ANNEX D

Estimation of Exponential Cooling Rates for Shell Eggs

INTRODUCTION

From lay to consumption, eggs are stored in various environments with different ambient temperatures. Growth kinetics of *Salmonella* depend on temperature; thus, to model the growth of *Salmonella* within eggs it is necessary to know the temperature of the eggs at all times. Consequently, models that can predict the temperatures of the eggs as a function of the ambient temperature are needed. This annex presents models that are used for determining the temperature of the eggs that cool down after lay and processing (i.e., washing, candling, grading, and packaging). The primary parameter that determines the changes of temperatures is the exponential cooling rate, *k*.

Exponential cooling rates of shell eggs are influenced by a number of factors, including air movement, ambient temperature, palleting methods, and packaging materials.[1] The information used to estimate exponential cooling rates was obtained from studies by Bell and Curley,[2] Anderson et al.,[3] Czarick and Savage,[1] Stadelman and Rhorer,[4] and Keener et al.[5] Exponential cooling rates were derived from each of these studies using the internal temperatures of eggs that were located in the center of packages (i.e., cases or pallets)[a] and assuming a constant ambient temperature. (Data were not available to provide estimates of the differences in internal temperatures of eggs throughout a pallet.) Because cooling is slowest at the center of the package, the derived temperature profiles would considerably overestimate the temperatures of many eggs not located in this area. The "worst case" would be inappropriate to use for describing temperature profiles for these eggs. Consequently, an adjustment is made to the derived temperature curves that accounts for the location of eggs in the packaging material.

ANALYSIS

For modeling temperature change over time, it was assumed, for the most part, that there is a log-linear relationship between temperature and time, which is described by a single parameter *k*, the exponential cooling rate. Specifically, it was assumed that, as a function of time, *t*, the temperature, $T(t)$ in °C of an egg at the center of the container could be described by

$$\ln[u(T(t)] = -kt \tag{D1}$$

and

$$u(T(t)) = \frac{T(t) - T_a}{T_i - T_a} \tag{D2}$$

[a]Description of egg case and pallet: The size of an egg case is 2 ft long x 1 ft wide x 14 in high. A pallet of eggs consists of 30 egg cases (2 x 3 cases per level and 5 levels). The rough outside dimensions are 3 ft wide x 4 ft long x 6 ft high, including the pallet. Cases are stacked centered on the pallet and then wrapped together with plastic wrap to keep them from shifting during transport.[3]

where $\mu(T(t))$ is the proportional change in temperature relative to initial temperature, T_i, of the egg, and the ambient temperature, T_a, of the surrounding environment (cooler).

With some exceptions, which are described below, values of k were estimated from data provided in the above-mentioned studies assuming Equation D1. Exceptional cases were identified because, when the data were plotted, the temperature profile did not follow what would be expected based on Equation D1. By examining the graphs of 13 cooling experiments from the above-mentioned studies, three patterns of temperature profile curves were noted. These are summarized below (the experiment number refers to that given in Table D1). Also, see figures D1-D8.

1. The temperature was observed to decline immediately, and this decline was adequately described by Equation D1 (experiments 2, 3, 5, 6, 7, 8, 9, 10, and 12-trial 1).

2. An increase in temperature was observed before temperature decline began. This can occur for eggs that have just been washed and candled, because when these eggs are packed, the packaging initially acts as insulation, thereby preventing decrease of egg temperatures, particularly for eggs in the center of the packaging material. To accommodate this, the following equation was used:

$$\ln [u(T(t)] = -kt - c(e^{-bt}-1) \qquad \text{(D3)}$$

   where k is the asymptotic exponential cooling rate, and b and c are parameters to be estimated. This equation, for appropriate values of b and c, captures the increase in temperature prior to the decline (experiment 1, 4, 11, 12-trial 2, and 13).

3. The result from experiment 1 in Table D1 represents a unique situation. Temperature decline was slow, and the observed temperature profile did not display the expected asymptotic behavior implied by Equation D1 but rather appeared linear with time. However, a linear relationship is unrealistic because, besides not conforming to theory, the projected temperatures for sufficiently large times would provide estimates of egg temperature that fall below the ambient temperature of the cooler, which is not possible. In these situations, it was not possible to derive unique solutions using Equation D3.

4. Rather, for experiment 1, the best-fit estimates of k and c were obtained by fixing the value of b to 0.1, which approximates the estimates obtained from experiments 4 and 13.

TABLE D1 THE ESTIMATED COOLING RATES OF EGGS WITH VARIOUS PACKING METHODS. FOR ANY DATA USED IN THE PREVIOUS RISK ASSESSMENT ON SE,[6] REFERENCE TO THAT ASSESSMENT IS NOTED IN PARENTHESES.

Experiment Number	Packing Method	Exponential Cooling Rate, *k* (min–max)	Note
1	Pallet of cardboard (in-line)[a] (Figure 4) **(SERA: Pallet, cardboard and fiber flats, In-line)* Pallet of cardboard cases (Figure 13)[b]	k = 0.00943 b = 0.100 c = 0.00946 n = 1 *(SERA: 0.0075)* Exponential: k = 0.0075	Equation D3 *b* was fixed as 0.1

2	*(SERA: Pallet, cardboard boxes)*	n = 1 *(SERA: 0.008)*	Equation D1
3	Individual case/basket temperature (Figure15)[b] *(SERA: Pallet, cardboard and fiber flats, Styrofoam)*	k = 0.0131 n = 1 *(SERA: 0.013)*	Equation D1 T_a = 7.22°C, T_i = 34.72°C
4	Pallet of cardboard (off-line)[a] (Constant ambient temperature) (Figure 6) *(SERA: Pallet, cardboard, off-line)*	k = 0.00626 b = 0.103 c = 0.279 n = 1 *(SERA: 0.035)*	Equation D3 T_a = 9.44°C, T_i = 24.44°C (T_{max} = 26.39°C)
5	Pallet of plastic basket cases (Figure 14)[b] (location 3) *(SERA: Single cardboard cases)*	k = 0.0524 n = 1 *(SERA: 0.052)*	Equation D1 T_a = 7.22°C T_i = 28.61°C
6	Plastic and fiber filler flats, fiber case, closed Formed and folded cartons, fiber case, closed (curve D)[c] *(SERA: Flats, closed)*	k = 0.0628 n = 1 *(SERA: 0.07)*	Equation D1
7	Formed and folded cartons, open stack Formed and folded cartons, wood case Plastic and fiber filler flats, wood case Plastic and fiber filler flats, fiber case, open (curve C[c]) *(SERA: flats, folded shut)*	k (range) = 0.08-0.1297 k (geometric mean) = 0.100 n = 3 *(SERA:0.08-0.14)*	Equation D1
8	Plastic and fiber filler flats, open stack (curve B)[c] *(SERA: Open stack)*	k = 0.19-0.39 k (geometric mean) = 0.275 n = 3 *(SERA:0.2-0.4)*	Equation D1
9	(1) Filler flats[d] (2) Fiberboard case (30-dozen) Foam cartons (closed top) (3) Fiberboard case (30-dozen) Foam cartons (slotted top) *(SERA: Fiber case, foam cartons with and without slots, moving air)*	(1) k = 0.240 (2) k = 0.216 (3) k = 0.231 k (geometric mean) = 0.228 n = 1 *(SERA: 0.24)*	Equation D1 This study concluded that the packaging material has no significant influence on cooling rate. The three k values are close enough to be considered as single data.
10	Fiber filler flats or fiber cases with forced air cooling through opening in cases (curve A)[c]*(SERA: Open stack, forced air)*	k = 0.39-0.97 k (geometric mean) = 0.615 n = 3 *(SERA:0.4-1.0)*	Equation D1
11	Pallet of cardboard cases (flats) (Figure 16)[b]	k = 0.0472 b = 6.218 c = 0.283 n = 1	Equation D3: Ta = 7.22°C Ti = 25°C (T_{max} = 30°C)

12	Pallet, cardboard cases (Traditional cooling)[e]	k_1 = 0.0160 k_2 = 0.0270 b = 4.078 c = 0.229 k (geometric mean) = 0.0215 n = 2	Trial 1: Equation D1(k_1) Trial 2: Equation D3 (k_2)
13	Pallet, cardboard (off-line)[a] (Figure D1) (Fluctuated ambient temperature)	k = 0.0064 b = 0.116 c = 0.360 n = 1	Equation D3 Ta = 10 °C Ti = 26.67°C (T_{max} = 30.00°C)

[a]Anderson et al.[3]
[b]Czarick and Savage.[1]
[c]Bell and Curley.[2]
[d]Stadelman and Rhorer.[4]
[e]Keener et al.[5]

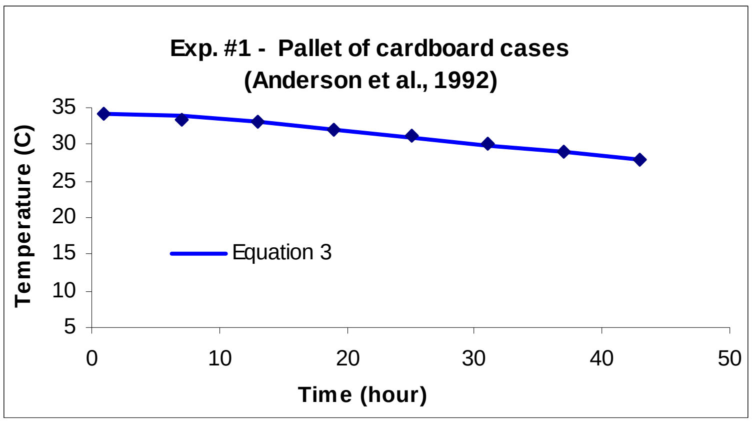

FIGURE D1 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET OF CARDBOARD CASES—EXPERIMENT #1.[3]

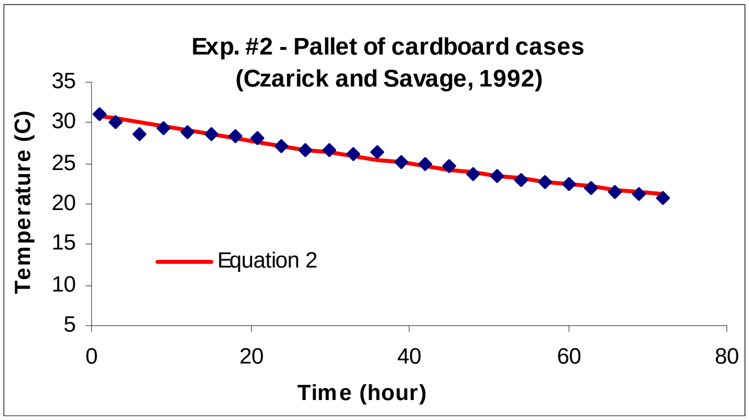

FIGURE D2 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET OF CARDBOARD CASES—EXPERIMENT #2.[1]

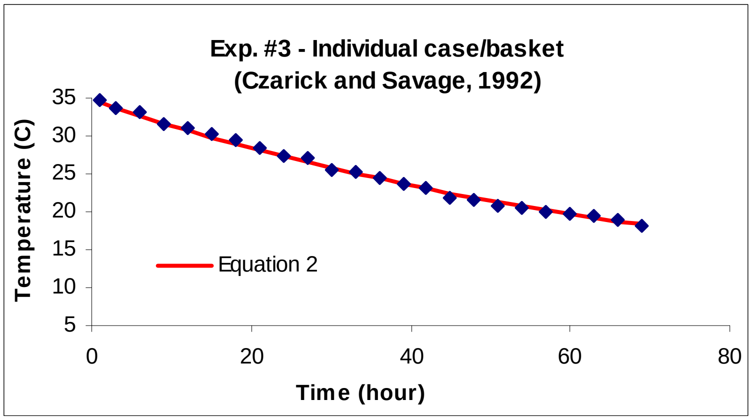

FIGURE D3 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: INDIVIDUAL CASE/BASKET—EXPERIMENT #3.[1]

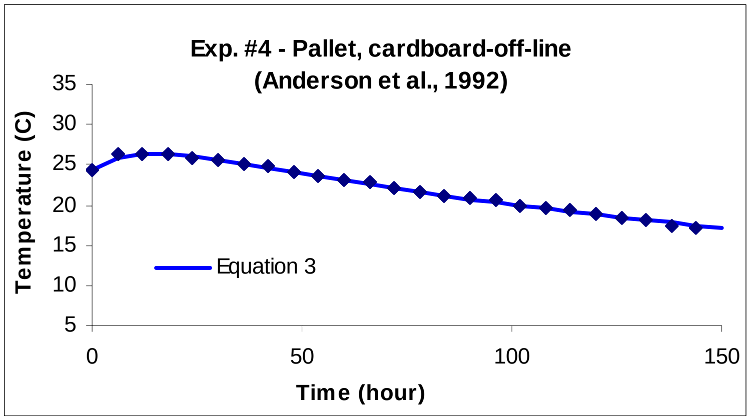

FIGURE D4 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET, CARDBOARD OFF-LINE—EXPERIMENT #4.[3]

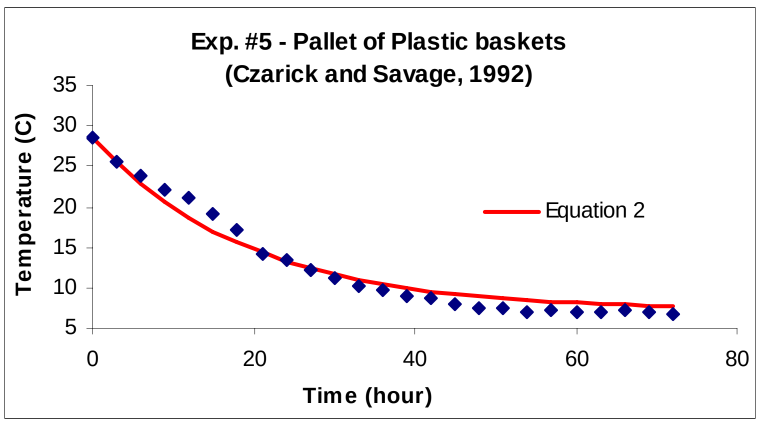

FIGURE D5 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET OF PLASTIC BASKETS—EXPERIMENT #5.[1]

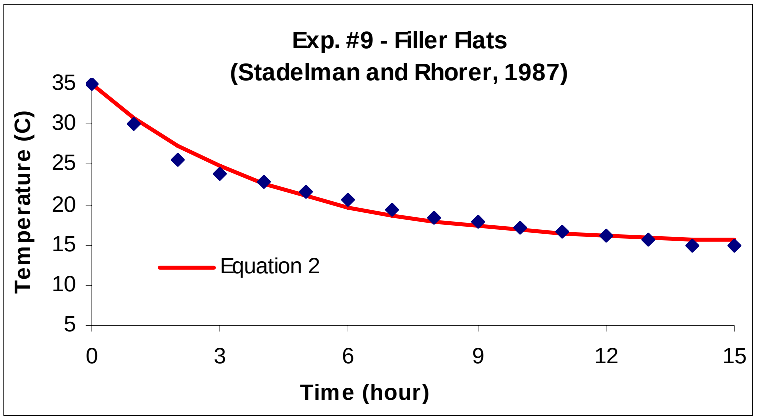

FIGURE D6 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: FILLER FLATS—EXPERIMENT #9.[4]

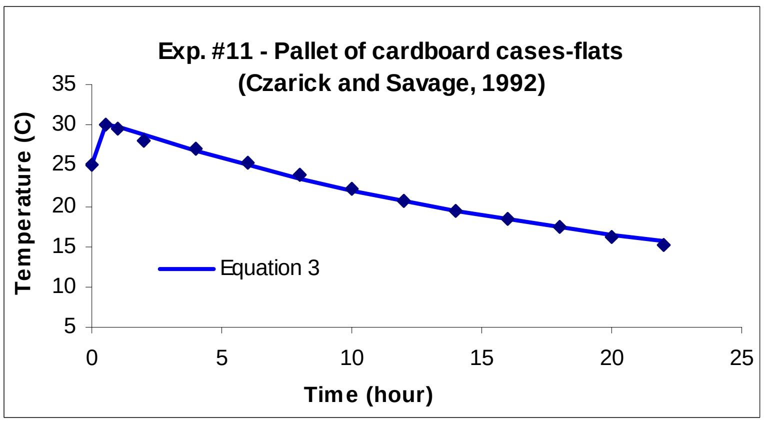

FIGURE D7 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET OF CARDBOARD CASES—FLAT—EXPERIMENT #11.[1]

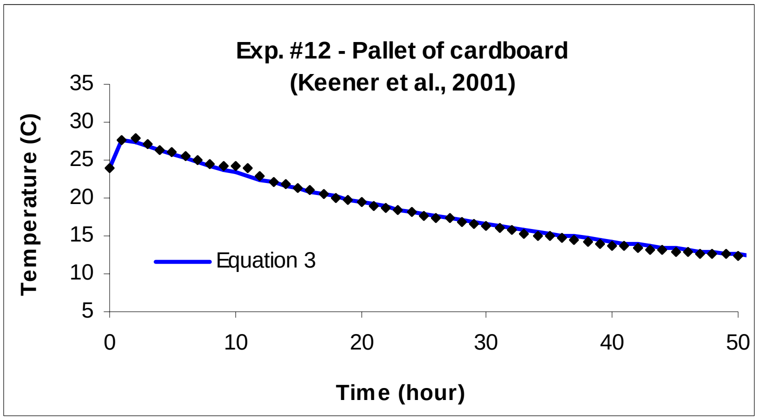

FIGURE D8 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET OF CARDBOARD—EXPERIMENT #12.[5]

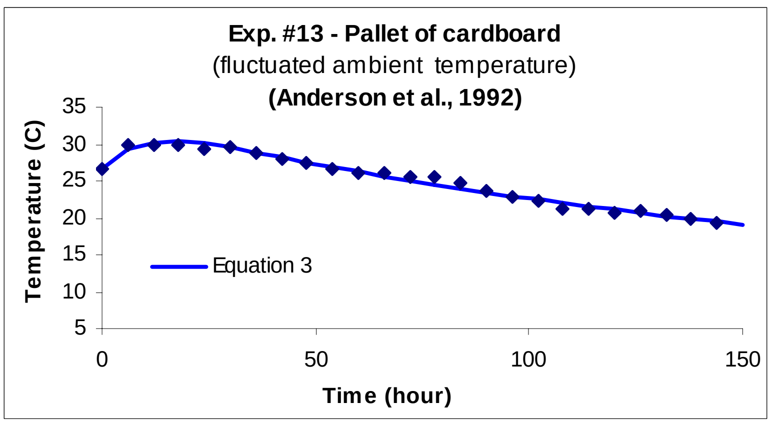

FIGURE D9 OBSERVED AND PREDICTED TEMPERATURES VERSUS TIME FOR AVAILABLE EXPERIMENTAL DATA: PALLET OF CARDBOARD—EXPERIMENT #13.[3]

THE DISTRIBUTION USED FOR MODELING EXPONENTIAL COOLING RATES

Most of the experiments consisted of a single trial; however, four experiments (experiments 7, 8, 10, and 12) included more than one trial or replicate. A fifth experiment (experiment 9) consisted of a comparison of three types of packaging material. Thus, initially, these three results were considered results from three replicates. In was assumed that, for similar conditions (i.e., the same packaging material), the variable k is random with a distribution F. Given the dearth of information, this assumption seems reasonable because there are many (random) factors that could influence values of k. Such factors are associated with the features of the eggs and of the packaging materials that were not explicitly accounted for in these experiments. More data in controlled experiments are needed to validate this assumption or to identify factors that influence values of k.

Individual estimated values of *k* were not given in studies that report results of experiments with more than two replicates; rather the maximum and minimum values were reported. In addition, the number of replicates was small, usually two or three. Thus, it is not possible to estimate *F* without some simplifying assumptions. Consequently, to estimate the values of the parameters that characterize the distribution of *F*, an underlying normal distribution was assumed. An analysis of the data indicated that the coefficient variances (CV) of the estimated *k* values (Table D2) were not correlated with the estimated mean values of *k*. This property, and the fact that *k* takes only positive values, suggests that the distribution of *k* can be assumed to be lognormal, or that the logarithm of *k*, ln(*k*), can be a normal distribution (μ, σ), where μ is the expected value and σ is the standard deviation of ln(*k*). Estimates of μ involve straightforward estimation procedures of a mean value, using the midrange when the maximum and minimum values were recorded. The remainder of this section describes the method used to estimate the variance (σ^2).

Because there are only small numbers of observations or replicates for each experiment (n = 2 or 3), there may be a large amount of uncertainty associated with the estimates of the mean value of ln(*k*). Therefore, the variance estimates were pooled over results from five different experiments with more than one replicate to increase the degrees of freedom associated with the estimates.

Three of these experiments (experiments 7, 8, and 10) are from Bell and Curley,[2] who reported the results as ranges. From this study, the maximum and minimum ln(*k*) values were estimated using the reported maximum and minimum times for eggs to cool to given temperatures, assuming the log-linear relationship given in Equation D1.[b] It was stated that typically, three replicates were performed, but for some unspecified occasions, more were done. Consequently, it was assumed that the number of replicates in these experiments was equal to 3 (a conservative assumption insofar as it entails the maximum amount of uncertainty that can be assigned given what only is known is that there were at least 3 replicates). The fourth experiment (exp. 12) had two replicates.[5] The fifth experiment (exp. 9) involved three different packaging materials/methods for which the estimates of the exponential cooling rates were close, so that the study concluded "no significant differences due to packaging" were evident.[4] Therefore, one might consider these three results as three realizations of independent replicate experiments. However, after examining the description of the experimental design, we realized the three results could not be justifiably considered as independent realizations. This was supported by a comparison of the variances derived from this experiment and the other four experiments. An *F*-test was performed, comparing the variance obtained by pooling results, as described below, over the other four experiments along with the unbiased estimate of variance of the three results from the fifth experiment. The *F*-statistic was found to be statistically significant at the 0.02 significance level. Consequently, we did not include this experiment when pooling the variances.

---

[b] Note: In one experiment the estimated *k* value was derived from graphical analysis.

TABLE D2 DATA AND CALCULATIONS USED TO DETERMINE EXPONENTIAL COOLING RATES, κ, AND STANDARD DEVIATIONS FOR LN(κ)

	Data # 10[2]	**Data # 8**[2]	**Data # 7**[2]	**Data # 12**[5]
n	3	3	3	2
Min ln (*k*1)	–0.944	–1.637	–2.553	–4.135
Max ln (*k*2)	–0.027	–0.944	–2.042	–3.611
Mean (ln*k*)	–0.486	–1.290	–2.298	–3.873
κ	0.274	0.274	0.274	0.274
range, *R*(*k*2 – *k*1)	0.916	0.693	0.511	
R^2	0.840	0.480	0.261	
Variance (R^2 X κ)	0.230	0.132	0.071	0.137
Variance (total), *v*	0.143	0.143	0.143	0.143
Standard deviation	0.379	0.379	0.379	0.379
k (predicted)	0.615	0.275	0.100	0.021

Pooled variances were calculated as follows: when $n = 3$, the estimates of the variances, *v*, were made by multiplying the square of the range (*R*) of ln(*k*) values by an appropriate value, κ, chosen such that the *v* would be an unbiased estimator of the variance, assuming an underlying normal distribution ($v = \kappa R^2$). Through simulation it was determined that $\kappa = 0.274$. This value could also be approximated by noting that the distribution of $(R/\sigma)^{1.96}$ is approximated by 1.725 times a chi-square with 2.05 degrees of freedom.[7] From this, it is approximated that the expected ratio of R^2 and σ^2 is about 0.276. This result, however, also shows that, in approximation, (v/σ^2) can be considered distributed as a chi-square with two degrees of freedom. Consequently, the pooled estimate of variance (*V*) was determined by taking a weighted average of the *v*'s, where the weights are equal to the degrees of freedom, $n - 1$. The distribution of *V* was examined through simulations: equating the first two moments of the simulated results to those of a chi-square distribution with υ degrees of freedom, it is derived that the distribution of *V* can be assumed proportional to a chi-square distribution with 6.9 degrees of freedom. The estimate *V* is 0.143; thus, the estimate of the standard deviation, *s*, is 0.379. Further details of calculations are given in Table D2, which contains maximum and minimum of ln(*k*) values, the estimated variances for each of the four experiments, and the estimated mean of ln(*k*) and the anti-log of the estimated mean ln(*k*). Table D1 includes the estimated *k* values for all experiments, which represent the anti-log of the mean of ln(*k*).

ASSUMPTIONS FOR MODELING THE EXPONENTIAL COOLING RATE

It was assumed that the packaging materials correspond to one of the entries listed in Table D1. It was assumed that ln(*k*) is normal distributed with mean, μ, obtained from Table D1, which gives values of *k*, and standard deviation, σ, equal to 0.379. The uncertainties of these values are realized by generating values, *m* and *s*, respectively, for the mean and standard deviation. Based on the above analysis, generated values are determined as follows.

1. A standard deviation, *s*, is generated assuming that σ^2/s^2 is distributed as a chi-square with 6.9 degrees of freedom.

2. A mean value of the ln(*k*), *m*, is generated by considering the statistic $\tau = n^{0.5}(\mu - m)/\sigma$. When μ is estimated by an average, then τ was assumed distributed as a *t*-distribution with 6.9 degrees of freedom. When μ is estimated by a midrange, then, based on a simulation (of 1,000,000) and fitting the first two moments of τ^2, it was assumed that τ is proportional to a *t*-distribution with proportionality constant, $c = 1.044$ and degrees of freedom, $v = 7.4$. The percentiles of τ^2 from the simulation agreed closely with those from the derived distribution: the observed 99th percentile was 12.83, the theoretical one was 12.86; for the 75th percentile, the observed was 1.701, and the theoretical one was 1.697.

3. For the cases where Equation D3 would apply (Table D1), it was assumed that *b* is constant, and that *c* varies directly proportional to *k*.

To determine temperatures for eggs that are not near the center of the pallet, box, or carton, a simplified approach is used. The basic heat transfer equation for conduction is

$$\frac{\partial u(T(t),x)}{\partial t} = \alpha \frac{\partial^2 u(T(t),x)}{\partial x^2} \qquad \text{(D4)}$$

with boundary conditions

$$u(T(0),x) = 1,\ u(T(t),x_c) = 0,\ \frac{\partial u(T(t),0)}{\partial x} = 0 \qquad \text{(D5)}$$

where x_c is some fixed distance to the boundary from the center ($x = 0$) of the object being cooled.[8] The above set of conditions imply that the initial temperature of the body is uniform, the ambient temperature is constant at all times and is the same as the temperature at the surface, and that cooling at the center of the pack occurs symmetrically, that is, the temperature is always greatest at the center of the pack. These assumptions probably do not hold precisely; however, they serve as convenient assumptions. The solutions for various shaped geometric bodies have been developed and are given by Zwietering and Hasting.[8] For a given geometric shape, for a sufficiently large time, the equation

$$u(T(t),x_c) = he^{-g\alpha t/x_c^2} \qquad \text{(D6)}$$

describes the temperature change in the center, where *h* and *g* are known constants. Comparing Equation D6 to Equation D1, for large *t*, α can be approximated as kx_c^2 / g, which suggests that for a distance *x* from the boundary, $u(T(t), x)$ be approximated in a simple and rough fashion as

$$u(T(t),x) = u(T(t),x_c)\exp(-kt(\left(\frac{x_c}{x}\right)^2 - 1)) \qquad \text{(D7)}$$

For packaging materials for which the cooling curves (for the egg at the center of the packaging material) were observed to be log-linear in temperature and time, Equation D7 simplifies to

$$u(T(t), x) = exp(-k(\tfrac{x_c}{x})^2) \qquad \text{(D8)}$$

The distance x cannot equal 0 insofar as the eggs are placed within boxes or cartons some distance from the boundary. For example, when x is half the distance between the edge of the boundary and the center, then the coefficient of kt in Equation D7 is –3, and if $u(T(t), x_c) = \exp(-kt)$, then $u(T(t), x) = \exp(-4kt)$; in effect, the exponential cooling rate increases by a factor of 4; if x is one-third the distance, then k in effect increases by a factor of 9.

The inputs needed to use Equation D7 or D8 are the type of packaging material, the location of the egg relative to the boundary and center of the packaging material, the ambient temperature, and the temperature of the egg when the ambient temperature changes. The temperature of the egg at lay was assumed approximately 41°C, which is the body temperature of the hen.

REFERENCES

1. Czarick M and Savage S. Egg cooling characteristics in commercial egg coolers. Journal of Applied Poultry Research 1992;1:258-70.

2. Bell DD and Curley RG. Egg cooling rates affected by containers. California Agriculture 1966;2-3.

3. Anderson KE, Jones TF, and Curtis PA. Legislation ignores technology. Egg Industry 1992;September/October:11-3.

4. Stadelman WJ and Rhorer AR. A comparison of quality of eggs from in-line and simulated oof-line production units. Egg Industry 1987;October:8-10.

5. Keener, K. M. , Curtis, P. A., Anderson, K. E., and Conner, D. E. Quantification of shell egg quality, shelf-life, and safety using rapid cooling with carbon dioxide gas. Report to U.S. Poultry and Egg Association and PRAXIR, Inc. 2001.

6. Food Safety and Inspection Service/Food and Drug Administration. *Salmonella* Enteritidis risk assessment. Shell eggs and egg products. Final Report. Avaialble at http://www.fsis.usda.gov/ophs/index.htm. 1998.

7. Johnson NL, Kotz S. Distribution in statistics, continuous univariate distributions-1. Ney York: John Wiley and Sons, 1970.

8. Zwiertering MH and Hasting APM. Modeling the hygienic processing of foods - influence of individual processing stages. Trans I ChemE 1997;75:168-73.