NASA Explorer Schools Pre-Algebra Unit

Lesson 3 Teacher Guide

Solar System Math

Comparing Planetary Travel Distances

http://quest.nasa.gov/vft/#wtd

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NOTE: A “session” is considered to be one 40-50 minute class period.

Solar System Math

Comparing Planetary Travel Distances

Lesson 3

How far can humans travel in our solar system?

Introduction

In this lesson, students will use the geometry of circles to calculate the distance a crew vehicle would travel to reach another planet or moon in our solar system. Using the speed at which a crew vehicle would be traveling, students will calculate how long each mission would take. Based on this information, students will decide which planets or moons are too far away for humans to visit.

Lesson 3 – OBJECTIVES, SKILLS, & CONCEPTS

Main Concept

Some planets are so far away that, with current technology, it would take a very long time for humans to reach them.

Instructional Objectives

During this lesson, students will:

- Use the geometry of circles to calculate the distances a crew vehicle would travel from Earth to other planets and moons.
- Use the speed of a crew vehicle to calculate the time a journey to each planet or moon would take.
- Combine the time of a roundtrip journey to one of the planets or moons with the length of a synodic period between Earth and that planet or moon to determine the total mission length.
- Use ratio and proportion, fractions, decimals, and percentages to compare mission lengths to average lifetimes and average careers.
- Choose data points to graph and be able to communicate why particular data points and types of graphs are chosen.
- Consider the different mission lengths and decide which planets or moons are too far away for humans to visit.

Major Focus Skills

Math topics covered in this lesson:

- Converting units
- Calculating speed using distance and time
- Solving speed problems for distance or time
- Graphing and data representation
- Ratio and proportion
- Converting metric units, customary units, and time units

Major Focus Concepts

Math

- Seconds, minutes, hours, days, weeks, and years are used to measure time.
- Algebra can be used to find the shortest distance from one planet to another planet, given the average distance from each planet to the Sun.
- The length of time it takes to travel from one planet to another planet can be calculated by dividing the distance by the average rate or speed at which a crew vehicle travels.
- One way to convert seconds to days is to first divide seconds by 60 seconds/minute to convert from seconds to minutes, then divide minutes by 60 minutes/hour to convert from minutes to hours, then divide hours by 24 hours/day to convert from hours to days.
- To find the total length of time needed for a mission's travel, you will need to add the amount of time it takes to travel to and from the planet, plus the amount of time for the mission (which will need to be a multiple of the amount of time between planetary alignments, i.e. the synodic period).

Science

- Speed is distance traveled per unit time and can be used to calculate the length of time it would take to travel a specified distance.
- The distances between the planets are constantly changing because they all orbit the Sun at different rates.
- The length of time it would take to travel from Earth to other planets or moons depends on the location of the planets and moons at the time of launch and arrival as well as the length of time between planetary alignments.

Prerequisite Skills and Concepts

Math Skills

- Ratio and proportion (Lesson 1)
- Units of time and how to convert from one time unit to another time unit (seconds, minutes, hours, days, years)
- Formula for circumference of a circle using diameter or radius (Lesson 2)
- Rounding decimals
- Basic calculator functions
- Concepts of and relationships between decimals, fractions, and percents
- Conversion between decimals, fractions, and percents

- Bar graphs—useful for comparing quantities such as distances or times (Lesson 1)
- Pie graphs and number lines—useful for comparing parts of a whole, such as fractions, decimals, or percents (Lesson 1)

Science Concepts

- The Earth is the third planet from the Sun in a system that includes Earth's Moon, the Sun, eight other planets and their moons, and smaller objects such as asteroids and comets—all of which vary greatly in terms of their size, distance from the Sun, mass, density, and composition. (Lessons 1 and 2)
- The planets in our solar system are not often in alignment with each other. (Lesson 1)

NATIONAL EDUCATION STANDARDS	
Fully Met	Partially Met
NCTM	NCTM
(3-5) Measurement #1.3	(3-5) Measurement #1.1
(3-5) Data Analysis and Probability #1.3	(3-5) Measurement #2.2
(6-8) Number and Operations #1.4	(6-8) Number and Operations #1.2
(6-8) Measurement #1.2	(6-8) Measurement #1.1
(6-8) Measurement #2.5	(6-8) Data Analysis and Probability #1.2
(6-8) Measurement #2.6	
Problem Solving #1	
Problem Solving #2	
Communication #2	
Connections #3	

SW = student workbook   TG = teacher guide   EG = educator guide

Lesson 3 – PRE-LESSON ACTIVITY

- **Estimated Time:** 2 sessions, 40 minutes each
- **Materials:**
  - Pre-Lesson Activity worksheet Part I (SW p.2)
  - Distance, Time, & Speed: Equations worksheet (SW p.3)
  - Distance, Time, & Speed: Problems I, II, III worksheets (SW pp.4-6)
  - Pre-Lesson Activity worksheet Part II (SW p.7)
  - Measuring tape, yardstick, or meterstick
  - Chalk or tape
  - Stopwatch
  - Calculators
  - Math Review: Converting Units Teacher Resource (TG pp.10-16)

Before beginning the lesson, it is important that students understand the concept of **speed** and how it applies to both **distance** and **time**.

Class Discussion

First, poll the class to see who considers themselves to be good runners? Then, ask those students to estimate how fast they think they can run a particular distance. Responses should include a *number and a unit for the distance* and a *number and a unit for the time*. (For example: 1 mile in 11 minutes or 100 meters in 16 seconds.) Next, have students complete the **Pre-Lesson Activity Part I** worksheet (SW p.2) addressing units of measurement for distance, time, and speed.

Misconception Alert!

Ask the students if they know of another word that means speed? (velocity) Speed and velocity both refer to distance traveled per unit time. However, in physics and mathematics, velocity is a vector, which means that it also takes direction into account; it is how fast an object moves away from a given point in the direction of another point. Some people may use the word velocity and speed interchangeably, but keep in mind that speed is a number, while *velocity represents a number as well as a direction.*

For the purposes of this lesson, **students will be using values of constant speed.** While the actual speed of the crew vehicle may vary at different times, we will use a constant value based on the average speed at which the crew vehicle will be traveling.

Next, direct students to recognize key unit ratios for the conversion of the following time units:

seconds to minutes	minutes to hours	hours to days	days to years
$\frac{\text{1 minute}}{\text{60 seconds}}$	$\frac{\text{1 hour}}{\text{60 minutes}}$	$\frac{\text{1 day}}{\text{24 hours}}$	$\frac{\text{1 year}}{\text{365 days}}$

The reciprocal of each of these unit ratios is also a unit ratio.

$\frac{\text{60 seconds}}{\text{1 minute}}$	$\frac{\text{60 minutes}}{\text{1 hour}}$	$\frac{\text{24 hours}}{\text{1 day}}$	$\frac{\text{365 days}}{\text{1 year}}$

The value on the top (numerator) of a ratio and the value on the bottom (denominator) of a ratio depend on the units you are converting.

Same Distance, Different Speed

Ask the students to imagine two cars on the freeway: one traveling at 55 miles per hour and the other traveling at 65 miles per hour. If both cars were traveling a distance of 50 miles, which car would complete the 50 miles first? Why? (The car traveling at the faster speed of 65 mph would arrive first because that car covers more miles in less time.)

Same Speed, Different Distance

Now ask the students to imagine two cars traveling at the same speed. Suppose Car A must travel 50 miles to its destination and Car B must travel 100 miles to its destination. If they start at the same time, which car arrives at its destination first? Why? (Since each car is traveling at the same speed, Car A would arrive first at its destination because it has fewer miles to travel.)

Distance, Time, & Speed Relationships

Use the **Distance, Time, & Speed: Equations** worksheet (SW p.3) to guide students in understanding the relationship between distance, time, and speed (rate).

Students should recognize that if two values are given in a problem involving distance, time, and speed, then they will be able to solve for the third value. If they can identify which quantity they are solving for, then they can identify the equation to use. Use the **Distance, Time, & Speed: Problems I, II, and III** worksheets (SW pp.3-6) to let students practice identifying, setting up, and solving the three equations. (Use the **Math Review: Converting Units Teacher Resource** to refresh students' skills if needed, TG pp.10-16.)

Hands-on Outdoor Activity

Divide students into groups of three comprised of 1) a measurer, 2) a timer, and 3) a runner. Using measuring tape, a yardstick, or a meterstick, have the “measurer” in each group mark off a length to run outside on the school grounds. (To make this activity easier to time, lengths should not be less than 100 meters or yards.) In measuring their distance, the groups may choose their own unit of measurement, but they must state what it is and why they chose that unit. Guide students to understand which units would be most appropriate for this exercise. For example, inches or centimeters would be inappropriate units for this activity because these small units may make it difficult to calculate an accurate speed. Students can record their data and calculations on the **Pre-Lesson Activity Part II** worksheet (SW p.7).

Next, have each “runner” run the marked distance as fast as they can while the “timer” uses a stopwatch to time the runner. The runners should run their lengths at top speed 3 times, and then the group should average the results. Next ask the runners to run their distances at a jog 3 times—they need to run slower but should try to keep a constant pace. Again, have the students average their group’s results.

Return to the classroom, and instruct each group to use the measured distance and averaged times to calculate their runner’s sprinting speed and jogging speed. (These speeds should be determined by using the speed equation: speed = distance ÷ time.)

Next, referring to key unit ratios and the three distance, time, and speed equations, discuss with the class how the measured distance and the average time it took to run that distance can be calculated in speeds of:

meters per second

feet per second

yards per second

Finally, have students apply what they have learned by comparing their two running speeds to the speed of a car by converting their speeds to miles per hour. Calculations and answers can be recorded on the bottom of the **Pre-Lesson Activity Part II** worksheet.

Math Review: Converting Units

Teacher Resource

In everyday life, we often need to convert from one unit to another unit. To do so, we can use **unit ratios**. When using unit ratios, we multiply to change one unit into another unit. This resource contains three sample problems for teachers to use as a demonstration in the classroom.

SAMPLE PROBLEM 1: How many inches are in 47 feet?

Equation: 47 feet = __ inches

Step 1: Identify the relationship between the unit you HAVE and the unit you WANT.

Relationship: 1 foot = 12 inches

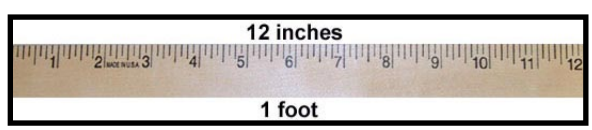
Step 2: Write the two unit ratios associated with the relationship.

$$\frac{1\text{ foot}}{12\text{ inches}} \quad \text{and} \quad \frac{12\text{ inches}}{1\text{ foot}}$$

Note that 12 inches is equal to 1 foot, so each unit ratio is actually equal to 1.

Step 3: We start with 47 feet and we want feet to cancel, so use the unit ratio that has feet (or foot) in the denominator.

Unit Ratio: $\frac{12\text{ inches}}{1\text{ foot}}$



This is the unit ratio you will use to solve the problem because it will leave you with inches.

Step 4: Multiply the number you have (47 feet) by the unit ratio.

$$47 \text{ feet} = \frac{47 \text{ feet}}{1} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$$

Step 5: When you multiply, the unit you HAVE (feet) will cancel and you will be left with the unit you WANT (inches).

$$\frac{47 \cancel{\text{feet}}}{1} \cdot \frac{12 \text{ inches}}{1 \cancel{\text{foot}}}$$

Step 6: Multiply the numerators and multiply the denominators.

$$\frac{47 \cdot 12 \text{ inches}}{1 \cdot 1}$$

$$\frac{564 \text{ inches}}{1}$$

Step 7: State your answer.

47 feet = 564 inches

Note: Because you multiplied by 1, you did not change the distance that you measured; you only changed the unit.

```
      47 feet  =  564 inches
|------------------------------------------|
                47 feet
|------------------------------------------|
               564 inches
```

SAMPLE PROBLEM 2: How many miles are in 8,000 feet?

Equation: 8,000 feet = __ miles

Step 1: Identify the relationship between the unit you HAVE and the unit you WANT.

Relationship: 1 mile = 5,280 feet

2 Unit Ratios: $\frac{\text{1 mile}}{\text{5,280 feet}}$ and $\frac{\text{5,280 feet}}{\text{1 mile}}$

Note that 1 mile is equal to 5,280 feet, so each unit ratio is actually equal to 1.

Step 2: We start with 8,000 feet and we want feet to cancel, so use the unit ratio with feet in the denominator.

Unit Ratio: $\frac{\text{1 mile}}{\text{5,280 feet}}$

This is the unit ratio you will use to solve the problem.

Step 3: Multiply the number you have (8,000 feet) by the unit ratio.

$$\text{8,000 feet} = \frac{\text{8,000 feet}}{1} \cdot \frac{\text{1 mile}}{\text{5,280 feet}}$$

Step 4: When you multiply, the unit you HAVE (feet) will cancel and you will be left with the unit you WANT (miles).

$$\frac{\text{8,000 } \cancel{\text{feet}}}{1} \cdot \frac{\text{1 mile}}{\text{5,280 } \cancel{\text{feet}}}$$

Step 5: Multiply the numerators and multiply the denominators.

$$\frac{\text{8,000} \cdot \text{1 mile}}{\text{5,280} \cdot 1} = \frac{\text{8,000 miles}}{\text{5,280}}$$

Step 6: State your answer: *8,000 feet ≈ 1.52 miles*

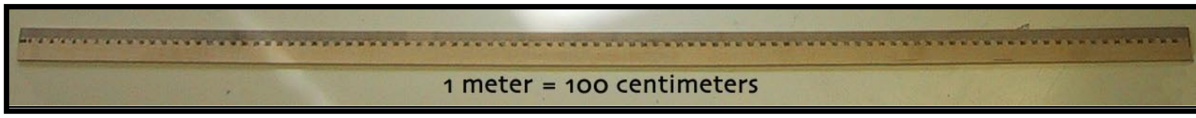
Sometimes you may need more than one unit ratio to change units.

SAMPLE PROBLEM 3a: How many inches are in 20 meters?

Equation: 20 meters = __ inches

Step 1: One approach is to first change meters to centimeters and then change centimeters to inches. We will first use the relationship 1 meter = 100 centimeters to change from meters (m) to centimeters (cm).

Relationship: 1 m = 100 cm



Two Unit Ratios: $\frac{1 \text{ m}}{100 \text{ cm}}$ and $\frac{100 \text{ cm}}{1 \text{ m}}$

Note that 1 m is equal to 100 cm, so each unit ratio is actually equal to 1.

Step 2: We start with 20 meters and we want meters to cancel, so use the unit ratio with meters (m) in the denominator.

Unit Ratio: $\frac{100 \text{ cm}}{1 \text{ m}}$

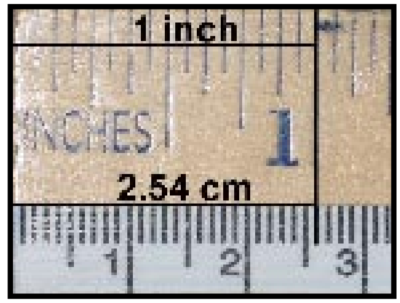
This is the **first** unit ratio you will use to begin converting meters to inches.

Step 3: Multiply the number you have (20 meters) by the unit ratio.

$$20 \text{ m} = \frac{20 \text{ m}}{1} \cdot \frac{100 \text{ cm}}{1\text{m}}$$

Step 4: When you multiply, the unit you HAVE (meters) will cancel and you will be left with the unit you WANT (centimeters).

$$\frac{20 \cancel{\text{m}}}{1} \cdot \frac{100 \text{ cm}}{1 \cancel{\text{m}}}$$



Step 5: Multiply the numerators and multiply the denominators.

$$\frac{20 \cdot 100\text{ cm}}{1 \cdot 1} = 2{,}000\text{ cm}$$

Step 6: Now we will use the *approximated* relationship 2.54 centimeters ≈ 1 inch to change from centimeters (cm) to inches (in).

Relationship: 2.54 cm ≈ 1 in

Two approximate unit ratios:

$$\frac{1\text{ in}}{2.54\text{ cm}} \quad \text{and} \quad \frac{2.54\text{ cm}}{1\text{ in}}$$

Note: 1 inch is approximately equal to 2.54 cm, so each unit ratio is approximately equal to 1.

Step 7: We start with 2,000 centimeters and we want centimeters to cancel, so use the unit ratio with centimeters (cm) in the denominator.

Unit Ratio: $\frac{1\text{ in}}{2.54\text{ cm}}$

This is the **second** unit ratio you will use to convert meters to inches.

Step 8: Multiply the number you have (2,000 centimeters) by the approximate unit ratio.

$$2{,}000\text{ cm} = \frac{2{,}000\text{ cm}}{1} \cdot \frac{1\text{ in}}{2.54\text{ cm}}$$

Step 9: When you multiply, the unit you HAVE (centimeters) will cancel and you will be left with the unit you WANT (inches).

$$\frac{2{,}000\text{ }\cancel{\text{cm}}}{1} \cdot \frac{1\text{ in}}{2.54\text{ }\cancel{\text{cm}}}$$

Step 10: Multiply the numerators and multiply the denominators.

$$\frac{2{,}000 \cdot 1\text{ in}}{1 \cdot 2.54} = \frac{2{,}000\text{ in}}{2.54}$$

$$\approx 787.4\text{ in}$$

Step 11: State your answer: *20 meters ≈ 787 inches*

When you get really good at converting from one unit to another unit using unit ratios, then you can solve this problem using two unit ratios at the same time.

SAMPLE PROBLEM 3b: How many inches are in 20 meters?

Equation: 20 meters = __ inches

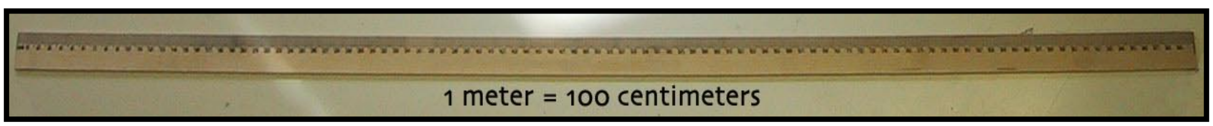
Step 1: Identify the relationships between the units you HAVE and the units you WANT.

Relationship A: 1 m = 100 cm

Relationship B: 2.54 cm ≈ 1 in

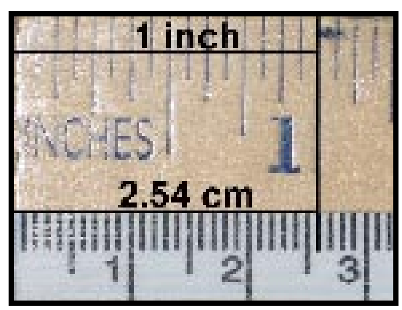
Step 2: Write the relationships as ratios. First we change meters (m) to centimeters (cm), so for the first unit ratio, meters is in the denominator.

Unit Ratio A: $\frac{100\text{ cm}}{1\text{ m}}$



Next we change centimeters (cm) to inches (in), so for the second unit ratio, centimeters is in the denominator. (Remember this is an *approximated* unit ratio.)

Unit Ratio B: $\frac{1\text{ in}}{2.54\text{ cm}}$



Step 3: Multiply the number you have (20 meters) by the unit ratios.

$$20\text{ m} = \frac{20\text{ m}}{1} \cdot \frac{100\text{ cm}}{1\text{m}} \cdot \frac{1\text{ in}}{2.54\text{ cm}}$$

Step 4: When you multiply, the units you HAVE (meters and centimeters) will cancel and you will be left with the unit you WANT (inches).

$$\frac{20\ \cancel{m}}{1} \cdot \frac{100\ \cancel{cm}}{1\cancel{m}} \cdot \frac{1\ in}{2.54\ \cancel{cm}}$$

Step 5: Multiply the numerators and multiply the denominators.

$$\frac{20 \cdot 100 \cdot 1\ in}{1 \cdot 1 \cdot 2.54}$$

$$= \frac{2{,}000\ in}{2.54}$$

$$\approx 787.4\ in$$

Step 6: State your answer.

20 meters ≈ 787 inches

SW = student workbook TG = teacher guide EG = educator guide

Lesson 3 – ENGAGE

• **Estimated Time:** 1 session, 50 minutes

• **Materials:**

– Transparency #1: Solar System Distances (TG p.19)
– Transparency #2: Circle Parts and Circumference (TG p.21)
– Transparency #3: Traveling to a Moving Target—Missed (TG p.23)
– Transparency #4: Traveling to a Moving Target—Reached (TG p.24)
– Transfer Orbits: Student Reading (SW pp.8-10) and Sample Problem (SW pp.11-12)
– Transfer Orbits: A Hands-on Proof (SW p.13)
– Transparency #5: Top View of the Solar System (TG p.27)

1. TRAVEL DISTANCES TO PLANETS AND MOONS

Remind the students of their final goal:

To determine where in the solar system NASA should send humans.

In Lesson 1, students observed the vast range of sizes of the planets in our solar system. In Lesson 2, students observed the mass and composition of planets and moons to determine which bodies had surface conditions acceptable for humans. Another factor that students must take into account as they decide where to send humans in our solar system is the time it takes to travel to other planets and moons, which will be addressed now in Lesson 3.

In Lesson 1, students made a scale model of the solar system to compare the size of the planets and the distances from the planets to the Sun. In Lesson 2, students made another scale model of the solar system to compare planetary volume (circumference), mass, and composition. Activate students' prior knowledge and review what they learned in Lessons 1 and 2 by asking the following questions:

Information from Lessons 1 and 2

- What are the largest and most massive planets in our solar system? (Jupiter, Saturn, Uranus, and Neptune—the gas giants)
- What are the smallest and least massive planets in our solar system? (Mercury, Venus, Earth, Mars, and Pluto)
- Which planets are closer to the Sun: the smaller or larger planets? (The smaller planets except for Pluto)
- Which are the most dense planets in our solar system? (Mercury, Venus, Earth, Mars, and Pluto)

- Which are the least dense planets in our solar system? (Jupiter, Saturn, Uranus, and Neptune—the gas giants)

Conclusions from Lessons 1 and 2

- Based on your experience with a scale model of our solar system, where do you think we should send humans in our solar system? Why?
- Due to surface conditions, which planets were eliminated as possibilities? Why? (Jupiter, Saturn, Uranus, and Neptune—the gas planets do not have a solid surface.)
- Which planets or moons are still possible destinations for human travel? (Mercury, Venus, Earth's Moon, Mars, Titan, Io, Europa, Triton, and Pluto)
- What else should you know about the planets before you make your final decision?

The students began Lesson 1 by imagining that they were planning a family vacation and correlating that to planning a human mission to a planet or moon in our solar system. Factors that students said they would need to consider before determining a destination included 1) cost, 2) distance, 3) activities and entertainment, and 4) weather (temperature/atmosphere).

> What else do students need to know before they can decide where to send humans in our solar system?
>
> While in Lesson 1 they studied the distances from the planets to the Sun, now ***the students need to calculate how far the planets and moons are from Earth and how long it would take to travel to each planet or moon.*** In this lesson the students will calculate a close approximation for how long it would take a mission from Earth to reach each planet and moon.

Based on the scale model and/or graphs in Lesson 1, students should have an approximate idea of the distances between the planets when aligned in a straight line from the Sun. Now they will need to look at the *distances from the Earth to the planets and moons*, taking into consideration the movement of the planets along their orbital paths. Show students **Transparency #1: Solar System Distances** (TG p.19) and discuss the following five questions:

1. Name the 2 planets closest to Earth: Venus & Mars
2. Name the 2 planets furthest from Earth: Neptune & Pluto
3. Is the picture of the planets to scale for the size of the planets, the distances of the planets to the Sun, or both? The planets are to scale for distance from the Sun but *not* for size.
4. Estimate how long you think it would take to travel from Earth to each of the planets and moons. Answers will vary.
5. What unit of time makes the most sense for you to use in your estimate? Months or Years

Transparency #1: Solar System Distances

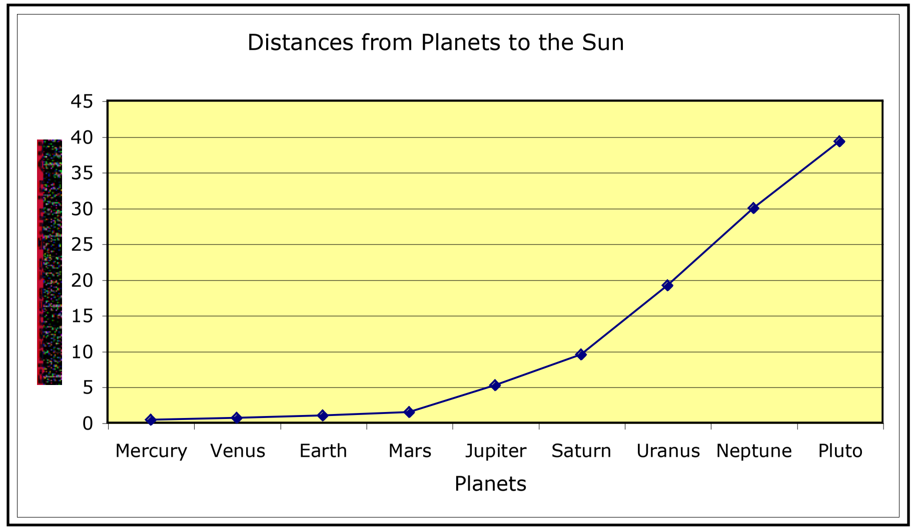
1. Name the 2 planets closest to Earth:

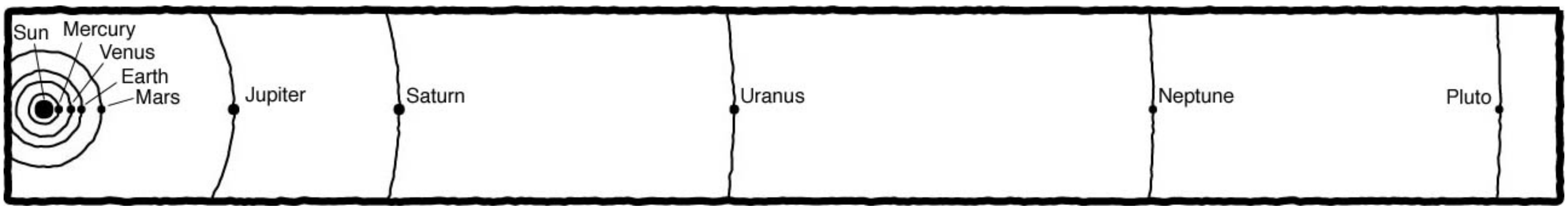
______________ & ______________

2. Name the 2 planets furthest from Earth:

______________ & ______________

3. Is the picture of the planets below to scale for the size of the planets, the distances of the planets to the Sun, or both?





Planetary Body	Mercury	Venus	Earth (Moon)	Mars	Jupiter (Io, Europa)	Saturn (Titan)	Uranus	Neptune (Triton)	Pluto
Average Distance to Sun (km)	58 million km	108 million km	150 million km	228 million km	778 million km	1,429 million km	2,875 million km	4,504 million km	5,900 million km
Average Distance to Sun (AU)	0.4 AU	0.7 AU	1.0 AU	1.5 AU	5.2 AU	9.5 AU	19.2 AU	30.0 AU	39.3 AU

4. Estimate how long you think it would take to travel from Earth to each of the planets and moons.

5. What unit of time makes the most sense for you to use in your estimates?

2. CIRCLE GEOMETRY

In the EXPLORE section of this lesson students will be using circle geometry to solve distance problems. They will need to understand the definition of a circle and how the various terms that define a circle can be used to calculate the circumference of a circle.

Recall that *a **circle** is the collection of points that are all the same distance from a fixed point.* The fixed point is called the center of the circle. That same *distance from the center of the circle to each point on the circle is called the **radius*** of the circle.

In the EXTEND section of Lesson 2, students had the opportunity to derive an appropriate value for π (pi) using circles and spheres. To remind students of this activity, first review the parts of a circle using **Transparency #2: Circle Parts and Circumference** (TG p.21).

> **Note:** If your students did not complete the EXTEND activity in Lesson 2, then they can perform a hands-on activity to find the value of pi by measuring the circumference and diameter of any circle and calculating the ratio shown below. One way to do this activity is to use a tin can and string.
>
> $$\pi = \frac{\text{circumference of the circle}}{\text{diameter of the circle}}$$

Ask the students if they can write an equation to give a formula for the *circumference* of a circle using the above ratio for π. By solving the ratio for the circumference, they should derive the following formula:

$$\text{Circumference of the circle} = \pi \cdot \text{diameter of the circle}$$

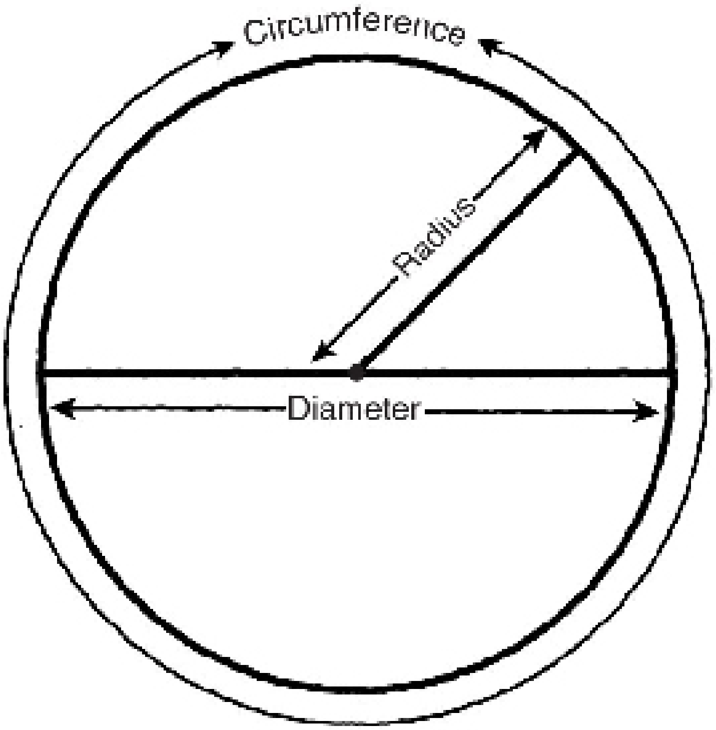
We can write this as:
$C = \pi \cdot d$
where C is the circumference of the circle and d is the diameter of the circle.

The diameter (d) of a circle is equal to two times the length of the radius (r) of a circle, so the formula for circumference can also be written as follows:

$$\text{Circumference of the circle} = \pi \cdot 2 \cdot \text{radius of the circle}$$

Since we can interchange the 2 and the r, we can write this as:
$C = 2 \cdot \pi \cdot r$
where C is the circumference of the circle and r is the radius of the circle.

Transparency #2: Circle Parts and Circumference



Radius (r) is the length of a line that extends from the center of the circle to any point on the circle.

Diameter (d) is the length of a line that passes through the center of the circle and extends to opposite points on the circle.

The length of the diameter of the circle is equal to two times the length of the radius of the circle.

Circumference (C) is the distance around the circle.

The ratio of the circumference of a circle to the diameter of a circle is equal to a number called **pi**, represented by the Greek letter π.

$$\pi = \frac{C}{d}$$

Pi (π) is an irrational number that is greater than 3. It is often *approximated* by the decimal number **3.14** or approximated by the fraction $\frac{22}{7}$.

By solving the above equation for the **circumference**, we obtain the following formula:

$$C = \pi \cdot d$$

Since the diameter (d) of a circle is equal to two times the length of the radius (r) of a circle, the formula can also be written as:

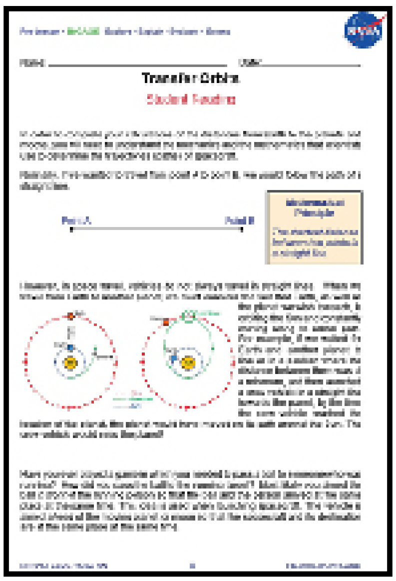
$$C = \pi \cdot 2 \cdot r$$

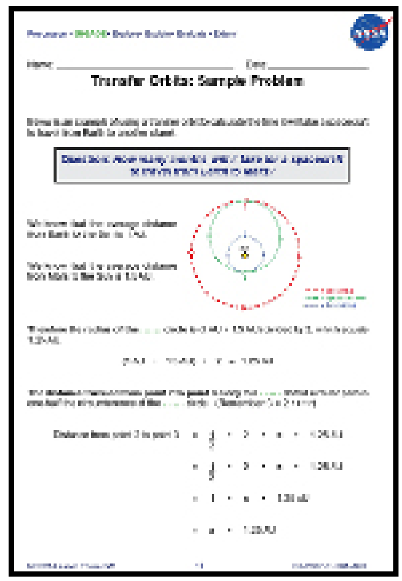
$$= 2 \pi r$$

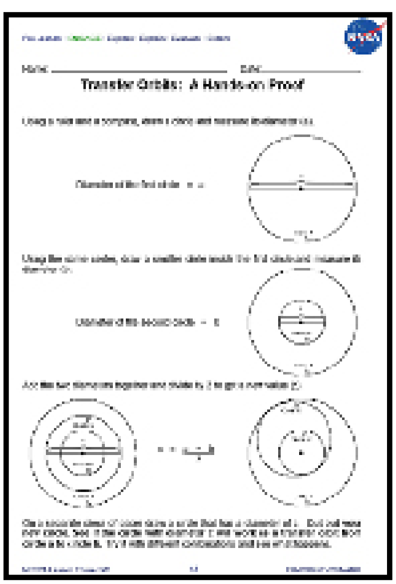
3. TRANSFER ORBITS

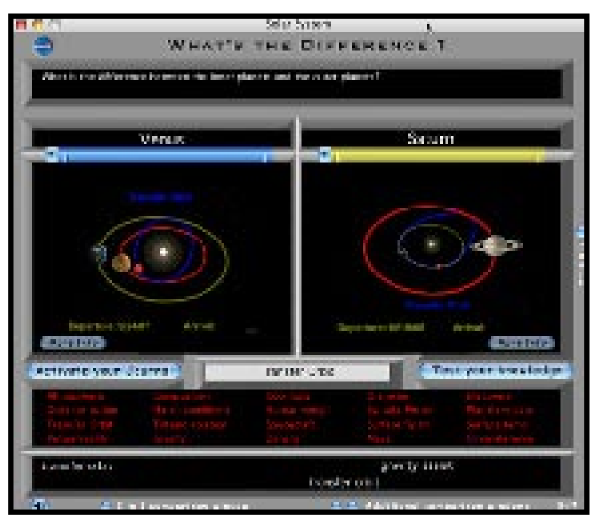
In the EXPLORE section of this lesson, students will be calculating the distance traveled by crew vehicles to planets and moons by following transfer orbits. The calculations of these orbits and distances rely on the geometry of circular and elliptical orbits. The calculations and principles are explained in detail in the student reading, **Transfer Orbits** (SW pp.8-10), and in the illustrations on **Transparency #3** (TG p.23) and **Transparency #4** (TG p.24).

Use the **Student Reading** (SW pp. 8-10), **Sample Problem** (SW pp.11-12), and **Hands-on Proof** (SW p.13) to help students understand the use of transfer orbits and to prepare them for calculating travel distances in the EXPLORE section of this lesson. Students can also view an animated illustration of transfer orbits for each planet and moon in the **"Transfer Orbit" attribute of *What's the Difference***.


Transfer Orbits
Student Reading

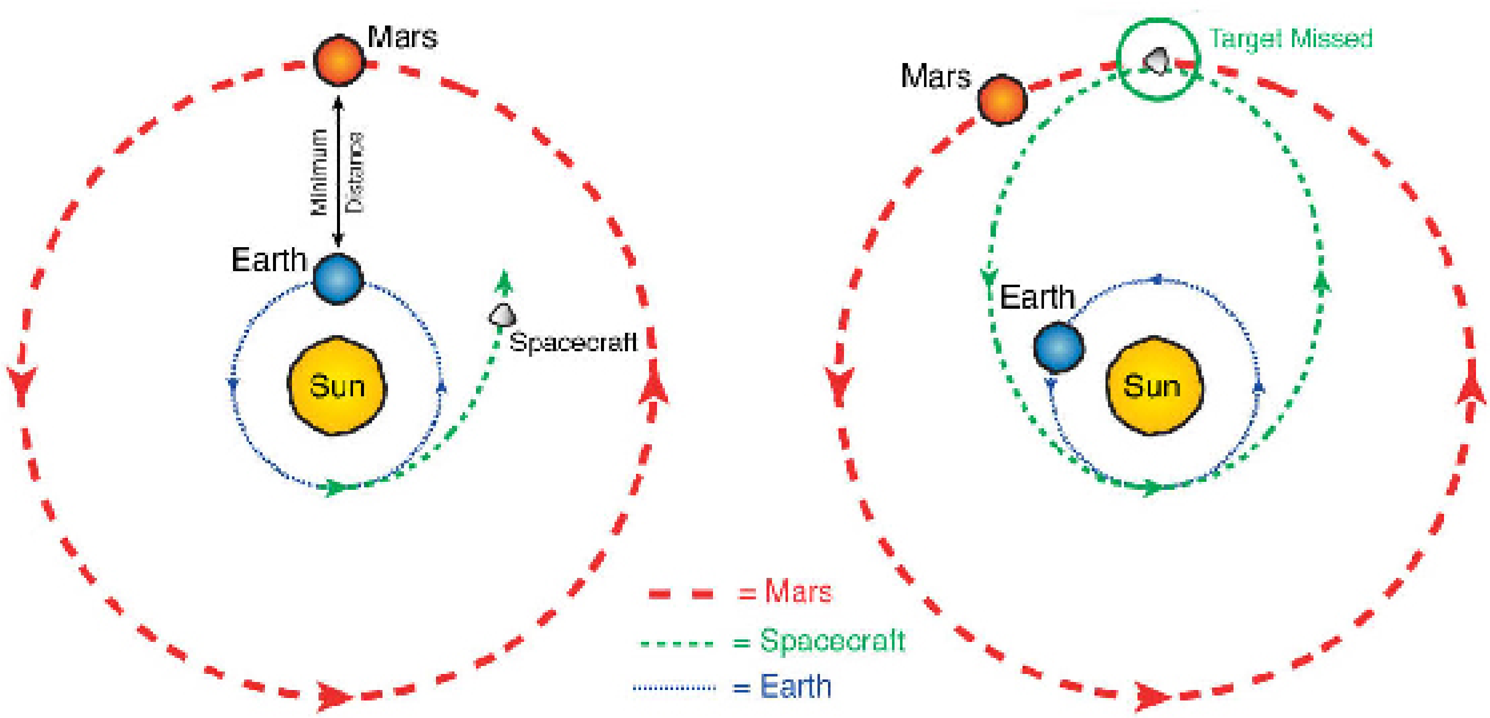

Transfer Orbits: Sample Problem


Transfer Orbits: A Hands-on Proof



Transparency #3: Traveling to a Moving Target — Missed

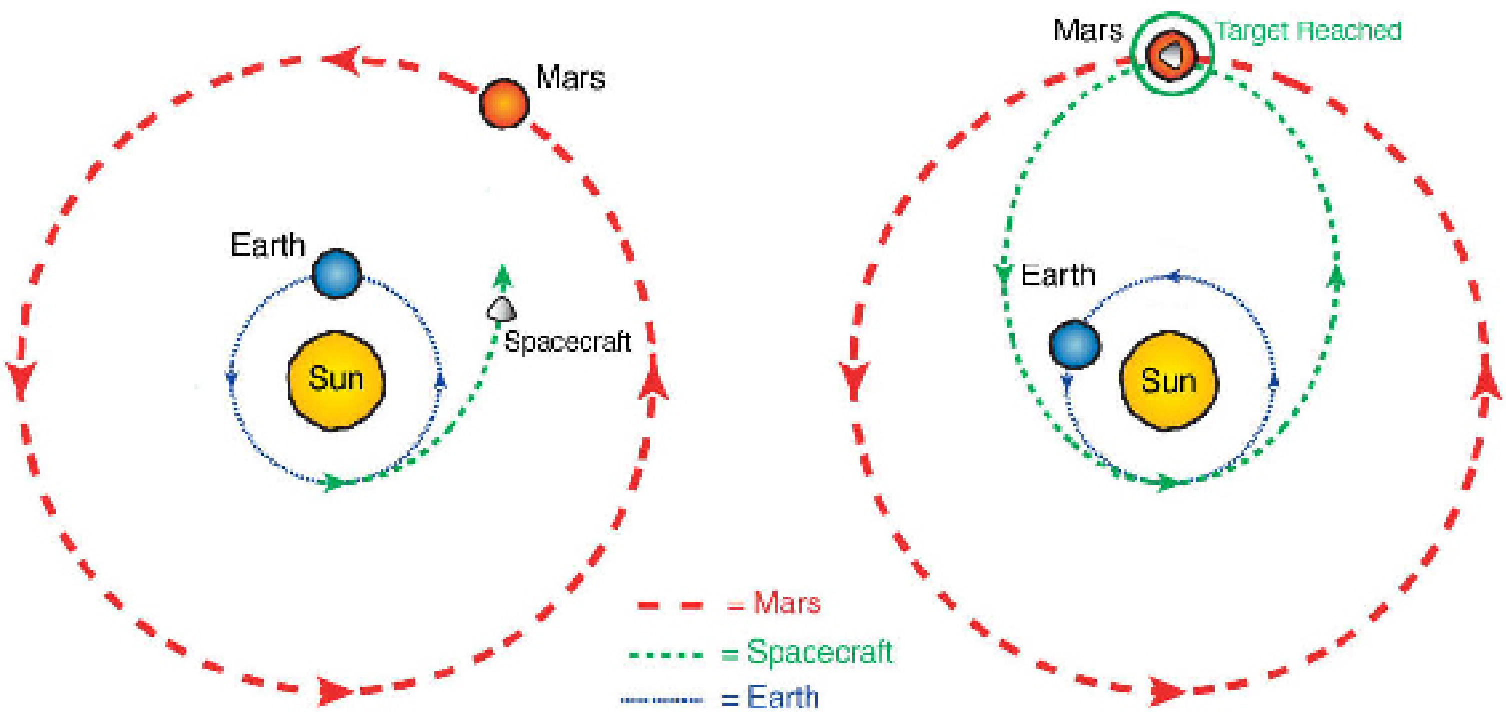
In space travel, vehicles do not always travel in straight lines. If we waited for two planets to line up in a position where the distance between them was at a minimum and then launched a crew vehicle in a straight line towards the planet, then by the time the crew vehicle reached the location of the planet, the planet would have moved on its orbital path around the Sun. The crew vehicle would miss the planet!



** The objects in the picture are not to scale.*

Transparency #4: Traveling to a Moving Target — Reached

In this scenario, the movement of the planets has been accounted for and the target is successfully reached.



** The objects in the picture are not to scale.*

4. AVERAGES, ESTIMATIONS, AND APPROXIMATIONS

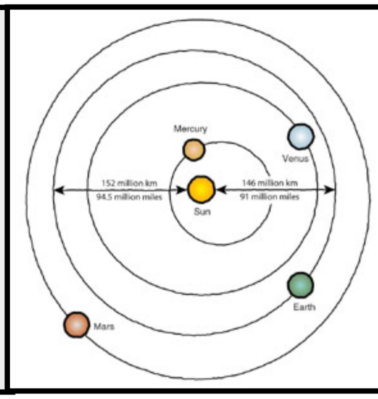
Discuss with students when it is important to calculate exact measurements and when it is appropriate to use approximations. Use the example of calculating how far away a planet is from the Sun and **Transparency #5: Top View of the Solar System** (TG p.27) to aid students in this discussion.

Q: Is Earth always the same distance from the Sun?

A: No, it's distance from the Sun changes as it moves along its orbital path.

Q: Does Earth orbit in a perfect circle around the Sun?

A: No, it travels in an elliptical path that is nearly circular.



Averages

Students should realize that Earth and the other planets are not always the same distance from the Sun. Earth is closest to the Sun (perihelion) in January and furthest from the Sun (aphelion) in July. It's perihelion distance is about 146 million km or 91 million miles, and it's aphelion distance is about 152 million km or 94.5 million miles. Because of Earth's varying distance from the Sun along its orbit, we often refer to the *average distance from Earth to the Sun* which is about 150 million km or 93 million miles—more commonly referred to as *1 AU*.

Estimations

Q: When would it be important to know the exact distance to the Sun at a given point in time?

A: Exact distances are needed when planning a mission launch window, speed, and direction. For example, when NASA is preparing for an actual mission, the exact location and the exact measurements are used so as to ensure accuracy and to optimize success and safety of the mission.

Q: When is an estimate okay to use?

A: Estimates are fine to use when getting a general sense of how long a mission might take in the early planning stages.

Q: Why do we use AUs instead of using the exact distances from the Sun to the planets?

A: The distance between a planet and the Sun varies and would require multiple mathematical calculations for each distance. *Using an average simplifies the math, requiring only one calculation and providing a general estimate that is acceptable in many situations.* By using a reasonably accurate average, the results of our calculations should also be fairly accurate.

Approximations

Often in science and mathematics, we are allowed to use approximations of measurements instead of exact measurements. As long as the calculations are performed with a reasonable degree of accuracy, the results are comparable to calculations that would be performed with exact measurements. The degree of accuracy of an approximation that is allowed for a particular measurement will depend on the degree of accuracy needed for the calculation being performed. ***For our purposes, we will use averages to calculate reasonable approximations of distances at any given time.***

Q: What degree of accuracy do you think you will need for calculating the length of a mission?

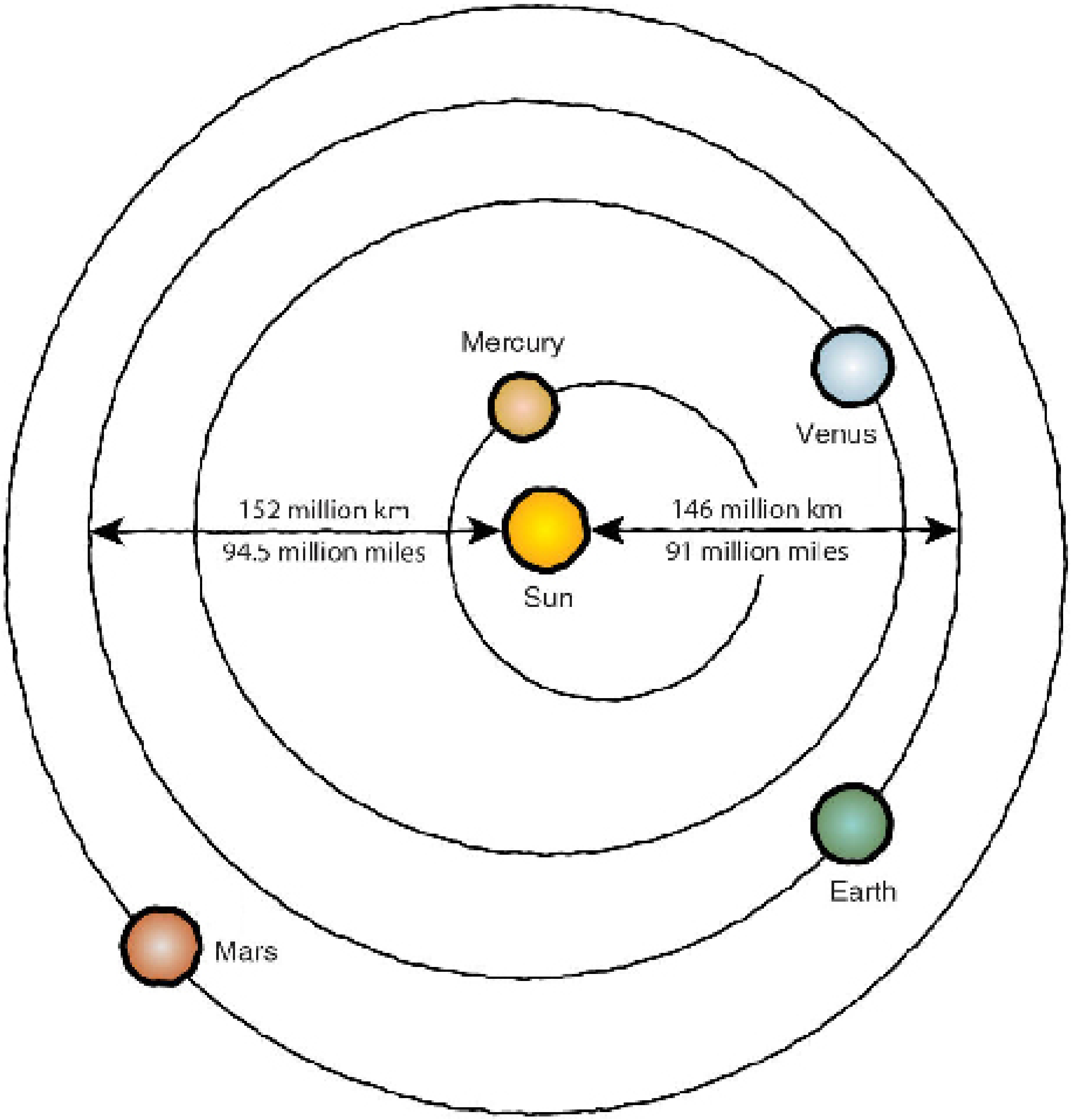
A: The following approximations will be acceptable for these calculations:

1) **Orbital circumferences—**We will be approximating the circumference of an ellipse using the *circumference of a circle*. While the circumferences of ellipses and circles do vary, the calculation of the circumference of a circle is much easier than the calculation of the circumference of an ellipse. *In this project we will approximate any ellipse with a circle.* Again, the calculations will be comparable to actual values and, for our purposes, will be accurate enough for students to gain an insight into the distances from Earth to other planets and moons.

2) **Value of pi—**Students will be calculating the circumference of a circle using an approximate value of pi. Recall that pi is represented by the Greek letter π and is an irrational number, whose decimal representation continues indefinitely without repetition. Students can use either *the fraction 22/7* or *the decimal 3.14* as an approximation of pi, which will give the students reasonably accurate results.

3) **Place value to which results are rounded—**In most cases, it will be sufficient for students to round results to the *hundredth or thousandth place* to obtain the accuracy required for this project.

Transparency #5: Top View of the Solar System



** The Sun and planets are not drawn to scale.*

SW = student workbook TG = teacher guide EG = educator guide

Lesson 3 – EXPLORE

- **Estimated Time:** 3 sessions, 50 minutes each
- **Materials:**
  - Transparency #6: November 2005 Positions of Earth and Mars (TG p.30)
  - Transparency #7: May 2006 Positions of Earth and Mars (TG p.31)
  - Transparency #8: Calculating Transfer Orbits (TG p.35)
  - Travel Distance Using Transfer Orbits student worksheet (SW p.14)
  - Transparency #9: Calculating Earth's Revolution Rate (TG p.37)
  - Crew Vehicle Speed student worksheet (SW p.15)
  - Travel Time student worksheet (SW p.16)
  - Math Review: Converting Units (TG pp.10-16)
  - Synodic Period (SW p.17)
  - Computers with *What's the Difference* Solar System dataset and orrery tool
  - Total Mission Length (SW p.18)
  - Percent of a Lifetime and Career student worksheet (SW p.19)
  - Calculators

The EXPLORE section of this lesson contains 7 parts:

1. Distance from Earth to other planets/moons
2. Travel distance using transfer orbits
3. Speed of the crew vehicle
4. Travel time
5. Synodic period (using *What's the Difference* orrery tool)
6. Total mission length
7. Percent of lifetime and career

These 7 sections build on one another and follow a logical sequence. However, if time does not permit your students to do all seven tasks, then you may choose to divide the class into 7 teams, assigning each team a separate task, or you may pick and choose 2 or 3 of the tasks for the class to complete.

1. DISTANCE FROM EARTH TO OTHER PLANETS AND MOONS

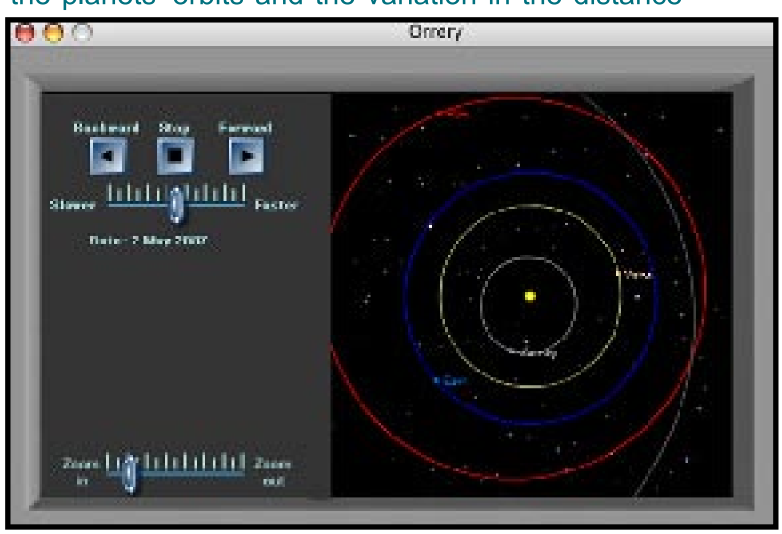
The distances from Earth to the other planets and moons are constantly changing due to the planets' orbiting the Sun at different rates.

Misconception Alert!

Students may reason that they can calculate the distance between Earth and another planet by taking the difference of the distance of the given planet to the Sun and 1 AU (the distance between Earth and the Sun). For example, if they subtract the distance from Earth to the Sun (1 AU) from the distance from Mars to the Sun (1.5 AU), then supposedly that will give the distance between Earth and Mars (0.5 AU). In some sense, this is correct. However, while this approach would give the distance between Earth and Mars when Earth and Mars are lined up, it would *not* give the distance between the two planets when they are not aligned.

Use **Transparency #6: November 2005 Positions of Earth and Mars** (TG p.30) and **Transparency #7: May 2006 Positions of Earth and Mars** (TG p. 31), to help students visually see why calculating the difference between two planets' distances from the Sun is an unreliable approach to solving the distance between the two planets.

Note: The **orrery** found in the "Tools" section of *What's the Difference* allows students to view the inner planets' relative speed. The rate of the model can be sped up or slowed down and specific dates can be viewed using the slider bar. Although it may not be entirely accurate, this model does illustrate the difference in the rates of the planets' orbits and the variation in the distance between them.

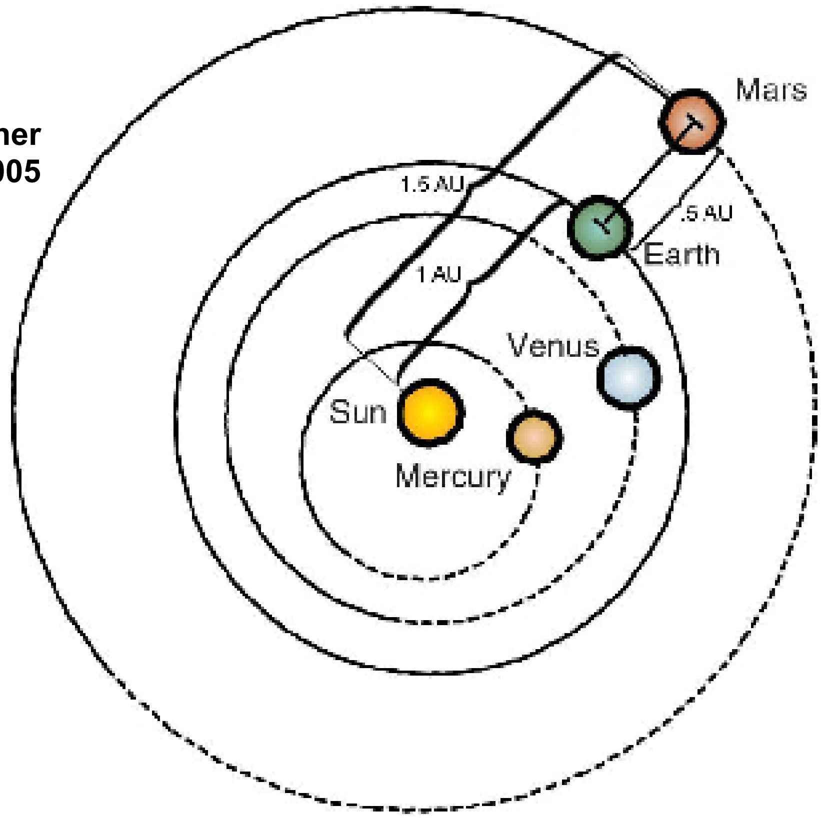


Transparency #6: November 2005 Positions of Earth and Mars

Actual positions of the inner planets on November 10, 2005 (17:04:44).

On this date, Earth and Mars were aligned with the Sun and at a minimum distance from each other.

Note: The size of each planet and the Sun are not drawn to scale.



Unreliable approach to calculating interplanetary distances:

What is the distance between Earth and Mars?

Distance from Earth to Sun = 1 AU

Distance from Mars to Sun = 1.5 AU

Distance from Mars to Sun – Distance from Earth to Sun = Distance from Earth to Mars

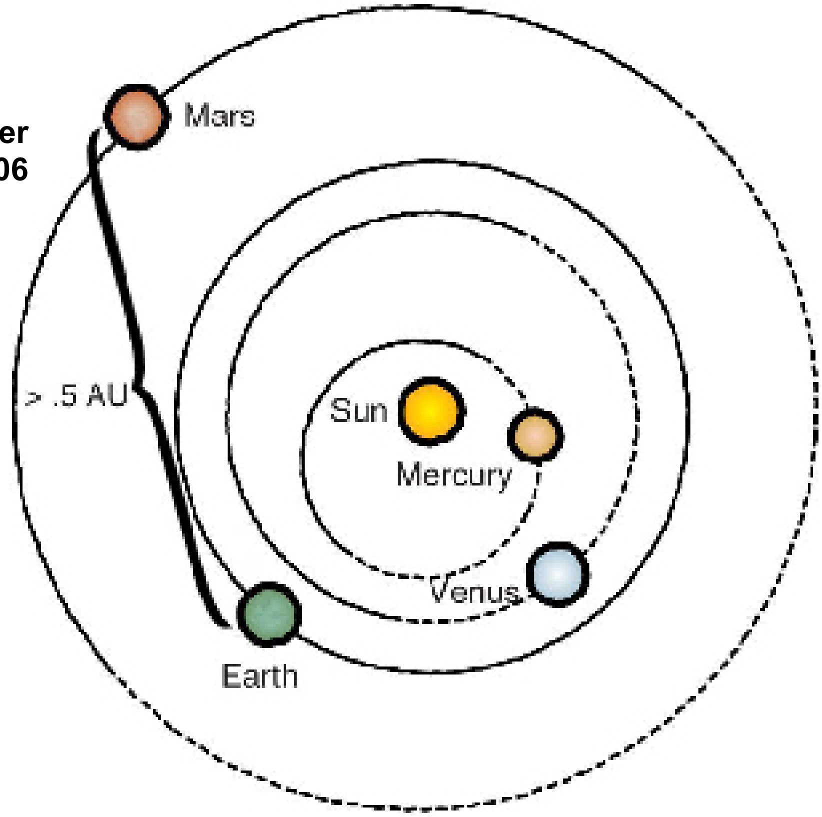
1.5 AU – 1 AU = 0.5 AU

Transparency #7: May 2006 Positions of Earth and Mars

Actual positions of the inner planets on May 10, 2006 (17:04:44)

On this date, Earth and Mars were not aligned with the Sun and were at a distance greater than 0.5 AU.

Note: The size of each planet and the Sun are not drawn to scale.



This picture shows the position of the planets six months later on May 10, 2006.

Is the distance between Earth and Mars still 0.5 AU?

No, the distance between Earth and Mars is now greater than 0.5 AU. The distance between the two planets has changed because the planets travel around the Sun at different speeds.

For example, Mercury is the fastest planet, traveling about 10 times faster than Pluto, which is the slowest planet. As the planets orbit the Sun, the distance from one planet to the next varies.

2. TRAVEL DISTANCE USING TRANSFER ORBITS

As discussed in the **Transfer Orbits** student reading (SW pp.8-10), the shortest distance between the planets is not the actual distance that the crew vehicle will travel. Rather, the spacecraft will use the orbital motion of Earth to propel itself in the direction of the other planet or moon. To calculate an approximation of the distance traveled by the crew vehicle, students will use the geometry of circles and average distances. For the purposes of this lesson, we will use a circular orbit (rather than an elliptical orbit) to approximate the distance traveled by a spacecraft on its way to another planet or moon. The mathematics is explained and demonstrated in the student reading as well as illustrated for each planet and moon in the **"Transfer Orbit" attribute of *What's the Difference***.

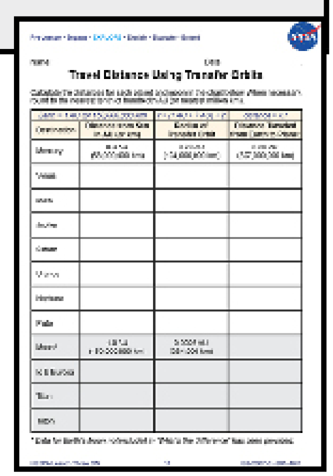
Transparency #8: Calculating Transfer Orbits (TG p.35) shows the orbit of Earth, the orbit of the destination planet, and the path the crew vehicle will take. To determine the roundtrip distance that the crew vehicle will have to travel from Earth's orbit to the orbit of the destination planet and back home again, students will have to calculate the *circumference of the crew vehicle's path*. In order to do this, students first need to calculate *the radius of that path*.

The **Travel Distance Using Transfer Orbits** student worksheet (SW p.14) includes a chart that can be used to record data as the students make their calculations. The distances to the moons, except for Earth's Moon, will be the same as the planets that the moons orbit. For example, the distances calculated for Jupiter will be used for Jupiter's moons Io and Europa. ***Note:*** *The radius of the transfer orbit for Earth's Moon will be 384,000 km.*

Example: Calculating Distance Traveled from Earth to Mars

To get the radius of the circular transfer orbit, we average the distance from Earth to the Sun and the distance from Mars to the Sun.

$$\text{Radius of transfer orbit to Mars} = \frac{\text{distance from Mars to Sun} + \text{distance from Earth to Sun}}{2}$$



Note: Ask the students what unit would be easiest to use in calculating the distances? Students should remember from Lesson 1 that Astronomical Units (AU) are easier to manage because the value for the distance from Earth to the Sun is 1 AU. However, the distances can be calculated in kilometers (or miles) if desired. The class should agree on what unit they will use to calculate the distances so that their results can be compared.

Using astronomical units:

$$\text{Radius of transfer orbit to Mars} = \frac{\text{distance from Mars to Sun} + \text{distance from Earth to Sun}}{2}$$

$$= \frac{1.5\ \text{AU} + 1\ \text{AU}}{2}$$

$$= \frac{2.5\ \text{AU}}{2}$$

$$= 1.25\ \text{AU}$$

Using kilometers:

$$\text{Radius of transfer orbit to Mars} = \frac{\text{distance from Mars to Sun} + \text{distance from Earth to Sun}}{2}$$

$$= \frac{228{,}000{,}000\ \text{km} + 150{,}000{,}000\ \text{km}}{2}$$

$$= \frac{378{,}000{,}000\ \text{km}}{2}$$

$$= 189{,}000{,}000\ \text{km}$$

Using *miles:

$$\text{Radius of transfer orbit to Mars} = \frac{\text{distance from Mars to Sun} + \text{distance from Earth to Sun}}{2}$$

$$= \frac{142{,}000{,}000\ \text{miles} + 93{,}000{,}000\ \text{miles}}{2}$$

$$= \frac{235{,}000{,}000\ \text{miles}}{2}$$

$$= 117{,}500{,}000\ \text{miles}$$

** Miles will not be the units used in this lesson; answers will be given primarily in AU and km.*

The circumference of the circular transfer orbit is 2 π times the radius.

$$C = 2 \pi r$$

The distance traveled by a crew vehicle to reach Mars is half of the circumference of the entire transfer orbit.

$$\text{Distance traveled} = \frac{1}{2} C$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2} \pi r$$

$$= \pi r$$

So the distance traveled by a crew vehicle is pi times the radius of the transfer orbit.

$$\text{Distance traveled} = \pi r$$

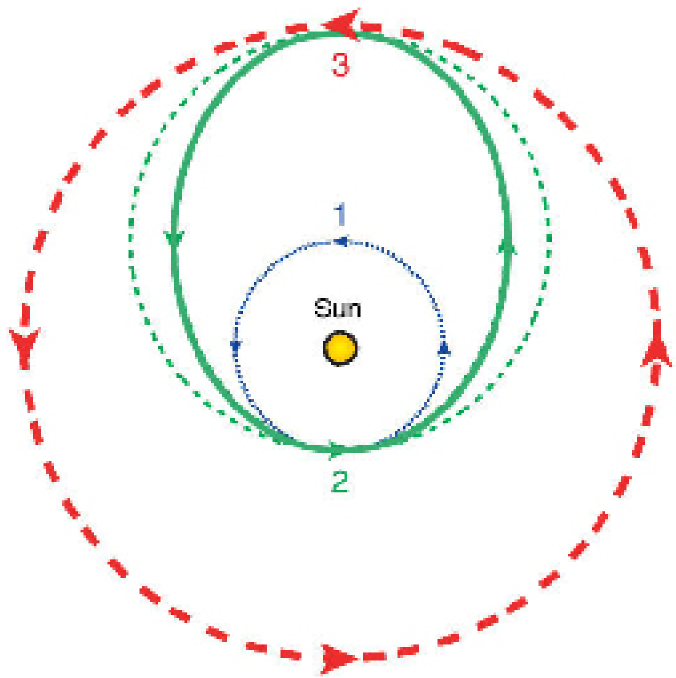
Distance traveled to Mars using astronomical units: $= \pi \cdot$ radius of transfer orbit to Mars $= \pi \cdot 1.25$ AU ≈ 3.93 AU
Distance traveled to Mars using kilometers: $= \pi \cdot$ radius of transfer orbit to Mars $= \pi \cdot 189{,}000{,}000$ km $\approx 594{,}000{,}000$ km
Distance traveled to Mars using *miles: $= \pi \cdot$ radius of transfer orbit to Mars $= \pi \cdot 117{,}500{,}000$ miles $\approx 369{,}000{,}000$ miles

** Miles will not be the units used in this lesson; answers will be given primarily in AU and km.*

Students can use this same process to calculate the travel distance from Earth to each planet and/or moon using transfer orbits. *(Note: the distance to Earth's Moon will be π 384,000 km.)*

Note: Distances from the Sun can be gathered from the Lesson 1 Planet Data Sheets, the *What's the Difference* program, or from Transparency #1 (TG p.19).

Transparency #8: Calculating Transfer Orbits



The ········ circular orbit represents the path of Earth around the Sun.

The – – – circular orbit shows the path of the outer planet around the Sun.

The - - - - orbit shows the circular path of a crew vehicle leaving Earth and traveling to the outer planet.

NOTE: In reality, the crew vehicle would follow the solid —— elliptical path, but for our purposes we approximate this with a - - - - circular path.

1. What do we need to calculate to find the distance the crew vehicle will travel on a round trip mission from Earth to the outer planet?

The circumference of the - - - - circle

2. What information do we need in order to make our calculations?

Circumference - - - - = 2 • π • radius - - - -

Distance traveled from Earth to the target planet = $\frac{1}{2}$ C - - - -

= π • radius - - - -

Radius - - - - = $\frac{\text{radius ········ + radius – – –}}{2}$

3. SPEED OF THE CREW VEHICLE

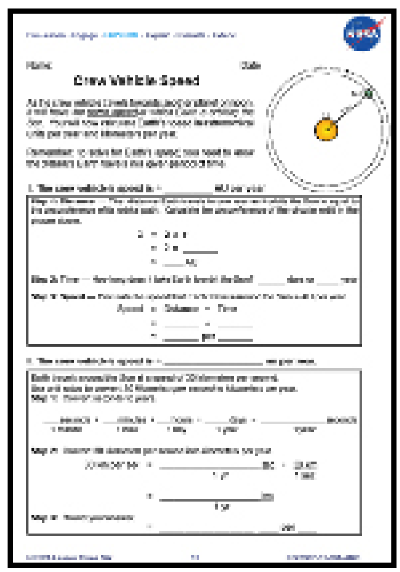
In section 2, students calculated the *distance* the crew vehicle will travel to reach each planet or moon. Now, in order to calculate the length of time it will take for the crew vehicle to travel from Earth to each destination, students first need to calculate the *speed* of the crew vehicle.

As the crew vehicle travels towards another planet or moon, it will have the same speed with which Earth is orbiting the Sun. **The Earth is traveling around the Sun at a rate of 30 kilometers per second.** (Students can calculate this speed in the EXTEND section of this lesson.)

If the travel distance was calculated in AU in section 2, then the speed of Earth as it orbits the Sun should be calculated in AU per year using the speed equation derived in the Pre-Lesson Activity (SW p.3).

Speed = Distance ÷ Time

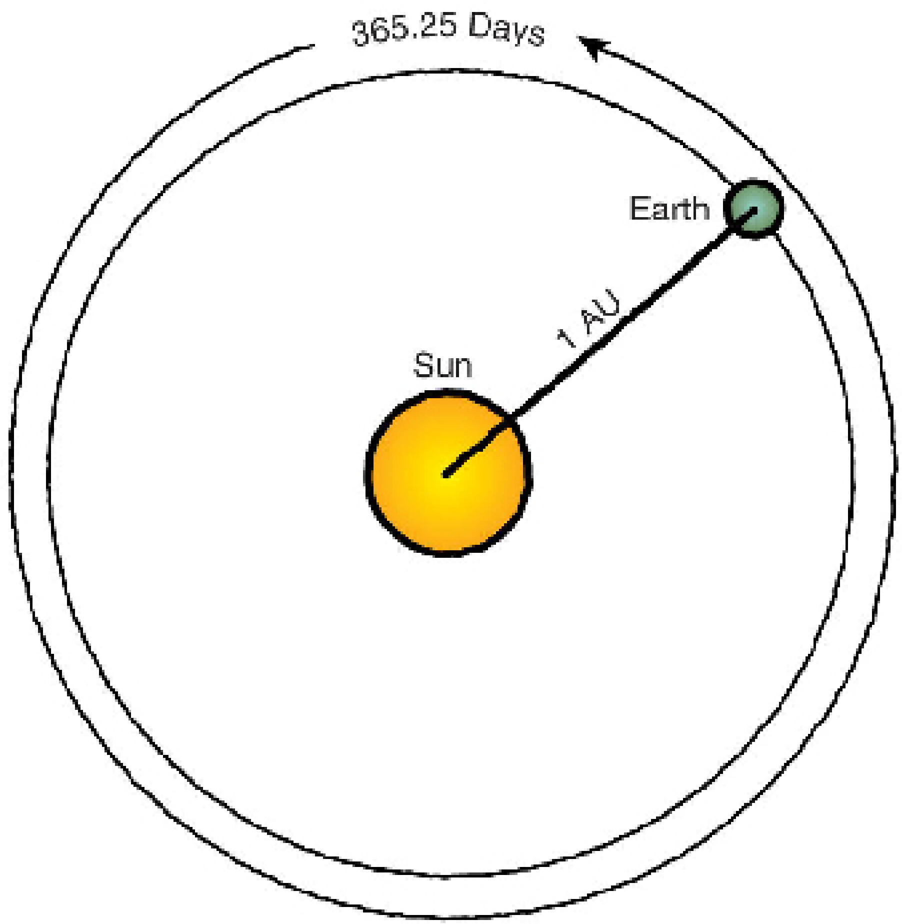
However, ***if the travel distance was calculated in km*** in section 2, then the speed of Earth as it orbits the Sun should be converted in km per year based on Earth's given rate of 30 kilometers per second.


Crew Vehicle Speed

Use **Transparency #9: Calculating Earth's Revolution Rate** (TG p.37) to guide students in calculating the two speeds on the **Crew Vehicle Speed** worksheet (SW p.15).

Students can cross-check their two answers (6.28 AU/year and 947,000,000 km/year) by dividing 947,000,000 km by 150,000,000 km (or 1 AU). Results will show that 947,000,000 km is approximately equal to 6.3 AU. Likewise 6.28 AU, when rounded, is approximately equal to 6.3 AU.

Transparency #9: Calculating Earth's Revolution Rate



The Earth and Sun sizes are not drawn to scale.

The *distance* Earth travels in one year as it orbits the Sun is equal to the *circumference* of its orbital path.

The radius of Earth's orbital path is equal to the distance from Earth to the Sun, or one astronomical unit:

$$r = 1\ AU$$

$$(\text{or } r = 150{,}000{,}000\ km)$$

It takes 365.25 days, or 1 year, for Earth to complete one revolution around the Sun.

4. TRAVEL TIME

Once students have calculated the *distance* the crew vehicle must travel to reach a planet or moon and the *speed* at which the crew vehicle will be traveling, then they can calculate the *time* it will take for the crew vehicle to reach its destination.

Refer students to the three equations they derived in the Pre-Lesson Activity (SW p.3). Which equation will they use to calculate the time it will take to travel from Earth to another planet?

Time = Distance ÷ Speed

The **Travel Time** student worksheet (SW p.16) includes examples in astronomical units and kilometers as well as a chart for students to record data as they make their calculations. Decide as a class which units of measurement to use: AU or km. ***Note****: It is not necessary to do calculations for both units.*

Regardless of which unit the students decide to use (AU or km), they should be consistent. For example, if they use "km" for distance, then they should use "km per year" for speed. Likewise, if they use "AU" for distance, then they should use "AU per year" for speed.

Note: Students can calculate the travel times individually or in small groups. They can calculate all or some of the travel times and then compare their results. However, the class as a whole will need to have the data for <u>all</u> of the possible destinations for the EXPLAIN part of the lesson.

Optional Extension Activity: If students do not wish to use a fraction of a year, then you can have them convert the travel time from years to days (or years to months, weeks, etc) using unit ratios. Refer to **Math Review: Converting Units** (TG pp.10-16).

Example: Travel time from Earth to Mercury in days

$$0.35 \text{ year} = 0.35\ \cancel{\text{year}} \cdot \frac{365 \text{ days}}{1\ \cancel{\text{year}}}$$

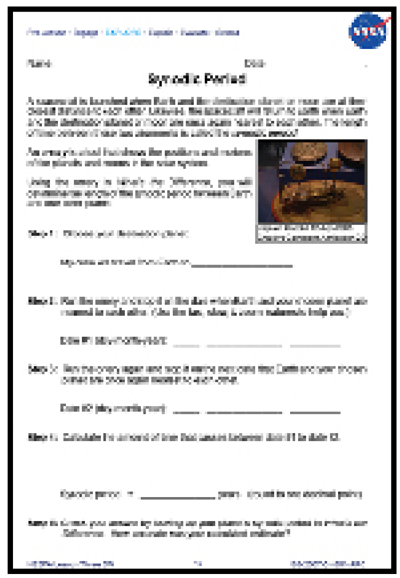
$$\approx 128 \text{ days}$$

5. SYNODIC PERIOD

The appropriate day and time when a mission can launch and when a crew vehicle can return to Earth is based on when Earth and the destination planet or moon are at their closest distance to each other. Once astronauts reach their destination, they must remain there until the planets are once again nearest to each other. This length of time is called the synodic period, and it must be added to the entire length of the mission. *The synodic period will serve as the length of time that the crew will remain at their destination to conduct their mission research.*

> *The* **synodic period** *is the time that it takes for a planet or moon to return to the same point in the sky, relative to the Sun and Earth; i.e. return to the same elongation. It is the time between successive conjunctions of two planets (or moons).*

Scientists and mathematicians use a formula to calculate the synodic period. However, in this lesson, students will use a guess-and-check approach to find the minimum distance between Earth and one of the other planets. This process is outlined on the **Synodic Period** student worksheet (SW p.17).



The **orrery tool in *What's the Difference*** allows students to view planetary positions according to selected dates. Students may wish to start with the current date and change the date forward or backward to observe the general movement of the planets. Students can then focus on the location of Earth and their chosen planet to find how much time passes between the initial alignment of Earth and that planet and the next alignment of Earth and that planet.

> **Note**: The "slower/ faster" and "zoom in/ zoom out" slider bars will help students observe their selected planet.

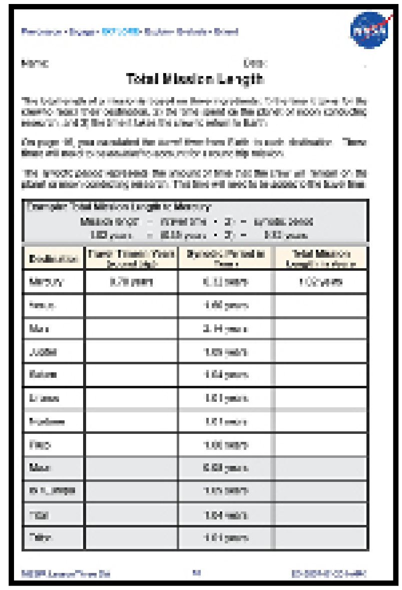
> **Note**: The synodic periods for the moons of Jupiter, Saturn, and Neptune will be the same as the planets themselves.

When finished, students should share their strategies. Did they find a pattern or a system that worked for them? Did they move the date by any set amount (a month at a time, two months, six months?) or did they just try random dates? What worked best? What did not work?

In section 6, students will be provided with the synodic periods between Earth and the other planets and moons. Meanwhile, they can compare their estimated calculation with the data in the **"Synodic Period" attribute of *What's the Difference***.

6. TOTAL MISSION LENGTH

Now that the students have calculated the time a crew vehicle would need to reach each of the planets or moons (section 4), they will need to calculate the total length of time it would take a crew to travel to each destination, conduct their mission research, and return from each destination.


Total Mission Length

Students already calculated the length of time it would take to reach each destination, so they know it would take the same length of time to return to Earth. Students now need to add in the minimum length of time the crew must stay on the planet or moon, which is based on the synodic period.

Using their solutions on SW p.16, students will use the **Total Mission Length** student worksheet (SW p.18) to calculate the minimum length (in years) of a mission to each planet and moon.

Example: Minimum length of a mission from Earth to Mars

Minimum mission length = roundtrip travel time to Mars + synodic period for Earth and Mars

= (Earth to Mars + Mars to Earth) + synodic period

= (0.63 years + 0.63 years) + 2.14 years

= 1.26 years + 2.14 years

= 3.40 years

7. PERCENT OF A LIFETIME AND CAREER

As a means of analyzing the value of each mission, the length of the missions can be calculated as percentages of a lifetime or percentages of a career.


Percent of a Lifetime and Career

An average lifetime is considered to be 74 years. The average length of a career is 40 years.

To calculate the percent of a lifetime (career) that each mission would take, the students will divide the mission length (time) by the average value for a lifetime (career). Then, they will multiply their result by 100%. The **Percent of a Lifetime and Career** student worksheet (SW p.19) contains a chart for students to record their formulas and answers.

Example: Percent of a lifetime required by a mission to Mercury

% of a lifetime = (mission length to Mercury ÷ average lifetime) • 100%

= (1.02 years ÷ 74 years) • 100%

≈ 0.013 • 100%

= 1.3%

≈ 1%

Example: Percent of a career required by a mission to Mercury

% of a career = (mission length to Mercury ÷ average career) • 100%

= (1.02 years ÷ 40 years) • 100%

≈ 0.025 • 100%

= 2.5%

≈ 3%

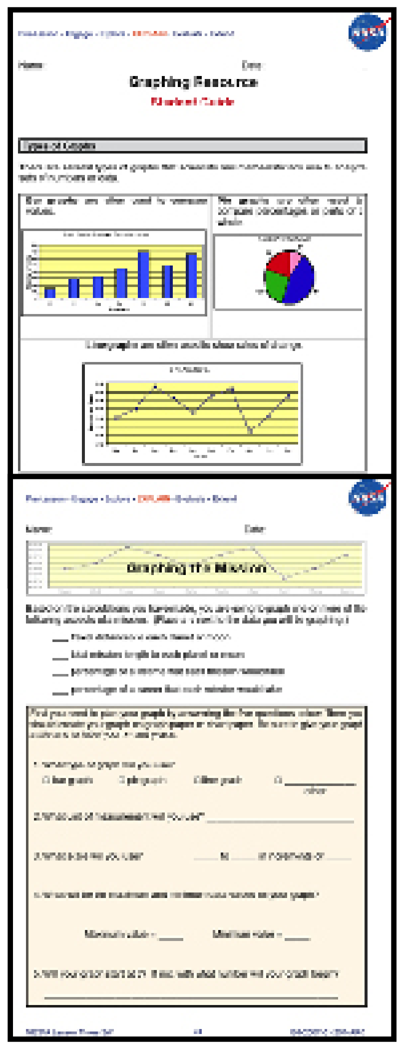
SW = student workbook TG = teacher guide EG = educator guide

Lesson 3 – EXPLAIN

- **Estimated Time:** 1 session, 50 minutes
- **Materials:**
  - Students' notes from EXPLORE section
  - Graphing Resource—Student Guide (SW pp.20-23)
  - Graphing the Mission student worksheet (SW p.24)
  - All Things Considered student worksheet (SW p.25)
  - Graph paper/ adding machine tape, or large chart/ poster paper
  - Calculators, rulers, markers
  - Graphing Rubric (TG p.45)

1. GRAPHING OPTIONS: Different Methods of Representing Results

Now that students have calculated the *distance* that the crew vehicle will travel to each destination and the amount of *time* a roundtrip mission to each destination will take, it will be useful for them to *graphically represent their data* so they can interpret and discuss the results.



Divide the students into groups and task them to graph one or more of the following:

- travel distances to each planet or moon based on transfer orbits
- total mission length to each destination based on synodic periods & roundtrip travel
- percentage of a lifetime that each mission would take
- percentage of a career that each mission would take.

Students can choose from pie graphs, line graphs, bar graphs, or number lines to represent their data. It would be helpful to have two types of graphs for each set of data for students to compare. For example, "percentage of a lifetime" could be represented in both a pie chart and a bar graph.

Before beginning the activity, review the **Graphing Resource—Student Guide** (SW pp.20-23) with the class. Students should use this resource to help them choose appropriate graphs for their data set. Each graph must have labels, a title, and a scale, as described in the Graphing Resource. With guidance from the **Graphing the Mission** student worksheet (SW p.24), have each group complete a sketch of their graphs for you to check before making a final copy on a poster or chart paper.

When each group has completed their graph(s), have the students present their work to the class. Students can compare graphs and note the similarities and differences between the different representations. Ask the following questions to ensure that each group communicates all of the important information:

- How did you decide to use this particular data set and graph? Explain.
- How do you know your graph is accurate?
- Do other students have questions about how you graphed the data? Does anyone disagree with your graph?
- Does your graph make sense? (For example, does Pluto stand out as the planet that is the furthest away and therefore would require the longest mission?)
- How do different students' strategies for graphing the data compare? Which strategy do you like best? Why?
- How does the graph represent the distances to the planets and moons, the time it would take to reach each destination, or the percent of a lifetime/career?

Have students reflect on which type of graph is most effective for communicating the data. Which type of graph makes the most impact? (For example, the fact that a trip to Pluto would take more than 50% of a person's career should make an impact on whether or not humans would want to travel there.) Student graphs may be assessed using the **Graphing Rubric** (TG p.45). **Sample graphs** are included on pages 46-50 of this guide.

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2. INTERPRETING RESULTS: Class Discussion

Now that students have graphical representations of the various time requirements of the different missions, they can begin to decide which planets they think humans should visit. When all of the groups have presented their graphs, open a class discussion:

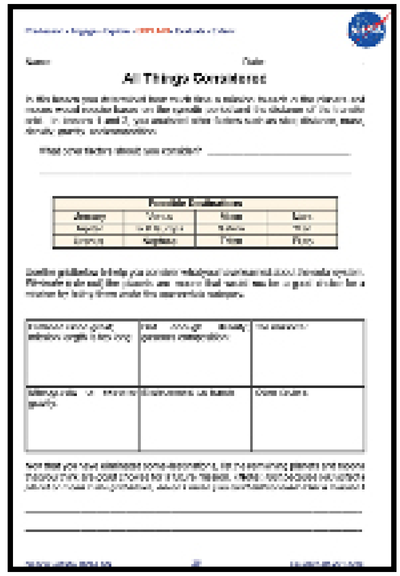
- Which planets or moons do you think humans should visit? Why?
- Which destinations have you ruled out?
- Why do you think a particular planet or moon should not be visited?

Students should use the results of their calculations and their graphs to support their claims. Encourage students to express different opinions, as long as they can support their statements with data and evidence.

If students support the idea of sending humans on very long missions (i.e., Pluto), then they should also discuss what adaptations they think should be made to accommodate humans on such a long voyage. For example, the effects of microgravity on the human body for time periods longer than a year are not known. How would students make sure astronauts stay healthy on longer missions?

3. OTHER FACTORS: Eliminating Possibilities

Before students make their decision as to where in our solar system we should send humans, they will need more information about planets and moons that humans may not be able to visit because of safety factors. Students can use the **All Things Considered** student worksheet (SW p.25) to help them process their thoughts and draw conclusions. Use the paragraphs below to guide students' thinking and to spark class discussion.


All Things Considered

While Venus is very close to Earth and would not take a long period of time to reach, Venus is not an acceptable destination for humans. Most pictures of Venus that students have seen probably do not show the surface of the planet. Instead, most pictures show the carbon dioxide atmosphere that covers the planet. Due to the greenhouse effect caused by Venus's thick atmosphere, the surface temperature is approximately 870 degrees Fahrenheit, which is hot enough to melt lead. Even with the protective space suits that NASA provides its astronauts, humans could not survive in such extreme heat.

Students determined in Lesson 2 that Jupiter is a gas planet and has no solid surface on which to land. While the moons of Jupiter may be more realistic destinations in terms of surface conditions, high levels of radiation are trapped near Jupiter by Jupiter's strong magnetic field. This radiation is extremely dangerous for humans and makes its moons, Io and Europa, difficult places for humans to visit. If humans were sent to one of these moons, major adaptations would have to be developed in order to keep humans alive. A large amount of water or shielding would have to accompany the astronauts, and these items would take up a great deal of weight and room in the spacecraft. With our current technologies, safely traveling to these destinations would be very difficult to accomplish. Students should still consider these moons as possible destinations, but they will have to take into consideration adaptations for safety.

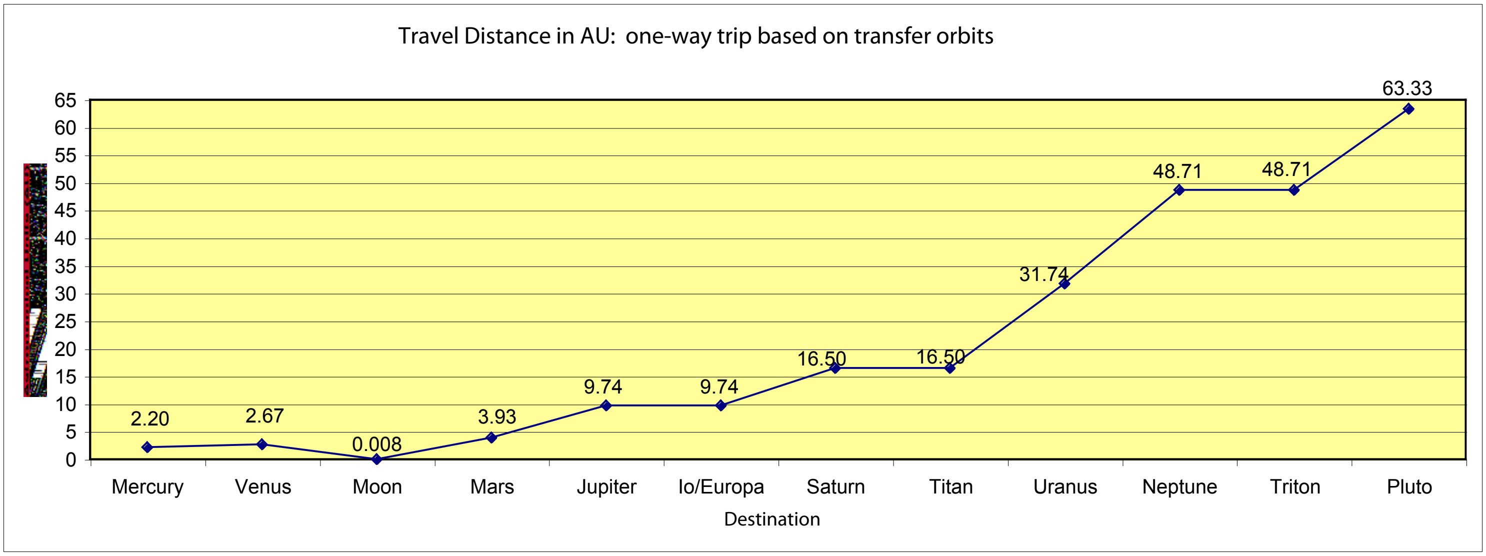
During the next lesson, students will be performing calculations based on one destination. If possible, each student or small group should be assigned one destination for which they will complete the calculations. Students can choose from Mercury, Venus, Moon, Mars, Io, Europa, Titan, Triton, and Pluto. It would be best if the calculations for each destination were performed by two students or two groups so that their results can be compared.

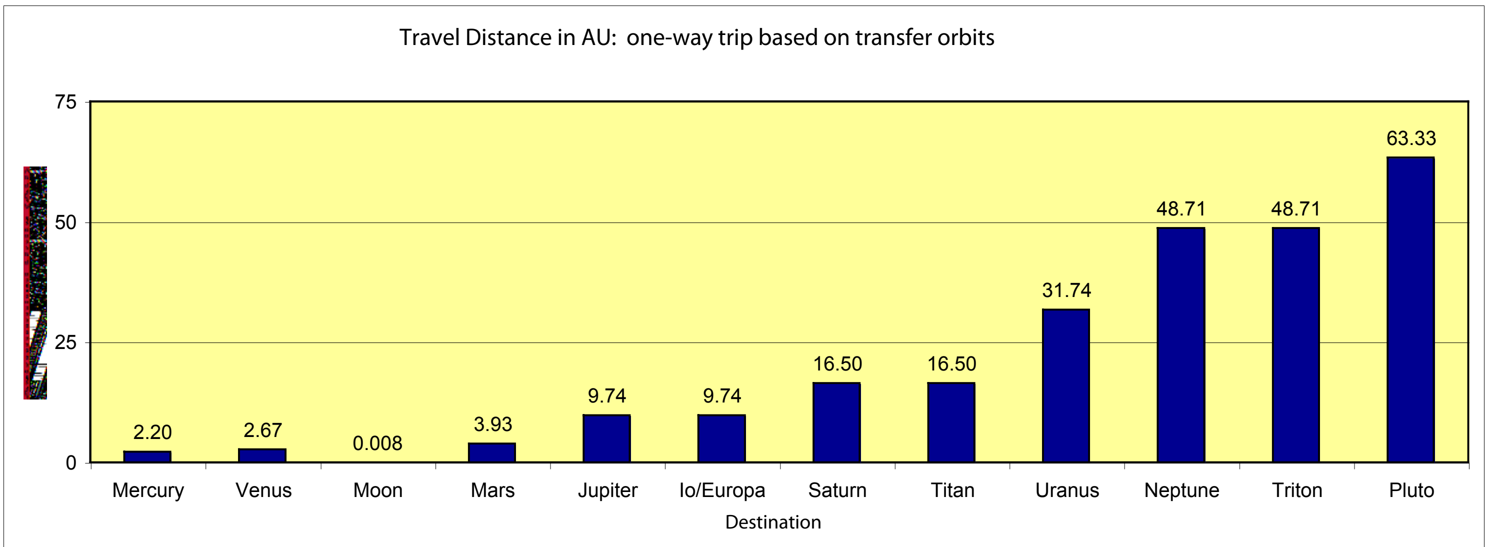
Graphing Rubric

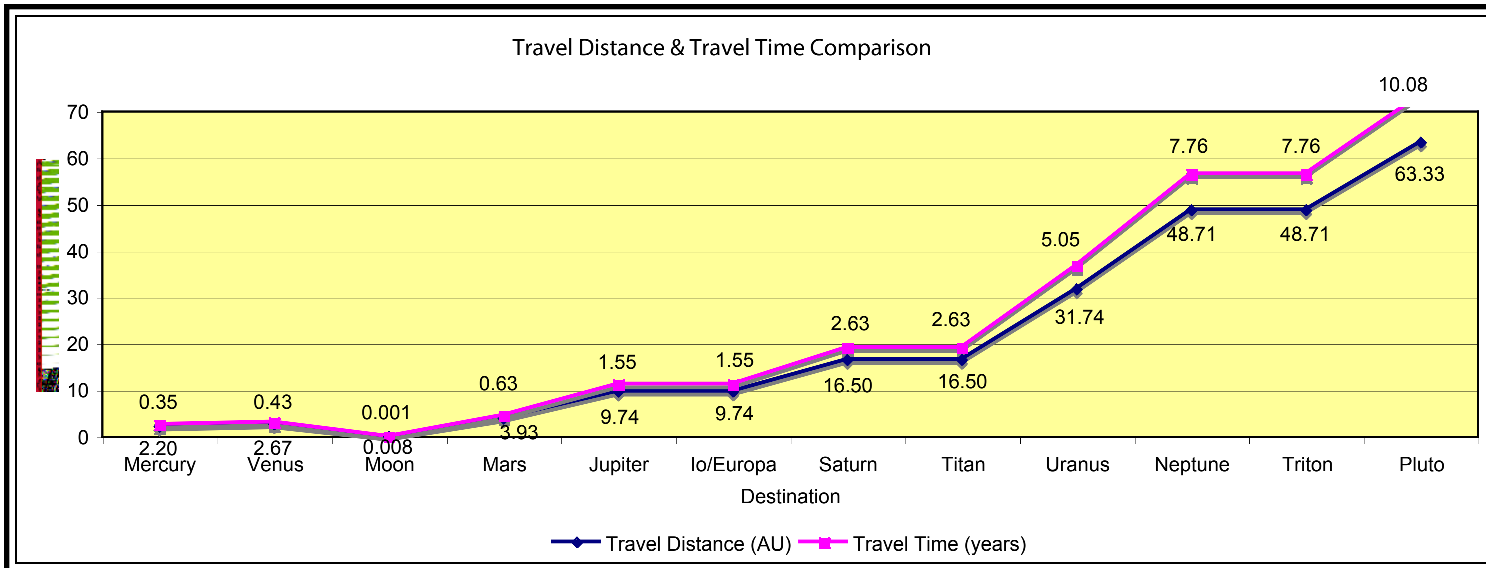
Student graphs can be assessed with the following rubric. Sample graphs are included on pages 46-50 of this guide.

4	• All data is graphed extremely accurately. Decimals and fractions are taken into account. • Graph is titled and all axes are correctly and neatly labeled. • Graph includes a consistent scale on the y-axis. • Graph type is appropriate for data used. • Choices for graph type, scale, and units are fully justified and related to the data.
3	• All data is graphed accurately. Decimals and fractions were rounded to whole numbers. • Graph is titled and all axes are labeled. • Graph includes a consistent scale on the y-axis. • Graph type is appropriate for data used. • Choices for graph type, scale, and units are justified and may be related to the data.
2	• Data is graphed somewhat accurately. Decimals and fractions were ignored. • Graph is missing either title or axis labels. • Graph includes a consistent scale on the y-axis. • Graph type is somewhat appropriate for data used. • Choices for graph type, scale, and units are not justified and/or may not be related to the data.
1	• Data is not graphed accurately. • Graph does not have a title or axis labels. • Graph does not have a consistent scale for y-axis. • Graph type is inappropriate for data used. • Choices for graph type, scale, and units are not justified and are not related to the data.

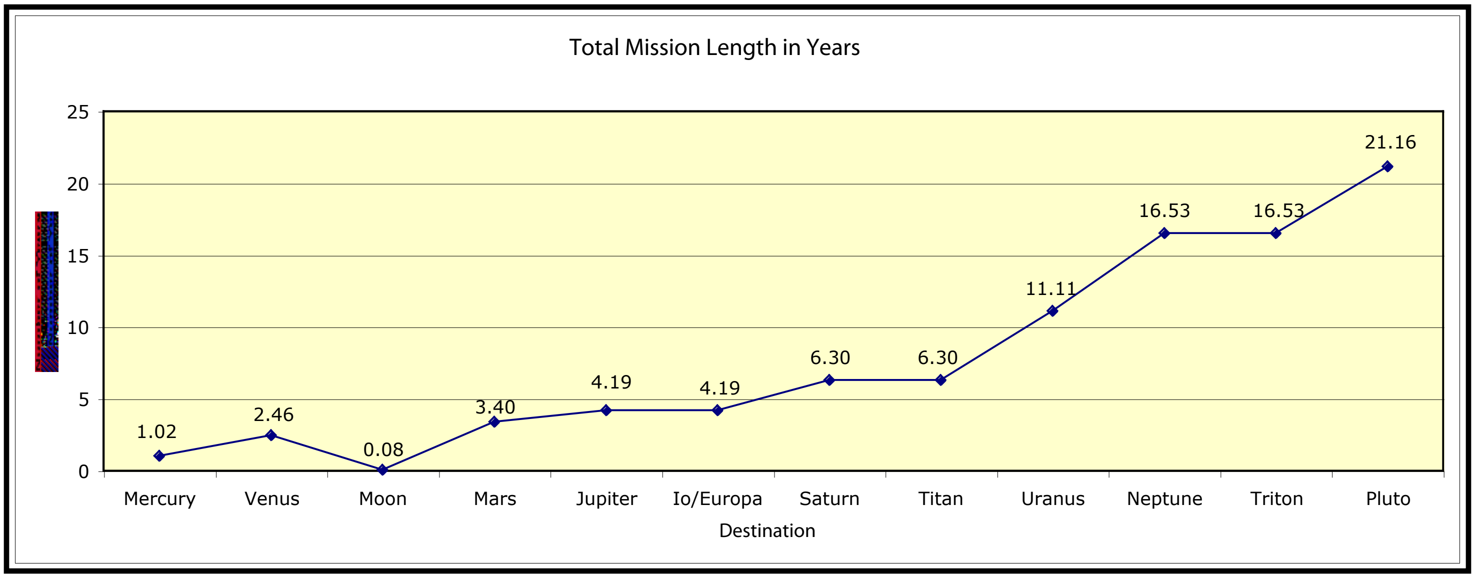
Sample Graphs: Travel Distance

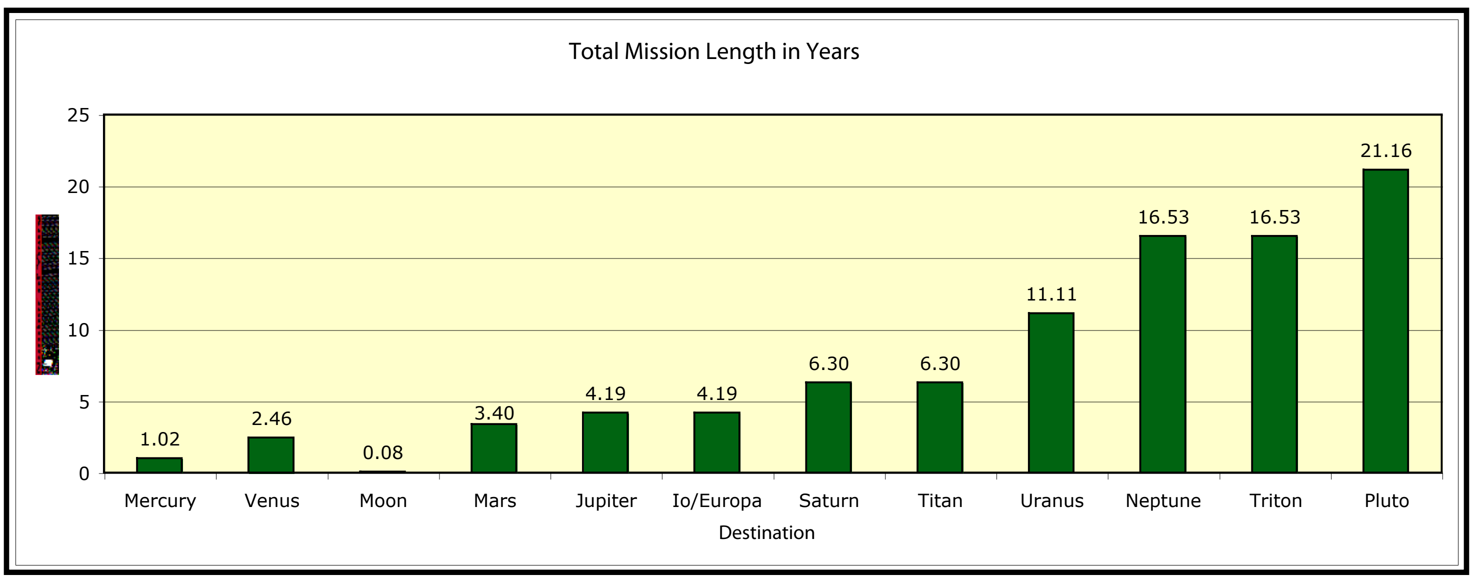




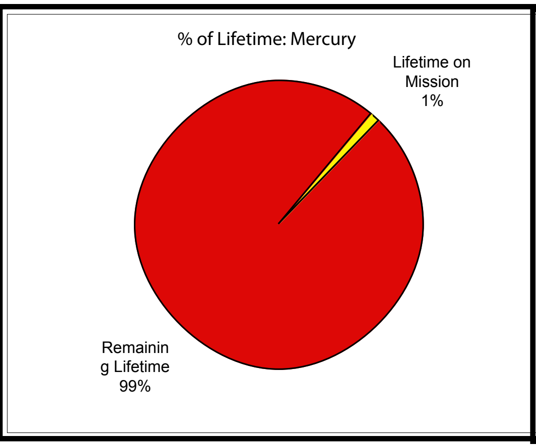


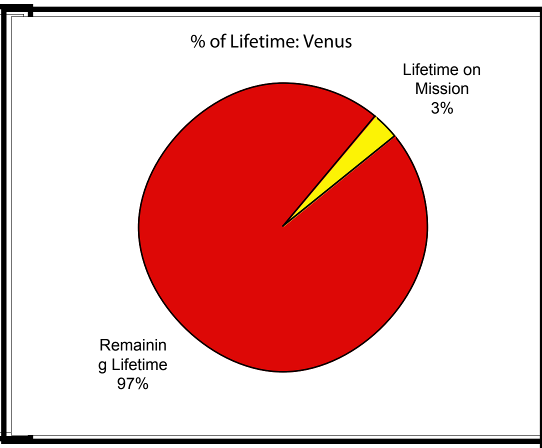
Sample Graphs: Mission Length

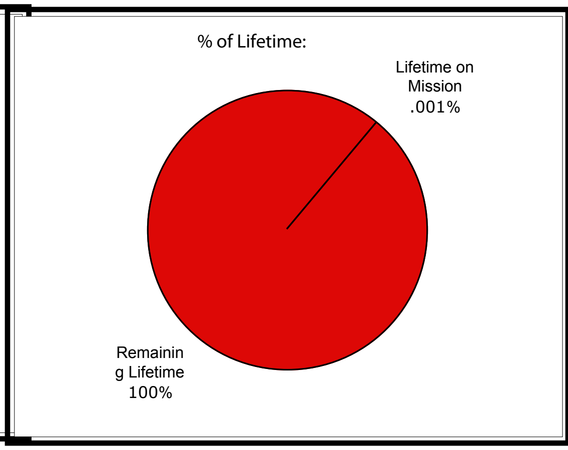


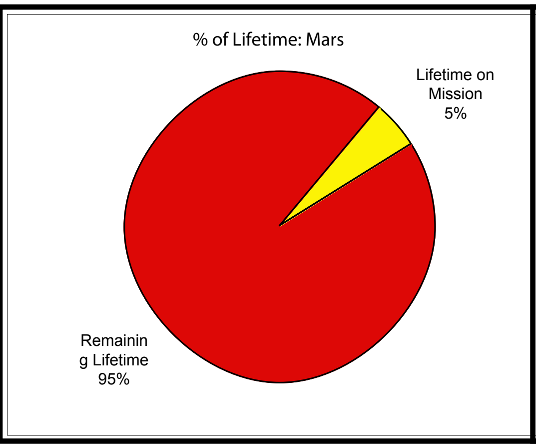


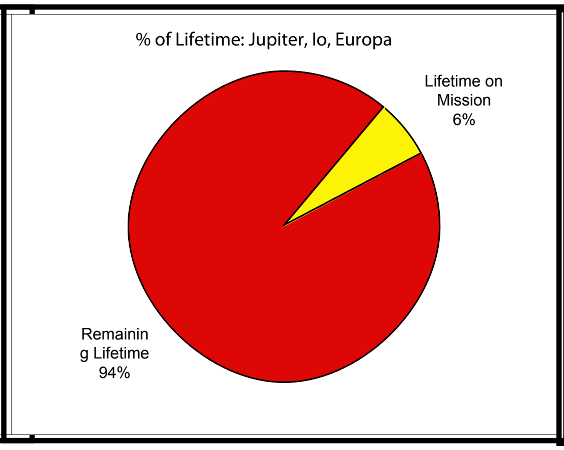
Sample Graphs: Percent of a Lifetime

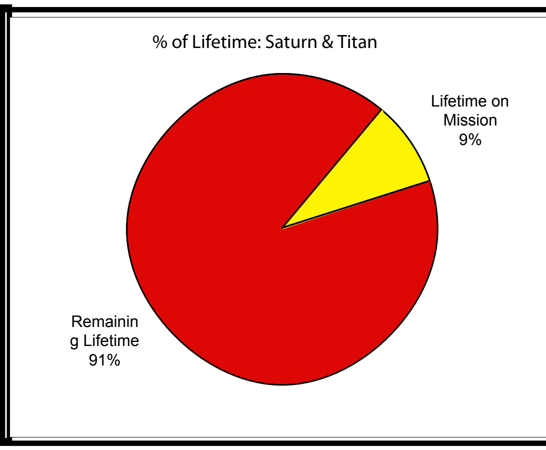


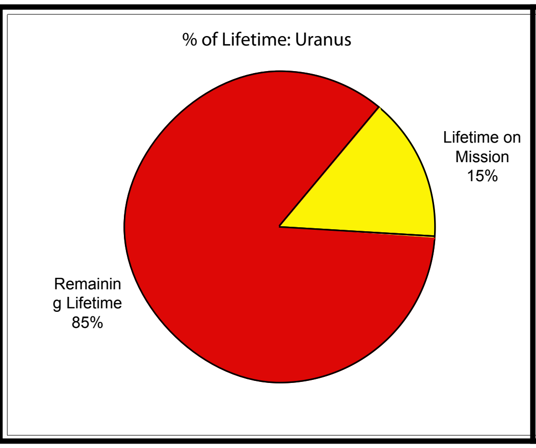


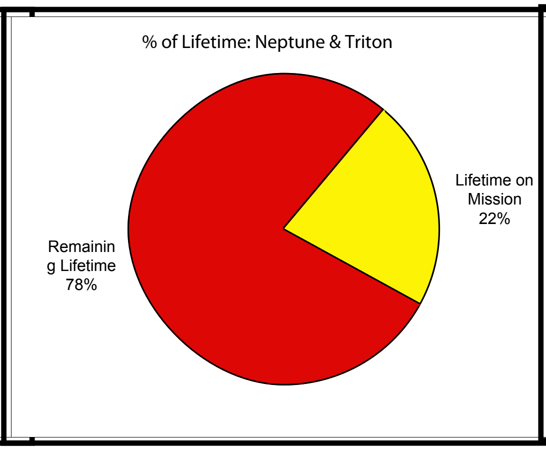


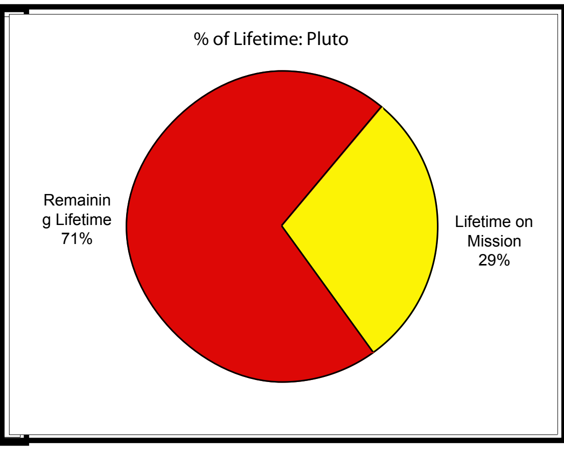




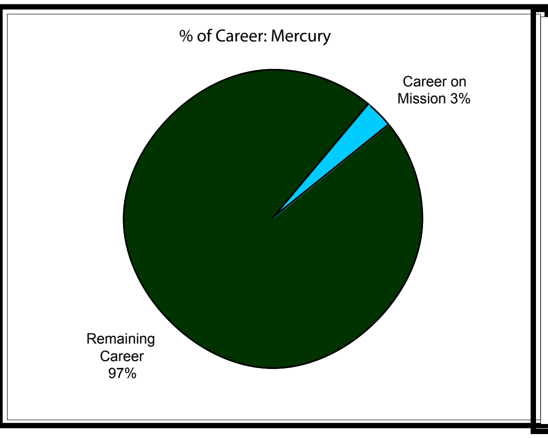


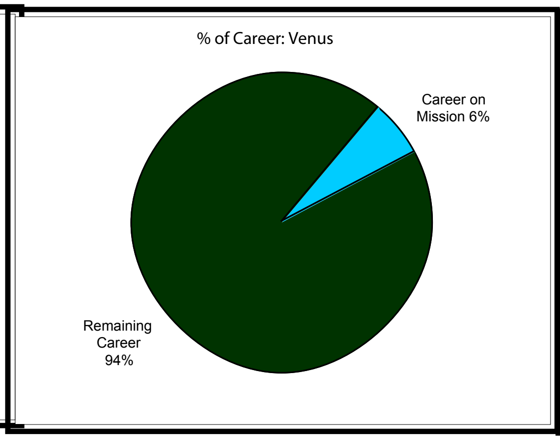


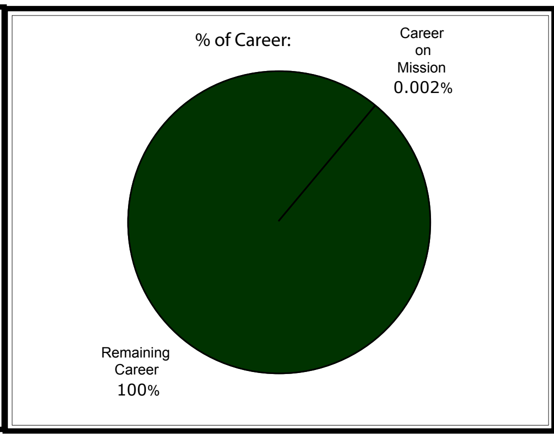


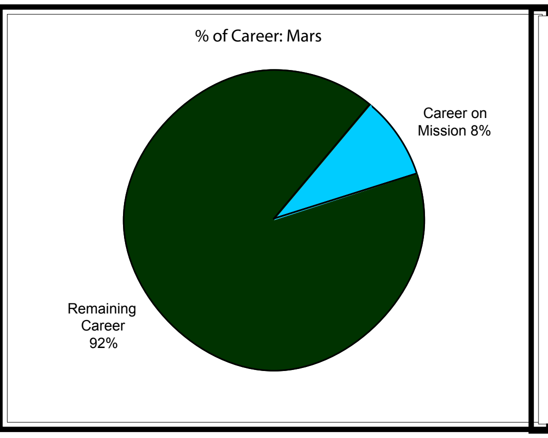


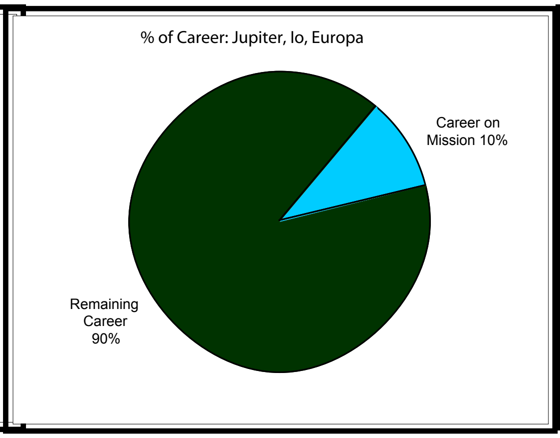
Sample Graphs: Percent of a Career

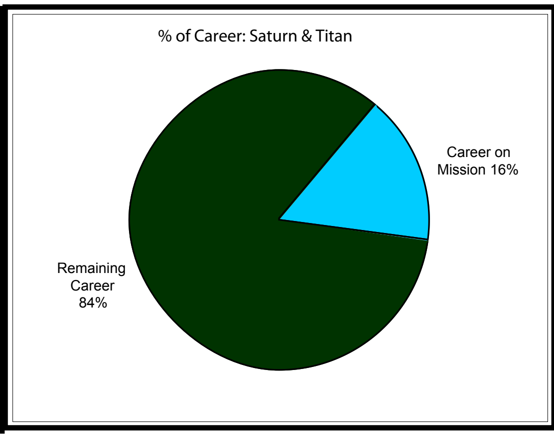


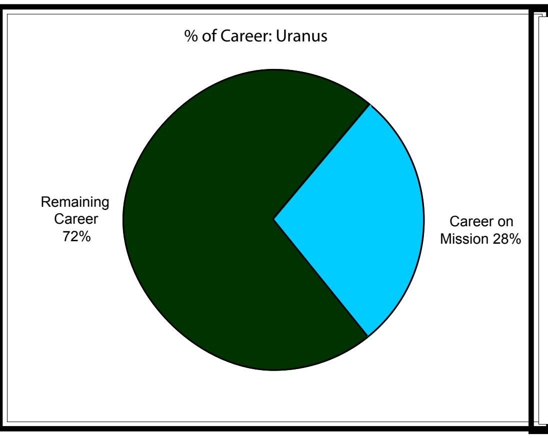


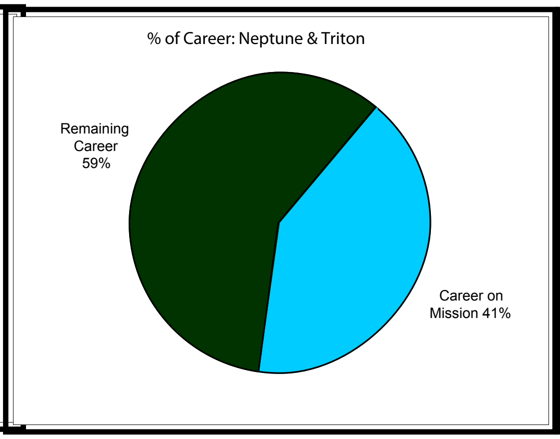


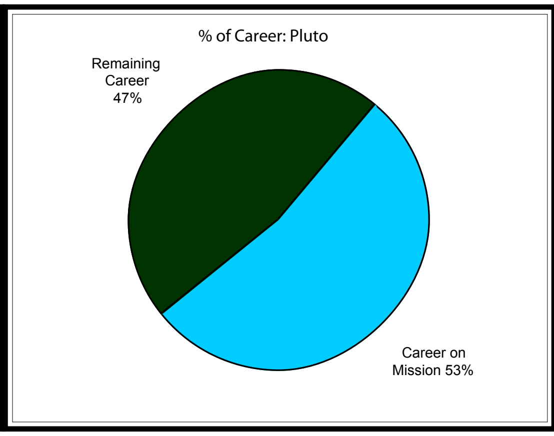






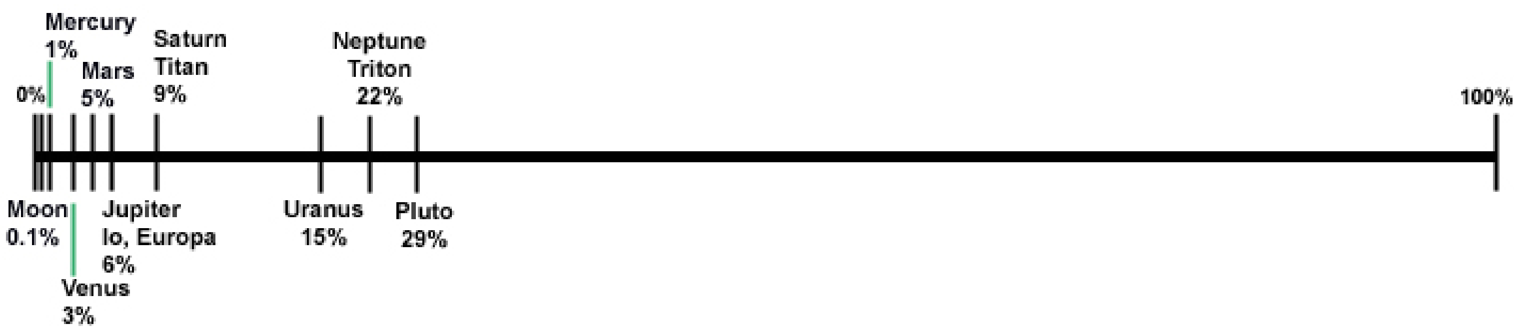




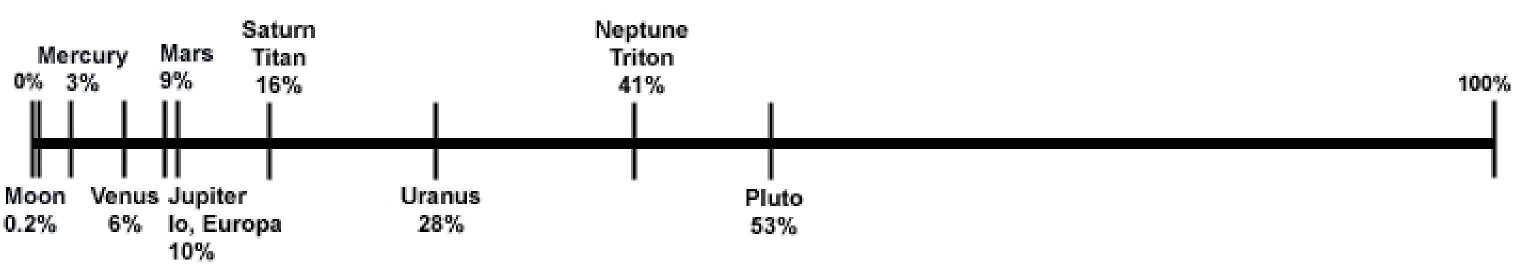


Number Lines

Number Line: Percent of Lifetime



Number Line: Percent of Career



SW = student workbook TG = teacher guide EG = educator guide

Lesson 3 – EVALUATE

- **Estimated Time:** 1 session, 40 minutes
- **Materials:**
  - Student notes, observations, and graphs
  - Problem Solving Rubric (TG p.53)

To reflect on and review the lesson, lead the class in the following discussion.

Check for understanding:

1. What units do we use to measure time?
2. What units do we use to measure speed?
3. How are speed, time, and distance related? Can you give an equation that allows you to solve for one of the variables in terms of the other variables?
4. Which planets would take the greatest amount of time to reach?
5. Which planets would take the least amount of time to reach?

Reflection:

1. Which planets or moons should be ruled out as possible locations for humans to visit? Why?
2. Which planets or moons do you think are the best places to send humans? Why?
3. What else will you need to know about a mission before you decide where to send humans?

Note: If your students completed lessons 1 and 2, then this would be a good time to review what students said they would need to know at the end of each of those lessons.

A **Problem Solving Rubric** (TG p.53) is provided for evaluating students' work throughout this lesson.

Preparing to move on:

In the next lesson, students will calculate how much mass would be added to a crew vehicle in the form of supplies for a trip to another planet or moon. Students will also calculate how much it will cost to launch such a mission from Earth.

1. Astronauts need to take all of their supplies with them into space. Make a list of the resources that are most important for survival.
2. If humans are sent on missions that last several years, estimate how much of each resource the spacecraft will need to hold.
3. What challenges will this create for very long missions?
4. What are some possible solutions? What can be done to reduce the amount of supplies astronauts would need to take with them?

Brief closing assignment:

Lesson 4 will have students focus their calculations on the supplies and fuel needed to reach a planet or a moon. At this point, students must choose the planet or moon for which they will calculate the cost of a mission.

The following can be given as a brief, one paragraph writing assignment. Students can respond on index cards (which keep responses concise) or in a journal. Alternatively, students can discuss their answers in pairs or small groups and report their answers back to the class.

- What did you learn during this lesson?
- Based on your experience in this lesson, where do you think we should send humans in our solar system?
- What else do you need to know in order to make a recommendation?
- How will you gather that information?

Problem Solving Rubric

Problem-solving assignments and presentations can be assessed with the following rubric.

4	• Answers were calculated correctly to an appropriate degree of accuracy (rounded to a decimal place or whole numbers where specified). • Answers are fully explained and justified in detail. • All steps of the problem are explained in detail. • Information supplied by the students is accurate and the source of the information is given. • Picture that accompanies problem is relevant, labeled, and demonstrates how the problem was solved. • Written explanation completely outlines the problem and the solution.
3	• Answers were calculated correctly, but to an inappropriate degree of detail (rounded to whole numbers or not rounded where it was appropriate). • Answers are explained and justified. • All steps of the problem are explained. • Information supplied by the students is accurate, but the source of the information is not given in detail. • Picture that accompanies problem is somewhat relevant, may or may not be labeled, and somewhat demonstrates how the problem was solved. • Written explanation outlines the problem and the solution.
2	• Answers were mostly calculated correctly. • Answers are stated clearly but not explained or justified • All steps of the problem are not fully explained. • Information supplied by the students may not be accurate and the source of the information is not given. • Picture that accompanies problem is not relevant, is not labeled, or does not demonstrate how the problem was solved. • Written explanation does not clearly outline the problem and the solution.
1	• Answers were not calculated correctly. • Answers are not stated clearly and are not explained or justified. • Steps of the problem are not explained. • Information supplied by the students is not accurate. • No picture. • Written explanation does not outline the problem or the solution.

SW = student workbook TG = teacher guide EG = educator guide

Lesson 3 – EXTEND & APPLY (optional portion of lesson)

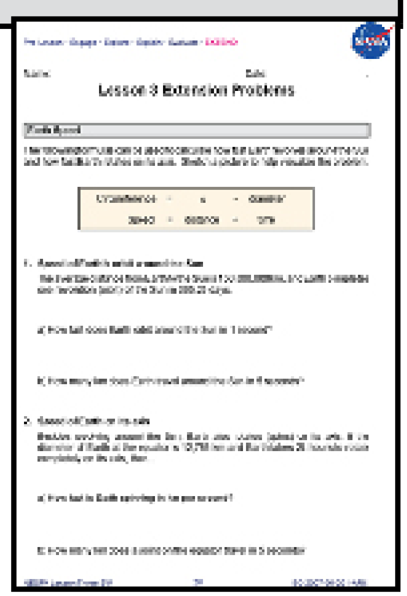
- **Estimated Time:** 1 session, 40 minutes
- **Materials:**
  - Lesson 3 Extension Problems (SW pp.26-32)
  - Problem Solving Teacher Resource (TG pp.57-59)
  - Graphing Resource—Student Guide (SW pp.20-23)
  - Paper for student work
  - Calculators (optional)

Have students work on the **Lesson 3 Extension Problems** (SW pp.26-32). These problems are multi-step open-ended challenges. Some will require the students to apply what they know about scale and ratio and proportion. For these problems, students may choose the units they work with, as long as they are appropriate. The problems can be done individually, in groups, or as a class.

You may want students to accompany each solution with a written and graphical explanation of how the problem was solved. Review the **Problem Solving—Teacher's Resource** and sample write up (TG pp.57-59) with your students before having them complete their own write up.

Students can also refer to the **Graphing Resource—Student Guide** (SW pp.20-23) for guidance in creating a pie chart for section "g" of the Percent of a Lifetime extension activity (SW p.29).

Earth Speed (SW p.26)


Lesson 3 Extension Problems

Students should know that Earth is orbiting the Sun and that Earth is spinning on its axis as it travels around the Sun. The following two problems ask students to calculate both the speed of Earth's *revolution* and the speed of Earth's *rotation* using the formulas for circumference ($C = \pi \cdot d$) and speed (rate = distance ÷ time).

Guidance for solving these two problems are included on the following page. Students should also *sketch* the problems as part of their strategy for finding a solution. The answers are located on page 26 of the Answer Key.

1. The speed of Earth's orbit around the Sun

Using the distance from Earth to the Sun and the length of time it takes for Earth to complete one revolution, students can calculate the rate at which Earth is moving on its orbital path.

A. Use the distance from the Earth to the Sun (150,000,000 km or 1.5×10^8 km) and the formula for the circumference of a circle to calculate the distance Earth travels in one complete orbit around the Sun. (Remember that the distance from Earth to the Sun is the radius of the circle, so this value will need to be doubled to get the diameter.)

B. Earth completes one orbit of the Sun in 365.25 days. Use this value and the distance traveled in that time (calculated in part A) to calculate the speed at which the Earth is traveling. (rate = distance ÷ time)

C. The speed calculated in part B will be in km/day. If students use 3 steps to convert km/day into km/sec, then they will need to convert their answer to seconds by first dividing the rate by 24 hours, then by 60 minutes, and finally by 60 seconds.

D. Students then need to multiply this rate by 5 to get the distance traveled in 5 seconds.

E. To get a sense of this speed, count out 5 seconds with the class and discuss how far the Earth traveled in that amount of time. Does this surprise them? Why or why not?

2. The speed of Earth on its axis

Using the diameter of Earth at the equator, and the amount of time it takes for Earth to complete one rotation about its axis, students can calculate the rate at which a point on the equator is traveling.

A. Use the diameter of Earth at the equator (12,755 km) and the formula for the circumference of a circle to calculate the distance a point on the equator travels in one rotation.

B. It takes the Earth 24 hours to rotate completely on its axis. Use this value and the distance traveled (calculated in part A) to calculate the speed at which the point on the equator is traveling. (rate = distance ÷ time)

C. This speed calculated in part B will be in km/hr, so students will need to convert their answer to km/sec. One way to do this is by first dividing the rate by 60 minutes and then by 60 seconds.

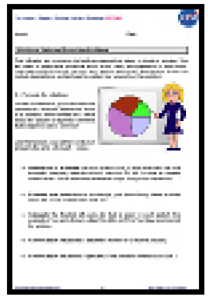
D. Students then need to multiply this rate by 5 to get the distance traveled in 5 seconds.

E. To get a sense of this speed, count out 5 seconds with the class and discuss how far a point on the Earth traveled in that amount of time. Does this surprise them? Why or why not?

Additional Ratio and Proportion Problems (SW pp.27-32)

Pages 27-32 of the student workbook contain three sets of extension activities that students can complete individually, in groups, or as a class:

1. Percent of a Lifetime
2. Travel Time—Part I
3. Travel Time—Part II



Think About It / Write About It / Discuss It Questions

The following questions can be used for brief writing assignments or mini class discussions.

1. Which planets would take the longest amount of time to reach? Which planets would take the shortest amount of time to reach? What is the maximum length of time you would be willing to travel to reach another planet or moon?

2. Which planets or moons do you think are too far away to send humans? Why? If we did send humans to the outer moons or planets, what kinds of adaptations would we need to make for very long missions?

3. If you left today for a mission to Pluto, how old would you be when you returned? What do you think you would have missed the most? What would all of your friends be doing in that time? What would be the benefits of going on a mission to another planet? What would be the benefits of staying on Earth? Based on these benefits, which of the following do you think would be better: staying on Earth or traveling to Pluto? Why?

4. When astronauts travel in space, they have a very small amount of room for personal items. If you were leaving Earth for many years, what items would you take? Why? Remember, everything you take would have to fit into a small suitcase.

5. What do you think would be some challenges of planning a very long mission to another planet or moon?

Problem Solving

Teacher's Resource

During the course of this unit, students will be presented with multi-step, open-ended challenges. The problems can be solved in a variety of ways, and there will often be multiple solutions. The problems can be done individually, in groups, or as a class.

Each problem can be accompanied by a written explanation and a picture explaining how the problem was solved. Students can use the following outline to explain their work in written form:

1. Restate the problem. What are you trying to find out?
2. What information do you have? What information do you need to find your answer? Explain how you got the information and record it.
3. Estimate what you think the answer will be. How do you know your estimate is reasonable?
4. Show your work. Include all calculations you made in order to solve the problem—even the ones that did not work.
5. Explain HOW you solved the problem. Step-by-step, what did you do? Use transitions like first, next, then, and finally.
6. State your answer. Explain HOW you know it is correct. Does it make sense? Why?
7. Draw a picture to go along with the problem. Label sizes and distances.

When you finish, read over your work. Pretend you are explaining this problem to someone younger than you.

- Is it clear?
- Does it make sense?
- Did you explain the problem and the answer well?

Example: Scale Movie Stars

Some fantasy characters, such as Hobbits from Lord of the Rings, or Hagrid from the Harry Potter series are on different scales than humans. The following calculations will demonstrate how an everyday object would need to be changed to fit the scale size of a character.

Hobbits are known as Halflings. They are about half the size of a human. Hagrid, however, is half-giant because he had a Giantess Mother. He is about twice the size of a human.

If your teacher became a Hobbit, estimate how tall he or she would be. Estimate how tall your teacher would be if he or she were Hagrid's size. Measure your teacher and calculate his or her Hobbit and Hagrid heights. If possible, mark the Hobbit height, Hagrid height, and actual height of your teacher on the wall or chart paper.

Sample Write Up:

1. I am going to calculate the height my teacher would be if she was a Hobbit or if she was a half-giant like Hagrid.

2. I know that Hobbits are half the size of humans, and I know that Hagrid is twice the size of a human. In order to solve the problem, I need to know my teacher's height. I will use a meter stick and measure her. My teacher is 1.75 meters tall.

3. I estimate that as a Hobbit my teacher will be less than a meter tall because Hobbits are much smaller. I think that as Hagrid my teacher will be over 3 meters tall because Hagrid is much bigger.

4. Hobbit Height:

   1.75 meters • 1/2 = teacher's Hobbit height

   1.75 meters • 0.5 = 0.875 m

   My teacher's Hobbit height = 0.875 m

   Hagrid Height:

   1.75 m • 2 = teacher's Hagrid height

   1.75 m • 2 = 3.5

   My teacher's Hagrid height = 3.5 m

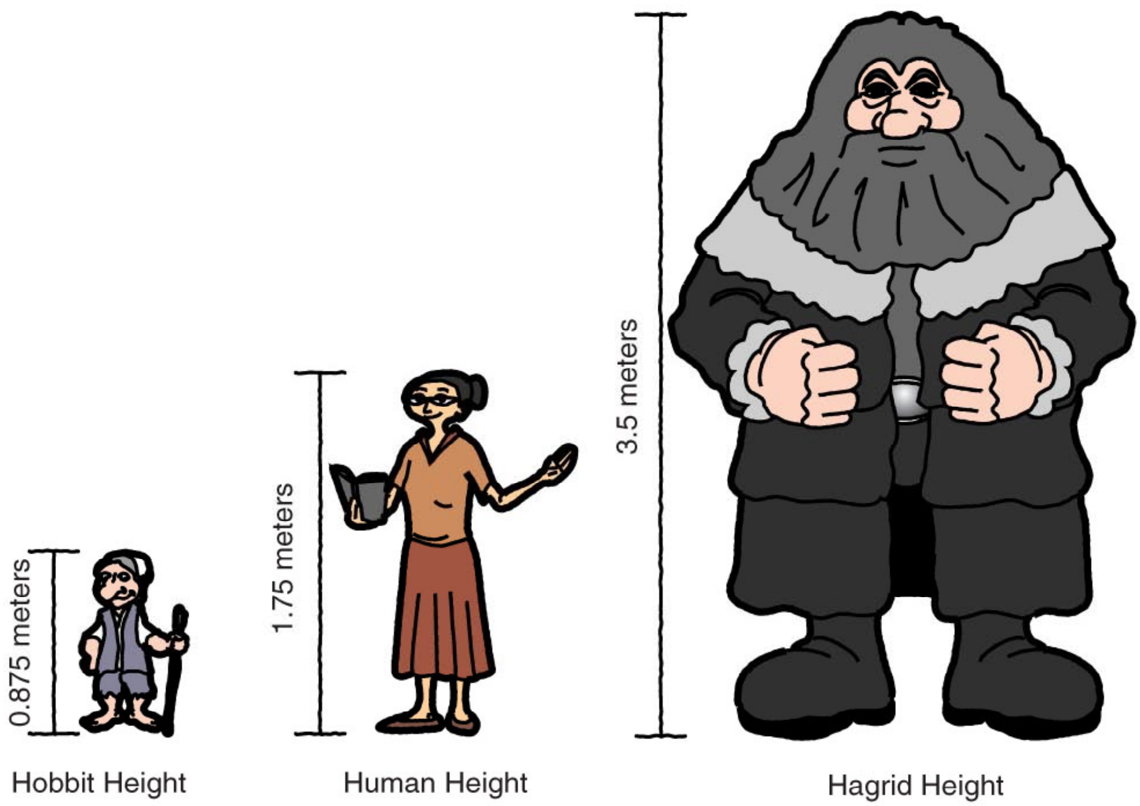
5. I solved the first part of the problem by multiplying my teacher's height by one-half. I solved the second part of the problem by multiplying my teacher's height by two.

   First, I solved for her Hobbit height. Hobbits are half the size of humans, so to get my teacher's Hobbit height I multiplied her normal height by one-half. I decided it would be easier to multiply decimals, so I multiplied 1.75 meters by 0.5 because 1/2 is equal to 0.5.

   Next, to get my teacher's Hagrid height, I multiplied her normal height by 2, because Hagrid is twice the size of a human.

6. I found that if my teacher were a Hobbit, she would be 0.875 meters tall because this is one-half of her normal height. I also found that if my teacher were like Hagrid, she would be 3.5 meters tall because this is two times her normal height. This makes sense because as a Hobbit she would be much smaller than her normal size, and as Hagrid she would be much bigger than her normal size. My estimates were pretty close. I was not off by that much.

7.



0.875 meters   1.75 meters   3.5 meters