

On The Copenhagen Interpretation of Quantum Mechanics
Symposium on "The Copenhagen Interpretation: Science and History on Stage"
National Museum of Natural History of the Smithsonian Institution
John Marburger
Director, Office of Science and Technology Policy
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In Michael Frayn's fascinating play on the intersection of physics, personalities, and social issues, the central character never appears on stage. Neils Bohr and Werner Heisenberg, and their observer Margrethe Bohr, traverse the circular arena like electrons in Bohr's early picture of the atom. The forces in their environment are well-defined, but their manifest responses to them follow the exasperating rules of microscopic Nature that we call quantum mechanics. The fourth character, whose personality dominates the play, is the one whose name appears in the title – the "Copenhagen Interpretation" of quantum mechanics. My aim here is to explain as clearly as I can why quantum mechanics needs interpreting at all, and what picture of the world the Copenhagen Interpretation leads us to. Despite the technical complexity in which it was born, requiring the genius of a Heisenberg to lay its framework, the picture of the quantum world we have today is not particularly difficult – but it is surprising, and deeply unsatisfying for reasons that are perfectly clear.

Physicists know that the most important contact between Bohr and Heisenberg was not the awkward meeting in September 1941, the focal point of Frayn's play, but the intense period in 1926 when the two collaborated at Bohr's institute in Copenhagen to understand the meaning of quantum mechanics – and to bring the Copenhagen Interpretation into existence. Abundant records exist of the first encounter (none by Bohr himself) but few of the second. Frayn has scrutinized the earlier documentation to shed light on the later curious and somewhat ominous return of Heisenberg to Copenhagen during the early phase of Hitler's aggression in Europe. The best account I know of both periods is Abraham Pais's "*Niels Bohr's Times*," which also sketches the scientific and historical context.

Why does an "Interpretation" deserve chief billing here, and not the quantum mechanics itself, or the people who created it? Why is an interpretation necessary at all? That is a question that Heisenberg might have asked of Bohr in 1926, and one to which Frayn alludes in an exchange interrupted by a quantum jump of forty pages in the text of the play: (page numbers refer to the 2000 Anchor Press edition of "Copenhagen")

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Bohr ... As long as the mathematics worked out you were satisfied.

Heisenberg If something works, it works.

Bohr But the question is always, What does the mathematics mean, in plain language? What are the philosophical implications?

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Heisenberg What something means is what it means in mathematics.

Bohr You think that so long as the mathematics works out, the sense doesn't matter.

Heisenberg Mathematics *is* sense! That's what sense is!

Bohr But in the end, in the end, remember, we have to be able to explain it all to Margrethe!

We cannot much blame Heisenberg for his reluctance to philosophize. Modern science began, after all, with Galileo, who insisted that "the book of Nature lies open to all, and it is written in the language of mathematics." And in the next generation, Newton set forth the mathematical description of gravity with the disclaimer "I make no hypotheses", meaning that he does not speculate on the origin of gravity, or its "meaning", but only gives a mathematical description.

The mathematics of Galileo and Newton, however, and of every physical theory up to Heisenberg and Schrödinger, stated the relationships among things that could be measured: forces, masses, distances and times, velocities and accelerations. Each physical property has its traditional units that convert a tangible magnitude in Nature to a number. The mathematics expresses relations among these numbers, or among symbols that signify them. Thus Newton's law of gravity: *the force between two objects is proportional to the product of their masses and inversely proportional to the square of the distance between them.* Or his second law of motion: *an object accelerates in the presence of a force in proportion to the force and inversely as its mass.* No interpretation is needed to use these relations. And what the mathematics means, in Bohr's plain language, is clear enough. This is how Nature works. What else do we need to know?

Heisenberg was the first to discover a systematic mathematical theory that describes the small scale behavior of Nature, as opposed to the large scales that Galileo and Newton could observe with the naked eye. The border between large and small is defined, roughly speaking, by the delicacy of the system we are trying to perceive. If the object of our attention is so unsubstantial as to be displaced by our very scrutiny, then we need new approaches to describe the link between the underlying reality and what we see.

Heisenberg's new mathematics accounted for many phenomena that had been explained up to that time with an awkward patchwork that mixed the older Newtonian mechanics with new ad hoc quantum rules, the most important of which had originated with Bohr himself. Heisenberg's formulation, called by the now obsolete term "matrix mechanics," replaced the patchwork with a coherent mathematical theory which, however, had one disturbing feature. Unlike all preceding physical theory, the basic entities in Heisenberg's formulas did *not* signify measurable quantities to which numerical values could be assigned. In today's technical language, the basic entities in matrix mechanics are "operators," not ordinary numbers. Operators can be represented by *matrices*, or square arrays of numbers. Heisenberg's matrices were *infinite* square arrays of numbers.

Apart from that one unfortunate difference, the basic formulas of matrix mechanics looked strikingly like those of Newtonian mechanics. That is, if the infinite

arrays are summarized by letters, the equations for the letters are close in appearance to Newton's equations. They become identical to Newton's equations if the mysterious "quantum of action" – Planck's constant – is set to zero. At the time, that was enough for Heisenberg. He had found the mathematical link between the old physics and the new. His equally brilliant colleague Wolfgang Pauli and their collaborators immediately set about applying the theory to new phenomena. The theory, then as now, worked.

As a formulation of quantum theory, however, matrix mechanics is difficult to describe to a non-technical audience. Nor was this kind of mathematics part of the education of physicists at the time. Fortunately, just after Heisenberg's discovery, Erwin Schrödinger published a second, apparently very different, theory that also accounted for all the previous ad hoc results. This is the theory to which Heisenberg refers in the play as "Schrödinger's nonsense."

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Bohr Nonsense? Come, come. Schrödinger's wave formulation?

Margrethe Yes, suddenly everyone's turned their backs on your wonderful new matrix mechanics.

Heisenberg No one can understand it.

Margrethe And they *can* understand Schrödinger's wave.

Heisenberg Because they'd learnt it in school! We're going backwards to classical physics! And when I'm a little cautious about accepting it . . .

Bohr A little cautious? Not to criticize, but . . .

Margrethe You described it as repulsive!

Heisenberg I said the physical implications were repulsive. Schrödinger said my mathematics were repulsive.

Permit me to let Schrödinger speak for himself: (quoted in Pais, p 281)

Schrödinger "I, of course, knew of his [Heisenberg's] theory but was scared away, if not repulsed, by its transcendental algebraic methods which seemed very difficult to me."

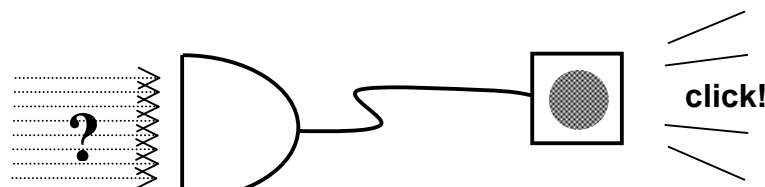
The great value of Schrödinger's theory was that it did not introduce unfamiliar mathematical objects, but looked very much like the equations all physicists had indeed learned in school to describe things like fluids in motion, or electric fields that pervaded space. Motions in fields are well described by waves, and so Schrödinger's theory was called *wave mechanics*. Not long afterward – and this is significant – Schrödinger proved that wave mechanics and matrix mechanics were mathematically equivalent. That is, they were just different mathematical ways of expressing the same underlying structure. Please do not ask me to explain here how a theory for a thing that waves can be equivalent to a theory based on infinite arrays of numbers. Mathematically it is clear enough, and today we teach it to undergraduate electrical engineers and chemists, as well as physicists. Since Schrödinger's theory was more familiar, most physicists subsequently took his formulation as their starting point when attacking a new problem.

Appealing though Schrödinger's version was, it too possessed an alarming feature. The thing that waves in wave mechanics is not a real number that can easily be related to anything that can be measured in Nature. Schrödinger's wave is a mathematical object called a *complex number*, that is to say, its symbol represents *two* numbers, an *amplitude* and a *phase*. It was the first time such an entity had appeared as an intrinsic feature of a physical theory. And its appearance sharpened the issue. Unlike all previous quantities in physics, Schrödinger's wave did not appear to signify anything that could be measured directly. The equations were there, and they worked, but what is this wave? That is the question Bohr pursued so relentlessly when he invited Schrödinger to his institute in 1926. Bohr's gentle inquisition is a famous story, told nicely by Frayn through the three actors whose real-life counterparts participated in it. It was a milestone in the creation of the Copenhagen Interpretation.

The Copenhagen Interpretation applies not only to Schrödinger's wave, for which a lucid explanation was given already by Max Born in 1926, but to the entire theory – actually to the picture of Nature that incorporates phenomena at the smallest scale. No one felt the need for such an interpretation more acutely than Bohr, and no one thought more deeply about it. At this point, I must turn away from the play and the historical account, and tell you as directly as I can how physicists, or at least this physicist, think of the theory of microscopic Nature today. It may seem an odd and unsatisfactory picture, but it has not been possible to account for experiments in any other way without introducing equally odd and unsatisfactory assumptions. No exceptions or contradictions to the picture have ever been discovered, despite Herculean efforts.

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We experience the microscopic world through instruments that are best called *detectors* (see Figure below.) These are very much like mouse-traps that snap shut when tickled by the whisker of a microscopic influence. Bohr called the snapping a *registration*. I like to call it a *click*, after the sound of the familiar Geiger counters used to detect radioactivity. Each click registers the presence of some property of Nature, and different properties can be observed with different kinds of detectors. Accepting Newton's wisdom, we "frame no hypotheses" about what makes the click. The temptation to call it "particles" is strong but wrong, or at best an approximation to Nature's ultimate stuff. We speak of *tuning* a detector across a band of properties such as energy, or momentum, or position. (You tune across the position spectrum by picking up the detector and moving it to another place.) Detectors are big objects that are described very accurately by the Newtonian mechanics of large-scale Nature. But they are our windows onto the small-scale Nature that triggers them.



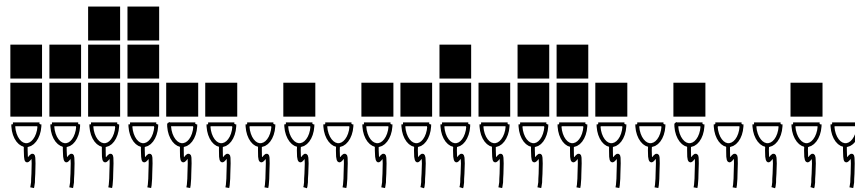
One of the aims of physics research is to discover all the different kinds of detectors Nature can trigger. During the past century, we discovered how to detect exotic

properties like “weak isospin” that define the so-called “Standard Model” of matter. We already knew how to detect things like position, momentum, and electric charge. These are all fundamental properties of Nature that are defined by the way we go about measuring them. For large scale matter, the measuring procedures might involve rulers, scales, springs, and electrical circuits. At the microscopic scale, think of Geiger counters, or photodetectors that “click” in response to radiation. All direct knowledge of microscopic stuff comes from such devices.

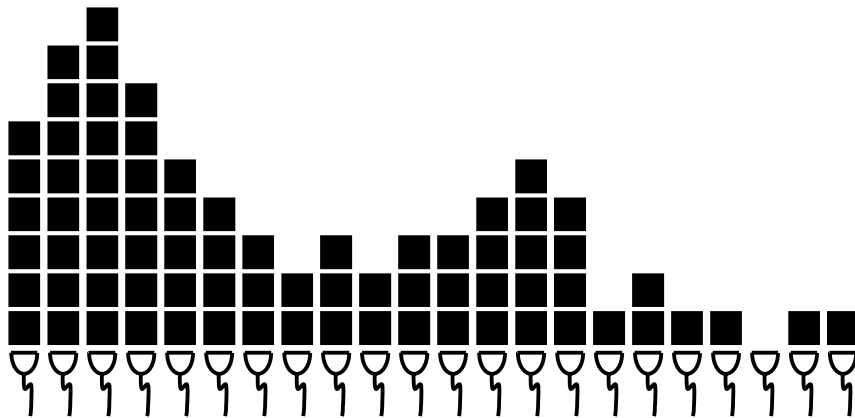
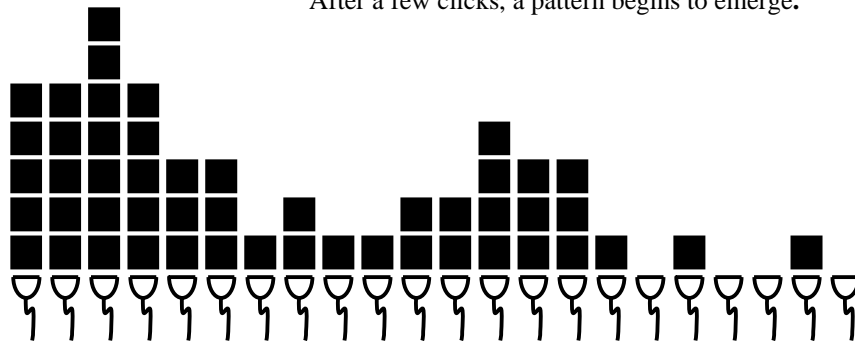
Already you can see a problem emerging from this way of looking at things. You might be able to set out multiple detectors for different properties but you only get one click at a time. Suppose you build an apparatus that produces some kind of microscopic stuff - electron stuff, for example. Then you set up a detector that measures, say, position. That is, the detector is tuned to a definite position q . At a certain time after you turn on the apparatus you hear the detector click. Good. You know that at that time the electron stuff has position q . Now you set up a different kind of detector that will click when it is tickled by electron stuff with momentum p . Then you turn on the apparatus again and wait the same amount of time. If you hear a click you know that the electron stuff has momentum p . You can see that because measurements of the two properties require two different kinds of detectors, you cannot simultaneously detect the position and momentum of the stuff produced by the apparatus. It’s logically impossible to have one click on two different detectors.

This is the gist of Bohr’s concept of *complementarity*. It is not so much that the underlying stuff does not possess position and momentum simultaneously, it is that we will never have any way of verifying it. This has nothing to do with waves and particles. It has to do with how we get knowledge about the microscopic world. We do not have this problem with large-scale things because the incompatibility of detectors for position and momentum is a tiny effect. For microscopic things, we can choose what we want to measure about Nature, but some properties are incompatible in that they simply cannot be measured at the same time, no matter how hard we try. Bohr says that such properties are *complementary*.

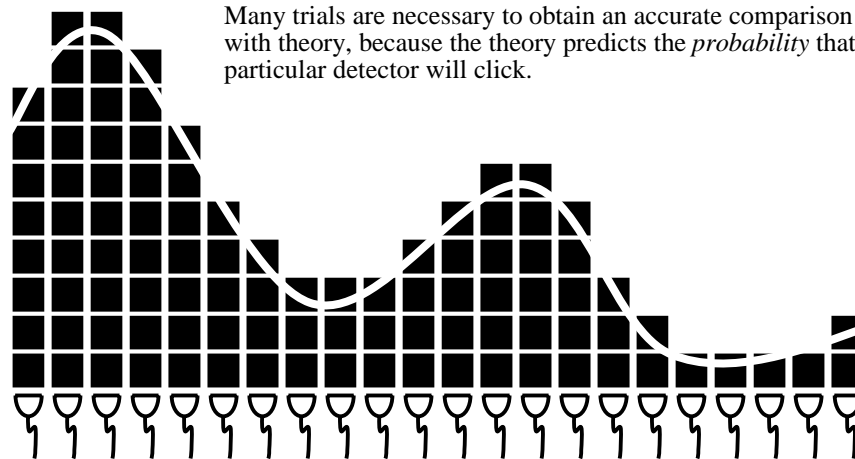
Let us set up a whole array of position detectors (as in the figures below) so we can tell where the electron-stuff triggers a click each time we turn on the apparatus. Unless the apparatus has been constructed in a special way we find that a different detector clicks on each trial. Repeat the experiment many times. Make a bar chart, a histogram, showing a bar at each detector position whose height is the number of times that detector clicked in, say, a total of a thousand trials. For a large number of trials, the ratio of this number to the total becomes the *probability* that a trial will trigger that detector. The tops of the bars trace a curve in space. The curve could have a single hump, or two or three irregular humps, or it could look something like a wave. *This* is the basic wave-like phenomenon of quantum theory. Plotted versus position, it is a wave of click-probability in space.



After a few clicks, a pattern begins to emerge.



Many trials are necessary to obtain an accurate comparison with theory, because the theory predicts the *probability* that a particular detector will click.



$q_1 q_2 q_3 q_4 q_5 \dots$

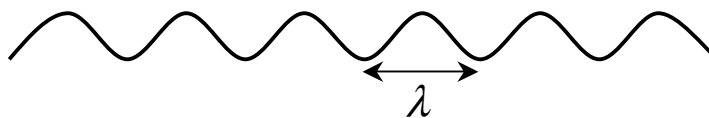
Each square corresponds to a single click in the detector below it. Schrödinger's wave amplitude (squared) predicts the curve through the tops of the columns of squares.

This kind of experiment may seem cumbersome, but it is natural if your apparatus creates stuff continuously. The big particle accelerators at the Department of Energy laboratories such as Fermilab, Brookhaven, and Stanford produce streams of matter that strike a target, much like the beam of light that illuminates the sample in a microscope. Arrays of detectors register position, momentum, energy, charge, and so forth of the emerging beam. From the pattern of their clicks on multiple trials emerges an image of the scattered stuff that is then compared with the theoretical image that quantum mechanics predicts.

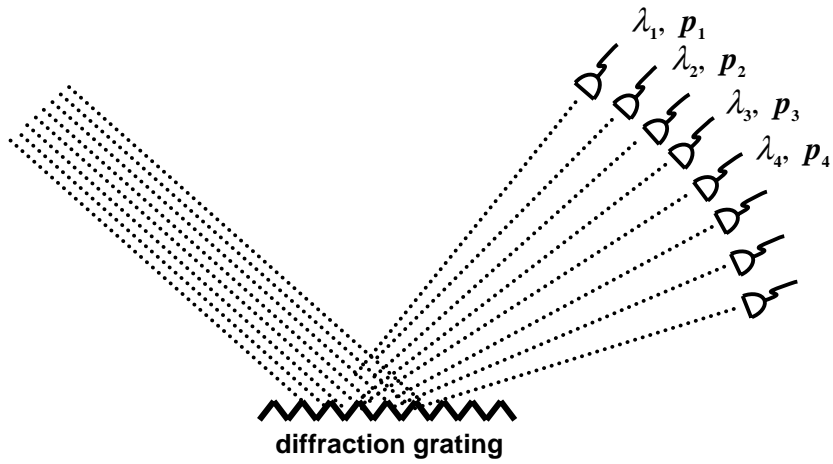
And this is where the mysterious wave-like object of Schrödinger comes in. The bar graph of the detector clicks matches the amplitude of Schrödinger's complex wave, multiplied by itself. Max Born was the first to state it clearly: the *amplitude of Schrödinger's wave is a probability amplitude for detector clicks*.

Please note that I have said nothing of whether the underlying stuff is waves or particles. In the Copenhagen Interpretation of microscopic Nature, there are *neither waves nor particles*. There is an underlying non-visualizable stuff that has the power to make clicks in different kinds of detectors. We can use the mathematics of Schrödinger or of Heisenberg to predict how frequently a particular kind of detector, tuned to a particular value of the property it detects, will be triggered by a given apparatus. A complete description of the underlying stuff that makes the clicks, according to Bohr, can only be obtained by repeated observations of different complementary properties.

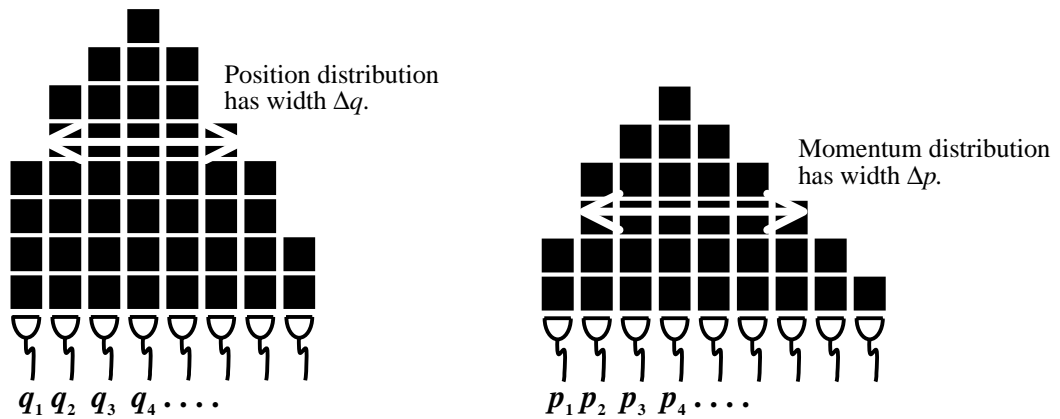
Why all the talk about waves and particles? That has to do, in part, with the different kinds of detectors. You can make an apparatus that triggers a *momentum* detector consistently at the same value p – so its histogram just has one tall bar reaching to 100% at p . It is a fact of Nature that the same apparatus will trigger an array of *position* detectors in a wave pattern with a definite wavelength precisely equal to h/p , where h is the tiny “Planck's constant”. (Multiple trials have to be run to generate the histogram. See the figures on the next page.) *The wave is not in the underlying stuff, it is in the spatial pattern of detector clicks*. We do not – cannot – measure waves in the underlying stuff. We can only measure detector clicks. But when we hear the click we say “there's an electron!” We cannot help but think of the clicks as caused by little localized pieces of stuff that we might as well call particles. This is where the particle language comes from. It does not come from the underlying stuff, but from our psychological predisposition to associate localized phenomena with particles. That predisposition is reinforced by the fact that in large scale Nature there *are* particles whose trajectories we can trace to an accuracy limited by the size of Planck's constant. This is how the Copenhagen Interpretation frames the wave versus particle issue.



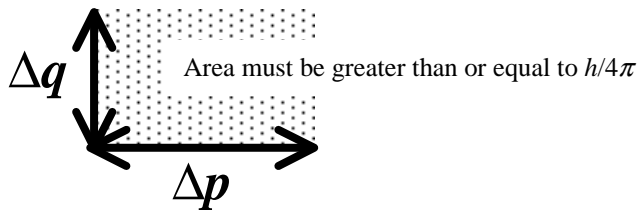
One of the empirical facts of quantum mechanics is that a phenomenon with definite momentum p has a Schrödinger wave with a well-defined wavelength $\lambda = h/p$. The amplitude of the wave depicted here is constant, but its phase advances in proportion to distance. Only the "real part" of the wave is shown. The actual Schrödinger wave cannot be drawn because it is a complex number.



A diffraction grating disperses waves of different wavelengths, and therefore different momenta. Detectors arranged as shown will produce a histogram of clicks for different momentum components present in the incoming stuff.



Schrödinger waves can appear in "position space," or "momentum space," or whatever quantity the detector array is set to measure. The two sets of detector clicks shown have characteristic spreads conventionally denoted Δq and Δp .



Heisenberg's uncertainty relation requires the area of the rectangle formed by Δq and Δp to equal or exceed a minimum value proportional to Planck's constant h .

It is not true that the underlying stuff sometimes behaves like a wave and sometimes like a particle. It always behaves like itself, but we sometimes choose to measure one property, sometimes another. When we choose to measure momentum, we find momentum clicks. When we choose to measure position, we find position clicks. If the clicks cluster near a particular position we say the position is defined up to a certain spread that measures the width of the cluster. Similarly with momentum. For a particular apparatus, we expect in general to find spreads in both position and momentum clicks. The fact that a well defined momentum click-pattern leads to a wavy pattern of position clicks with definite wavelength defines a purely mathematical relation between results from the two kinds of detectors. One of the consequences of that relation is that the spread of the position cluster multiplied by the spread of the momentum cluster always exceeds a certain number proportional to Planck's constant. This is the precise meaning of Heisenberg's famous uncertainty relation: $\Delta q \Delta p \geq h/4\pi$. (Δq and Δp signify precise statistical measures of cluster spreads.)

The uncertainty relation is embedded in the mathematics of quantum mechanics, and therefore in both Heisenberg's matrix mechanics and Schrödinger's wave mechanics. The paper Heisenberg prepared in Bohr's absence at the beginning of 1927, which is mentioned in Frayn's play, contained not only an approximate form of the relation, but also a discussion of its physical significance. Heisenberg analyzed schemes to measure the position and momentum of a particle, and noted that the physical act of measurement would perturb the particle motion consistent with the relation. For example, to measure the position of a particle, you must illuminate it with a beam of light whose collision will unavoidably impart some of its momentum, causing the observed particle to recoil. Through such models Heisenberg perceived the origin of the uncertainty as the inevitable disturbance of the observed object by the act of observation.

When Bohr returned from his ski trip to an ecstatic Heisenberg, his manner was cautious. It is true that he found a mistake in Heisenberg's analysis, and it was a revealing one. Heisenberg had treated the recoil disturbance mentioned above assuming light is made of particles – photons – whereas the actual origin of the collision-induced momentum uncertainty arises from the *wave* nature of light. This indicated to Bohr that Heisenberg had not yet grasped that both aspects were necessary for a complete treatment of matter. Heisenberg was impatient, but added a postscript acknowledging Bohr's point. In reading Heisenberg's paper, and the one subsequently published by Bohr on complementarity, we can see a profound difference in style and depth of thought. Bohr realized that the mathematics harbored a much greater departure from past ideas of waves and particles than Heisenberg was willing to contemplate at that time. In a crucial statement about halfway through Act 2, Frayn has Bohr exclaim

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Bohr It starts with Einstein. He shows that measurement – measurement, on which the whole possibility of science depends – Measurement is not an impersonal event that occurs with impartial universality. It's a human act, carried out from a specific point of view in time and space, from the one particular viewpoint of a possible observer. Then, here in Copenhagen in those three years in the mid-twenties we discover that there is no

precisely determinable objective universe. That the universe exists only as a series of approximations. Only within the limits determined by our relationship with it. Only through the understanding lodged inside the human head.

I would demur only on the last sentence. It is not through our understanding that the universe exists, but through countless mousetrapping events – mostly accidental and unobserved, but including some we make by conscious choice – that magnify the obscure restlessness of microscopic Nature and capture her in somewhat stable forms that all can test objectively with grosser instruments. Reality at human scale may be approximate, but that approximation, with rare exceptions, is independent of human consciousness.

The Copenhagen Interpretation is disappointing because it does not give us a concrete picture of reality. In a physics experiment, the concreteness comes from the detectors we choose to set up to observe the action. In ordinary life, however, concreteness comes from the accidental arrangements into which the fundamental parts of Nature fall in the course of time. In most cases those arrangements are stable – mousetrapped by Nature's own intrinsic detectors and subsequently available to anyone for objective study. Stars and planets, animals and artifacts, and even the tiny molecules of life are frozen into stable structures effectively immune to the ambiguities that plagued the quantum physicists. There remains the possibility, however, of creating special circumstances where quantum effects intrude into our ordinary macroscopic world. Other talks in today's symposium will deal with these intriguing phenomena.

My job is done, and I will end by thanking Michael Frayn for bringing the core issues of this beautiful aspect of science to such a large audience. The discovery of quantum mechanics and the unravelling of its significance is a singular event in human intellectual history. Many stories can be told of its struggles and their consequences, but I doubt that many will rise to the standard set by Frayn's "Copenhagen."