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Electromagnetic Effects Due to Spin-Orbit Coupling

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The existence of strong spin-orbit coupling in the single-particle model of the nucleus implies the existence of a term

$$-f(r)\ (\vec{\sigma}\,\vec{\ell}) = -f(r)(\vec{\sigma}\cdot[\vec{r}\times\vec{p}]) \qquad (1)$$

in the single-particle Hamiltonian. This gives rise to an interaction of charged nucleons with external electromagnetic fields with vector potentials A. Replacing the momentum in the Hamiltonian by $\vec{p} - \frac{e}{c}\vec{A}$, we obtain from the spin-orbit coupling term the interaction energy:

$$\frac{e}{c}\, f(r)\ .\ (\vec{\sigma}\,[\vec{r}\times\vec{A}]). \qquad (2)$$

One consequence of this effect is that the magnetic moments of odd-proton nuclei should deviate from the Schmidt lines.[1] In a constant magnetic field with $\vec{A} = \frac{1}{2}[\vec{H}\times\vec{r}]$ the interaction term (2) becomes:

$$\frac{e}{2c}\, f(r)\ (\vec{\sigma}[\vec{r}\times[\vec{H}\times\vec{r}]]) = \frac{e}{2c}\, f(r)\ \left\{ r^2\,(\vec{\sigma}\cdot\vec{H}) - (\vec{r}\,\vec{\sigma})(\vec{r}\cdot\vec{H}) \right\} \quad (3)$$

For a state with orbital angular momentum ℓ, total angular momentum $j = \ell \pm\frac{1}{2}$, and magnetic quantum number m_j the expectation value of the interaction energy is:

$$\pm\ \frac{e}{2c}\ .\ \overline{r^2\, f(r)}\ \cdot \frac{2\,j+1}{2\,j+2}\ \frac{m_j}{j}\ \cdot H \qquad (4)$$

with the positive sign for $j = \ell + \frac{1}{2}$, the negative sign for $j = \ell - \frac{1}{2}$. The factor in front of H, for $m_j = j$, is the negative additional magnetic moment. The effect consequently places the magnetic moments inbetween the Schmidt lines, as is experimentally observed. Taking the spin-orbit splitting energy, $\overline{f(r)}(2\ell + 1)\cdot\hbar$, to be about 2 Mev at $\ell = 4$, A = 100,

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1. The fact that velocity dependent two-body forces give rise to additional moments has been pointed out by Blanchard, Avery, and Sachs, Phys. Rev. 78, 292 (1950).

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we estimate the magnitude of the additional magnetic moment to be 0.25 $\frac{2j+1}{2j+2}$ nuclear magnetons, which is too small to explain the observed deviations. If we assume[2] the average $\overline{f(r)}$ to be proportional to $A^{-2/3}$, then $\overline{r^2 f(r)}$ and consequently the additional magnetic moment is independent of A.

Another consequence of (2) are additional radiative transition probabilities which are important only in cases where the ordinary transitions have vanishing matrix elements. For example, in transitions from a state with $j = \rho - \frac{1}{2}$ to a state with $j' = \rho' + \frac{1}{2}$, the interaction (2) gives a non-vanishing matrix element for magnetic L-pole transitions with $\Delta j = L$ and $\Delta \rho = L + 1$, which otherwise would be forbidden. As a special example we mention the transition $d\ 3/2 \rightarrow S\ 1/2$ for which we calculated for the magnetic dipole radiation a transition probability which corresponds to the radiation of an oscillating classical magnetic dipole with an amplitude of 0.25 nuclear magnetons. The electric transitions turn out not to be affected by (2), in agreement with a general theorem given by Sachs and Austern.[3]

As stated above, both effects are present for odd protons only. They are closely connected with the more general "interaction moments" as defined and discussed by Sachs[3], which according to Sachs might give additional moments of a somewhat greater magnitude than those discussed in this paper, and of equal character for odd neutrons and odd protons. We wish to express our thanks to R. G. Sachs for helpful discussions.

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2. M. Goeppert Mayer, Phys. Rev. **78**, 21 (1950).
3. R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).