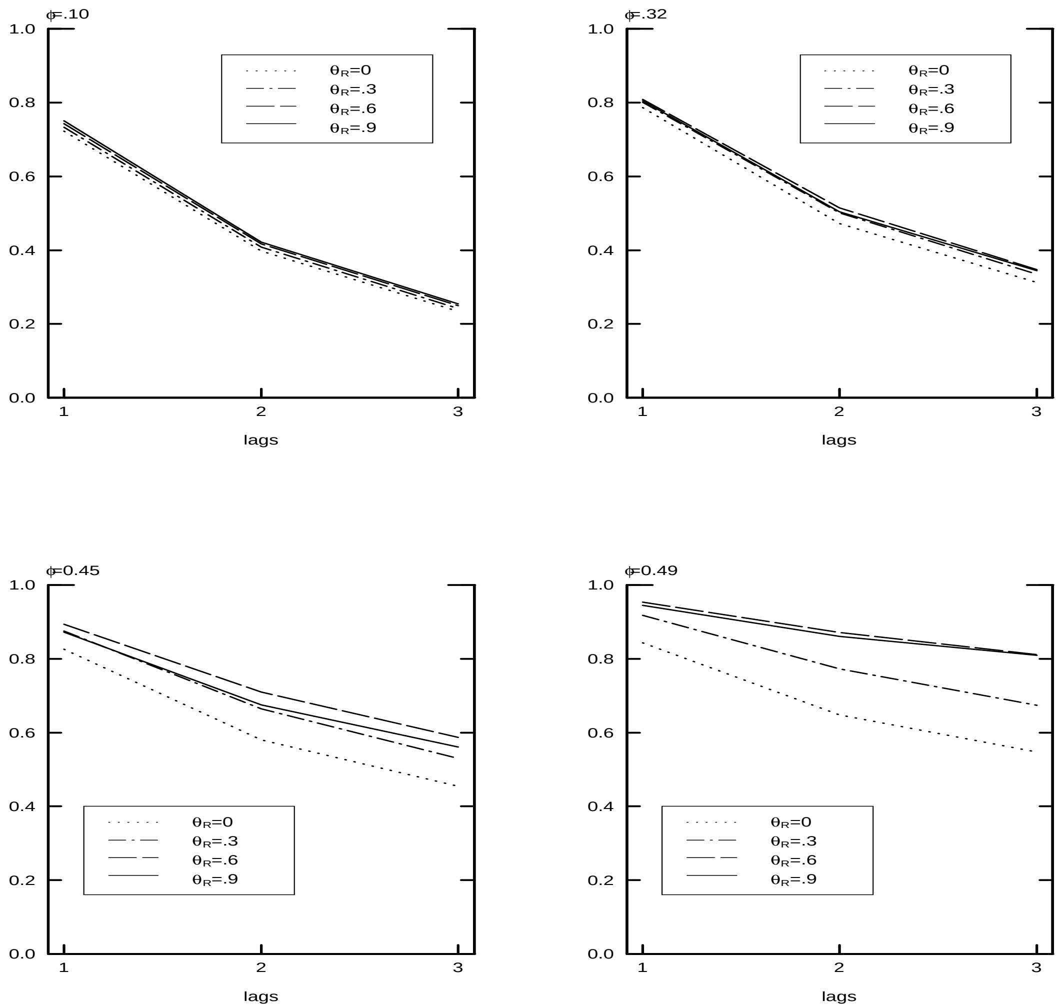
Figure 3
Model with Strategic Complementarity, but no misperceptions about the persistence of the technology shock*



*** For different values of θ_R, the figure shows the autocorrelation function of aggregate output.**

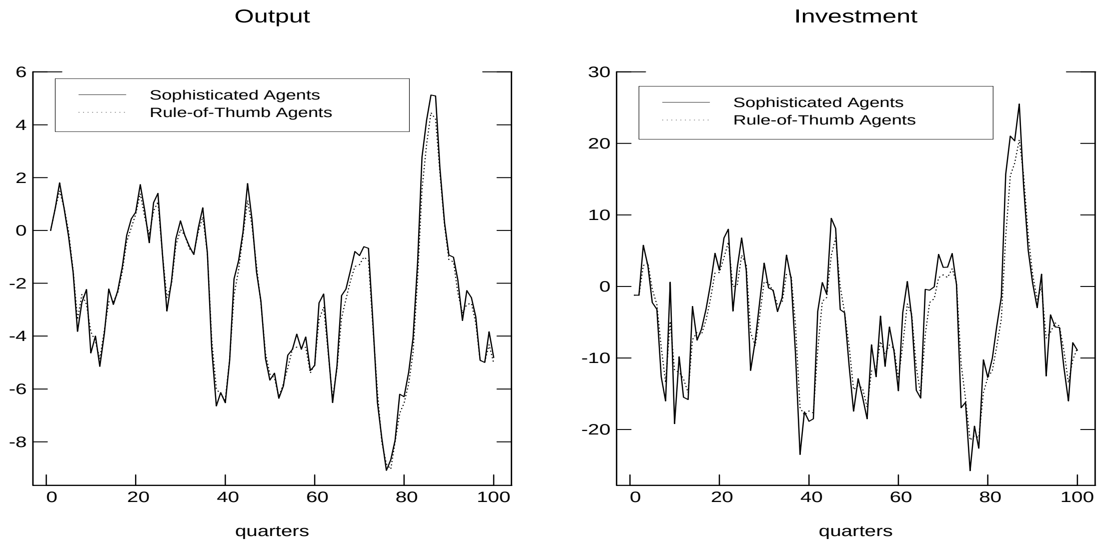
Parameters not mentioned here are set according to table 1, which also defines all parameters.

Figure 4
The aggregate effects of Expectational Heterogeneity*
($\alpha = 0.70$, $\rho = 0.50$)



*** The figure shows the autocorrelation function of aggregate output. Parameters not mentioned here are set according to table 1, which also defines all parameters.**

Figure 5
Comparing the behavior of Sophisticated and Rule-of-Thumb agents



*** The charts show the output and investment decisions of representative sophisticated and rule-of-thumb agents.**

The parameterization is the same described for table 2b. ($\alpha = \rho = 0.70$, $\phi = 0.49$, $\theta_R = 0.30$)