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Metallurgical Project
A. H. Compton, Project Director

* * *

Metallurgical Laboratory
S. K. Allison, Director

* * *

PHYSICS RESEARCH

E. Fermi, Division Director; Gale Young, Section Chief

* * *

RECALCULATION OF THE CRITICAL SIZE AND MULTIPLICATION CONSTANT OF A HOMOGENEOUS UO_2 - D_2O MIXTURES

E. P. Wigner, A. M. Weinberg, J. Stephenson

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ABSTRACT

The multiplication constant and optimal concentration of a slurry pile is recalculated on the basis of Mitchell's recent experiments on resonance absorption. The smallest chain reacting unit contains 45 to 55 m^3 of D_2O.

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The present report is a recalculation of the homogeneous heavy water slurry pile based on the recent measurements of Mitchell.

The method of calculation is exactly the same as given in report CP-668 so that it is unnecessary to go into further details. However, the constants were slightly changed. The fission cross section of U for fission neutrons (in units of 10^{-24} cm^2) was assumed to be .4, which then gave for the fast effect:

$$\varepsilon - 1 = \frac{1}{6.9N + 4}$$

where N is the number of D_2O molecules per UO_2 molecule. The thermal absorption cross-section of U was assumed to be 7.6; that of the D_2O molecule to be .004 (instead of .0114 as in CP-668) and this gave for the thermal utilization:

$$p_2 = f = \frac{1}{1 + 5.27 \times 10^{-4} N}$$

The value of $\eta = 1.32$ was adopted.

Mitchell's data on resonance absorption measured with a Ga detector were used for the calculation of the resonance absorption. Only two corrections were applied to them. First, the scattering cross-section of U was reduced to 9×10^{-24} cm^2. Second, a correction was made for the epi-cadmium absorption on the basis of a thermal cross-section of 3×10^{-24} cm^2. This gives a $\int \sigma dE/E$ of 1.4×10^{-24} above .5 eV which was subtracted from the measured values. We convinced ourselves that there is no appreciable self-protection of the U above the cadmium limit.

Mitchell's curve after the corrections were applied is given in Figure 1 where the abscissa is the scattering cross-section associated with each U atom. The ordinate is the effective $\int \sigma dE/E$ due to the resonances. It is worthwhile to remark that a direct measurement of the resonance absorption of U in water, undertaken by Halban's group, gave good agreement for σ around 850.

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The following table gives $\epsilon - 1$, $1/p_1$, $1/f$, Δ and volume of critical sphere as function of the number N of D_2O molecules per U molecules and weight of U in grams per cc.

w (gm/cc)	N	ϵ	$1/p_1$	$1/f$	k	$M^2(cm^2)$	$-\Delta x\, 10^6 (cm^{-2})$	v (m^3)
0.22	60	1.0024	1.226	1.0316	1.0467	284	164	62
0.165	80	1.0018	1.190	1.0422	1.0662	338	196	47.5
0.132	100	1.0014	1.167	1.0527	1.0759	390	194	48.2
0.110	120	1.0012	1.152	1.0632	1.0795	439	181	53.3
0.095	140	1.0010	1.138	1.0736	1.0815	489	167	60.3
0.082	160	1.0009	1.129	1.0842	1.0794	535	148	72.2
0.073	180	1.0008	1.120	1.0948	1.0777	583	133	84.6
0.066	200	1.0007	1.112	1.1052	1.0739	632	117	102

In calculating M^2, the migration area, we have used 120 cm^2 for the age. Fig. II shows the variation of the multiplication constant k and of the volume of the chain reacting sphere as function of the concentration of the slurry in grams U per cm^3 One sees that the smallest chain reacting sphere has a volume of about 47 m^3 even at ordinary temperatures. This is considerably larger than the values obtained in earlier calculations. It should be remarked however, that the γ - n and the n - 2n contributions to the multiplication constant were not included and the cross-section of D_2O was assumed to be somewhat higher than the measured value.

The reliability of the present calculations depends strongly on the accuracy of the resonance absorption measurement. A set of calculations based on a curve through the highest points of Mitchell in Fig. 1 gave 56 m^3 instead of 47 m^3 as the minimum critical size; on the other hand, an old set of calculations based on the resonance curve of CP-668 but using .004 for the D_2O thermal absorption cross-section gave 30 m^3. The uncertainty in the critical size can therefore very easily be as much as 10 tons; the uncertainty in the position of the optimum is probably much less, however.

In more recent measurements, Mitchell finds that the resonance absorption may have been over estimated in the above and all previous measurements by as much as 20%. If this is substantiated, $\ln 1/p_1$, would have to be reduced by this amount. In addition, since the same correction applies also to all previous measurements, the value of η would have to be reduced to about 1.28. Naturally, the two corrections compensate approximately. However, for $1/p_1 > 1.15$, the change in p_1 prevails and the multiplication constant is somewhat higher than given in the Table. For $1/p_1 < 1.15$, the change in η is more important and the multiplication constant becomes somewhat lower than the figures of the Table. No major changes are expected, however, although the optimal concentration of U would again be shifted to slightly higher values.

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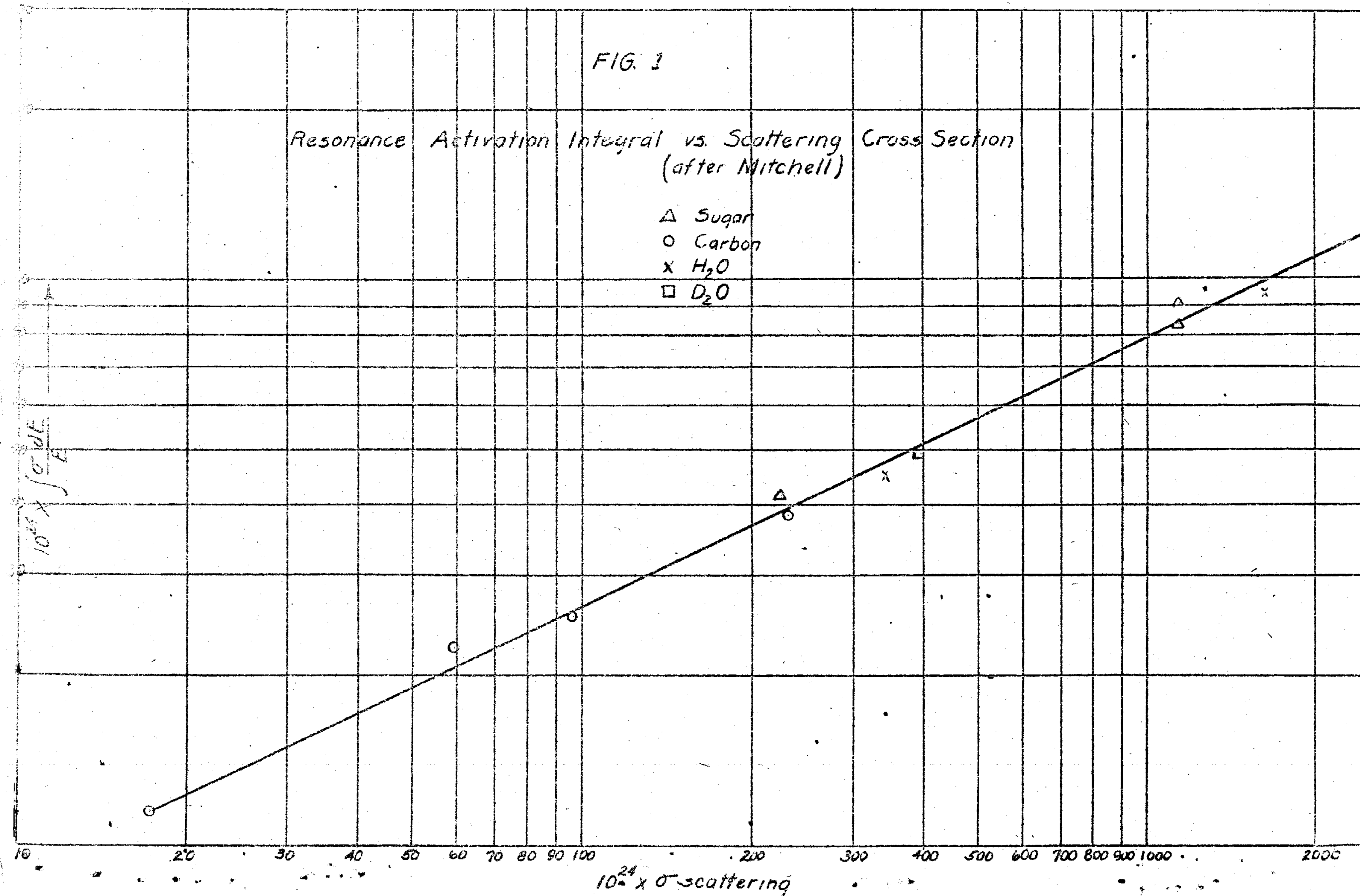

FIG. 1
Resonance Activation Integral vs. Scattering Cross Section
(after Mitchell)
△ Sugar
○ Carbon
x H_2O
□ D_2O
$10^{24} \times \int \sigma \frac{dE}{E}$
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
$10^{24} \times \sigma$ scattering

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FIG. 2

Critical Size and Reproduction Factor in Homogeneous P-9 Systems