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MEMORANDUM TO FILE

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FROM: E. P. WIGNER

## THE MAGNITUDE OF THE $\eta$ EFFECT

Present information on the magnitude of the $\eta$ effect originates from CP-1381 by Bragdon, Hughes and Marshall. Their value, slightly corrected because of improved values of the cross sections, is $\delta\eta/\eta$ $5.0 \times 10^{-5}/^{\circ}C$. Actually, what is recorded in CP-1381 is a change in the ratio of the activation cross sections of U-235 and U-238. Assuming that the capture by U-235 does not lead to activation, the ratio in question is also the ratio of the fission cross section to the capture cross section of U-238. Since

$$\eta(\text{natural } U) = \frac{\sigma_f(U\text{-}235)\nu}{\sigma_f(U\text{-}235) + \sigma_c(U\text{-}235) + 1.39\,\sigma_c(U\text{-}238)}$$

A correction may be applicable to the value of CP-1381 if $\alpha$ also changes with temperature since

$$(1) \quad \frac{1}{\eta}\frac{d\eta}{dT} = -\frac{\eta}{\nu}\left[\frac{d\alpha}{dT} + 139\,\frac{d}{dT}\left(\frac{\sigma_c(U\text{-}238)}{\sigma_f(U\text{-}235)}\right)\right]$$

The existence of radiative capture in U-235 was not known at the time CP-1381 was issued.

If we assume that $\alpha$ changes with temperature, $d\eta/dT$ can also be calculated from the formula $\eta = (\nu/(1+\alpha))\,\sigma_a(U\text{-}235) \times [\sigma_a(U\text{-}235) + 139\,\sigma_a(U\text{-}238)]^{-1}$

$$(2) \quad \frac{1}{\eta}\frac{d\eta}{dT} = -\left[\frac{1}{1+\alpha}\frac{d\alpha}{dT} + \frac{139\,\sigma_a(U\text{-}235)}{\sigma_a(U\text{-}235) + 139\,\sigma_a(U\text{-}238)}\,\frac{d}{dT}\,\frac{\sigma_a(U\text{-}238)}{\sigma_a(U\text{-}235)}\right]$$

if we can obtain the temperature variation of the ratio of the absorption cross sections of U-238 and U-235. In the above formula,

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which is except for notation identical with (1),

$$\sigma_a(U\text{-}238) = \sigma_c(U\text{-}238)$$

(3)

$$\sigma_a(U\text{-}235) = \sigma_f(U\text{-}235) + \sigma_c(U\text{-}235) = \sigma_f(U\text{-}235)(1+\alpha).$$

The $\sigma_a$(U-235) as function of the energy E is given in a report by Glenn M. Roe to H. Hurwitz, 3-30-51. From it, one obtains for the low energy region (up to about to .2 ev)

(4) $$\sqrt{E}\,\sigma_a(U\text{-}235) = 110(1 - 2.95E + 11E^2).$$

All energies are given in electron volts, all cross sections in barns.

Dancoff gives for the absorption cross section of U-238 at the resonance of 7 ev

(5) $$\sqrt{E}\,\sigma_a(U\text{-}238) = \frac{38}{(3.25\times 10^{-2})^2 + (E-7)^2}$$

This gives 4.9b for the cross section at thermal temperature instead of the actual cross section of 2.8b. Expanding (5) in the low energy region but replacing the constant term by a smaller one so as to give the observed cross section gives

(5a) $$\sqrt{E}\,\sigma_a(U\text{-}238) = .44(1 + .5E + .11E^2).$$

If one reduces the 38 in Dancoff's formula to 25 to give the proper low energy cross section, one has

(5b) $$\sqrt{E}\,\sigma_a(U\text{-}238) = .44(1 + .28E + .06E^2).$$

Both (5a) and (5b) are supposed to be valid only in the low energy region (up to about 1 ev) and, for the rest of the present calculation, it hardly matters which one one uses. Actually (5a) was used.

From the above, the absorption as function of the temperature $T = 1/k\vartheta$ can be obtained by integrating $c\sqrt{E}\,\sigma_a \times \sqrt{E}\,e^{-\vartheta E}$ with respect to E. One obtains for

(6) $$\frac{139\,\sigma_a(U\text{-}238)}{\sigma_a(U\text{-}235)} = \frac{.44 + .33kT + .18(kT)^2}{.792 - 3.5kT + 32.57(kT)^2} = .555(1 + 5.17kT - 18(kT)^2)$$

if one keeps all terms up to $(kT)^2$. This gives when inserted into (2)

(7)

$$\frac{1}{\eta}\frac{d\eta}{dT} = -\frac{1}{1+\alpha}\frac{d\alpha}{dT} - .64(1 - 1.85kT) \times .555(5.17k - 36k^2T)$$

$$= -\frac{1}{1+\alpha}\frac{d\alpha}{dT} - 1.85k(1 - 8.8kT).$$

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The value of k must be inserted in ev/°C which is k = .856 x $10^{-4}$. Hence we have

$$(7a)\quad \frac{1}{\eta}\frac{d\eta}{dT} = -\frac{1}{1+\alpha}\frac{d\alpha}{dT} - 1.58\times10^{-4}(1-.75\times10^{-3}T).$$

At 300°k at which the measurements of CP-1381 were made, this is $-12 \times 10^{-5}$/°C, i.e. more than twice greater than the measured value if one neglects $d\alpha/dT$.

It is possible to blame the above discrepancy on a temperature variation of $\alpha$. If one does this, one has to assume $d\alpha/dT = -22\times10^{-5}$/°C. and obtains a positive $d\eta/dT = 6.4 \times 10^{-5}$/°C. Actually, it is more than doubtful that such an interpretation is correct: a positive temperature coefficient of $\eta$ instead of the anticipated negative one would surely have been noticed at Hanford or at Oak Ridge. In addition, the $-d\alpha/dT$ is hardly likely to be quite as large as $22 \times 10^{-5}$. The fact remains, however, that the assumption $d\alpha/dT = 0$ leads to a discrepancy between two measurements, one of which seems to be quite good. Hence, the value of $d\eta/dT$ cannot be considered to be established but may be anywhere between -12 and $6 \times 10^{-5}$.

Direct and accurate measurements of $\sigma_a$(U-233) would yield a value for $d\alpha/dT$ and hence also for $d\eta/dT$. The only remaining assumption would then be that $\nu$ does not depend on the energy of the neutron inducing fission.

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EPW:s

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