THE DEVELOPMENT OF CONFIDENCE LIMITS FOR FATIGUE STRENGTH DATA*†

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ABSTRACT

Over the past several years, extensive databases have been developed for the S-N behavior of various materials used in wind turbine blades, primarily fiberglass composites. These data are typically presented both in their "raw" form and curve fit to define their average properties. For design, confidence limits must be placed on these descriptions. In particular, most designs call for the "95/95" design values; namely, with a 95 percent level of confidence, the designer is assured that 95 percent of the material will meet or exceed the design value. For such material properties as the ultimate strength, the procedures for estimating its value at a particular confidence level is well defined if the measured values follow a normal or a log-normal distribution. Namely, based upon the number of sample points and their standard deviation, a commonly-found table may be used to determine the survival percentage at a particular confidence level with respect to its mean value. The same is true for fatigue data at a constant stress level (the number of cycles to failure N at stress level S_1). However, when the stress level is allowed to vary, as with a typical S-N fatigue curve, the procedures for determining confidence limits are not as well defined. This paper outlines techniques for determining confidence limits of fatigue data. Different approaches to estimating the 95/95 level are compared. Data from the MSU/DOE and the FACT fatigue databases are used to illustrate typical results.

INTRODUCTION

The derivation of fatigue-life curves, commonly called S-N curves for the stress level S that produces failure at N cycles, is typically based on suites of test data that cover a wide range of stress levels. Typically, these data are then used with curve-fitting techniques to develop the "average" fatigue behavior of the material over an appropriate range of stress levels.

There is always scatter in the test data, indicating that some of the material has lower strength and some has higher strength than the average. In fact, there is usually a distribution of strengths underlying the scatter. Designers therefore cannot use the average behavior, because, by definition, approximately half of the material cannot meet or exceed the average strength. Thus, the designer must use a "design" level at which acceptably high percentages will not fail. This leads to the search for a so-called safe strength level, extracted from available test data, that designers can use with confidence. Thus, we use the term "confidence limit."

If the distribution of the test data about the average is known exactly, the desired safety level could be determined by simply picking the strength that corresponds to an acceptable probability of failure. However, while the test data help to determine the underlying distribution of strengths, they cannot define it perfectly because they are limited in extent. Thus, there is uncertainty about the distribution. To account for the imperfect knowledge of the true strength distribution, a confidence limit is developed for the data. This limit permits a conservative estimate of an acceptable probability of failure.

Confidence limits can be created in a number of ways, most of which are well documented in the literature. This paper attempts to help the wind turbine designer apply appropriate standard techniques to the specific problem of fatigue-life curves.

Preliminaries and Definitions

When dealing with a random variable, such as the static strength of a material, the design engineer typically uses a value for the strength that is "guaranteed" by the manufacturer. What the manufacturer is actually guaranteeing is the probability P that the fraction γ of all future tests of this material will exceed the

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guaranteed strength X^*. And, this statement is made with a confidence level of (1-α).[1] These two probabilities are usually described as follows: "with a (1-α) confidence level, we expect that at least γ of all future strength tests will exceed X^*." [‡]

This one-sided tolerance limit has been computed and tabulated for the normal and the log-normal[§] distributions by a number of authors, e.g., see Natrella.[2] Typically, these tabulations take the following form:

$$X^* = \bar{X} - c_{1-a\,,\,g}\; s_x \quad , \qquad [1]$$

where the sample average $\bar{X}$ is given by

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \quad . \qquad [2]$$

$c_{1-\alpha,\gamma}$ is a multiplier (factor) tabulated as a function of the confidence level (1-α), probability γ and the number of data points n. The standard deviation σ_x is given by:

$$s_x = \left[\frac{\sum_{i=1}^{n} \left(X_i - \bar{X} \right)^2}{(n-1)} \right]^{\frac{1}{2}} = \left[\frac{n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i \right)^2}{n\,(n-1)} \right]^{1/2} \quad . \qquad [3]$$

A typical set of values of $c_{1-\alpha,\gamma}$ for various tolerance limits is given in Table I. These multipliers are based upon a normal distribution of the data. The 95/95 level is the one typically used in the wind industry for design (i.e., with a 95 percent confidence level, we expect that at least 95 percent of all future strength tests will exceed X^*).

TYPICAL DATA SET

A typical data set is used for illustration. The data, shown in Fig. 1, were taken from the MSU/DOE Database.[3] They are from materials called DD5 and DD5P in the database, which are fiberglass with polyester matrix. Their composition is 72 percent 0° fibers, and the remaining 28 percent fibers are oriented at ±45°. The DD5 has a volume fraction of 38 percent fibers, and the DD5P has a 36 percent volume fraction. The tests were conducted at an R value of 0.1 (tension). The data set has a total of 45 data points, of which 6 are static strength and the remaining 39 are ε-N fatigue data.

Additional data sets are evaluated later in the paper.

[‡] For example, for γ = 0.95 and (1-α) = 0.9, one would say that with 90 percent confidence that more than 95 percent of all samples will exceed the guaranteed strength.

[§] A log-normal distribution is a distribution of X when log(X) is normally distributed. Thus, log(X) may be analyzed using methods based on the normal distribution.

Table I. Multiplier for One-Sided Tolerance Limits for Normal Distributions.

Number of Samples	Multiplier		
	95/90	95/95	99/99
3	5.310	7.655	-
5	3.40	4.202	-
8	2.755	3.188	5.811
10	2.568	2.911	5.075
15	2.329	2.566	4.224
20	2.208	2.396	3.832
30	2.080	2.220	3.446
50	1.965	2.065	3.124

CURVE FITTING S-N OR e-N DATA

The problem of defining a confidence level for stress-life (S-N) or strain-life (ε-N) data is that a random function, rather than a random variable, must be used in the description of this material property.

ASTM[1] offers a "Standard Practice" for this class of analysis. Although their analysis is directed at "two-sided" tolerance limits, their guidance offers important insights into the analysis of one-sided tolerance limits.

To facilitate the analysis of this function, a set of simplifying assumptions is made. The first is that the relationship between the log of the measured life (N

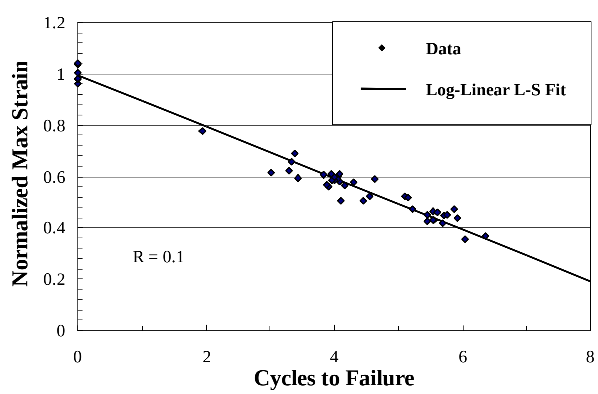

Figure 1. Typical e-N Data , Material DD from the MSU Database.

Table II. ASTM Standard Practice[1] Recommended Sample Size.

Type of Test	Minimum Number of Specimens
Preliminary & Exploratory	6 to 12
Research and Development	6 to 12
Design Allowables	12 to 24
Reliability	12 to 24

Table III. ASTM Standard Practice[1] Replication Percentage.

Type of Test	Percent Replication (minimum)
Preliminary & Exploratory	17 to 33
Research and Development	33 to 50
Design Allowables	50 to 75
Reliability	75 to 88

cycles) is a linear function of the strain (stress) or the log of the strain. The second assumption is that the distribution function of the residuals about the mean line is homogeneous; i.e., it does not depend on the strain level.

Sample Size

The ASTM Standard Practice[1] offers guidance for the minimum number of ε-N (S-N) data points that should be included in the statistical analysis. Their recommendation assumes that the data are based on random samples of the material and that the test data contain no run-outs[**] or suspended tests. Their recommendations are summarized in Table II.

With 39 ε-N data points, the data shown in Fig. 1 contain sufficient samples for statistical analysis.

Replication

In addition to number of specimens, ASTM[1] offers replication guidelines. If the percent replication *R* is defined as

$$R = 100 \left[1 - \frac{l}{n} \right] \quad , \qquad [4]$$

where *l* is the number of different strain (stress) levels in the test data, then the minimum replication percentage is given in Table III.

With 7 levels and 39 ε-N data points, the replication level of 82 percent is within the reliability guidelines.[††]

The authors are not sure why the replication rate is included in the ASTM Standard Practice.[1] One would surmise that data spread over the entire data range are better than data clustered at several points. Perhaps this is an attempt to insure that the data points do not contain a systematic error. If so, the "Distribution of Residuals," discussed below and shown in Fig. 3, is a better indicator of systematic variations about the mean.

[**] The specimen did not fail at a specified number of cycles.

[††] There are some minor variations in the maximum strain within each of these levels for the current data set, see Fig.1.

Curve Fitting

The ASTM Standard Practice[1] assumes that the strain-life curve is fit with a straight line of the form

$$Y = A + mX \quad , \qquad [5]$$

where X is the independent variable, Y is the dependent variable, A is the intercept, and m is the slope of the linear curve fit.

Sutherland[4] provides a complete discussion of the various forms of Eq. 5.

Independent and Dependent Variable

Despite the normal form of plotting ε-N data shown in Fig. 1, the stress or strain (or log stress or log strain) is taken as the independent variable X, and log life (i.e., log N) is taken as the dependent variable Y.

Curve Fit

For typical ε-N (S-N), a linear fit may be obtained using log(N) and ε or log(ε). Before fitting, the data should be plotted and a decision made as the proper form of the equation and its appropriate range. For the data presented in Fig. 1, a log-linear fit is appropriate. The appropriate range includes all of the ε-N data and the static strength as well. As discussed below, it may or may not be appropriate to include the static strength in the fit. Only a plot of the data can serve as a guide.

The fitting technique should find the best-fit of a straight line through the data. A least-squares curve fit (L-S Fit), a function included in many spreadsheets, works well for this purpose. The line shown in Fig. 1 is a log-linear fit of the ε-N and the static strength data using a least-squares curve fitting routine. In this case, the independent variable, normalized strain ($\varepsilon/\varepsilon_o$, where ε_o is ultimate tensile strain of the materials), is fit to the dependent variable of log(N). For this fit, A equals 0.9897 and m equals -9.943. The "R-squared" measure of the goodness-of-fit is 0.967.

The ASTM Standard Practice[1] recommends a maximum likelihood estimator for A and m of the form

$$\hat{A} = \bar{Y} - m\bar{X} \quad , \qquad [6]$$

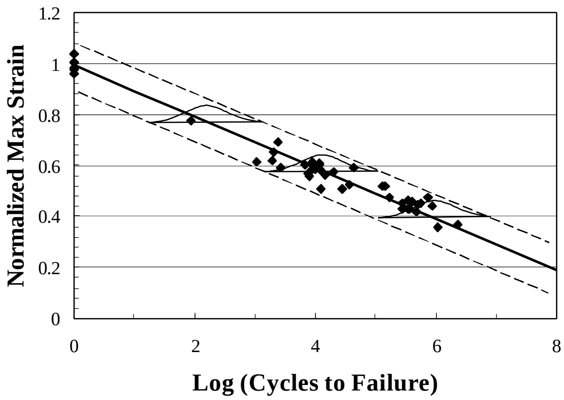

Figure 2. Distribution About the Mean Line Remains Constant.

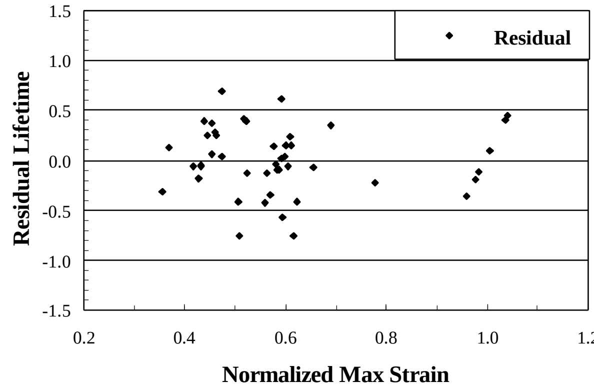

Figure 3. Distribution of Residuals.

and

$$m = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2} \quad . \qquad [7]$$

$\bar{X}$ and $\bar{Y}$ are the mean values of the X and Y test data, respectively. This technique yields an $\hat{A}$ of 0.9897 and an m of –9.943, the same as the least-squares curve fit.

<u>Adequacy</u>

ASTM Standard Practice[1] offers a testing procedure to determine if a linear model is adequate, but it is not reproduced here.

Distribution of Residuals

The added difficulty of finding tolerance limits for ε-N data over that of static strength is that the former requires a curve fit while the later requires only a single value. If the data are fit with a linear equation, see Eq. 7, both the A and m coefficients could be treated as correlated random variables. However, as the dashed lines in Figure 2 suggest, a simple, one-degree-of-freedom model might be sufficient and is in fact often used. It assumes the slope of the line is known but the intercept is uncertain.

The distribution about the mean line is determined by aggregating the residuals of the data with respect to the linear fit from different strain levels into a common pot. The residuals are defined by

$$R_i = Y_i - \hat{Y}_i \quad , \qquad [8]$$

where

$$\hat{Y}_i = \hat{A} - mX_i \quad \text{and} \qquad [9]$$

$\hat{A}$ is defined in Eq. 6.

The standard deviation of the residuals of Y, i.e., the residuals about the linear fit, is given by

$$\sigma_y = \left[\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n-2}\right]^{1/2} \quad . \qquad [10]$$

For the data cited above, $\bar{X}$ equals 0.597, $\bar{Y}$ equals 3.965 and σ_y equals 0.336. The mean of the residuals, as is typically the case, is nearly zero.

The distribution of residuals about the mean line is assumed to be independent of the maximum strain, see Fig. 2. To evaluate this assumption, the residuals are plotted against the maximum strain. Figure 3 illustrates that for this data set, the residuals are distributed about the maximum strain in no apparent pattern; i.e., there is not a systematic variation of the residuals about the mean. If there were, the straight-line fit on a log-linear plot would be in question and another fit to the data would be required (e.g., log-log, or bilinear).

Typically, the form of the distribution about the mean is taken to be either log-normal or Weibull. A graphical approach may be used to ascertain the functional form of this distribution. In this approach, the residuals, see Eq. 10, are computed for n points in the data record. They are then sorted in ascending order. The residual plot, shown in Fig. 4a, for the current data set is obtained by using the inverse normal distribution function available in most spreadsheets or by using normal (Gaussian) graph paper.[5] If this distribution is normal (log-normal) the residuals will plot as a straight line. Recall that Y is the log of the

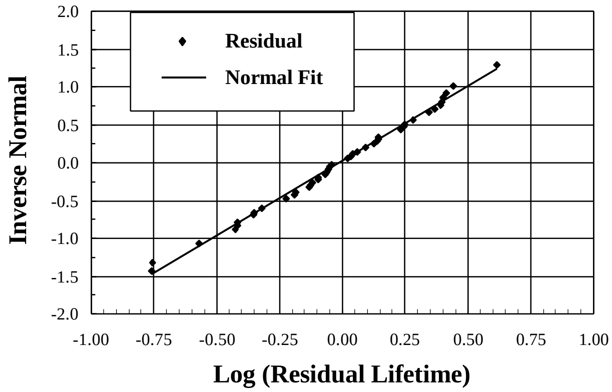

Figure 4a. Log-Normal Distribution.

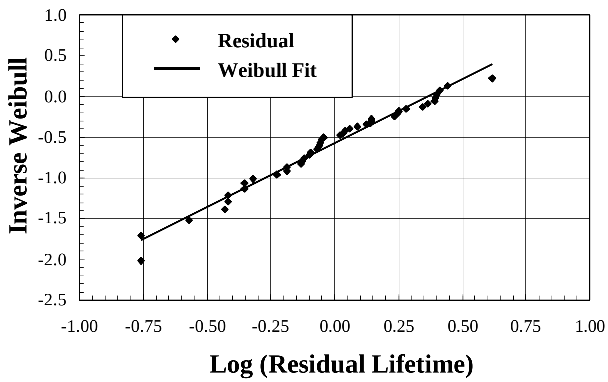

Figure 4b. Weibull Distribution.

Figure 4. Distribution of Residuals.

cycles to failure, so when the residuals of Y are tested for normality, the log-normal distribution of fatigue life is really being evaluated. As shown in Fig. 4a, the plot is very close to a straight line, with an R-squared goodness-of-fit of 0.996.

Another distribution commonly used for ε-N data is the Weibull distribution. A Weibull plot of the residuals is shown in Fig. 4b.[‡‡] This plot may also be obtained using Weibull graph paper.[5] For these data, the Weibull plot appears to be less appropriate, with an R-squared goodness-of-fit of 0.959.

Although R-squared is a good measure of the quality of the distribution fit, it does not tell the whole story. More importantly, a systematic deviation of the

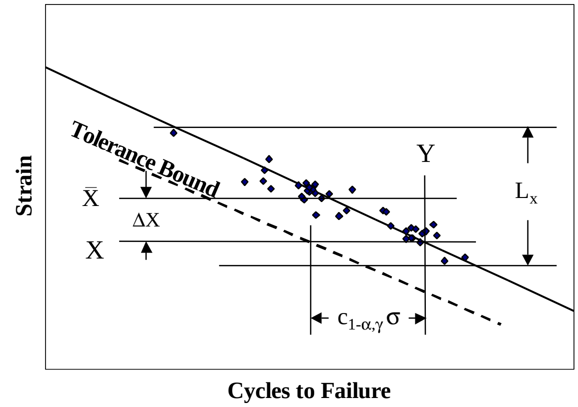

Figure 5. Graphical Definitions of Tolerance Parameters.

residuals from a straight line on either the normal or Weibull scales could indicate an inappropriate choice of distribution. For this case, neither the normal nor the Weibull fit can be rejected. Little[5] notes that a minimum of 35 data points is required to discriminate adequately between a normal and a Weibull distribution.

TOLERANCE LIMITS

Once the functional form of the distribution of residuals is known, the one-sided tolerance limit can be computed. For a linear fit of the ε-N data, the tolerance limit is determined using a variation of Eq. 5. Namely,

$$Y = \hat{A} + m\,X - c_{1-\alpha,\lambda}\ \sigma_y \quad . \qquad [11]$$

In this case, σ_y is defined in Eq. 10. A graphical view of this reduction in life is shown in Fig. 5. Also defined in this figure are L_x, the data range, and ΔX, the differential strain about the mean. The former is the difference between the maximum and the minimum strain (stress) in the data set, and the latter is the absolute value of the difference between the current strain and mean strain of the data set, see Eq. 12 below. For log-log fits, these two variables are defined as differences in the log(strain).

Within the Data Range

In its simplest form, the multiplier $c_{1-\alpha,\gamma}$ remains a function only of the number of tests, n. The result is a one-degree-of-freedom model of the uncertainty and a constant value for $c_{1-\alpha,\gamma}$. The ASTM Standard Practice[1] recommends that the tolerance bounds be restricted to the range of the data, namely L_x in Fig. 5. For this data set, the range of the normalized strain is from 1 to approximately 0.35; see Fig. 1.

‡‡ Because the Weibull distribution is fit to the actual lifetimes instead of the log of lifetimes, the *ratio* of test life to the linear fit is the best residual. However, because a Weibull plot requires a logarithmic axis, and the log of the ratios is the same as the difference of the logs, the result is a plot of the residuals defined in Eq. 10.

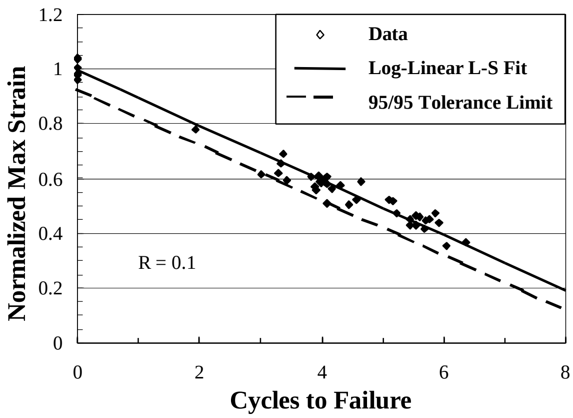

Figure 6. 95/95 Tolerance Limits for the e-N Data from Material DD with a Log-Linear Fit.

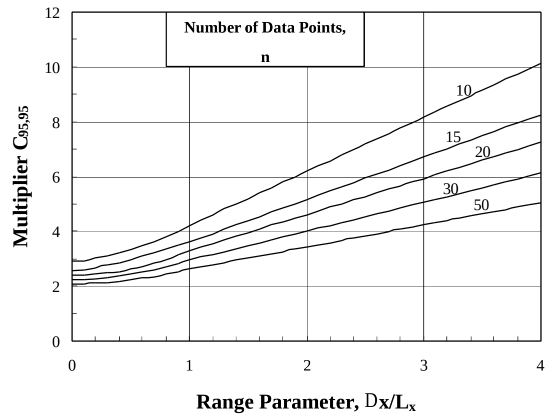

Figure 7. Tolerance Multiplier for 95/95 Tolerance Limits, after Echtermeyer, Hayman, and Ronold.[6,7]

For a normal distribution of residuals, the multiplier $c_{1-\alpha,\gamma}$ is equivalent to the one-sided tolerance limits for a single variable shown in Table I for various values of (1-α), γ, and n. For our example case, with 45 total data points, the multiplier is 2.092 at the 95/95 level. The resulting tolerance bound is shown in Fig. 6 as the long-dashed line. As a reminder, the ASTM Standard Practice recommends that this bound should not be used for normalized strain values that are less than approximately 0.35.

At a confidence level of 95/90, the multiplier would be 1.986 (namely with a 95 percent confidence level, we expect that at least 90 percent of all future ε-N tests will lie above the tolerance bound line defined in Eq. 11). At the 90/95 level the multiplier would be 1.669, and at a 99/99 level, the multiplier would be 3.181.

Outside the Data Range

Unfortunately, the recommendation of ASTM[1] to limit the tolerance bounds to the data range is not appropriate (or of much use) for wind turbine applications. Wind turbines are subjected to a wide range of fatigue cycles that is simply not covered by the current material databases. They probably never will be because of the excessively long test times required to obtain fatigue data at or above 10^8 cycles.

To extrapolate the tolerance bound outside the range of data requires a detailed statistical analysis that examines the joint distribution of the two variables A and m (a two-degree-of-freedom model of uncertainty). For normal distributions, Echtermeyer, Hayman, and Ronold[6,7] conducted this analysis. Their graphical description of the multiplier $c_{1-\alpha,\gamma}$ at a 95/95 level is shown in Fig. 7. As shown in this figure, the value of $c_{95/95}$ varies with the number of data points and with the normalized distance from the mean of the data set. The data in Fig. 7, can be approximated using the following equation:

$$c_{95,95} = 1.645 + 2.567\ (n-2)^{-0.71} + \frac{5.588}{\sqrt{n-2}}\frac{\Delta x}{L_x} \quad , \qquad [12]$$

$$\text{for } \begin{cases} \Delta X / L_x > 1.0 \\ n \geq 10 \end{cases} \quad .$$

The intercepts at $\Delta X/L_x$ equal to zero, for the various values of n are identical to those shown in Table I at the 95/95 level.

When this technique is applied to our data set, the

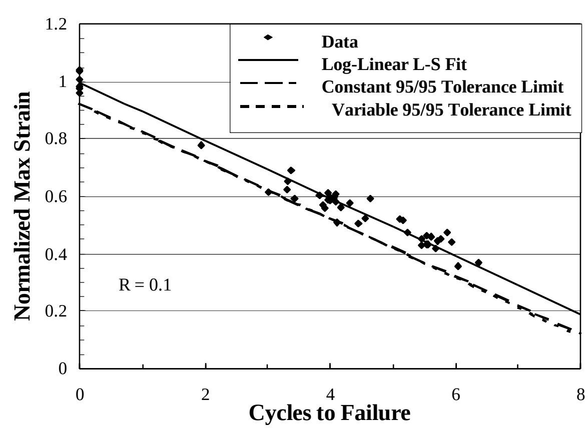

Figure 8. 95/95 Constant and Variable Tolerance Limits for the e-N Data from Material DD with a Log-Linear Fit.

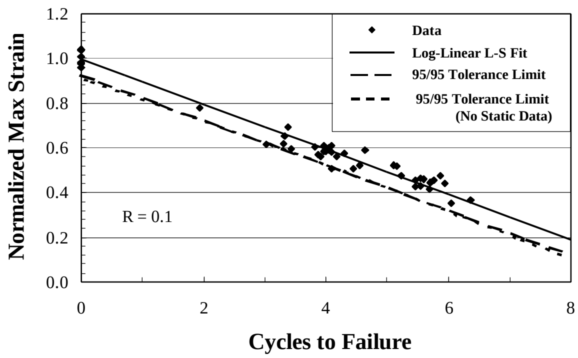

Figure 9. 95/95 Tolerance Limits for the e-N Data from Material DD, with and without the Static Strength Data.

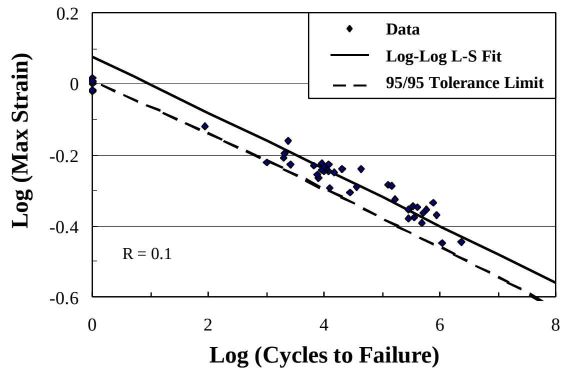

Figure 10a. Mean and 95/95 Tolerance Limits without the Static Data.

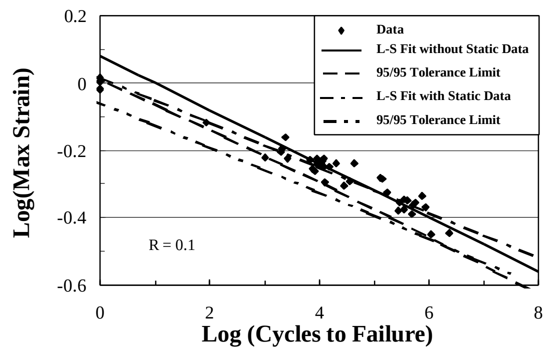

Figure 10b. Mean and 95/95 Tolerance Limits with and without the Static Data.

Figure 10. Log-Log Fit for Materials DD.

results are shown in Fig. 8 as the short dashed line. As seen in this figure, the two tolerance lines lie essentially one on top of the other, with a small deviation at the extreme of 10^8 cycles.

Inclusion of Static Strength

In this analysis, we have included the quasi-static strength data with the fatigue data in the curve fitting procedure. For comparison, we eliminated from consideration the static strength data shown in Fig. 1 and fit in Fig. 6. The results are shown in Fig. 9. In this figure, the fit is compared to the tolerance limit computed using the ASTM technique. As shown in this figure, the inclusion or exclusion of the static strength data does not significantly affect the predicted 95/95 tolerance limit; the two tolerance lines lie essentially one on top of the other with small deviations at the low and high cycle ends of the curve.

Log-Log vs. Log-Linear Fits

A major assumption made in estimating the 95/95 tolerance limits for the above illustration is that the best fit for these data is log-linear. With an R-squared value of 0.967, the fit is indeed very good. If the ε-N data (excluding the static strength data) are fit with log-log scales, an R-squared value of 0.879 results; see Fig. 10a.

Figure 10b compares the fit without the static data to that with the static data. As anticipated the fit including the static data is significantly better at the static data and the R-squared value is increased to 0.953.[§§] However, the fit to the ε-N fatigue data is significantly poorer. And the tolerance limit is lower (perhaps excessively) than that predicted without using the static data in the fit. Although the log-log fit including the static data cannot be rejected on a purely mathematical basis, our judgement indicates that the log-log fit without the static data is the proper choice.

A plot of the residuals about the log-log mean line yields R-squared values of 0.997 and 0.942 for the normal and Weibull fit of the residuals, respectively. The fit of the residuals to a normal distribution is shown in Fig. 11. Again, neither the normal nor the Weibull fit of these data can be rejected, although the normal fit is better.

The 95/95 tolerance limits obtained by using the techniques described by Echtermeyer, Hayman, and Ronold,[6,7] are also shown in Fig. 10.

In a direct comparison of the log-linear and log-log fits of the data, the two fits agree over the range of the fatigue data; namely, the normalized strain range of

[§§] This increase in R-squared is to be anticipated because the fit with static data passes through essentially two clusters of data while the fit without static data passes through one. The former typically produces a larger value for R-squared than the latter.

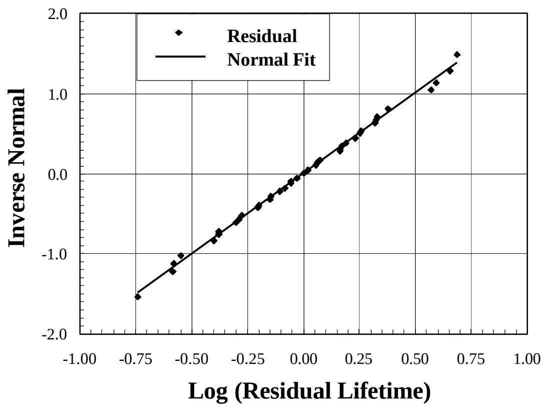

Figure 11. Log-Normal Distribution of Residuals for Material DD, Log-Log Fit.

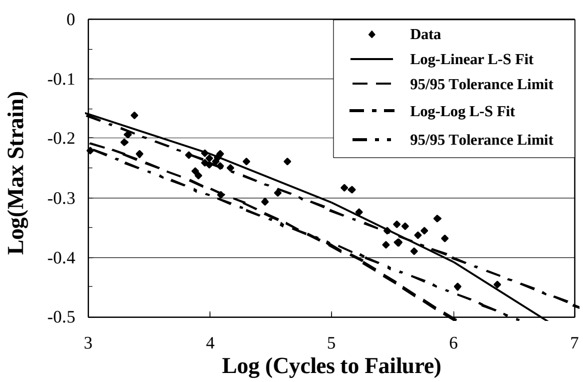

Figure 12. 95/95 Tolerance Limits for the e-N Data from Material DD, Log-Linear and Log-Log Fits.

approximately 0.8 to 0.4; see Fig. 12. Thus, within this range, either data fit works equally well.

However, when the log-linear and log-log fits are extrapolated beyond the test range, the two diverge significantly from one another. As shown in Fig. 13, the log-log fit yields a prediction for the static strength that is 20 percent high. The static strength data indicate that the log-linear fit is more appropriate for the entire range of this data set. If a log-log fit is used for these data, a bi-linear fit is indicated, with the first fit covering the normalized strain range of approximately 1.0 to 0.8 and the second covering the normalized strain range of approximately 0.8 to 0.4. Likewise, below a normalized strain of approximately 0.4, the curves diverge. In this case, the log-linear fit looks suspicious because its extension indicates that at approximately 10^{10} cycles, the material will fail at zero strain, which is unlikely. As fatigue data are not available above approximately 10^{7} cycles to failure, the log-log fit may or may not be

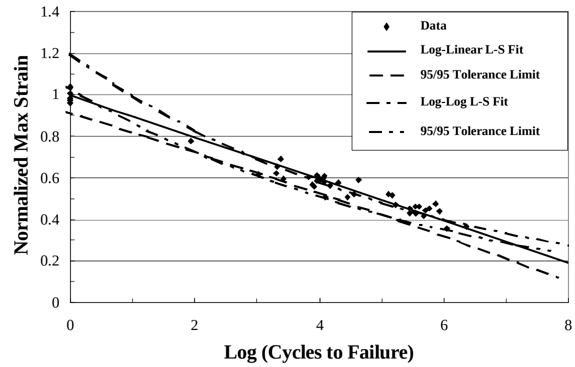

Figure 13a. Log-Linear Plot.

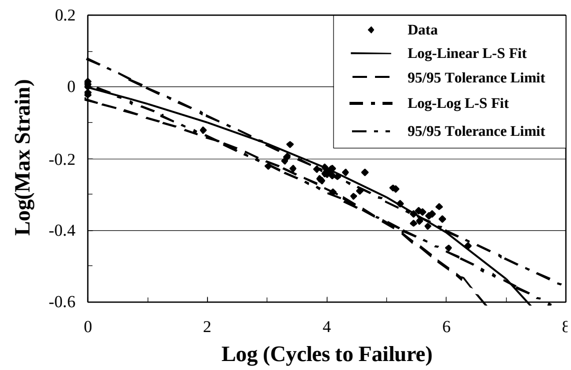

Figure 13b. Log-Log Plot.

Figure 13. 95/95 Tolerance Limits for the e-N Data from Material DD.

any better than then the log-linear fit. Again, a bi-linear (or tri-linear) fit is warranted. As discussed in the ASTM Standard Practice,[1] one should always be extremely careful when extrapolating data.

POTENTIAL PITFALLS

As with all illustrations, well-behaved data produce well-behaved results. Unfortunately, the application of the techniques discussed above can lead to erroneous conclusions concerning design curves. Several of these potential pitfalls are discussed and illustrated in this section of the paper.

Log-Log vs Log-Linear

Unfortunately, the ASTM Standard Practice[1] does not offer insights into the choice of log-linear or log-log fit of the data. To explore this choice, let us now consider the ε-N data for a composite from the FACT (FAtique of Composite for wind Turbines) database.[8] For this illustration, the ε-N data extracted from the database was for a uniaxial composite tested at an R ratio of 0.1 (tension). The material chosen has a 36.9

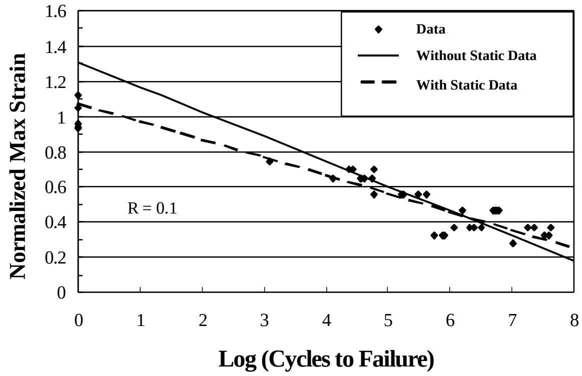

Figure 14. Log-Linear Fits for the e-N Data for a Uni-axial Composite, with and without the Static Strength Data (FACT Database).

percent volume fraction of fibers, and all specimens were tested at DLR (German Aerospace Research Establishment). There are 32 ε-N data points for this material and 5 quasi-static strength data points, see Fig. 14.

First, these data were fit using a log-linear, least-squares curve fit. One fit included the static data and the second did not. The fits had R-squared values of 0.888 and 0.741, respectively. As shown in Fig. 14, the two fits provide very different results, with the no-static-data fit significantly overpredicting the static strength by over 30 percent (this result is equivalent to that shown in Fig. 10b). When the two fits differ, several options are open. As discussed above, the first would be to use the static-data fit, which does not represent the mean of the fatigue data very well. The second, and preferred, is bi-linear fit. For these data, the first segment would cover the range from approximately 1.0 to 0.8 and the second from approximately 0.8 to 0.2.

When tolerance limits are fit to these data, additional problems become apparent. As shown in Fig. 15a, the 95/95 tolerance limit (using the techniques of Echtermeyer, Hayman, and Ronold[6,7]) yields a prediction of zero strain producing failure at approximately $10^{7.5}$ cycles. Thus, even a bi-linear log-linear fit should not be applied to normalized strain values of less than approximately 0.2.

When a log-log fit is used (R-squared of 0.690), the results are better for relatively low strain values, but very poor for relatively high strain values, see Fig. 15. As shown in this figure, the log-log fit overpredicts the static strength by a factor of approximately 2.5. However, it does not predict a finite life at zero strain.

Thus, with the normalized strain range of approximately 0.8 to 0.2, either fit will yield equivalent results. Above that range, a log-linear fit appears to be

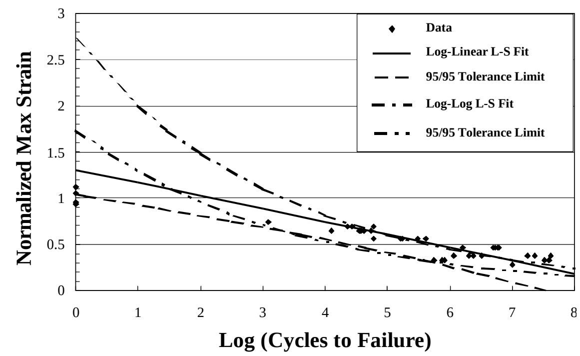

Figure 15a. Log-Linear Plot.

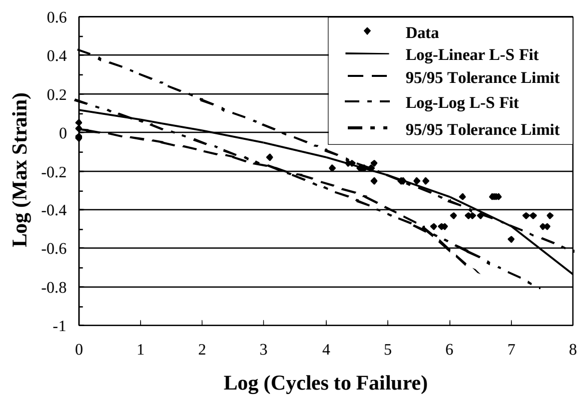

Figure 15b. Log-Log Plot.

Figure 15. Tolerance Limits for Uniaxial Material from FACT Database.

the best choice, and below, a log-log fit appears best. However, without data, a conclusive statement cannot be made. And, ASTM[1] does not recommend a form for the equation either.

Distribution of Residuals

In the examples above, the distributions of residuals for the various curve fits were fit best with a normal or log-normal distribution, based on their respective R-squared goodness-of-fit. In all cases, see Figs. 4a and 10b, the R-squared goodness-of-fit was at least 0.964 for the normal distribution. When the data are plotted on Weibull scales, see Fig. 4b, the linear fits are somewhat less accurate, ranging down to R-squared values of 0.795. However, in both cases, the number of data points is below the minimum number required to differentiate between a normal and a Weibull distribution, i.e., the minimum of 35 data points noted by Little.[5] Thus, neither distribution can be rejected as the proper form for the distribution.

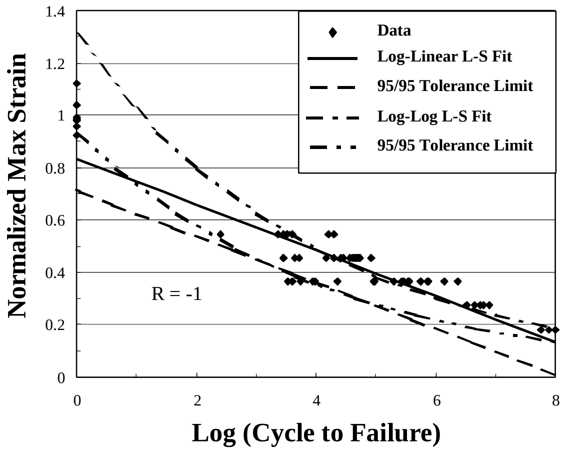

Figure 16a. Log-Linear Plot.

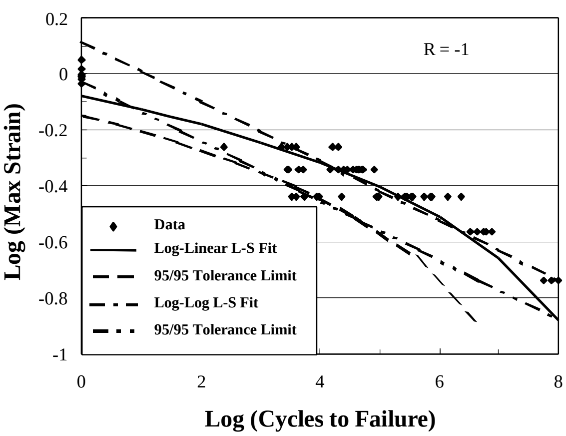

Figure 16b. Log-Log Plot.

Figure 16. Tolerance Limits for Multi-axial Material from FACT Database.

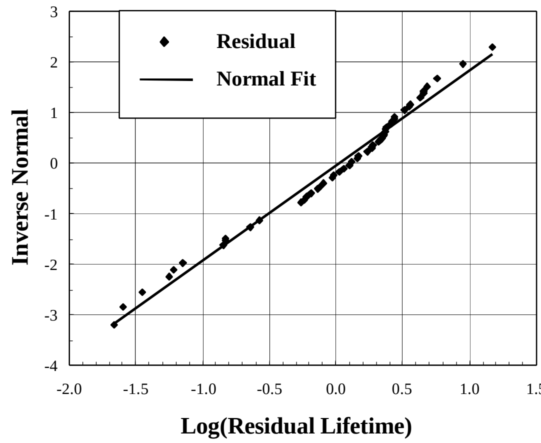

Figure 17a. Log-Normal Distribution.

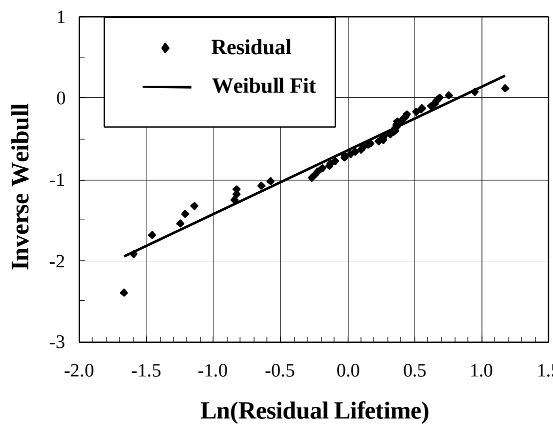

Figure 17b. Weibull Distribution.

Figure 17. Distribution of Residuals.

To expand our discussion, let us consider another material tested at DLR to examine the nature of distribution in more detail. This material is similar to the DD5 material discussed above, with 0° and ±45° layers of fibers. Its layup schedule is [[+45(280),-45(280)WR]1,[0(425)]2]s. The ±45° layers are 280 fabric and the 0° fibers are 425 fabric. Its volume fraction of fibers is 38 percent. The FACT database[8] reports 54 ε-N data points for testing an R value of -1 (tension/compression). Five quasi-static strength data points are also reported. As shown in Fig. 16, the fatigue data are best fit with a log-log fit that does not include the static data.

When the residuals are plotted, see Fig. 17, the results are still inconclusive. Both the log-normal and the Weibull models appear to fit the data equally well, with R-squared values of 0.984 and 0.962 for the log-normal and the Weibull distributions, respectively.

Thus, for a graphical analysis of the three data sets examined here, neither the normal (log-normal) nor the Weibull distribution may be rejected as the actual form of the distribution of residuals, although the log-normal distribution did consistently better.

Additional analysis techniques can be conducted to determine which distribution is appropriate; see D'Agostino and Stephens.[9] If the distribution is Weibull, then the determination of tolerance limits cannot be determined from a table. Rather, a detailed statistical analysis of the data is required. A description of one such technique is provided by Little.[5] A numerical approach can be found in Efron and Tibshirani.[10]

However, one must ask if a detailed evaluation of the distribution is warranted for our application. In particular, when dealing with wind turbines, most variations in the tolerance limits between a normal and a Weibull distribution will be minor when compared to those associated with the randomness of the input loads and the uncertainties associated with cumulative damage laws. Thus, the simplicity of determining tolerance limits using a normal (log-normal) distribution makes it the distribution of choice. However one warning should be sounded, as the probability level increases (γ>0.95), the log-normal distribution becomes increasingly non-conservative compared to the Weibull.

CONCLUDING REMARKS

In this paper, we illustrate the techniques and pitfalls of determining tolerance limits for fatigue data. A large number of figures are presented to illustrate the options. Unfortunately, this thoroughness may lead to some confusion. The following recommendations and observations may help.

When confronted with a set of fatigue data, first graph the data on log-linear and log-log scales; see Figs. 1 and 10. Include quasi-static strength data if available. Based on these plots, fit the data with one or more linear fits that cover the range of data and that can be extrapolated to the entire range of interest without violating physical constraints (i.e., strain cycles with zero amplitude should not produce failure). In the low-cycle range, a log-linear curve will probably provide the best fit. In the high-cycle range, a log-log curve is probably best.

The distribution of residuals about the best-fit line should be examined using plots similar to Figs. 3 and 4. Unless there are overriding circumstance, the normal distribution of residuals (log of cycles-to-failure) should be assumed.

If the extrapolation range is less than half the range of the test data, $(\Delta x/L_x) < 1.0$, then a constant $c_{1-\alpha,\gamma}$ may be determined from a table; see Table I and/or Ref. 2. Otherwise, use the non-linear evaluation of $c_{1-\alpha,\gamma}$ shown in Fig. 7.

Compute the tolerance limit and plot the resulting line with the original data. Examine the plot to insure the tolerance limit is consistent with the data.

This relatively simple set of procedures produces a reasonable estimation of the tolerance limit for fatigue data used in the evaluation of damage for wind turbine applications.

It should be noted that of the issues related to estimating a fatigue life curve at a given confidence level, the most likely to produce large differences in estimated lifetime is the choice of a linear or logarithmic axis for stress or strain.

Once established, the confidence level formulation of fatigue strength provides the designer with properties that can be used, with confidence, in design. However, the authors would be remiss if they did not remind the reader that these strength properties do not account for such design details as joints, size effects and environmental degradation. These design details must be handled outside of the confidence level formulation with additional safety (knock-down) factors.

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