Forecasting Long- and Short-Horizon Stock Returns in a Unified Framework

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Abstract

If stock prices do not follow random walks, what processes do they follow? This question is important not only for forecasting purpose, but also for theoretical analyses and derivative pricing where a tractable model of the movement of underlying stock prices is needed. Although several models have been proposed to capture the predictability of stock returns, their empirical performances have not been evaluated. This paper evaluates some popular models using a Kalman Filter technique and finds that they have serious flaws. The paper then proposes an alternative parsimonious state-space model in which state variables characterize the stochastic movements of stock returns. Using equal-weighted CRSP monthly index, the paper shows that (1) this model fits the autocorrelations of returns well over both short and longer horizons and (2) although the forecasts obtained with the state-space model are based solely on past returns, they subsume the information in other potential predictor variables such as dividend yields.

Considerable evidence has shown that stock returns are predictable and stock prices do not follow random walks. If stock prices do not follow random walks, what processes do they follow? This question is important not only for forecasting purpose, but also for theoretical analyses where a tractable model of the movement of underlying stock price is needed. For example, to solve an optimization problem of dynamic portfolio selection and consumption decision (such as in Merton's ICAPM), we have to know the movement of stock prices; to price a option, we need to specify the stochastic process of the underlying stock price. Unfortunately, little progress has been made in this aspect. Some popular models, though proposed based on the observation of historical data, have never been tested empirically. For this reason, the academic analysis of predictability has not yet generated a profound impact on financial practice and the Random Walk assumption still dominates.

This article tries to fill this gap. It first evaluates the existing popular models using a standard data set: CRSP monthly returns. The evaluation results suggest that these models have serious shortcomings. Popular models are usually too restrictive to capture long and short horizon properties together. Specifically, models which capture the properties of long-horizon returns typically do not have prediction power for short-horizon returns and do not fit high frequency data; while models which can fit high frequency data generally do not capture the characteristics of long-horizon returns. This paper proposes a more flexible state-space model in which two state variables characterize the stochastic behavior of stock returns (prices). Although the model is highly parsimonious, it forecasts stock returns over both short and longer horizons surprisingly well.

The approach used in this paper is different from several others which have been popular in the literature for detecting and/or characterizing the predictability of asset returns. The first approach, which I will call the general regression approach, regresses asset returns on some plausible predictors such as dividend yields, interest rates, and some macroeconomic variables (Breen, Glosten and Jagannathan 1989, Culter *et al.* 1989, Fama and French 1988a, Fama and Schwert 1977, Ferson 1991, Keim and Stambaugh 1986, and Zhou 1995). This approach is typically used to predict asset returns for the next period but is not used to characterize the stochastic movements of expected returns (e.g. the relationship between today's expected returns and tomorrow's expected returns).

Auto-regression is another popular approach (Bekaert and Hodrick 1992, Campbell 1991, Campbell and Ammer 1993, Campbell and Hamao 1992, Fama and French 1988a, Kandel and Stambaugh 1988), including univariate regressions and vector auto-regressions (VARs). This approach regresses realized asset returns on their own lags, and possibly, some other predetermined variables.

One practical problem with this approach is that realized asset returns consist of at least two components: expected returns and unexpected innovations. The two components may have rather different relations with future returns, but the auto-regression approach does not distinguish between these components. For this reason, the predictions based on this approach may be sub-optimal or even conditionally biased. The next section discusses this in detail.

Another problem associated with this approach (and the general regression approach too) is the lack of a structural interpretation. For example, dividend yields help to predict stock returns according to this approach, but the approach does not reveal causality between these variables.

The third approach is what I call the direct testing approach (Lo and MacKinlay 1988 and Poterba and Summers 1988). This approach uses some kind of statistical measures like variance-ratio tests to test if stock prices follow "random walks." However, this approach does not tell us what processes prices follow if the "random walk" hypothesis is rejected.

The work of Conrad and Kaul (1988) is also relevant to this paper. Conrad and Kaul use a state-space approach to study time-variation of expected returns. They focus on the predictability of high frequency weekly returns and they use a less flexible model than proposed in this paper. As a result, their model does not fit the low frequency data and especially, does not generate the typical pattern of negative autocorrelations of stock returns over longer horizons (say one year). The next section will discuss their model in details.

The rest of this paper is organized as follows. The next section evaluates a few popular models of expected stock returns which have not been well examined empirically. Section 2 provides a new state-space model of stock returns to fit the return process over both long and short horizons. Section 3 contains concluding remarks.

1 Evaluating Existing Popular Models

This section evaluates several popular models empirically. For the sake of comparability with previous literature, I use a standard data set: the Center for Research in Securities Prices (CRSP) equal-weighted monthly returns in this study. The data set runs from 1926 to 1994, but I reserve the first year to construct the dividend-price ratios so that my sample period is from January 1927 to December 1994. All returns are transformed in logarithms and are multiplied by 100 to express them in percent per month. To obtain real returns, I deflate the nominal returns by the monthly Consumer Price Index reported by Ibbotson Associates.

Since the CRSP data set has been well documented, I will not discuss this data set in detail. For the sake of comparison, I just report some relevant summary statistics–the first-order auto-correlations of stock returns over various horizons in Table I. In the table, k represents the period length (in months), and $r_{t,t+k}$ denotes k-period compounded return (in logarithm) from time t to time $t+k$.

For all four series, the auto-correlations are typically positive over short periods and become negative over longer horizons. This is a very important stylized fact of U.S. stock market. Culter *et al.* (1988) calls it the 'characteristic autocorrelation function' of stock returns.

1.1 Model I

This subsection considers the simplest model for the time-variation in expected returns. That is, the expected returns follow a first-order auto-regressive process as described in the following state-space representation.

Model I:

$$r_t = \mu_{t-1} + \epsilon_t,$$

$$\mu_t = \phi\mu_{t-1} + \eta_t.$$

In the above expressions, r_t is realized log return from time $t-1$ to t, μ_{t-1} is the corresponding expected log return conditional on market participants' information at time $t-1$,[1] $\epsilon_t \sim$ i.i.d. $N(0, \sigma_\epsilon^2)$, $\eta_t \sim$ i.i.d. $N(0, \sigma_\eta^2)$, and $|\phi| < 1$.

ϵ_t and η_t may or may not be correlated with each other. A non-zero correlation means that an innovation in current stock price will affect the expectations of future stock returns. Empirically, we find very little evidence that ϵ_t and η_t are correlated. The likelihood ratio tests ($\chi^2(1) \ll 1$) suggest that the zero correlation between ϵ_t and η_t cannot be rejected even at a very high significance level (say 20%). For this reason, we will assume $\text{Corr}(\epsilon_t, \eta_t) = 0$ to simplify our investigations.

In this paper, we assume that economic agents know the true values of model parameters such as ϕ, σ_ϵ, and σ_η unless there are other explicit specifications.

Model I is attractive in several aspects. First, it has a very simple form. Second, like the random walk, AR processes have been well studied. Third, the model is very similar to some popular single-factor term structure model. (See Vasicek (1977)). If interest rates (expected returns to riskfree bonds) follow some kind auto-regressive process, it seems natural to model the expected returns of other assets in this way. As for the basic characteristics of model I, please see Conrad and Kaul (1988) and Campbell (1991) for discussions.

Model I was first used in the empirical study by Conrad and Kaul (1988). Conrad and Kaul (1988) focus on the predictability of stock returns at a very short horizon. Using weekly returns of 10 sized-based portfolio returns, they show that the assumption of constant expected returns is strongly rejected.

The purpose of this subsection, however, is rather comprehensive. Besides the performance of the model at short horizons, we are also interested in the performance of the model at longer horizons. The results of this subsection will provide a basis for the comparison

[1] We assume that μ_{t-1} is known by market participants but not by econometricians.

between model I and some other models.

The estimation results for Model I are reported in Table II. The estimates of auto-regression parameter ϕ lie between 0 and 1 and are significantly different from 0 and 1. The estimates capture the positive auto-correlations of stock returns at short horizons. Actually, the small estimates of the standard deviation σ_ϵ reported in the table suggest that expected returns μ_t are very close to realized returns r_t.

To assess the performance of Model I, and other models, we use the following criteria: (1) conditional forecast unbiasedness at various horizons, that is, an intercept close to zero and a slope coefficient close to one for the regression of returns over any time horizon on the corresponding forecasted returns; (2) high goodness-of-fit; (3) forecasts that fit the important properties of the observed data such as autocorrelations of stock returns; (4) high informational efficiency, that is, the forecasts incorporate important information available to investors such as dividend yields, past stock returns, and past model forecast errors.

Now we evaluate how well Model I meets these criteria.

First, let's test the relationship between *ex post* returns $r_{t,t+k}$ and the corresponding *ex ante* forecasts $E_t(r_{t,t+k})$ by a simple linear regression

$$r_{t,t+k} = b_0 + b_1 E_t(r_{t,t+k}) + u_{t,t+k}. \tag{1}$$

If $E_t(r_{t,t+k})$ is a conditionally unbiased forecast of $r_{t,t+k}$, we will have $b_0 = 0$ and $b_1 = 1$.

Table III reports the regression results of equation (1) over horizons k from as short as one month to as long as 10 years. For the longer-horizon returns, the observations in the data overlap. The standard errors in the table have been adjusted for this overlap using the method of Hanson and Hodrick (1980).

The most striking aspect of the Table is that Model I has reasonable one-period ahead prediction power for stock returns, but has no prediction power at all for stock returns over longer horizons (say over six months.) The R^2 of the one month returns is close to 3 percent.[2] The R^2's for longer-horizon returns, however, are almost zero. Moreover, the estimated

[2]This value is much smaller than the values obtained by Conrad and Kaul (1988) for weekly returns but is consistent with those values typically found in studies with monthly data.

slopes become considerably negative and forecast errors $u_{t,t+k}$ become significantly auto-correlated (the auto-correlations of forecast errors are not reported here to save space) for longer-horizon returns.

Table IV presents the implied auto-correlations for the model. Comparing the results in this table with those in Table I, we see that the model can reasonably match the first order auto-correlation of monthly returns, but is absolutely incapable of matching auto-correlations of long-horizon stock returns.

To test the information content of extracted expected returns, we regress the realized one-month returns on the corresponding extracted expected returns and predetermined dividend-price ratios simultaneously in the following model:

$$r_t = \beta_0 + \beta_1 E_{t-1}(r_t) + \beta_2 dp_t + u_t, \tag{2}$$

where dp_t are demeaned log dividend-price ratios. The dividend-price ratio for each date is defined as the total dividends paid over past three months over the current stock price and is extracted from the differences between cum-dividend returns and ex-dividend returns from CRSP monthly index. The choice of dividend yields here is mainly based on the findings in other work that dividend yields have considerable power to predict future stock returns (see, for example, Fama and French 1988 and Campbell 1991).

If Model I is a complete model of expected returns, then $\beta_1 = 1$ and $\beta_2 = 0$. The findings reported in Table V are again unfavorable to Model I. Different from what is found by Conrad and Kaul for weekly returns, the regressions here show that dividend-price ratios are still significantly related to stock returns even when the extracted expected returns are included as regressors.

To summarize, we find that Model I, a simple AR(1) model of expected returns, has reasonable capacity to capture the movements of stock returns over short-horizons but does a poor job in characterizing the movements of expected stock returns over longer-horizons. As a matter of fact, besides what are reported above, there is another serious problem with the model. Expected returns in Model I, according to Table II, are almost as volatile as realized returns and are very likely to become negative. Although some theoretical models such as Merton (1973), Breeden (1979), and Cox, Ingersoll, and Ross (1985) do not rule out negative expected returns, the over volatile expected returns and high probability of negative expected (nominal) returns and negative expected market equity premia are still hard to be accept theoretically.

1.2 Model II

In the above subsection, we have shown that Model I, a first-order auto-regressive model, does not match the properties of historical data. Now we turn to some other alternative models.

Model II assumes that log prices have two components: one permanent component plus one transitory component, as suggested by Fama and French (1988), Poterba and Summers (1988), and Summers (1986). Since the ability of the model to fit the actual data has not been documented in the literature, this subsection is going to evaluate the model empirically.

Let p_t denote the logarithm of detrended asset price at time t. This model can be written as

$$\begin{aligned} p_t &= q_t + z_t, \\ q_t &= q_{t-1} + \epsilon_t, \\ z_t &= \phi z_{t-1} + \eta_t, \end{aligned} \tag{3}$$

where $\epsilon_t \sim$ i.i.d.$N(0, \sigma_\epsilon^2)$, $\eta_t \sim$ i.i.d.$N(0, \sigma_\eta^2)$, and $|\phi| < 1$. ϵ and η are assumed to be mutually independent.

Ignoring any dividend yields, we can easily find that asset returns can be expressed in the following state-space form

Model II:

$$r_t \equiv \Delta p_t = \Delta z_t + \epsilon_t,$$

$$z_t = \phi z_{t-1} + \eta_t.$$

Model II looks very similar to Model I. The only difference is that in Model I, the expected return μ_t is specified as an AR(1) process, while in the current model, Δz_t follows an ARMA(1,1) process

$$\Delta z_t = \phi \Delta z_{t-1} + \eta_t - \eta_{t-1}. \tag{4}$$

.

Based on the model specification (Model II), we have

$$\begin{aligned} \rho_z(k) &\equiv \operatorname{Corr}(z_{t+k} - z_t, z_t - z_{t-k}) \\ &= \frac{2\phi^k - \phi^{2k} - 1}{2 - 2\phi^k}, \end{aligned} \tag{5}$$

$$\begin{aligned} \rho(k) &\equiv \operatorname{Corr}(r_{t,t+k}, r_{t-k,t}) = \operatorname{Corr}(p_{t+k} - p_t, p_t - p_{t-k}) \\ &= \rho_z(k) \cdot \frac{2(1-\phi^k)\sigma_\eta^2/(1-\phi^2)}{2(1-\phi^k)\sigma_\eta^2/(1-\phi^2) + k\sigma_\epsilon^2}. \end{aligned} \tag{6}$$

It is easy to show that $\rho_z(k)$ approaches -0.5 and $\rho(k)$ approaches $-1/(2+k\cdot s)$ for large k, where $s = (1-\phi^2)\sigma_\epsilon^2/\sigma_\eta^2$. If the stock price does not have a random-walk component, i.e., if $\sigma_\epsilon = 0$, $\rho(k)$ equals to $\rho_z(k)$ and approaches -0.5 eventually. Otherwise, $\rho(k)$ will diverge from $\rho_z(k)$ gradually and approach 0 for large k. This implies that if the random walk component is present, the autocorrelations of asset returns at long horizon will disappear gradually and $\rho(k)$ will have a typical U-shape. This fact has also been documented by others (see, for example, Fama and French 1988a).

We now use model II to characterize cum-dividend returns in this subsection, though it is originally derived for returns without allowing for dividends.[3] The parameter estimates of the model for various return series are reported in Table VI. We can find from the Table that ϕ for each equal-weighted return series is close to but less than one. The small standard deviations suggest that all ϕ's are precisely estimated and are significantly different from zero and one. The relative large standard deviation σ_η also tells us that the stationary components in stock prices are very important.

Table VII shows the regression results *of ex post* returns on the corresponding *ex ante* expected returns based on Model II. Comparing this Table with the related Table obtained from model I, we find that the new results are much better than those in Table III for

[3]The results for stock index with dividends and those for stock index excluding dividends are almost the same. The reason is that the variation in dividends is relatively small.

longer-horizon returns. The R^2 values for returns over 4 to 5 years can be as high as 30-40 percent. Table VIII shows that the model can well capture the long-horizon negative auto-correlations of stock returns.

For short period returns, however, Model II is not so appealing. The R^2's for one month returns are so small that we must exclude Model II as an effective model for characterizing return movements at short periods. Tables VIII and IX show the same problem of Model II for explaining predictability of short-horizon returns. The implied auto-correlations of short-horizon returns are slightly negative but the data suggest positive autocorrelation, the forecast errors for short-horizon returns are significantly and positively auto-correlated, and moreover, after including dividend yields dp as an explanatory variable, the expected returns $E_{t-1}(r_t)$ extracted from Model II become insignificant in explaining realized return variations.

In summary, Model II does a good job of forecasting long-horizon returns, but is not flexible enough to match the behavior of expected returns at short horizons.

2 A New State-space Model

To capture the positive auto-correlations of short-horizon returns and negative auto-correlations of long-horizon returns, this section extends Model II to a more general model, Model III. In Model III, the transitory component z_t of price not only depends on its own past value z_{t-1}, but also relies on another underlying state variable x_{t-1}. Formally, Model III can be written as:

$$\begin{aligned} p_t &= q_t + z_t, \\ q_t &= q_{t-1} + \epsilon_t, \\ z_t &= \phi z_{t-1} + \gamma x_{t-1} + \eta_t, \\ x_t &= \lambda x_{t-1} + \xi_t, \end{aligned} \tag{7}$$

where x_t is some kind of state variable, $\xi_t \sim$ i.i.d.$N(0, \sigma_\xi^2)$, and ϕ and λ are constants such that $|\phi| \leq 1$ and $|\lambda| < 1$. All other variables are the same as those in Model II. For simplicity, we assume that noise terms ξ, ϵ and η are mutually independent.

We can rewrite the above state-space model in terms of stock returns:

<u>Model III:</u>

$$r_t = z_t - z_{t-1} + \epsilon_t,$$

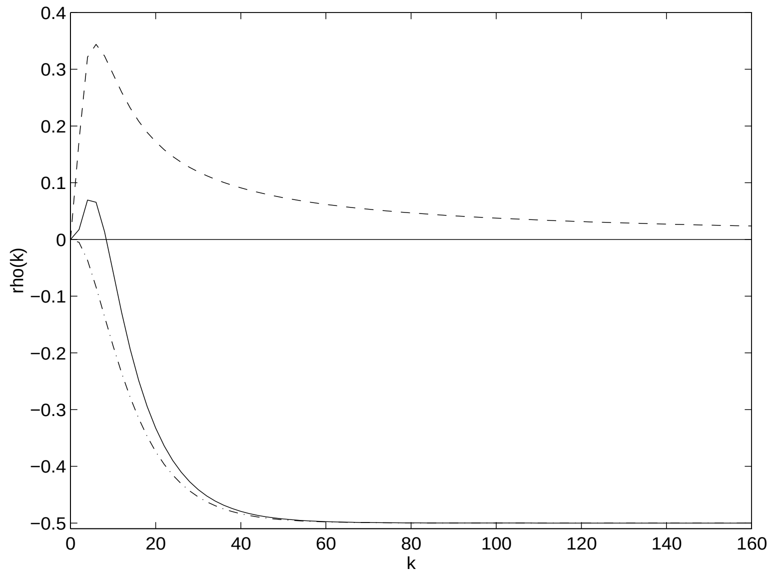
$$z_t = \phi z_{t-1} + \gamma x_{t-1} + \eta_t,$$

$$x_t = \lambda x_{t-1} + \xi_t.$$

At first glance, Model III is a simple variation of Model II. However, as we shall see below, the inclusion of the additional state variable x_t provides the model additional flexibility to fit the return process over both long and short horizons.

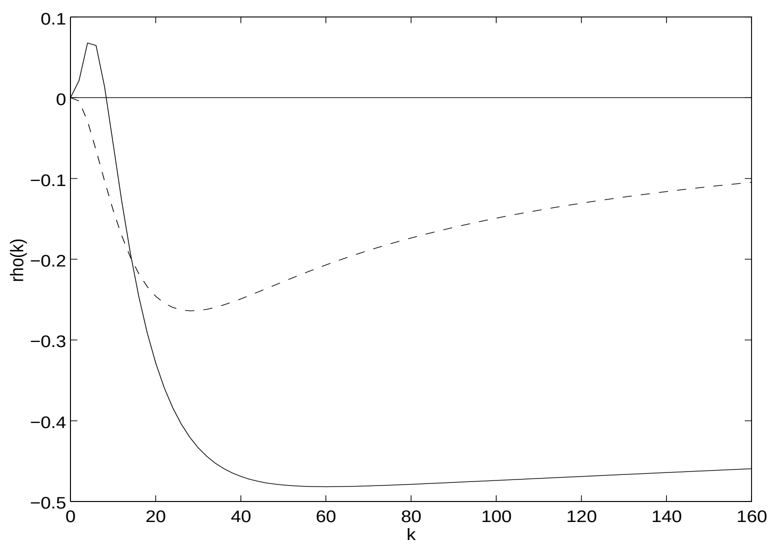
The intuition of Model III is quite straightforward. Similar to Model II, log stock price has two components: a random walk and a transitory part. Unlike Model II, the transitory part of stock price, z_t, is now no longer a univariate AR(1) process; it also depends on some state variable x, which can be interpreted in many ways, preferences, fashions, fads, economic states, etc. One-period stock returns now can be expressed as $r_{t+1} = -(1-\phi)z_t + \gamma x_t + \epsilon_{t+1} + \eta_{t+1}$. It is easy to see that at short horizon, x_t may play a very important role in characterizing the movements and the auto-correlations of stock returns, especially when ϕ is very close to 1. Since x_t is positively auto-correlated if $\lambda > 0$, it is not surprising that short horizon returns can be positively correlated. At long horizon, x_t become less important in affecting auto-correlations of stock returns and the properties of Model III are similar to those of Model II. Therefore, although Model III is parsimonious, it has very rich dynamics at both short and long horizons. Figures 1 and 2 illustrate several possible first-order auto-correlation curves implied by Model III. These figures show that Model III is able to capture a wide range of auto-correlation patterns. For a technical discussion of the properties of Model III, please see the Appendix.

If x_t is an unobservable state variable, it cannot be identified in state-space model III without a restriction on parameters. For this reason, we will normalize variable x_t so that $\sigma_\xi^2 = 1$.



—: $\phi = 0.9$, $\gamma = 5$; − −: $\phi = 1$, $\gamma = 1$; − · −: $\phi = 0.9$, $\gamma = 0$

Figure 1: **First-order Auto-correlations of k-period Returns Generated by Model III.** The figure plots autocorrelation curves $\rho(k) = \text{Corr}(r_{t,t+k}, r_{t-k,t})$ based on the following parameter values: $\sigma_\xi = 1$, $\sigma_\eta = 2$, $\sigma_\epsilon = 0$, $\lambda = 0.7$.



—:$\phi = 0.9$, $\gamma = 5$; − −: $\phi = .9$, $\gamma = 0$

Figure 2: **First-order Auto-correlations of k-period Returns Generated by Model III.** The figure plots auto-correlation curves $\rho(k) = \mathrm{Corr}(r_{t,t+k}, r_{t-k,t})$ based on the following parameter values: $\sigma_\xi = 1$, $\sigma_\eta = 2$, $\sigma_\epsilon = 1$, $\lambda = 0.7$.

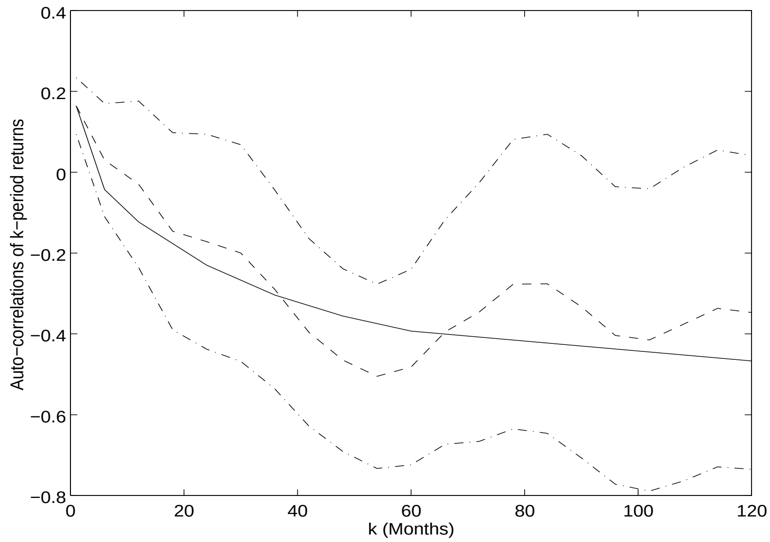
To evaluate Model III more carefully, we will apply the model not only to the data in full sample period 1927–1994, but also to the data in postwar period 1946–1994. Since we find that the results for the postwar period are very similar to those for the full sample period, in the following discussions, we are going to focus on the results for the full sample period.

Table X shows the estimated parameters. The large values of γ demonstrate that state variable x plays a very important role in characterizing stochastic movements in stock returns. As we know, Model III nests Model II with two more free parameters γ and λ. If we use likelihood ratio test, we will always strongly reject Model II with values of a $\chi^2(2)$ statistic larger than 25. (The test statistics are not reported in tables.)

The regression results of *ex post* returns on extracted expected returns are presented in Table XI. We can find that the R^2's for longer-horizon returns are similar to those in Table VII. However, the R^2's for short-horizon returns are typically larger than those from Models I and II. A four percent R^2 for one-month returns is much higher than the corresponding R^2's obtained from previous models and is also higher than the typical R^2 values obtained by VAR models with several variables. Table XI also shows that the constant intercepts are close to zero and that the slopes of expected returns in regression is very close to one. This is exactly what we expect from a good model. Actually, the slopes here are closer to 1.0 than slopes in any previous models. Their standard errors are also much smaller.

Table XII shows again that Model III is a good forecasting model. The estimated serial correlations of forecast errors $u_{t,t+k}$ are consistently very close to zero and are almost always smaller than the standard errors of the estimated auto-correlations.

Table XIII presents the implied autocorrelations of the model. The implied autocorrelations match the sample autocorrelations in Table I very well for both short-horizon returns and longer-horizon returns. To illustrate this point more clearly and more intuitively, we also present several plots here. Figures 3 and 4 exhibit implied auto-correlations and corresponding sample auto-correlations for both nominal and real return series. We see that implied auto-correlation curves and sample auto-correlation curves are very close to each



—: Autocorrelations Implied by Model; − −: Sample Autocorrelations;

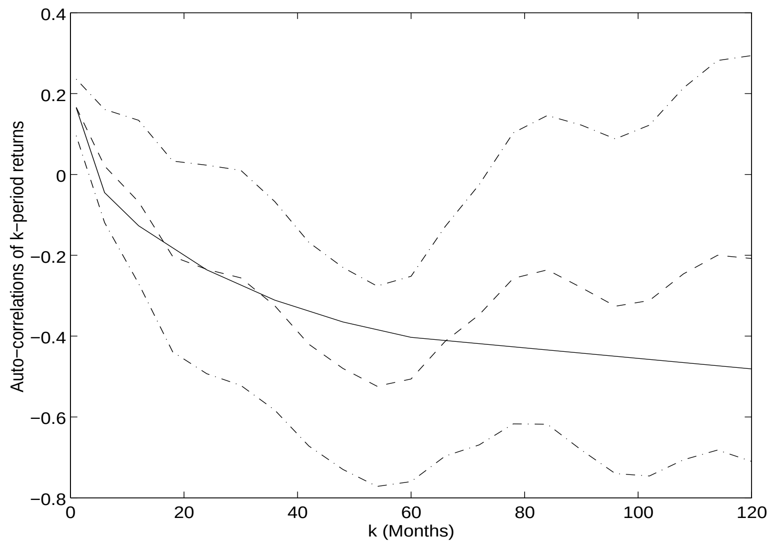
− · −·: 95% Confidence Interval of Sample Autocorrelations;

Figure 3: **Implied and Sample First-order Auto-correlations of k-period Stock Returns.** The figure plots autocorrelation curves $\rho(k) = \mathrm{Corr}(r_{t,t+k}, r_{t-k,t})$ based on Model III and CRSP Equal-Weighted Nominal Returns (1927–1994).

other and that implied auto-correlations always lie in the 95% confidence intervals of sample autocorrelations.

The information efficiency of extracted expected returns is reported in Table XIV. We see that the dividend yields almost fully lose their predicting power for returns after the extracted expected returns are included in regressors. This finding is exciting, but is not very surprising given the already high R^2's for one-month returns.

In summary, the results in this section suggest that Model III meets all four criteria for good models. This finding is striking: *with only the past returns, a parsimonious state space model characterizes the movements in returns at both short and longer-horizons excellently.*



—: Autocorrelations Implied by Model; – –: Sample Autocorrelations;

– · –·: 95% Confidence Interval of Sample Autocorrelations;

Figure 4: **Implied and Sample First-order Auto-correlations of k-period Stock Returns.** The figure plots autocorrelation curves $\rho(k) = \text{Corr}(r_{t,t+k}, r_{t-k,t})$ based on Model III and CRSP Equal-Weighted Real Returns (1927–1994).

3 Concluding Remarks

This paper compares various models for characterizing stochastic movements in asset returns and prices. It shows that the existing popular models typically have serious flaws. The paper then proposes a parsimonious model (Model III) with two state-variables to characterize the stochastic behavior of asset returns. Using equal-weighted CRSP monthly returns, we find this model outperforms other models.

The major findings of this paper are (1) the time-variation of expected returns can be well characterized by the structural state-space model III, which captures the autocorrelations of returns excellently over both short and long horizons; (2) although the forecasts obtained with state-space model III based solely on past returns, they subsume the information in other potential predictor variables such as dividend yields; (3) the state-space model III not only predicts the short-horizon returns quite well, but also predicts longer-horizon returns very successfully; and (4) the extracted expected returns can explain a substantial proportion of the variation in realized returns. At a horizon of 2 to 3 years, this proportion reaches about 20 to 25 percent; at a horizon of 8 years, the proportion can be as high as 60 percent. The paper also shows that Model III, a parsimonious model in which two state variables characterize the stochastic behavior of stock returns outperforms other models such as VAR, a first-order auto-regression model of expected returns, and the model in which the transitory component follows a simple AR(1) process.

Similar results can also be obtained for value-weighted returns. However, value-weighted returns are generally less predictable than equal-weighted returns.

The findings of this article may have interesting implications for Intertemporal CAPM, asset allocation, and even option pricing because they depend on expected returns and risks. A better understanding of the behavior of stock returns (and therefore related risks) at long and short horizons should help in our evaluating of these models. Future research is needed to explore these implications.

Appendix: A Technical Analysis of Model IV

The appendix discusses the properties of Model IV in Section 6 technically.

Model IV:

$$r_t = z_t - z_{t-1} + \epsilon_t,$$

$$z_t = \phi z_{t-1} + \gamma x_{t-1} + \eta_t,$$

$$x_t = \lambda x_{t-1} + \xi_t.$$

First, let's consider the case where $|\phi| < 1$. By definition, the correlation of $z_{t+k} - z_t$ and $z_t - z_{t-k}$, the first-order auto-correlation of k-period change in z_t, is

$$\rho_z(k) = \frac{\text{Cov}(z_{t+k} - z_t, z_t - z_{t-k})}{\text{Var}(z_{t+k} - z_t)}. \tag{8}$$

The numerator covariance is

$$\text{Cov}(z_{t+k} - z_t, z_t - z_{t-k}) = -\sigma_z^2 + 2\text{Cov}(z_{t+k}, z_t) - \text{Cov}(z_{t+2k}, z_t), \tag{9}$$

and the variance in the denominator is

$$\text{Var}(z_{t+k} - z_t) = 2\sigma_z^2 - 2\text{Cov}(z_{t+k}, z_t). \tag{10}$$

On the other hand, we can prove by recursion that

$$\text{Cov}(z_{t+k}, z_t) = \phi^k \sigma_z^2 + \gamma(\sum_{i=0}^{k-1} \lambda^i \phi^{k-1-i})\text{Cov}(z_t, x_t). \tag{11}$$

As a result,

$$\rho_z(k) = \frac{-\sigma_z^2 + 2\phi^k\sigma_z^2 - \phi^{2k}\sigma_z^2 + \gamma(2\sum_{i=0}^{k-1}\lambda^i\phi^{k-1-i} - \sum_{i=0}^{2k-1}\lambda^i\phi^{2k-1-i})\text{Cov}(z_t, x_t)}{2\sigma_z^2 - 2\gamma(\sum_{i=0}^{k-1}\lambda^i\phi^{k-1-i})\text{Cov}(z_t, x_t)} \tag{12}$$

where

$$\text{Cov}(z_t, x_t) = \frac{\lambda\gamma \cdot \sigma_x^2 + \text{Cov}(\xi_t, \eta_t)}{1 - \lambda\phi}. \tag{13}$$

It is now straightforward to verify that $\text{Cov}(z_{t+k}, z_t)$ approaches zero and $\text{Var}(z_{t+k} - z_t)$ approaches $2\sigma_z^2$ for large k whenever $|\phi| < 1$ and $|\lambda| < 1$. This means that $\rho_z(k)$ approaches -0.5 for large k as in Model III.

If the changes in the random walk and stationary components are independent, we have

$$\begin{aligned}\rho(k) &\equiv \mathrm{Corr}(r_{t,t+k}, r_{t-k,t}) = \mathrm{Corr}(p_{t+k} - p_t, p_t - p_{t-k}) \\ &= \rho_z(k)\frac{\mathrm{Var}(z_{t+k} - z_t)}{\mathrm{Var}(z_{t+k} - z_t) + \mathrm{Var}(q_{t+k} - q_t)} \\ &= \rho_z(k)\frac{2\sigma_z^2 - 2\mathrm{Cov}(z_{t+k}, z_t)}{2\sigma_z^2 - 2\mathrm{Cov}(z_{t+k}, z_t) + k\sigma_\epsilon^2}. \end{aligned} \tag{14}$$

The asymptotic property of $\rho(k)$ for large k in the current model is the same as that in Model III if $|\phi| < 1$ and $|\lambda| < 1$. If $\sigma_\epsilon = 0$, $\rho(k)$ is the same as $\rho_z(k)$ and approaches -0.5. Otherwise, since

$$\mathrm{Cov}(z_{t+k}, z_t) = \phi^k \sigma_z^2 + \gamma(\sum_{i=0}^{k-1} \lambda^i \phi^{k-1-i})\mathrm{Cov}(z_t, x_t)$$

approaches zero and $k\sigma_\epsilon^2$ goes to infinity as $k \to \infty$, the random walk component dominates for large k and $\rho(k)$ approaches 0.

If $\phi = 1$, we can prove that

$$\mathrm{Var}(z_{t+k}, z_t) = k(\sigma_\eta^2 + \gamma^2\sigma_x^2) + 2\gamma^2(\sum_{i=1}^{k-1}(k-i)\lambda^i)\sigma_x^2, \tag{15}$$

and that

$$\mathrm{Cov}(z_{t+k} - z_t, z_t - z_{t-k}) = \gamma \cdot \frac{(1-\lambda^k)^2}{1-\lambda} \cdot \mathrm{Cov}(z, x) > 0. \tag{16}$$

This ensures that $\rho_z(k) > 0$ and $\rho(k) > 0$ for any horizons k.

References

[1] Bekaert, G. and R.J. Hodrick(1992): "Characterizing Predictable Components in Excess Returns on Equity and Foreign Exchange Markets," *Journal of Finance* 47, 467-509.

[2] Breeden, D.(1979): "An intertemporal asset pricing model with stochastic consumption and investment opportunities," *Journal of Financial Economics* 7, 265-296.

[3] Breen, W., L.R. Glosten, and R. Jagannathan(1989): "Economic significance of predictable variations in stock index returns," *Journal of Finance* 44, 1177-1190.

[4] Campbell, J.Y.(1987): "Stock returns and the term structure," *Journal of Financial Economics* 18, 373-400.

[5] Campbell, J.Y.(1991): "A variance decomposition for stock returns," *Economic Journal* 101, 157-179.

[6] Campbell, J.Y. and J. Ammer(1993): "What moves the stock and bond market? A variance decomposition for long-term asset returns," *Journal of Finance* 48, 3-37.

[7] Campbell, J.Y. and Y. Hamao(1992): "Predictable stock returns in United States and Japan: A study of long-term capital market integration," *Journal of Finance* 47, 43-69.

[8] Campbell, J.Y. and R. Shiller(1988): "The dividend-price ratio and expectations of future dividends and discount factors," *The Review of Financial Studies* 1, 195-228.

[9] Cochrane, J.H.(1988): "How big is the random walk component of GNP?" *Journal of Political Economy* 96, 893-920.

[10] Cochrane, J.H.(1992): "Explaining the variance of price-dividend ratios," *The Review of Financial Studies* 5, 243-280.

[11] Conrad, J. and G. Kaul(1988): "Time-variation in expected returns," *Journal of Business* 61, 409-425.

[12] Cox, J., J. Ingersoll, and S. Ross(1985): "An intertemporal general equilibrium model of asset prices," *Econometrica* 53, 363-384.

[13] Culter, D.M., J.M. Poterba, and L.H. Summers(1989): "What moves stock prices?" *Journal of Portfolio Management* 15, 4-12.

[14] Fama, E.F. and K.R. French(1988a): "Permanent and temporary components of stock prices," *Journal of Political Economy* 96, 246-273.

[15] Fama, E.F. and K.R. French(1988b): "Dividend yields and expected stock returns," *Journal of Financial Economics* 22, 3-25.

[16] Fama, E.F. and G.W. Schwert(1977): "Asset returns and inflation," *Journal of Financial Economics* 5, 115-146.

[17] Ferson, W.E.(1989): "Changes in expected security returns," *Journal of Finance* 44, 1191-1217.

[18] Ferson, W.E. and R.A. Korajczyk(1995): "Do arbitrage pricing models explain the predictability of stock returns?" *Journal of Business* 68, 309-349.

[19] Hanson, L.P. and R.J. Hodrick(1980): "Forward exchange rates as optimal predictors of future spot rates: An econometric analysis," *Journal of Political Economy* 88, 829-853.

[20] Hanson, L.P. and K.J. Singleton(1983): "Stochastic consumption, risk aversion, and the temporary behavior of asset prices," *Journal of Political Economy* 96, 246-273.

[21] Jegadeesh, N.(1990): "Evidence of predictable behavior of stock returns," *Journal of Finance* 45, 881-898.

[22] Kandel, S. and R. Stambaugh(1988): "Modeling expected stock returns for long and short horizons," Rodney L. White Center Working Paper, No. 42-88, Wharton School, University of Pennsylvania.

[23] Keim, D.B. and R.B. Stambaugh(1986): "Predicting returns in the stock and bond markets," *Journal of Financial Economics* 17, 357-390.

[24] Lo, A.W. and A.C. MacKinlay(1988): "Stock market prices do not follow random walks: Evidence from a simple specification test," *The Review of Financial Studies* 1, 41-66.

[25] Merton, R.(1973): "An intertemporal capital asset pricing model," *Econometrica* 41, 867-888.

[26] Poterba, J.M. and L.H. Summers(1988): "Mean reversion in stock prices: Evidence and implications," *Journal of Financial Economics* 22, 27-59.

[27] Roll, R.(1988): "R^2," *Journal of Finance* 43, 541-66.

[28] Summers, L.H.(1986): "Does the stock market rationally reflect fundamental values?" *Journal of Finance* 41, 591-601.

[29] Vasicek, O.(1977): "An equilibrium characterization of the term structure," *Journal of Financial Economics* 5, 177-188.

Table I: **Summary Statistics of Equal-Weighted Monthly CRSP Returns.** The table reports the first-order auto-correlations of stock returns at various time horizons. The standard errors of auto-correlations are adjusted for the overlap of observations on longer-horizon returns with the method of Hanson and Hodrick (1980).

k (Months)	1	6	12	24	36	48	60	120
				Panel A: 1927–1994				
Nominal	0.164	0.030	-0.030	-0.172	-0.290	-0.465	-0.482	-0.347
(S.E.)	(.035)	(.070)	(.103)	(.133)	(.123)	(.113)	(.121)	(.194)
Real	0.166	0.021	-0.068	-0.235	-0.325	-0.480	-0.506	-0.208
(S.E.)	(.035)	(.070)	(.101)	(.129)	(.129)	(.125)	(.127)	(.251)
				Panel B: 1946–1994				
Nominal	0.166	-0.047	-0.100	-0.208	-0.053	-0.197	-0.411	-0.707
(S.E.)	(.041)	(.084)	(.116)	(.152)	(.190)	(.239)	(.240)	(.316)
Real	0.182	-0.019	-0.072	-0.219	-0.064	-0.141	-0.314	-0.463
(S.E.)	(.041)	(.084)	(.115)	(.150)	(.185)	(.238)	(.249)	(.299)

Table II: **Estimates of Model I** (Full Sample)

Model I

$$r_t = \mu_t + \epsilon_t$$

$$\mu_t = \phi\mu_{t-1} + \eta_t$$

	ϕ (SE)	σ_ϵ (SE)	σ_η (SE)
Nominal	.166	.728	7.27
	(.034)	(.030)	(.180)
Real	.168	.729	7.29
	(.034)	(030)	(.180)

Table III: **Estimates of Regressions of Realized k-period Returns on Corresponding Forecasted Returns:** $r_{t,t+k} = b_0 + b_1 E_t(r_{t,t+k}) + u_{t,t+k}$ (Full Sample).

Model I

k (Months)	1	6	12	24	36	48	60	120
				Panel A: Nominal Returns				
R^2	0.027	0.000	0.005	0.000	0.000	0.001	0.004	0.002
b_0	-0.000	0.019	0.044	-0.114	-0.458	-0.153	1.118	9.825
	(.256)	(1.67)	(3.44)	(6.30)	(8.57)	(9.23)	(8.36)	(9.83)
b_1	1.001	0.179	1.328	-0.433	-0.500	-1.228	-2.116	-1.364
	(.211)	(.516)	(.753)	(1.08)	(1.19)	(1.38)	(1.49)	(1.06)
				Panel B: Real Returns				
R^2	0.028	0.000	0.004	0.001	0.001	0.003	0.005	0.003
b_0	-0.000	0.000	-0.010	-0.299	-0.833	-0.810	-0.001	4.568
	(.257)	(1.67)	(3.38)	(5.88)	(7.76)	(8.34)	(7.84)	(12.8)
b_1	1.001	0.153	1.179	-0.801	-0.887	-1.667	-2.278	-1.651
	(.208)	(.507)	(.732)	(1.03)	(1.09)	(1.24)	(1.30)	(.900)

Table IV: **Implied Auto-correlations of Stock Returns.** The implied statistics are functions of the estimated parameters of the model (Full sample).

Model I

k (Months)	1	6	12	24	36	48	60	120
Nominal	0.164	0.030	0.015	0.007	0.005	0.004	0.003	0.001
Real	0.166	0.030	0.015	0.007	0.005	0.004	0.003	0.001

Table V: **Estimates of Regressions of Realized One-month Returns on Forecasted Returns and Dividend Yields:** $r_t = \beta_0 + \beta_1 E_{t-1}(r_t) + \beta_2 dp_t + u_t$ (Full Sample).

Model I

	β_0 (SE)	β_1 (SE)	β_2 (SE)	R^2
Nominal	-0.000	1.005	1.522	0.032
	(.256)	(.210)	(.731)	
Real	-0.002	1.003	1.616	0.033
	(.256)	(.208)	(.733)	

Table VI: **Estimates of Model II** (Full Sample)

Model II

$$r_t = \Delta z_t + \epsilon_t$$

$$z_t = \phi z_{t-1} + \eta_t$$

	ϕ (SE)	σ_ϵ (SE)	σ_η (SE)
Nominal	0.983	7.377	0.179
	(.007)	(.182)	(.033)
Real	0.982	7.392	0.100
	(.007)	(.280)	(19.52)

Table VII: **Estimates of Regressions of Realized k-period Returns on Corresponding Forecasted Returns:** $r_{t,t+k} = b_0 + b_1 E_t(r_{t,t+k}) + u_{t,t+k}$ (Full Sample).

Model II

k (Months)	1	6	12	24	36	48	60	120
Panel A: Nominal Returns								
R^2	0.005	0.042	0.086	0.166	0.219	0.285	0.345	0.393
b_0	0.137	1.021	2.065	3.628	4.354	5.801	7.713	13.43
	(.267)	(1.73)	(3.60)	(6.55)	(8.91)	(10.3)	(10.7)	(10.1)
b_1	1.059	1.173	1.360	1.506	1.480	1.543	1.521	1.061
	(.500)	(.478)	(.548)	(.546)	(.526)	(.487)	(.426)	(.294)
Panel B: Real Returns								
R^2	0.006	0.050	0.103	0.194	0.249	0.311	0.371	0.406
b_0	0.192	1.447	3.000	5.471	6.805	9.004	11.55	15.65
	(.273)	(1.76)	(3.59)	(6.25)	(8.27)	(9.51)	(10.0)	(12.5)
b_1	1.067	1.194	1.392	1.505	1.436	1.469	1.457	1.019
	(.466)	(.444)	(.501)	(.481)	(.452)	(.419)	(.378)	(.299)

Table VIII: **Implied Auto-correlations of Stock Returns.** The implied statistics are functions of the estimated parameters of the model (Full sample).

Model II

k (Months)	1	6	12	24	36	48	60	120
Nominal	-0.008	-0.048	-0.092	-0.167	-0.229	-0.279	-0.319	-0.435
Real	-0.009	-0.053	-0.100	-0.180	-0.244	-0.295	-0.336	-0.447

Table IX: **Estimates of Regressions of Realized One-month Returns on Forecasted Returns and Dividend Yields:** $r_t = \beta_0 + \beta_1 E_{t-1}(r_t) + \beta_2 dp_t + u_t$ (Full Sample).

Model II

	β_0 (SE)	β_1 (SE)	β_2 (SE)	R^2
Nominal	0.109	0.842	1.141	0.008
	(.268)	(.522)	(.772)	
Real	0.155	0.867	1.259	0.010
	(.274)	(.482)	(.766)	

Table X: **Estimates of Model III**

Model III

$$r_t = z_t - z_{t-1} + \epsilon_t$$

$$z_t = \phi z_{t-1} + \gamma x_{t-1} + \eta_t$$

$$x_t = \lambda x_{t-1} + \xi_t$$

	ϕ (SE)	γ (SE)	λ (SE)	σ_ϵ (SE)	σ_η (SE)
Panel A: 1927–1994					
Nominal	0.183	7.203	0.973	0.821	0.299
	(.050)	(.407)	(.011)	(1.53)	(2.00)
Real	0.973	7.181	0.186	0.017	1.145
	(.008)	(.176)	(.034)	(.022)	(.037)
Panel B: 1946–1994					
Nominal	0.181	5.145	0.974	0.017	0.010
	(1.20)	(.721)	(2.04)	(1.19)	(1.01)
Real	0.978	5.206	0.195	0.226	0.004
	(.009)	(.143)	(.040)	(.384)	(1.51)

Table XI: **Estimates of Regressions of Realized k-period Returns on Corresponding Forecasted Returns:** $r_{t,t+k} = b_0 + b_1 E_t(r_{t,t+k}) + u_{t,t+k}$.

Model III

k (Months)	1	6	12	24	36	48	60	120
			Panel A: Nominal Returns, 1927–1994					
R^2	0.036	0.042	0.092	0.167	0.221	0.288	0.348	0.370
b_0	0.152	0.923	1.962	3.360	4.046	5.492	7.415	13.32
	(.256)	(1.69)	(3.57)	(6.54)	(8.92)	(10.4)	(10.9)	(10.3)
b_1	1.010	0.787	0.969	1.087	1.114	1.211	1.234	0.930
	(.184)	(.313)	(.376)	(.392)	(.392)	(.377)	(.342)	(.267)
			Panel B: Real Returns, 1927–1994					
R^2	0.037	0.050	0.108	0.192	0.248	0.312	0.372	0.395
b_0	0.224	1.405	3.027	5.361	6.669	8.856	11.37	15.34
	(.258)	(1.72)	(3.56)	(6.23)	(8.26)	(9.52)	(10.1)	(12.6)
b_1	1.015	0.820	1.002	1.092	1.084	1.150	1.176	0.901
	(.180)	(.297)	(.351)	(.351)	(.342)	(.328)	(.305)	(.268)
			Panel C: Nominal Returns, 1946–1994					
R^2	0.038	0.059	0.103	0.182	0.244	0.331	0.455	0.612
b_0	0.095	0.636	1.377	3.016	4.391	6.105	7.782	9.665
	(.214)	(1.45)	(2.80)	(4.84)	(6.70)	(8.17)	(9.14)	(6.50)
b_1	1.018	0.915	0.962	1.008	1.021	1.110	1.340	1.525
	(.213)	(.363)	(.400)	(.393)	(.394)	(.388)	(.378)	(.344)
			Panel D: Real Returns, 1946–1994					
R^2	0.042	0.053	0.094	0.172	0.234	0.314	0.419	0.600
b_0	0.122	0.923	2.051	4.574	6.816	9.351	12.19	18.85
	(.217)	(1.54)	(3.02)	(5.18)	(7.07)	(8.72)	(9.98)	(10.8)
b_1	1.028	1.006	1.072	1.121	1.114	1.169	1.369	1.500
	(.041)	(.082)	(.116)	(.155)	(.189)	(.244)	(.276)	(.316)

Table XII: **The Auto-correlations of Forecast Errors.** The table presents estimates of first-order auto-regressions of forecast errors $u_{t,t+k}$: $u_{t,t+k} = \alpha_0 + \alpha_1 u_{t-k,t} + e_{t,t+k}$

Model III

k (Months)	1	6	12	24	36	48	60	120
				Panel A: Nominal Returns, 1927–1994				
α_0	0.000	-0.035	-0.266	0.336	4.849	7.232	7.284	7.863
	(.255)	(1.61)	(3.40)	(6.18)	(6.47)	(7.83)	(8.45)	(6.34)
α_1	-0.001	0.061	0.086	0.043	0.008	-0.036	0.009	-0.176
	(.035)	(.069)	(.104)	(.142)	(.132)	(.139)	(.151)	(.187)
				Panel B: Real Returns, 1927–1994				
α_0	-0.001	-0.049	-0.330	0.084	3.825	5.073	3.903	0.448
	(.256)	(1.61)	(3.35)	(5.98)	(7.03)	(8.92)	(10.1)	(14.4)
α_1	-0.002	0.053	0.054	-0.004	-0.026	-0.076	-0.023	0.041
	(.035)	(.069)	(.104)	(.140)	(.145)	(.163)	(.183)	(.295)
				Panel C: Nominal Returns, 1946–1994				
α_0	-0.013	0.189	0.621	1.515	1.729	1.381	1.271	-0.734
	(.213)	(1.41)	(2.69)	(4.74)	(6.63)	(8.72)	(10.3)	(8.03)
α_1	0.001	0.000	0.021	-0.011	0.117	0.038	-0.068	-0.172
	(.041)	(.083)	(.119)	(.159)	(.197)	(.249)	(.274)	(.346)
				Panel D: Real Returns, 1946–1994				
α_0	-0.014	0.273	0.848	1.926	2.004	1.731	1.753	-3.642
	(.215)	(1.44)	(2.76)	(4.91)	(6.79)	(9.02)	(11.0)	(9.75)
α_1	-0.002	0.016	0.038	-0.030	0.104	0.059	-0.037	-0.100
	(.041)	(.082)	(.116)	(.155)	(.189)	(.244)	(.276)	(.316)

Table XIII: **Implied Auto-correlations of Stock Returns.** The implied statistics are functions of the estimated parameters of the model.

Model III

k (Months)	1	6	12	24	36	48	60	120
				Panel A: 1927–1994				
Nominal	0.164	-0.043	-0.123	-0.230	-0.304	-0.356	-0.393	-0.467
Real	0.165	-0.045	-0.127	-0.236	-0.311	-0.365	-0.403	-0.481
				Panel B: 1946–1994				
Nominal	0.166	-0.041	-0.120	-0.226	-0.300	-0.354	-0.393	-0.477
Real	0.181	-0.027	-0.100	-0.197	-0.268	-0.322	-0.363	-0.463

Table XIV: **Estimates of Regressions of Realized One-month Returns on Forecasted Returns and Dividend Yields:** $r_t = \beta_0 + \beta_1 E_{t-1}(r_t) + \beta_2 dp_t + u_t$

Model III

	β_0 (SE)	β_1 (SE)	β_2 (SE)	R^2
		Panel A: 1927–1994		
Nominal	0.148	0.981	1.077	0.038
	(.257)	(.185)	(.733)	
Real	0.215	0.981	1.134	0.040
	(.259)	(.182)	(.735)	
		Panel B: 1946–1994		
Nominal	0.074	0.933	0.972	0.041
	(.214)	(.220)	(.670)	
Real	0.099	0.946	0.893	0.044
	(.217)	(.213)	(.685)	