Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

Imperfect Information and Monetary Models: Multiple Shocks and their Consequences

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2008-58

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Board of Governors of the Federal Reserve System

October 2008

[1]Thanks go to Alberto Alesina, Thomas Baranga, Robert Barro, Davin Chor, Eyal Dvir, Emmanuel Farhi, Benjamin Friedman, Gita Gopinath, Kai Guo, Oleg Itskhoki, Kenneth Kasa, Mike Kiley, David Laibson, Guido Lorenzoni, Greg Mankiw, Emi Nakamura, Kristoffer Nimark, Christina Romer, David Romer, Giacomo Rondina, Philippa Scott, Ross Scott, Jón Steinsson, Aleh Tsyvinski and participants at numerous seminars. This paper is a revised version of the second chapter of my Ph.D. dissertation at Harvard University and was partly written while I was a visiting scholar at the University of Sydney. All errors are my own. The analysis and conclusions set forth are those of the author and do not indicate concurrence by other members of the staff, by the Board of Governors, or by the Federal Reserve Banks.

Abstract

This paper examines the role of multiple aggregate shocks in monetary models with imperfect information. Because agents can draw mistaken inferences about which shock has occurred, the existence of multiple aggregate shocks profoundly influences macroeconomic dynamics. In particular, after a contractionary monetary shock these models can generate an initial increase in inflation (the "price puzzle") and a delayed disinflation (a "hump"). A conservative numerical illustration exhibits these patterns. In addition, the model shows that increased price flexibility is potentially destabilizing.

1 Introduction

This paper incorporates multiple aggregate shocks into monetary models with imperfect information. After a contractionary monetary shock these models exhibit: a) an initial increase in inflation, i.e. the "price puzzle"; and b) a delayed disinflation, i.e. a "hump". These patterns are found in many empirical studies, for example see Christiano et al. (2005). With multiple aggregate shocks, imperfect information implies that agents are not only unsure of the *magnitude* of the aggregate shock, but they are also unsure *which* shock has hit. In the New Keynesian variant of the models I present, when a contractionary monetary shock occurs agents see the interest rate increasing. They conclude that this is either due to a monetary disturbance or due to rising inflationary pressures, and so put some weight on both of these explanations. The weight they place on the "inflationary pressures" explanation then acts as a force inducing firms to increase their price. As time passes agents are better able to determine what has happened and start to cut prices in response to falling demand, generating a hump. I generate this pattern with very conservative parameter values.

It is important to explain what happens after a monetary shock because it influences how we view the monetary transmission mechanism and its associated delays. Empirically, the existence of the price puzzle is contentious. In vector autoregression exercises it is often suggested that augmentation of models with commodity prices eliminates the problem, see for example Christiano et al. (1999). Hanson (2004) argued that this is most likely a coincidence. Hanson also pointed that out that some authors have proposed informal specification tests where a model is rejected if the price puzzle is found. Such specification tests are flawed if the price puzzle is indeed a structural part of the economy. By contrast the existence of the hump is widely accepted (eg. Romer and Romer (2004), Bernanke et al. (2005)) but how it is modeled often lacks microfoundations. Typically it is achieved by assuming some sort of indexing behavior (see Woodford (2003b)).

In addition, I show that increased price flexibility can lead to more monetary non-neutrality. This runs counter to the results of traditional models, where the very source of non-neutrality is price stickiness. The recent controversy over the degree of stickiness in the United States (Bils and Klenow (2004), Nakamura and Steinsson (Forthcoming)) has certainly been viewed through this conventional prism. The findings in this paper question that presumption.

Monetary models with imperfect information have experienced a revival of late. Initially they were popularized by Lucas (1972), who expanded upon some ideas of Friedman (1968) and Phelps (1969), but their use waned after the 70s. Woodford (2003a) noted that this was, at least partially, because the simplifying assumptions needed to solve the models implied short-lived fluctuations. For example Lucas (1972) assumed everything became common knowledge after one period's duration. However Woodford (2003a) was able to get around such problems, obtaining long-lived responses to shocks. This, along with Sims's work on rational inattention (Sims (2003)), sparked much of the revival. One notable characteristic of this new work is the introduction of situations where agents have different information sets. This can introduce higher order expectations into models, where agents' actions depend upon what they believe other agents believe. This was the crucial mechanism in Woodford (2003a). Other topics have been investigated in this literature. For example Maćkowiak and Wiederholt (Forthcoming) looked at how agents allocate their attention between aggregate and idiosyncratic shocks. Gorodnichenko (2008) analyzed a menu cost model where information gets "trapped" when firms do not reset their price. I have a different focus. Uncertainty regarding which aggregate shocks have hit the economy drives the results. It also illustrates a broader methodological point. In imperfect information models all significant aggregate shocks should be included in order to get accurate model predictions.

The paper proceeds as follows. Section 2 introduces a simple static model that demonstrates some of the main points of the paper. Section 3, introduces imperfect information into a traditional New Keynesian framework. Sharply different results are generated than would be the case if information were perfect, even though agents are very good at determining what is occurring. Section 4 considers the stabilizing role of sticky prices, showing that greater flexibility can induce more volatility. Section 5 concludes.

2 Simple Static Model

The model presented in this section is static and hence cannot replicate the delayed disinflation. However, it is usually the case that anything that enhances the price puzzle enhances the hump in the model of Section 3. Therefore I view the static exercise as informative. There are a continuum of firms, indexed by j on the interval $[0,1]$. A firm's price is a positive function of their expectation of the aggregate price level, the level of economic activity, and a common markup shock. Formally, each firm sets the log of their price according to the following rule:

$$p_j = E_j(p + y + \nu)$$

where E_j is the expectation of agent j, p is the aggregate price level ($p = \int_0^1 p_j dj$), y is the log of output, and ν is the markup shock. This could be derived using a model of imperfect competition. Indeed pricing rules of this type are quite familiar in the New Keynesian literature (see Woodford (2003b)).

Monetary policy is conducted in the form of a money supply rule. Because the central bank dislikes the inflationary consequences of markup shocks, the money supply is cut when they occur. The money supply is also subject to its own shocks. Explicitly:

$$m = \psi - \frac{1}{\phi}\nu$$

where m is the log of money, ψ is a monetary shock, and $\phi > 1$ determines how aggressively the central bank reacts to a markup shock.[1] The response of the central bank to markup shocks is the key difference between this model and Lucas (1972). There is a fixed velocity of money, so that with appropriate normalization:

$$m = p + y$$

[1] $\phi > 1$ is necessary to ensure that the central bank's reaction is not too strong. If $\phi > 1$ and information were perfect, prices would fall after positive markup shock due to the severity of the monetary response.

The two shocks that hit the economy, ψ and ν, are uncorrelated and normally distributed:

$$X = \begin{bmatrix} \psi \\ \nu \end{bmatrix} \sim N(0, \Sigma)$$

$$\Sigma = \begin{bmatrix} \sigma_\psi^2 & 0 \\ 0 & \sigma_\nu^2 \end{bmatrix}$$

where X can be interpreted as the state of the economy.

Each firm receives an idiosyncratic noisy signal, H_j, of the money supply, m, and markup shock, ν, where the noise associated with each observation is normally distributed and uncorrelated with the other, i.e.:

$$H_j = BX + \mu_j$$

where:

$$B = \begin{bmatrix} 1 & -\frac{1}{\phi} \\ 0 & 1 \end{bmatrix}$$

and:

$$\mu \sim N(0, \Sigma_n),\ \Sigma_n = \begin{bmatrix} \sigma_{n,m}^2 & 0 \\ 0 & \sigma_{n,\nu}^2 \end{bmatrix}$$

These noisy signals could be motivated by the kind of information frictions that are introduced in Section 3.

Agents also know the parameters of the model and all firms have rational expectations, so:

$$E_j(x) = \int x f_x(x|H_j)dx$$

for any variable x, where f_x is the probability density function implied by the model. The equilibrium in the economy is defined to be the following:

Definition. *An equilibrium is a set of beliefs for firms (for any variable x, a pdf $f_x(: \mid :)$) prices*

($\forall j$, p_j), output (y), and government policies (m) such that:

1. *$f_x(:|:)$ are rational*
2. *Given H_j, $E_j x = \int x f_x(x|H_j)dx$, and $p_j = E_j(p+y+\nu)$*
3. *$m = \psi - \frac{1}{\phi}\nu$*
4. *$m = p + y$ where $p = \int_0^1 p_j dj$*

Appendix A shows that the equilibrium is unique. If the price level after a monetary shock is considered, the following results are obtained:

Claim 1. *After a unitary contractionary monetary shock, i.e. $\psi = -1$ and $\nu = 0$, the aggregate price level, p_c, satisfies:*

- $p_c > -1$
- $\frac{dp_c}{d\sigma_\nu^2} > 0$
- $p_c > 0$ *if* $\sigma_\psi^{-2} > \frac{\phi^2}{\phi - 1}(\sigma_{n,\nu}^{-2} + \sigma_\nu^{-2})$ *where* $\sigma_x^{-2} = (\sigma_x^2)^{-1}$

Proof. See Appendix A □

The first result, $p_c > -1$, indicates that p fails to fall by as much as the money supply. This should not come as a surprise. As a whole, agents are not sure if the decrease in the money supply that they see is due to their individual noise, or due to a movement in the aggregate, i.e. they are uncertain of the *magnitude* of the shock. This is a standard signal extraction result. Average expectations of the change in the money supply, and hence price, will be less than the actual change in the money supply. However, there is a further element to the inference problem. Firms are unsure *which* aggregate shock has hit the economy. On average, they see that the money supply has fallen, but this could be due to the response of monetary policy to a positive markup shock. They therefore put some weight on this possibility, a situation I shall call confusion. This places upward pressure on prices.

This mechanism relies on agents using their signal of the central bank's action to update their beliefs. This would imply that the central bank has some separate information that the public does not posses. This information structure finds empirical support in the study of Romer and Romer (2000), who conclude that "the Federal Reserve has considerable information about inflation beyond what is known to commercial forecasters".

If the variance of the markup shock is increased then firms will attribute more of what they see to a markup shock. Therefore firms will set a higher price than they otherwise would, which explains why $\frac{dp_c}{d\sigma_\nu^2} > 0$. Indeed, the confusion about which shock has occurred can be so severe that prices may go *up* after a contractionary monetary shock. This price puzzle result occurs if $\sigma_\psi^{-2} > \frac{\phi^2}{\phi-1}(\sigma_{n,\nu}^{-2} + \sigma_\nu^{-2})$, i.e. if the relative variance of the monetary shocks is sufficiently small.

3 A New Keynesian Model

The model above abstracts from many different issues. For example, central banks rarely use money supply as a policy instrument. Also, in the model firms get signals of exogenous variables, m and ν, and not endogenous variables such as prices, p. I address these and other issues in the more sophisticated model that follows.

3.1 The Model

The core components of this model follow a standard New Keynesian model, in the tradition of Woodford (2003b) and Clarida et al. (1999). The point of departure is that each agent has an idiosyncratic information set.

3.1.1 Private Agents

There are a continuum of private agents indexed by $j \in [0,1]$. In what follows, T and t index time periods. Each agent in the economy can be thought of as a yeoman farmer, producing a unique good, also indexed by j. The only input into production is labor with functional form:

$$Y_{j,T} = L_{j,T}^{\gamma},\ 0 < \gamma \leq 1$$

where $Y_{j,T}$ is the amount of good j produced and $L_{j,T}$ is the amount of the agent's labor used to produce it. They then sell $Y_{j,T}$ for price $P_{j,T}$.

Each period the agent consumes a finite number of goods, N. The identity of these goods is chosen randomly from the continuum in the economy. This is intended to limit the amount of information that the agent can infer from their consumption basket about the overall price level.[2] The per period utility function of agent j is a linearly separable function of a consumption index and labor:

$$U_{j,T} = \frac{C_{j,T}^{1-\sigma}}{1-\sigma} - L_{j,T}$$

where:

$$C_{j,T} = \left(\sum_{k=1}^{N} \frac{1}{N} C_{k,j,T}^{\theta_{j,T}}\right)^{\frac{1}{\theta_{j,T}}}$$

and $\{C_{1,j,T}, \ldots C_{N,j,T}\}$ is consumption of the N goods that agent j can consume. $\theta_{j,T}$ is a preference parameter that determines the elasticity of substitution between goods. There is a random, persistent, aggregate component to $\theta_{j,T}$ along with a transitionary, idiosyncratic component following:

$$\ln\theta_{j,T} = \ln\theta_T + \mu_{j,\theta,T},\ \mu_{j,\theta,T} \sim N(0, \sigma_{\mu,\theta}^2)$$

$$\ln\theta_T - \ln\theta_{ss} = \rho_\theta(\ln\theta_{T-1} - \ln\theta_{ss}) + \nu_{\theta,T},\ \nu_{\theta,T} \sim N(0, \sigma_\theta^2)$$

A random elasticity of substitution between products serves the same purpose as the markup shock in Section 2. A low elasticity of substitution (low θ) will lead to higher markups because the elasticity of demand will be low. Therefore shocks to θ will be referred to as markup shocks.

[2] As far as the author is aware, this method of decoupling the consumption basket from the overall economy has not been used before. Lorenzoni (2008) achieved a similar end by assuming that agents consumed a continuum of goods, but that the shock affecting those goods was correlated for each agent.

There are other ways to generate markup shocks in these models. For example, in a model with a labor market, employees could bargain over wages so bargaining shocks could be introduced. That path wasn't followed because it introduces another market, adding complexity to the model.

The only asset in the economy is a nominal one-period bond, so the budget constraint at time T is:

$$\sum_{k=1}^{N} C_{k,j,T} P_{k,j,T} + Q_T B_{j,T} \leq B_{j,T-1} + P_{j,T} Y_{j,T} \tag{1}$$

where $P_{k,j,T}$ is the price of the kth good in agent j's consumption basket, Q_T is the price of the nominal bond, and B_T is the number of bonds purchased. Ponzi schemes are ruled out:

$$\lim_{T\to\infty} E_{j,t} Q_{tT} B_{j,T} \geq 0 \tag{2}$$

for all points along a planned consumption path, where $Q_{tT} = \prod_{i=t}^{T} Q_i$.[3]

Appendix B shows that this implies:

$$C_{k,j,T} = \left(\frac{P_{k,j,T}}{\mathcal{P}_{j,T}}\right)^{\frac{1}{\theta_{j,T}-1}} C_{j,T} \tag{3}$$

and:

$$\sum_{k=1}^{N} C_{k,j,T} P_{k,j,T} = N \mathcal{P}_{j,T} C_{j,T} \tag{4}$$

with price index:

$$\mathcal{P}_{j,T} = \left(\sum_{k=1}^{N} \frac{1}{N} P_{k,j,T}^{\frac{\theta_{j,T}}{\theta_{j,T}-1}}\right)^{\frac{\theta_{j,T}-1}{\theta_{j,T}}} \tag{5}$$

The agent will sell to N other agents which are randomly allocated each time period. Therefore the

[3] As noted in Sims (2004c), when asset markets are incomplete, this condition seems to imply that running a risk that something will happen that makes debt become large is acceptable as long as it is offset by the possibility that assets will be built up in other states of the world. Such a situation could occur if the creditor, in this case the government, could examine agents' accounts to ensure that they are abiding by this constraint, and any previous commitments to accumulate assets. The government would not need to do this for every agent in every period. It could do so with some probability, inflicting a severe punishment if the agent were not doing so.

demand for agent j's product is:

$$Y_{j,T}=\sum_{k=1}^{N}\left(\frac{P_{j,T}}{\mathcal{P}_{j,k,T}}\right)^{\frac{1}{\theta_{k,T}-1}}\mathcal{C}_{j,k,T} \tag{6}$$

where $\mathcal{C}_{j,k,T}$, and $\mathcal{P}_{j,k,T}$ are the consumption and price indices of the k'th consumer in agent j's customer base for that period.

Prices are sticky using the Calvo (1983) specification. Specifically, there is a probability equal to α that the agent will have the opportunity to reset their price in any given period. The discount factor is β.

At the end of each time period, the agent observes their own $\theta_{j,T}$; how much they sell, $Y_{j,T}$; the prices in their consumption basket, $\{P_{1,j,T},...,P_{N,j,T}\}$; and the price of the nominal bond, Q_T. Agents get to see these only after they have made the decision on $P_{j,T}$ and $\mathcal{C}_{j,T}$.[4] Therefore the information used when making decisions on $P_{j,T}$ and $\mathcal{C}_{j,T}$ is:

$$I_{j,T}=\{\theta_{j,T-1},Y_{j,T-1},\{P_{1,j,T-1},...,P_{N,j,T-1}\},Q_{T-1},I_{j,T-1}\} \tag{7}$$

However, when deciding on $C_{k,j,T}$ the agent has information on the prices of the goods in their consumption basket. This arrangement could be thought of as having a consumer and a producer in the household. At the beginning of the period they decide upon $P_{j,T}$ and $\mathcal{C}_{j,T}$, and then the consumer goes out into the marketplace to decide how best to achieve $\mathcal{C}_{j,T}$.[5]

As a result, using the production function, the agent's problem can be written as choosing a plan for $\mathcal{C}_{j,T}$ and $P_{j,T}$ under each state of the world that maximizes:

$$E_t\left(\sum_{T=t}^{\infty}\beta^T\left[\frac{\mathcal{C}_{j,T}^{1-\sigma}}{1-\sigma}-(Y_{j,T})^{\frac{1}{\gamma}}\right]|I_{j,t}\right) \tag{8}$$

[4]The results are not sensitive to this assumption, but there is a timing issue of it is not made. If agents a and b are setting their prices simultaneously, and agent a consumes agent b's product, it is then unclear how agent's b's new price could be in a's information set.

[5]An alternative is to assume that the consumer also decides upon $\mathcal{C}_{j,T}$ so the decision on $\mathcal{C}_{j,T}$ is made with more information than the decision on $P_{j,T}$.

subject to Equations 1, 2, 6, and 7 with sticky prices à la Calvo (1983), where the hazard is α.

Agents are rational, so that the underlying probability density functions used to calculate expectations are those implied by the model.

3.1.2 The Central Bank

The easiest and most conventional characterization of monetary policy is a Taylor Rule:

$$i_T = \rho i_{T-1} + (1-\rho)(\phi_\pi \pi_T + \phi_y y_T) + \nu_{i,T},\ \nu_{i,T} \sim N(0, \sigma_i^2) \tag{9}$$

where i_t is the nominal interest rate (which sets the price of the nominal bond), π_t is the inflation rate, y_t is the output gap and $\nu_{i,t}$ is a monetary policy shock. The output gap is defined to be the log deviation of aggregate output from the steady state level:

$$y_T = \ln\left(\left(\int_0^1 Y_{j,T}^{\theta_T} dj\right)^{\frac{1}{\theta_T}}\right) - \ln Y_{ss}$$

where:

$$Y_{ss} = \left(\int_0^1 Y_{j,ss}^{\theta_{ss}} dj\right)^{\frac{1}{\theta_{ss}}},\ Y_{j,ss} = \text{The steady state level of individual } j'\text{s output}$$

Inflation is defined as the rate of change of the aggregate idealized price index:[6]

$$\pi_T = 100\left(\frac{\mathcal{P}_T}{\mathcal{P}_{T-1}} - 1\right) = 100\left(\frac{\left(\int_0^1 P_{j,T}^{\frac{\theta_T}{\theta_T - 1}} dj\right)^{\frac{\theta_T - 1}{\theta_T}}}{\left(\int_0^1 P_{j,T-1}^{\frac{\theta_{T-1}}{\theta_{T-1} - 1}} dj\right)^{\frac{\theta_{T-1} - 1}{\theta_{T-1}}}} - 1\right) \tag{10}$$

The addition of the lagged interest rate on the right hand side of Equation 9 reflects the large

[6]The variability of θ_T will not affect the level of output or the inflation rate directly because equations are log linearized around the symmetric, perfect information, zero inflation steady state where all prices and output levels are the same. That is, at those points $\frac{d\mathcal{P}_T}{d\theta_T} = \frac{dy_T}{d\theta_T} = 0$.

and significant value for ρ found in many studies, for example see Clarida et al. (1998). This dependence upon past rates is usually interpreted as "interest rate smoothing".

3.2 Equilibrium

I now define equilibrium:

Definition. *An equilibrium is a set of beliefs for agents (for any variable z, a pdf $f_z(:|:)$), prices ($\forall j, T$, $P_{j,T}$), allocations for consumers ($\forall j, T$, $\{C_{1,j,T}, ..., C_{N,j,T}\}$), and government policies (i_T) such that:*

1. $f_z(:|:)$ *are rational.*

2. *Each agent is randomly assigned N consumption goods and N customers.*

3. *Prices and allocations satisfy the agent's problem, i.e. Equation 8.*

4. $i_T = \rho i_{T-1} + (1-\rho)(\phi_\pi \pi_T + \phi_y y_T) + \nu_{i,T}$.

The model is log linearized around a perfect information, zero inflation steady state (Appendix C). The solution method is described in Appendix D.

3.3 Numerical Illustration

I begin my exposition of this model with the parameter values specified in Tables 1 and 2. I will argue that these parameter values are conservative, in the sense that parameter values that lead to a larger price puzzle and a more delayed disinflation can be justified. Table 1 covers parameter values that would affect the economy's reaction to a monetary shock even if information were perfect. A time period is specified to be a quarter, so the Calvo hazard parameter of 0.25 implies an average price duration of a year. This accords with some older estimates, see for example Blinder et al. (1998), but is slightly longer than some more recent work. For example, Nakamura and Steinsson (Forthcoming) find a median duration for prices of 8-11 months. The intertemporal elasticity of

Table 1: Traditional Parameters

Parameter	Symbol	Value
Calvo hazard parameter	α	0.25
Int. elasticity of substitution	σ	1
Taylor Rule inflation coeff.	ϕ_π	1.5
Taylor Rule output gap coeff.	ϕ_y	0.125
Interest rate smoothing	ρ	0.7
Discount Factor	β	1
Steady state θ	θ_{ss}	$\frac{3}{4}$
Elasticity of Y w.r.t L	γ	0.9

substitution is 1, following the real business cycle literature (Woodford (2003b), p. 165). I base the coefficients of the Taylor Rule on well known estimates (eg. Clarida et al. (2000)). The discount factor, β, is equal to 1.[7] This eliminates some wealth effects that would otherwise render the model difficult to solve, as described in Appendix C.

The parameters mentioned thus far do not affect the results significantly if varied over plausible ranges. That is not true for the last two parameters in Table 1. The steady state value of θ is $\frac{3}{4}$, which corresponds to an elasticity of substitution between goods of 4. This is on the low side of existing calibrations, see for example Chari et al. (2000), but it is consistent with the recent empirical work of Broda and Weinstein (2006). The elasticity of output with respect to the labor input, γ, is 0.9, so the model's labor share of income is 0.6.[8] As pointed out in Woodford (2003b), strategic complementarity in pricing, i.e. the situation where an agent wishes to increase their price in response to an increase in the price of others, is increasing in θ_{ss} and decreasing in γ. Therefore my values correspond to low levels of strategic complementarity relative to other work, see again Chari et al. (2000) and also Rotemberg and Woodford (1997). This is conservative in the sense that stronger strategic complementarity leads to a larger price puzzle and a longer delay in disinflation. This is discussed in Section 3.5.

[7] To avoid issues of infinite value functions, the limit of agents' behavior as $\beta \to 1$ is taken. See Appendix C.

[8] There is no labor market in this model, but a household can be thought to be made up of a laborer and a manager, with the laborer paid a wage equal to their marginal product. The steady state ratio of marginal cost to price is $\frac{\theta_{ss}-\frac{1}{N}}{1-\frac{1}{N}}$. Table 2 specifies $N = 4$. Therefore the labor share of income is $0.9\left(\frac{\frac{3}{4}-\frac{1}{4}}{1-\frac{1}{4}}\right) = 0.6$.

Table 2: Non-Traditional Parameters

Parameter	Symbol	Value
Markup shock variance	σ_{θ}^2	1
θ autocorr.	ρ_{θ}	0.95
Policy shock variance	σ_i^2	0.07
Variance of individual θ	$\sigma_{\mu,\theta}^2$	7
Number of goods consumed	N	4

Table 2 specifies parameter values that would not affect the economy's reaction to a monetary shock if information were perfect.[9] I can normalize the variance of the markup shock to 1 because ratios of variances determine agents' signal extraction calculations. The autocorrelation of θ is 0.95. I base this on Smets and Wouters (2007). This study estimated a structural model of the US economy using Bayesian methods and found that markup shocks are extremely persistent, with an autocorrelation of at least 0.9.[10]

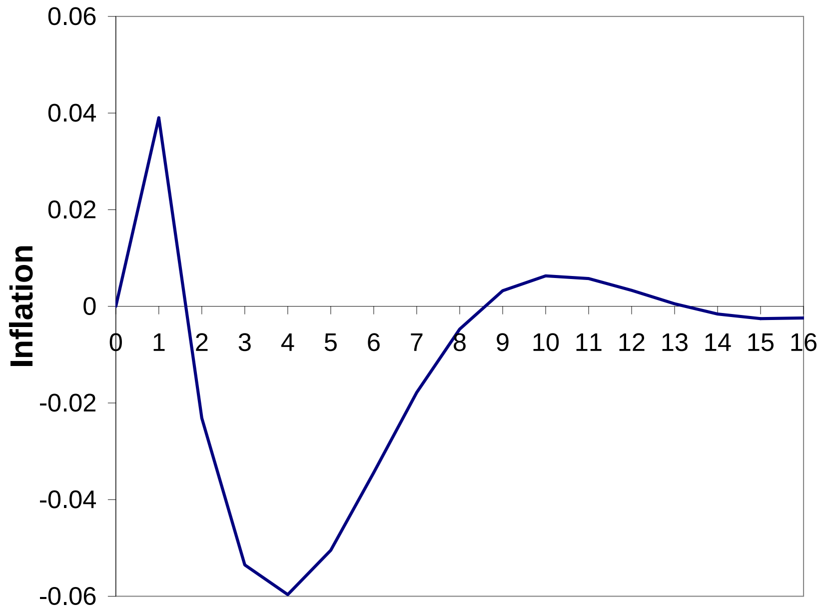
I choose the final three parameters, namely the variance of the monetary shock, the number of goods in the consumption basket, and the variance of the idiosyncratic θ shock, to give the price puzzle and a delayed disinflation. I shall discuss these parameters in Section 3.4. The resulting impulse response of inflation to a monetary shock is shown in Figure 1. By design, the inflation rate initially increases. This increase results from the same forces discussed in Section 2. Agents see the interest rate increasing and conclude that this is partly because the central bank is responding to a positive markup shock. Agents therefore increase their price.

The period of peak disinflation occurs 4 quarters after the shock. This hump is generated by the same forces generating the price puzzle. Agents are not sure which shock has hit the economy and as a result they do not cut prices initially. As time passes they gather more information and form a better picture of the shock. They then act upon the fall in demand and begin to cut their

[9] This is not strictly true of N, but over the parameter values I consider the dominant effect of N is through its determination of agents' signal precision.

[10] Note that keeping the variance of θ_T fixed, i.e. keeping $\frac{\sigma_\theta^2}{1-\rho_\theta^2}$ constant, a higher ρ_θ means the role of confusion is reduced. Increasing ρ_θ means σ_θ^2 has to decrease, so any short run variations will then be more likely ascribed to a monetary shock than a markup shock. Therefore this parameter is not driving any of the results.

Figure 1: Contractionary Monetary Shock Impulse Response ($\nu_i = 1$)



prices. This source of the hump is different to the source in Woodford (2003a) and other imperfect information work, for example Nimark (2008). That vein of literature produces a hump through a high autocorrelation of the shock and inertia in higher order expectations–beliefs of what other agents believe. In that framework, when the shock occurs agents form an opinion about the shock, but they believe other agents are less perceptive of it. Strategic complementarity then dictates that prices will not move quickly. However, as time passes higher order expectations adjust and agents begin to change their price more aggressively. This delayed response is aided by the autocorrelation of the shock. In contrast, the mechanism I emphasize has no reliance on these higher order expectations or autocorrelation of the monetary shock.

There is also another period of inflation beginning 9 quarters after the shock. This is part of some oscillatory behavior that dies down eventually. It appears to be driven by beliefs. At that point agents are still updating their beliefs of what happened in the past, but also of what happened relatively recently. In the periods leading up to the 9 quarter mark, agents are revising their beliefs of recent markup shocks upward.

Table 3: Variance Decomposition: Forecast Error Due to Monetary Shocks

4 periods	8 periods	16 periods
51%	22%	17%
(36,71)	(11,38)	(6,35)

Notes: These are the results from variance decompositions estimated from 10,000 vector autoregressions using simulated data that was 100 periods in length. The variables used were the output gap, inflation and the interest rate. Identification of the shocks was performed recursively, ordered: 1) inflation, 2) output gap, and 3) interest rate. This identifies monetary shocks because, in the model, the current monetary shock has no effect on the other variables. The average of the forecast error due to interest rate shocks over the 10,000 simulations is reported, and the brackets give the 5th and 95th percentiles.

3.4 The Final Three Parameters

I now focus attention on the parameters I chose in order to generate the patterns in Figure 1, namely the variance of the monetary shock, σ_i^2; the number of goods in the consumption basket, N; and the variance of the idiosyncratic θ shock, $\sigma_{\mu,\theta}^2$. While in some ways, estimating these parameters would have been desirable, computational issues rendered this impractical.

Section 2 demonstrated that the relative variance of the two shocks influences the price level. Table 2 specifies $\sigma_i^2 = 0.07$. I conduct a variance decomposition from simulated data as a guide to determine how this relates to the real world importance of this shock. The results are shown in Table 3.

In the model, monetary shocks account for over half of the four period ahead output forecast error. However, such a significant role for monetary shocks has little empirical support, see for example Kim (1999) and Kim and Roubini (2000). These studies concluded that monetary shocks are only responsible for 10-20 per cent of forecast errors. In the estimated model of Smets and Wouters (2007), the contribution of monetary shocks was either on the low side of this 10-20 per cent range or below it at all horizons. While the numbers from Table 3 are in the right ballpark in the longer run, it is the short horizon figures heavily influence learning in the first few periods after the shock, and so should be emphasized. Therefore even lower values of σ_i^2 can be justified, so in this sense $\sigma_i^2 = 0.07$ is conservative. However, the model has only two shocks. If other shocks were

Table 4: Mean Square Error of Output Gap Forecast over Output Gap Variance

Data	Model
0.186	0.0026

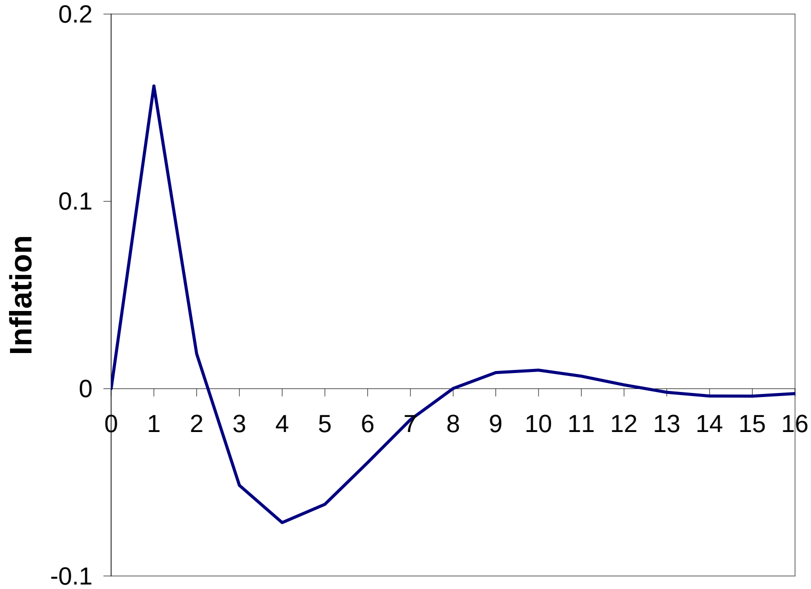
Notes: "Data" is calculated from the mean square error of the average forecast of current GDP from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters from 1990 Quarter 1 to 2007 Quarter 1. To avoid issues such as changing definitions, etc, the first revision numbers were used to determine this error. This is common in the literature on evaluating forecasts (for example see Zarnowitz and Braun (1993) and Bauer et al. (2003)). The output gap was calculated using the Congressional Budget Office's estimates of potential output.

added, then to keep the numbers in Table 3 stable, the variance of the markup shocks would have to be reduced. The rejoinder to this point is that most other major shocks that would be replacing the markup shock would induce the same sort of response. Take for example an aggregate demand shock. If added to the model, then when interest rates increase agents will attribute this partly to rising demand, which will place upward pressure on their price. Specifications that included demand shocks have been experimented with and the qualitative features of the results are not changed.

The other two parameters left to consider are $\sigma^2_{\mu,\theta} = 7$ and $N = 4$, i.e. the variance of the individual shock to θ and the number of goods consumed by each agent. These determine how well agents perceive the state of the economy because they govern the precision of their signals. While errors of beliefs regarding the actual shocks can't be measured, errors of beliefs regarding aggregates can from the Survey of Professional Forecasters. This survey includes a question on respondents' expectations of GDP for the current quarter. Models of perfect information imply that agents should know this exactly. Table 4 shows that they do not. The statistic used to show this is the ratio of the mean square error of the average forecast of the output gap to its variance. This is a variable that is negatively related to how well agents perceive current conditions. This statistic is 0.186 in the data, which is more than a factor of 70 greater than the model implies. Therefore, the results are not dependent upon assuming an unrealistic degree of uncertainty. In this sense $\sigma^2_{\mu,\theta} = 7$ and $N = 4$ are conservative.

Figure 2: Increasing Strategic Complementarity

$\theta_{ss} = \frac{6}{7}$ and $\gamma = 0.8$



3.5 Strategic Complementarity

The previous section made it clear that the degree of error in agents' beliefs is small. However, the consequences are large. A price puzzle is produced and there is a delayed response of inflation that would otherwise not occur. A reason for the significant consequences of a small amount of uncertainty is the strategic complementarity in pricing. Intuitively, strategic complementarity acts as a multiplier. If an agent thinks that a particular disturbance has pushed other agents' prices higher, more strategic complementarity induces this agent to set a higher price themselves. This multiplier role of strategic complementarity is familiar, see for example Cooper and John (1988). In my model, the multiplicative effect is large despite the lower level of strategic complementarity compared to other parameterizations in the literature. If parameters are altered in order to resemble this other work, with $\theta_{ss} = \frac{6}{7}$, and $\gamma = 0.8$,[11] then Figure 2 shows that the price puzzle and the delay

[11]These are approximately the same as the Rotemberg and Woodford (1997) estimates. Chari et al. (2000) use parameters that, in this setting, would imply even more strategic complementarity.

become even more pronounced.[12]

3.6 Comparative Statics

I will not document the sensitivity of the results to changes in all of the parameters from Section 3.3. I shall instead focus on the most interesting cases. These are σ_i^2 , N and $\sigma^2_{\mu,\theta}$. Variations in the remaining paramters over plausible values have little effect on the results. Figure 3 shows that increasing the variance of the interest rate shock and increasing the precision of the signals reduces the price puzzle and brings the peak effect on inflation forward. The left panel shows values for which the price puzzle is eliminated. The right panel shows the points at which the hump is eliminated.

Increasing σ_i^2 from 0.07 to 0.09 eliminates the price puzzle. This increase pushes the four period ahead forecast error due to monetary shocks up from 51% to 54%. Increasing the variance further to 0.13 eliminates the hump. In this case the forecast error due to monetary shocks is 60%.

Increasing N and decreasing $\sigma^2_{\mu,\theta}$ in integer steps eliminates the price puzzle at $N = 5$ and $\sigma^2_{\mu,\theta} = 3$. These parameters correspond to ratios calculated in Table 4 of 0.0025 and 0.0023 respectively. Continuing this process means the hump is eliminated when $N = 12$ and $\sigma^2_{\mu,\theta} = 1$. These parameters correspond to a ratio calculated in Table 4 of 0.0018 and 0.0015 respectively.

It can then be concluded that the presence of the price puzzle is quite sensitive to the parameters chosen. However, the hump is more robust.

4 Is Price Flexibility Stabilizing?

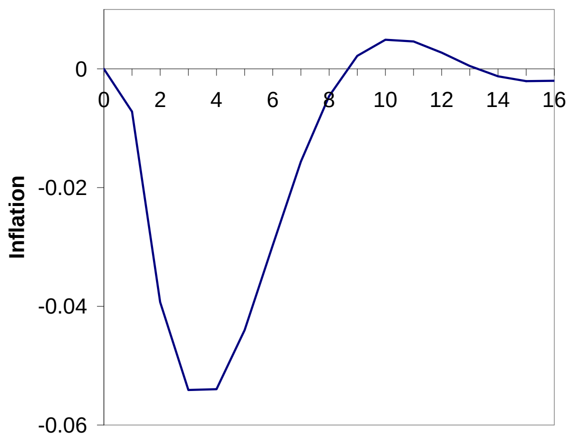
The degree of price stickiness in the United States has recently been subject to some debate. Bils and Klenow (2004) stimulated this debate by looking at data provided by the Bureau of Labor Statistics, arguing that the median length of price duration was a surprisingly short 4.3 months.

[12] L in Equation 20 is a function of θ_{ss}, γ and N which are all changed in this section and in Section 3.6. This changes the variance of $L\tilde{\theta}_T$, which given the results from Section 2, will affect the impulse responses in ways unrelated to the channels being considered. To avoid this effect clouding the results, σ_θ is normalized so as to keep the variance of $L\tilde{\theta}_T$ constant.

Figure 3: Robustness

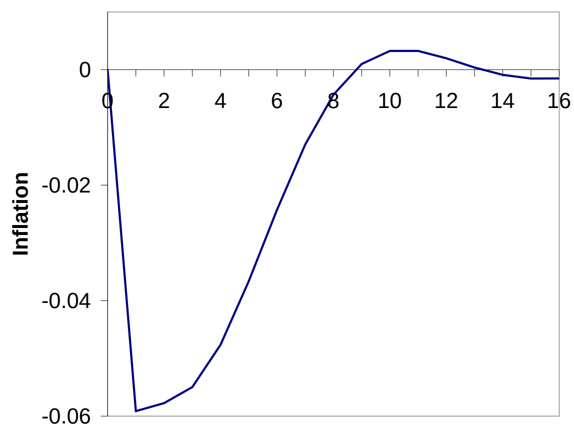
Price Puzzle Disappears

$\sigma_i^2 = 0.09$

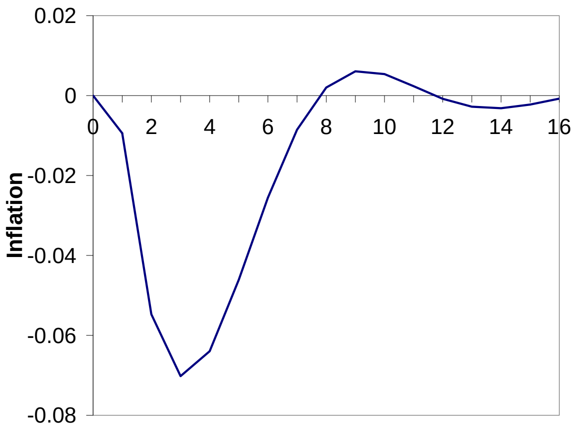


Peak Effect in First Period

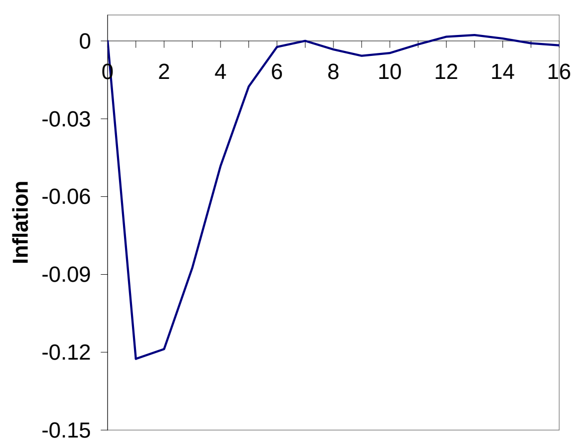
$\sigma_i^2 = 0.13$



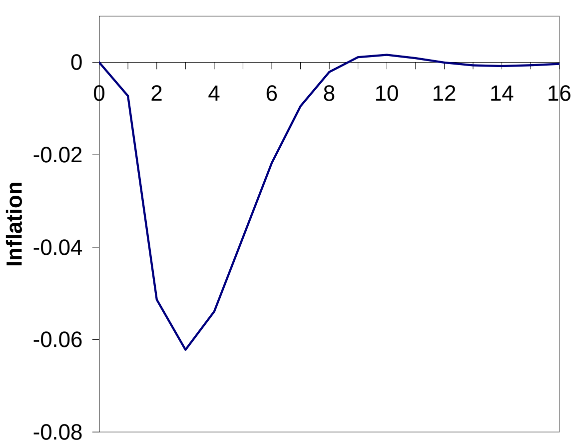
$N = 5$



$N = 12$



$\sigma_{\mu,\theta}^2 = 3$



$\sigma_{\mu,\theta}^2 = 1$

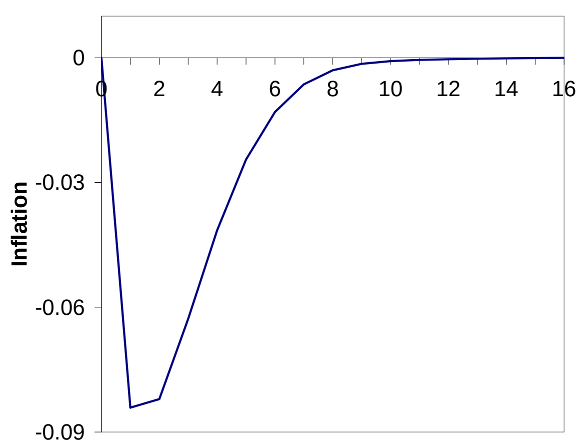
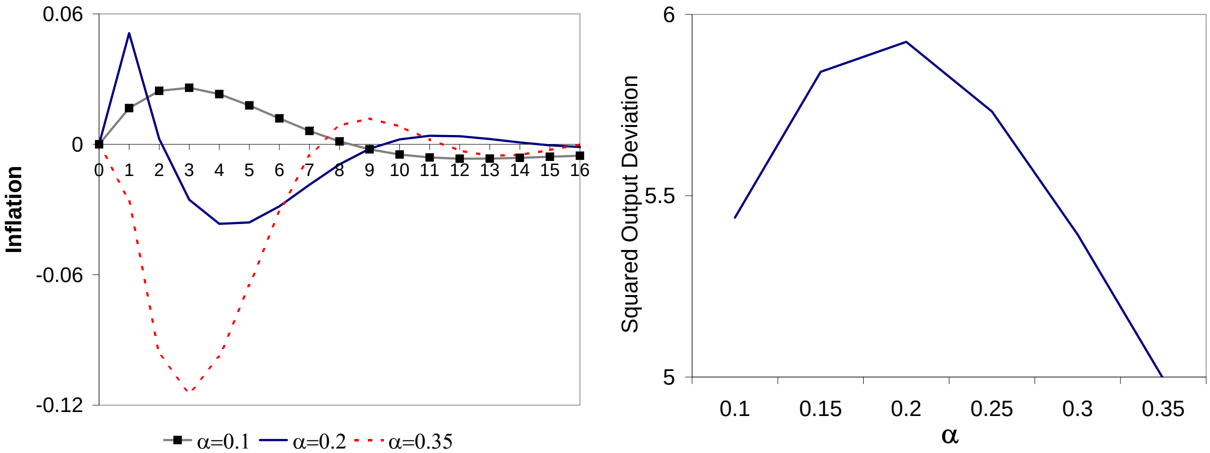


Figure 4: Changes in Price Flexibility



Since this research Nakamura and Steinsson (Forthcoming) have taken a different look at the data, generating longer estimates. This subject has received so much attention because more stickiness traditionally leads to more monetary non-neutrality. However, models have been constructed where this is not necessarily the case. For example, in a Keynesian model, DeLong and Summers (1986) showed that increased price flexibility can be destabilizing due to a "Mundell" effect. In their model increased price flexibility implied future prices were expected to fall further after a contractionary shock. The dependence of the forward looking IS curve on the real interest rate then dictated that agents would reduce their demand further, exacerbating the shock.

Similarly in this model, increased price flexibility can be destabilizing after a monetary shock, but for different reasons. Figure 4 shows that, with the baseline parameters from Section 3.3, output stability is increasing in price flexibility up to the point of $\alpha = 0.2$. The reason is simple, increased price flexibility means that the initial price puzzle is more pronounced and this destabilizes output. For price flexibility greater than $\alpha = 0.2$, the initial price puzzle is mitigated when α is increased. This is because a different influence dominates pricing decisions arising from the dynamic nature of price setting. Prices in the future will be lower if price flexibility is increased. Therefore, agents' new prices will fall immediately after the shock, because they may be stuck with that price for some time.

It should be noted however, that the price puzzle is not a necessary precondition for a positive relationship between price flexibility and output. There are other parameterizations where an initial fall in prices is tempered with greater price flexibility, which in turn leads to more output instability. The dominant force driving this comparative static is that flexibility leads to higher prices after a positive *markup* shock. In equilibrium, this means that more of the variability of prices and therefore interest rates is driven by markup shocks. This leads agents to place more weight on their markup shock beliefs after a monetary shock, which acts as a force tempering the initial price decrease.

5 Conclusion

This paper presented models where agents mistook a contractionary monetary shock for the endogenous response of monetary policy. As a result, they initially *increase* their own price, and by more if strategic complementarity in pricing is larger. However, the models illustrate a much broader point. Imperfect information models require a complete description of the important shocks that buffet the economy in order to get accurate model predictions. Agents don't know which shock has hit the economy and therefore will attribute some of what they see to all possible disturbances. This can have large effects and serves as a note of caution for further work.

The paper has shown that the mechanism that generates the price puzzle can also generate a hump in inflation's response to a monetary shock. Hump-shaped impulse responses turn up in many places in macroeconomics, presenting theoretical challenges. Indeed, the hump-shaped impulse response is what motivated Woodford (2003a). The mechanism creating the hump in Figure 3 is qualitatively distinct from any that the author has seen before. While Woodford (2003a) had a simple model that shared some features in common with the model employed here, the explanation for the hump is different.

The New Keynesian model developed could be enhanced further in order to investigate other challenges presented by the empirical literature. A potential finding of interest is the hump-shaped response of output to monetary shocks. While the model as it stands cannot replicate this, there are modifications that can. For example, if the interest rate shock is serially correlated, then hump-shaped responses are possible.

A Proof of Claim 1

Let $f(Y|Z)$ denote the probability density function of Y given Z. It is easy to show that $f(X|H_j)$ is symmetric around $X = (B'\Sigma_n^{-1}B+\Sigma^{-1})^{-1}B'\Sigma_n^{-1}H_j$, so

$$E_j(X) = (B'\Sigma_n^{-1}B+\Sigma^{-1})^{-1}B'\Sigma_n^{-1}H_j \tag{11}$$

Therefore integrating over all individual prices, $p_j = E_j m + E_j \nu$, gives:

$$\Rightarrow p = \int_0^1 p_j = [\begin{array}{cc} 1 & 1 \end{array}] B(B'\Sigma_n^{-1}B+\Sigma^{-1})^{-1}B'\Sigma_n^{-1}BX$$

As a result p is unique, so the equilibrium is unique. Computing this expression after a monetary shock gives:

$$p_c = -D\left(\sigma_{n,m}^{-2}(\sigma_{n,\nu}^{-2}+\sigma_\nu^{-2}) - \frac{(\phi-1)\sigma_{n,m}^{-2}\sigma_\psi^{-2}}{\phi^2}\right) \tag{12}$$

where:

$$D = \frac{1}{(\frac{\sigma_{n,m}^{-2}}{\phi^2}+\sigma_{n,\nu}^{-2}+\sigma_\nu^{-2})(\sigma_\psi^{-2})+(\sigma_{n,\nu}^{-2}+\sigma_\nu^{-2})(\sigma_{n,m}^{-2})} > 0$$

Therefore $p_c > -1$, and $p_c > 0$ if $\sigma_\psi^{-2} > \frac{\phi^2}{1-\phi}(\sigma_{n,\nu}^{-2}+\sigma_\nu^{-2})$. Differentiating Equation 12 gives:

$$\frac{dp_c}{d\sigma_\nu^{-2}} = -\frac{\sigma_{n,m}^{-2}\frac{\sigma_{n,m}^{-2}}{\phi^2}\sigma_\psi^{-2}+(\sigma_\psi^{-2}+\sigma_{n,m}^{-2})\frac{(\phi-1)\sigma_{n,m}^{-2}\sigma_\psi^{-2}}{\phi^2}}{((\frac{\sigma_{n,m}^{-2}}{\phi^2}+\sigma_{n,\nu}^{-2}+\sigma_\nu^{-2})(\sigma_\psi^{-2})+(\sigma_{n,\nu}^{-2}+\sigma_\nu^{-2})(\sigma_{n,m}^{-2}))^2} < 0 \Rightarrow \frac{dp_c}{d\sigma_\nu^2} > 0$$

B Derivation of Equation 3

If $B_{j,T}$ is taken as given then the Lagrangian for the problem is:

$$\mathcal{L}_{j,T} = \frac{C_{j,T}^{1-\sigma}}{1-\sigma} - L_{j,T} + \lambda\left(\sum_{k=1}^{N} C_{k,j,T}P_{k,j,T} + Q_t B_{j,T} \leq B_{j,T-1} + P_{j,T}Y_{j,T}\right)$$

Differentiating with respect to $C_{k,j,T}$ gives:

$$\frac{1}{N}C_{k,j,T}^{\theta_{j,T}-1}C_{j,T}^{1-\theta_{j,T}-\sigma} + \lambda P_{l,j,T}$$

$$\Rightarrow \ \forall k,l \ \left(\frac{C_{k,j,T}}{C_{l,j,T}}\right)^{\theta_{j,T}-1} = \frac{P_{k,j,T}}{P_{l,j,T}}$$

$$\Rightarrow \sum_{k=1}^{N} C_{k,j,T}P_{k,j,T} = C_{l,j,T}P_{l,j,T}^{\frac{1}{1-\theta_{j,T}}}\sum_{k=1}^{N} P_{k,j,T}^{\frac{\theta_{j,T}}{\theta_{j,T}-1}} \tag{13}$$

and:

$$\sum_{k=1}^{N} C_{k,j,T} P_{k,j,T} = C_{l,j,T}^{1-\theta_{j,T}} P_{l,j,T} \sum_{k=1}^{N} C_{k,j,T}^{\theta_{j,T}} \tag{14}$$

Dividing Equation 13 by Equation 14 and rearranging gives:

$$C_{l,j,T} = \left(\frac{P_{l,j,T}}{\mathcal{P}_{j,T}}\right)^{\frac{1}{\theta_{j,T^{-1}}}} \mathcal{C}_{j,T}$$

Substituting into Equation 14 then gives Equation 4.

C Log-linearized first order conditions for the private agent

Substituting in Equation 6 and Equation 4, the Lagrangian is:[13]

$$\begin{aligned}
\mathcal{L} =& E_{j,t} \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{\mathcal{C}_{j,T}^{1-\sigma}}{1-\sigma} - \left(\sum_{k=1}^{N} \left(\frac{P_{j,T}}{\mathcal{P}_{j,k,T}} \right)^{\frac{1}{\theta_{k,T^{-1}}}} \mathcal{C}_{j,k,T} \right)^{\frac{1}{\gamma}} \right] \\
&+ \lambda_{j,t} \left(N\mathcal{P}_{j,t-1}\mathcal{C}_{j,t-1} + Q_{t-1}B_{j,t-1} - B_{j,t-2} + P_{j,t-1} \sum_{k=1}^{N} \left(\frac{P_{j,t-1}}{\mathcal{P}_{j,k,t-1}} \right)^{\frac{1}{\theta_{k,t-1^{-1}}}} \mathcal{C}_{j,k,t-1} \right) \\
&+ E_{j,t} \sum_{T=t}^{\infty} \beta^{T-t+1} \lambda_{j,T+1} \left(N\mathcal{P}_{j,T}\mathcal{C}_{j,T} + Q_T B_{j,T} - B_{j,T-1} + P_{j,T} \sum_{k=1}^{N} \left(\frac{P_{j,T}}{\mathcal{P}_{j,k,T}} \right)^{\frac{1}{\theta_{k,T^{-1}}}} \mathcal{C}_{j,k,T} \right)
\end{aligned}$$

The first order condition with respect to $\mathcal{C}_{j,t}$ yields:

$$\mathcal{C}_{j,t}^{-\sigma} = -N\beta E_{j,t}\lambda_{j,t+1}\mathcal{P}_{j,t}$$

Log linearizing this around the steady state of perfect information with no shocks gives:

$$c_{j,t} = -\frac{1}{\sigma} E_{j,t}(\tilde{\lambda}_{j,t+1} + p_t) \tag{15}$$

where: $c_{j,t}$ is the log deviation of $\mathcal{C}_{j,t}$ from steady state (that there is a steady state is verified later), $\tilde{\lambda_{j,s}} = \ln \lambda_{j,s}$ and $p_t = \int_0^1 p_{j,t} dj,\ p_{j,t} = \ln P_{j,t}$.[14]

The first order condition with respect to $B_{j,t-1}$ yields:

$$\lambda_{j,t} Q_{t-1} = \beta E_{j,t} \lambda_{j,t+1}$$

Rearranging and log linearizing around the steady state gives:

$$i_t = E_{j,t+1}(\tilde{\lambda}_{j,t+1} - \tilde{\lambda}_{j,t+2})$$

[13]The method for solving this problem is taken from Kushner (1965), Sims (2004a), and Sims (2004b). In the notation of Sims (2004a) $(P_{j,t}, \mathcal{C}_{j,t}, B_{j,t-1}) = \bar{C}_t$.

[14]Everything is observed with a one period lag, so the expectation of the price level faced by the agent today is the expectation of the aggregate price level.

where i_t is the negative of the log deviation of Q_t from steady state, i.e. the nominal interest rate. Substituting this into Equation 15 and then substituting in Equation 15 that has been iterated forward gives:

$$c_{j,t} = E_{j,t}(c_{j,t} - \frac{1}{\sigma}(i_t - \pi_{t+1})) \tag{16}$$

where $\pi_{t+1} = p_{t+1} - p_t$.

The first order condition with respect to $P_{j,t}$ gives the following:

$$-E_{t|s\in\Omega}\sum_{T=t}^{\infty}\frac{(\beta(1-\alpha))^{T-t}}{\gamma(\theta_{k,T}-1)}\left(\sum_{k=1}^{N}\left(\frac{P_{j,t}^{\frac{2-\theta_{k,T}}{\theta_{k,T}-1}}}{\mathcal{P}_{j,k,T}^{\frac{1}{\theta_{k,T}-1}}}C_{j,k,T}-\frac{P_{j,t}^{\frac{1}{\theta_{k,T}-1}}}{\mathcal{P}_{j,k,T}^{\frac{\theta_{k,T}}{\theta_{k,T}-1}}}C_{j,k,T}\frac{d\mathcal{P}_{j,k,T}}{dP_{j,t}}\right)\right)$$

$$\left(\sum_{k=1}^{N}\left(\frac{P_{j,t}}{\mathcal{P}_{j,k,T}}\right)^{\frac{1}{\theta_{k,T}-1}}C_{j,k,T}\right)^{\frac{1}{\gamma}-1} = E_{t|s\in\Omega}\sum_{T=t}^{\infty}\sum_{k=1}^{N}\frac{(\beta(1-\alpha))^{T-t+1}\lambda_{j,T+1}C_{j,k,T}}{(1-\alpha)(\theta_{k,T}-1)}$$

$$\left[\theta_{k,T}\left(\frac{P_{j,t}}{\mathcal{P}_{j,k,T}}\right)^{\frac{1}{\theta_{k,T}-1}}-\left(\frac{P_{j,t}}{\mathcal{P}_{j,k,T}}\right)^{\frac{\theta_{k,T}}{\theta_{k,T}-1}}\frac{d\mathcal{P}_{k,T}}{dP_{j,t}}\right] \tag{17}$$

where Ω denotes states of the world where the agent has not had the opportunity to change their price. Each time period receives weight $(1-\alpha)^{T-t}$ because agents get to change price each period with probability α.

Equation 5 implies:

$$\frac{d\mathcal{P}_{j,k,T}}{dP_{j,t}} = \frac{1}{N}\left(\frac{P_{j,t}}{\mathcal{P}_{j,k,T}}\right)^{\frac{1}{\theta_{k,T}-1}}$$

Substituting this into Equation 17 and multiplying through by $P_{j,t}$ in order to get an expression that has a steady state, and then log linearizing gives:

$$E_{t|s\in\Omega}\left[\sum_{T=t}^{\infty}(\beta(1-\alpha))^{T-t}\left(\sum_{k=1}^{N}\frac{1}{N}\left(\left(\frac{-\theta_{ss}\tilde{\theta}_{k,T}}{\theta_{ss}-1}\right)+\frac{c_{j,k,t}}{\gamma}+\left(\frac{(\frac{1}{\gamma}-1)(N-1)+N-(\theta_{ss}-1)}{(N-1)(\theta_{ss}-1)}\right)\right.\right.\right.$$

$$\left.\left.\left.\left(p^*_{j,t}-p_{j,k,T}\right)\right)\right)\right] = E_{t|s\in\Omega}\sum_{T=t}^{\infty}(\beta(1-\alpha))^{T-t}\sum_{k=1}^{N}\frac{1}{N}\left[\tilde{\lambda}_{j,T+1}+c_{j,k,T}+p^*_{j,t}+\frac{-\theta_{ss}}{\theta_{ss}-1}\tilde{\theta}_{k,T}\right.$$

$$\left.+\frac{N}{N\theta_{ss}-1}\left(\left(\theta_{ss}\left(\tilde{\theta}_{k,T}+\frac{1}{\theta_{ss}-1}\left(p^*_{j,t}-p_{j,k,T}\right)\right)\right)-\frac{1}{N}\left(\frac{\theta_{ss}+1}{\theta_{ss}-1}\left(p^*_{j,t}-p_{j,k,T}\right)\right)\right)\right] \tag{18}$$

where $\tilde{\theta}$ is the log deviation of θ from steady state, $c_{j,k,t}$ is the log deviation of consumption of j's kth customer, $p_{j,k,T}$ is the log of the price index of j's kth customer and $p^*_{j,t}$ is the log of j's optimal price.

$$p^*_{j,t} - p_{j,k,T} = \ln \frac{P_{j,t}}{\left(\sum_{l=1}^{N} \frac{1}{N} P_{l,k,j,T}^{\frac{\theta_{k,T}}{\theta_{k,T}-1}} \right)^{\frac{\theta_{k,T}-1}{\theta_{k,T}}}}$$

where $P_{l,k,j,T}$ is the price of the lth good of j's kth's consumer's consumption basket. It must be that $P_{l,k,j,T} = P_{j,t}$ for an l. Without loss of generality suppose it is for $l = N$, so:

$$p^*_{j,t} - p_{j,k,T} = -\frac{\theta_{k,T}-1}{\theta_{k,T}} \ln \left(\frac{1}{N} + \sum_{l=1}^{N-1} \frac{1}{N} \left(\frac{P_{l,k,j,T}}{P_{j,t}} \right)^{\frac{\theta_{k,T}}{\theta_{k,T}-1}} \right) \approx \frac{1}{N} \sum_{l=1}^{N-1} \left(p^*_{j,t} - p_{l,k,j,T} \right) \tag{19}$$

From this point I look at the solution as $\beta \to 1$. This implies the wealth effect arising from resetting prices approaches zero. Therefore consumption becomes independent of whether the agent has had the opportunity to reset prices, i.e. $\bar{c}_{j,t} = 0$ so there is a steady state. All aggregates are also independent of whether the agent has reset prices. Therefore, substituting Equations 15 and 19 into Equation 18 and recognizing that $c_{j,k,T,s}$, $p_{l,k,j,T,s}$ and $\tilde{\theta}_{j,k,T,s}$ are i.i.d draws from the population yields:

$$p^*_{j,t} = \alpha E_t \sum_{T=t}^{\infty} (1-\alpha)^{T-t} \left(p_T + K c_T + L \tilde{\theta}_T + M c_{j,T} \right)$$

where:

$$K = \frac{1 - \frac{1}{\gamma}}{D}$$

$$L = \frac{N\theta_{ss}}{D\left(N\theta_{ss} - 1\right)}$$

$$M = \frac{-\sigma}{D}$$

$$D = \left(\frac{\left(\frac{1}{\gamma} - 1\right)(N-1) + N - \theta_{ss} - 1}{N\left(\theta_{ss} - 1\right)} \right) - \left(\frac{\left(N\theta_{ss} - \theta_{ss} - 1\right)(N-1)}{\left(N\theta_{ss} - 1\right)\left(\theta_{ss} - 1\right)(N)} \right) - 1$$

Ensuring supply equals demand gives:

$$c_T = \int_0^1 c_{j,T} dj = \int_0^1 y_{j,T} = y_T$$

$$\Rightarrow p^*_{j,t} = \alpha E_t \sum_{T=t}^{\infty} (1-\alpha)^{T-t} \left(p_T + K y_T + L \tilde{\theta}_T + M c_{j,T} \right) \tag{20}$$

D Solution Method

D.1 The initial guess of impulse responses

Denote the initial guess for the impulse response as M, a $(n+1,8)$ matrix.[15] Each column is the impulse response of a variable for the first $n+1$ periods to one of the shocks. It is assumed that the impulse response is flat after $n+1$ periods. Let the first two columns of M be the response of output (which equals consumption) to a $\tilde{\theta}$ shock and an interest rate shock. Let the following 2 columns be the response of price, then the next 2 be the response of interest rates and the next 2 the response of the preference parameter $\tilde{\theta} = \ln\theta_T - \ln\theta_{ss}$.

D.2 The state space

The true state space of the economy is postulated to be:

$$\tilde{X}_t = \begin{bmatrix} \nu_{\theta,t} & \nu_{i,t} \\ \nu_{\theta,t-1} & \nu_{i,t-1} \\ \vdots & \vdots \end{bmatrix}$$

If it is assumed that impulse responses are flat after $n+1$ periods, then each of the shocks further in the past than $n+1$ has the same effect on the aggregates. This means that the state space can be written as:

$$X_t = \begin{bmatrix} \nu_{\theta,t} \\ \nu_{\theta,t-1} \\ \vdots \\ \upsilon_{\theta,t} \\ \nu_{i,t} \\ \nu_{i,t-1} \\ \vdots \\ \upsilon_{i,t} \end{bmatrix} \tag{21}$$

Where $\upsilon_{x,t} = \upsilon_{x,t-1} + \nu_{x,t-n}$ and the state space follows the transition equation:

$$X_t = FX_{t-1} + \xi_t \tag{22}$$

where F contains a diagonal vector of 1's directly below the main diagonal, except at $(n+1, n+2)$. 1's are also located at $(n+1, n+1)$ and $(2n+2, 2n+2)$, and:

$$\xi_t' = \begin{bmatrix} \nu_{\theta,t} & 0 & \dots & \nu_{i,t} & 0 & \dots & 0 \end{bmatrix}$$

where the non-zero entries are placed at positions. $(1,1)$ and $(1, n+2)$.

[15] $n = 200$ was chosen for the calculations in this paper.

D.3 What is observed

From Equation 7, the following variables are used to update an agent's beliefs before making their decisions (converting these observations into their log linear counterparts):

$$\{\tilde{\theta}_{j,t-1}, y_{j,t-1}, \{p_{1,j,t-1}, ..., p_{N,j,t-1}\}, i_{t-1}\}$$

$\tilde{\theta}_{j,t-1}$ is normally distributed around $\tilde{\theta}_{t-1}$, and i_{t-1} is observed perfectly. $\{p_{1,j,t-1}, ..., p_{N,j,t-1}\}$ are N independent observations of the price level, but they are not necessarily normally distributed. This presents a problem for inference. It is overcome by assuming that for the purposes of filtering, each agent gets N independent signals of p_{t-1} that are normally distributed around p_{t-1} with a variance equal to the variance of the newly set prices, denoted σ_p^2. These N independent observations can be averaged so that one signal is received with a variance equal to $\frac{\sigma_p^2}{N}$.

Log linearizing Equation 6 around steady state gives:

$$y_{j,t-1} = \frac{1}{N}\sum_{k=1}^{N}\left(c_{j,k,t-1} + \frac{1}{\theta_{ss}-1}(p_{j,t-1} - p_{j,k,t-1})\right)$$

Inserting the approximation from Equation 19 gives:

$$y_{j,t-1} = \frac{1}{N}\sum_{k=1}^{N}\left(c_{j,k,t-1} + \frac{1}{\theta_{ss}-1}\left(\frac{1}{N}\sum_{l=1}^{N-1}(p_{j,t-1} - p_{l,k,j,t-1})\right)\right)$$

where $c_{j,k,t-1}$ and $p_{l,k,j,t-1}$ are independent draws from the population of consumption and prices. Therefore the signal received is as if they are receiving N separate signals, each of which is c_{t-1} plus $N-1$ signals of $\frac{p_{t-1}}{N(1-\theta_{ss})}$. $c_{j,k,t-1}$ is normally distributed around c_{t-1}[16] with variance σ_c^2 and the same assumption is made as above regarding the signals of p_{t-1}. These separate signals can be averaged so, for the purposes of inference, it is as if the agent receives a signal of:

$$c_{t-1} - \frac{1}{\theta_{ss}-1}\frac{N-1}{N}p_{t-1}$$

with a variance equal to $\frac{1}{N}$ multiplied by the variance of consumption in the economy plus $(\frac{1}{1-\theta_{ss}})^2\frac{N-1}{N^3}$ multiplied by the variance of the newly set prices in the economy.

As a result, if $\bar{M}$ is defined to be:

$$\bar{M} = \begin{bmatrix} M(1) & M(3) & M(5) & M(7) \\ M(2) & M(4) & M(6) & M(8) \end{bmatrix}$$

where $M(n)$ is the nth column of M, then each agent observes:

$$h_{j,t} = B\bar{M}'X_{t-1} + \eta_{j,t}$$

[16]This is postulated at the moment, but can be verified from the solution of the model.

where:

$$B = \begin{bmatrix} 1 & -\frac{1}{\theta_{ss}-1}\frac{N-1}{N} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and η_t is the observation noise with variance:[17]

$$\sigma_\eta^2 = \begin{bmatrix} \frac{1}{N}\sigma_c^2 + (\frac{1}{1-\theta_{ss}})^2\frac{N-1}{N^3}\sigma_p^2 & 0 & 0 & 0 \\ 0 & \frac{1}{N}\sigma_p^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\nu,\theta}^2 \end{bmatrix}$$

If this is the start of the recursion, any values of σ_c^2 and σ_p^2 can be assumed. If it is not, denote previous mean square error matrix used in the Kalman filter as Σ. In the algorithm, calculations were made for the mean beliefs of the population for the state space. Therefore the mean square error matrix can be calculated for average beliefs, denoted Σ_a. Let E_aX_t be the average expectation of the state space X_t. It is easy to show:

$$\Sigma = \Sigma_a + E((E_aX_t - E_jX_t)^2)$$

The last term is the disagreement of the population, which forms the basis for σ_c^2 and σ_p^2. Denote this $V = \Sigma - \Sigma_a$. Iterating Equation 16 forward gives:

$$c_{j,t} = E_{j,t}\left(\sum_{T=t}^{\infty}\frac{1}{\sigma}(i_T - \pi_{T+1})\right) + \bar{c}_{j,t}$$

where $\bar{c}_{j,t}$ is period t's expected deviation of consumption from steady state as T approaches infinity. This is affected by changes in permanent income. From this point I take the solution as $\beta \to 1$. Taking the limit rather than setting $\beta = 1$ avoids issues relating to infinite value functions. This implies that changes through time in $\bar{c}_{j,t}$ become arbitrarily small, so we can set $\bar{c}_{j,t} = 0\ \forall j,t$. Therefore:

$$c_{j,t} = E_{j,t}\sum_{s=0}^{\infty}(-\sigma(i_{t+s} - (p_{t+1+s} + p_{t+s}))) = E_{j,t}\sum_{s=0}^{\infty}\phi X_{t+s} = E_{j,t}\Phi X_t \tag{23}$$

where:[18]

$$\phi = -\sigma\begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}\bar{M} + \sigma\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}\bar{M}F,\ \Phi = \phi\sum_{s=0}^{\infty}F^s$$

Therefore the variance of $c_{j,t}$ is given by:

$$\sigma_c^2 = \Phi V \Phi'$$

[17] The covariances are zero because each observation is unrelated to the other

[18] Truncations of n periods were used to get around issues of infinite sums.

Price setters' optimal behavior is given by Equation 20. This can be written in the form

$$p^*_{j,t} = \alpha E_{j,t} \sum_{s=0}^{\infty} (1-\alpha)^{s+1} \tilde{\phi} X_{t+s} = E_{j,t} \Psi X_t \tag{24}$$

where $\tilde{\phi} = \left[\begin{array}{cccc} K & 1 & 0 & L \end{array} \right] + M\Phi$, $\Psi = \sum_{s=0}^{\infty}(1-\alpha)^{s+1}\tilde{\phi}F^s$, and K, L, M are defined as in Equation 20 and Φ comes from Equation 23. Therefore:

$$\sigma_p^2 = \Psi V \Psi'$$

D.4 Proof that the aggregate economy follows the same path as that chosen by an agent that observes no noise

Beliefs are determined by the Kalman filter[19] so:

$$\begin{aligned} E_{j,t}X_t &= E_{j,t-1}X_{t-1} + \kappa(B\bar{M}'X_t + \eta_{j,t} - B\bar{M}'FE_{j,t-1}X_{t-1}) \\ &= \sum_{s=0}^{\infty}(I - \kappa B\bar{M}'F)^s \kappa(B\bar{M}'X_{t-s} + \eta_{j,t-s}) \end{aligned} \tag{25}$$

where κ is the Kalman gains matrix and I is the identity matrix. Substituting this into Equation 23 or 24 gives:

$$x_{j,t} = \Lambda \sum_{s=0}^{\infty}(I - \kappa B\bar{M}'F)^s \kappa(B\bar{M}'X_{t-s} + \eta_{j,t-s}),\ \Lambda = \Phi, \Psi \tag{26}$$

For the agent that observes no noise, $\eta_{i,t-v}$ is a zero matrix for all v. If we integrate Equation 26 over all individuals then $\eta_{j,t-s}$ cancels out also.

D.5 Calculating a new impulse response function

From the result of Appendix D.4 only the stand in agent who observes no noise needs to be considered. I first consider an interest rate shock. For the period after the shock the stand in agent has seen all variables at their baseline level in the past (which was zero) and observes:

$$h_1 = B\bar{M}'X_1$$

where X_1 has a 1 at $(n+2,1)$ (corresponding to the interest rate shock) and zeros everywhere else. The agent then forms beliefs about the state space of the economy, $E_2X_2 = E_2FX_1$ using the Kalman filter and makes their decisions using Equations 23 and 24. This gives the first entry for

[19] Given an M there may or may not be long term effects on the variables. If less than two of them have long term effects, then there is less than two linearly independent contributions of the infinite sum parts of Equation 21 to what agents see. If that is the case, then the mean square error matrix used in the calculation of the Kalman filter will not converge. What can be done in this case is to remove from the state space these infinite sums and replace them with the linear combination of infinite sums that does affect the variables observed (if there is one, if there is not one, then all can be ignored).

price/output gap in the new impulse response, with aggregate price given by:

$$p_t = (1-\alpha)p_{t-1} + \alpha p_t^* \tag{27}$$

where the last term is given by Equation 24 for the stand in agent. This follows from Appendix E. The new value for the interest rate will be given by Equation 9.

This process is then repeated for the next $n-1$ periods and all shocks. This is the mapping that takes us from M to the new impulse response M^*, and is iterated until convergence. Sometimes to prevent destabilizing oscillations partial adjustment is required.

The adequacy of the state space postulated in Appendix D.2 is proven as Equations 23 and 24 are lienar functions of the postulated state space.

D.6 Proof that the fixed point is an equilibrium

Points 2 and 4 from the definition are trivially true because they are used in the construction at each iteration. Point 1 follows from the use of the Kalman filter by the agents, and that the actual motion of the economy follows the aggregate that the agents choose as a result of the fixed point of the iterative procedure. Point 3 follows because the new impulse responses are constructed using the solution to this problem at each iteration.

E Evolution of the aggregate price level

The price index, P_t, used to compute the inflation in Equation 10 is can be transformed to yield

$$P_t^{\frac{\theta_t}{\theta_t-1}} = \int_{j\in R} P_{j,t}^{\frac{\theta_t}{\theta_t-1}} dj + \int_{j\notin R} \int_0^1 P_{j,t}^{\frac{\theta_t}{\theta_t-1}} dj \tag{28}$$

where R is the set of firms who get to reset their price in period t.

$\int_{j\notin R} P_{j,t}^{\frac{\theta_t}{\theta_t-1}} dj = (1-\alpha)\int_0^1 P_{j,t-1}^{\frac{\theta_t}{\theta_t-1}} dj$ because the agents that get to reset their prices are chosen randomly. Therefore Equation 28 becomes

$$P_t^{\frac{\theta_t}{\theta_t-1}} = \alpha e^{(\frac{\theta_t}{\theta_t-1})p_{R,t}} + (1-\alpha)P_{t-1}^{\frac{\theta_t}{\theta_t-1}} \tag{29}$$

If we approximate around the point where, $p_{R,t} = p_{t-1}$ and $\theta_t = \theta_{ss}$, and where information is perfect, then we get with some rearranging of the above:

$$p_t = \alpha p_{R,t} + (1-\alpha)p_{t-1} = (1-\alpha)p_{t-1} + \int_{j\in R} p_{j,t}^*$$

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