NASA Explorer Schools Pre-Algebra Unit

Lesson 2 Student Workbook

Solar System Math

Comparing Mass, Gravity, Composition, & Density

What interval of values for the mass of a planet or moon has surface conditions suitable for human exploration?
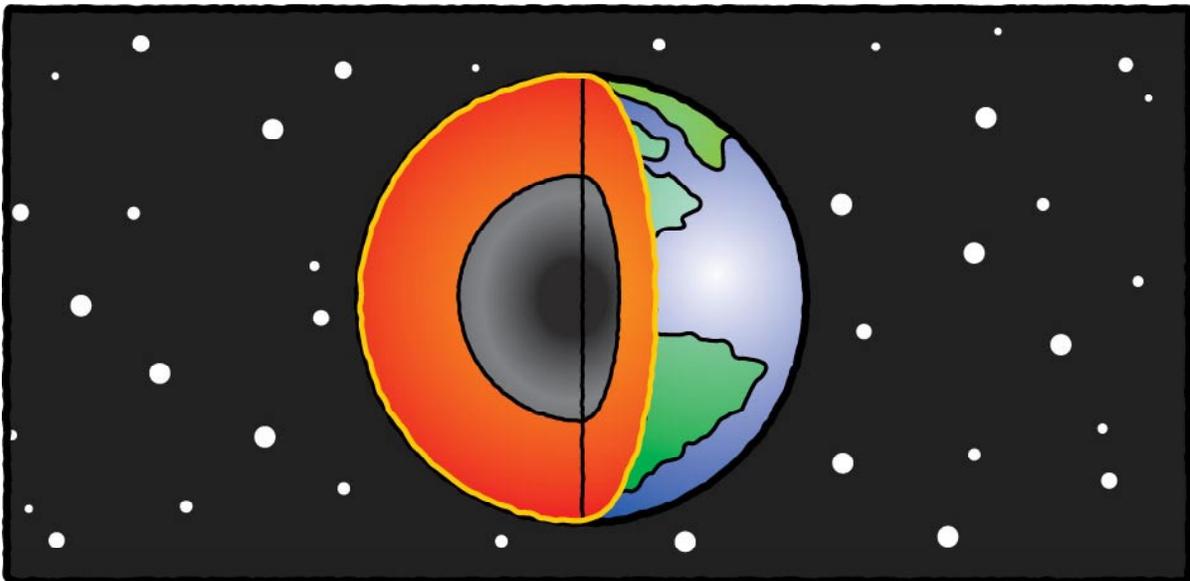
Name: _____   Date: _____

Pre-Lesson Activity

Make Observations!

1. *Without touching them*, look at the three containers and notice their similarities and differences. List your observations below: _____ _____ _____	2. Now observe the containers more closely by touching them and moving them, but *do not open them*. Write your new observations below: _____ _____ _____

3. Measure the *weight* of each of the three containers. Be sure to record your unit of measurement.

     _____   _____   _____

     Container A   Container B   Container C

4. Using a *single finger*, try moving each container one at a time. Record which container is easiest and which is hardest to move.

     _____   _____   _____

     Easiest to move   In between   Hardest to move

5. Imagine you are on a spacecraft orbiting Earth with the same containers. If you had to move these containers with just one finger, *which container* (if any) *would be the most difficult to move and which would be the easiest?*

     _____   _____   _____

     Easiest to move   In between   Hardest to move

Name: _____ Date: _____

Mass vs. Weight
MASS is the amount of "stuff" (MATTER) in an object.
WEIGHT is a measurement of the force of gravity on the mass of an object.

6. Would the *weight* of these three containers change in space?

Yes, because _____

No, because _____

7. Would the *mass* of these three containers change in space?

Yes, because _____

No, because _____

Draw Conclusions!

8. Objects that have the same size and shape may weigh different amounts and have different masses based on their _____ .

9. An object's weight will (change / stay the same) in space and on other planets, and its mass will (change / stay the same) in space and on other planets.

10. Objects of greater mass are (easier / harder) to move than objects of lesser mass.

11. Statement 10 would be (true / false) in space and on other planets and moons.

Make a Guess!

12. Based on the observations you have made and the conclusions you have drawn, guess the composition (or content) of each container.

_____   _____   _____
Container A   Container B   Container C

Name: _____   Date: _____

Planetary Mass and Human Exploration

Before you begin gathering data about the planets, you should make a hypothesis (or educated guess) as to which planets or moons have a mass that will support human exploration. Use the following questions to help you formulate your hypothesis.

1. Why is the mass of a planet or moon important in regard to human exploration?

   _____

   _____

   _____

2. Which planets do you think have the most mass?

   _____

   _____

   _____

3. Which planets do you think have the most gravity?

   _____

   _____

   _____

4. Which planets do you think would be best for humans to visit:

   ☐ planets with more mass
   ☐ planets with less mass

5. Predict which planets or moons have an appropriate amount of mass for humans to visit.

   _____

   _____

Name: _____ Date: _____

Minimum and Maximum Values

Imagine you are in Chicago in January. During one day, you check the outside temperature each hour between 7am and 7pm.

> The temperature readings in degrees Fahrenheit are:
>
> 6º, 5º, 7º, 10º, 11º, 11º, 14º, 16º, 18º, 20º, 16º, 12º, 10º
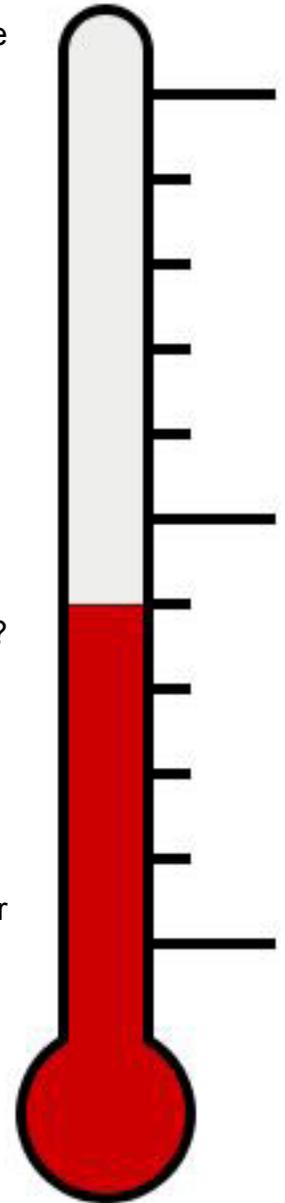
1. What is the *minimum* (least) temperature reading? Why?

    minimum reading: _____ º

2. What is the *maximum* (greatest) temperature reading? Why?

    maximum reading: _____ º

3. Based on the *minimum* and *maximum* temperatures, write a number interval that contains all of the temperature readings.

    _____ ≤ x ≤ _____

4. Create a number line showing your interval of values. Remember to label your line in uniform (equal) segments.

Name: _____   Date: _____

Circles and Spheres

Think about the differences and similarities between circles and spheres, and then answer the questions below.

1. List three objects that are spheres.

    _____   _____   _____

2. Draw two lines: one line to indicate the *diameter* of the circle and one line to indicate the *diameter* of the sphere.

    Circle   Sphere
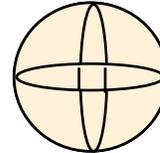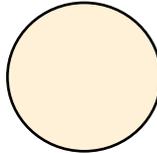
3. Draw two lines: one line to indicate the *circumference* of the circle and one line to indicate the *circumference* of the sphere.

    Circle   Sphere
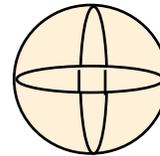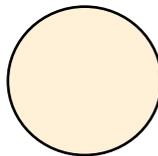
4. A *great circle* is a circle on the surface of a sphere that divides the sphere into two equal hemispheres. Great circles have the same _____ as the sphere.

Longitude   Latitude
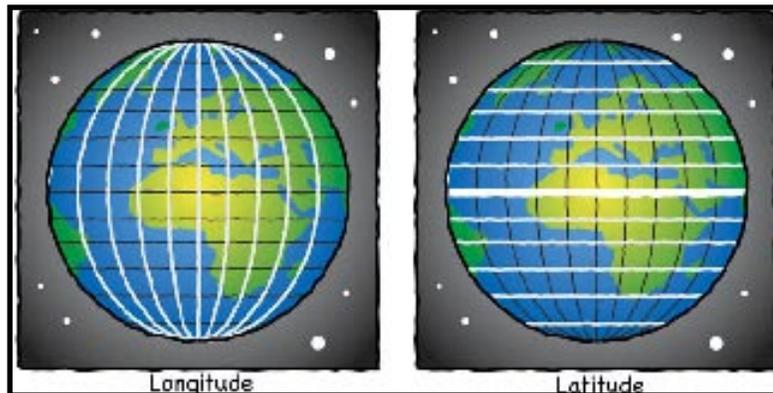
5. ALL lines of _____ are great circles.

6. ONE line of _____ is a great circle. It is called_____.

Name: Date: .

Lesson 2 Planet Data Sheet — Inner Planets
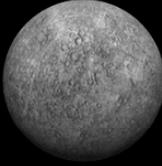
	Mercury	Venus	Earth	Mars
Planet	**Mercury**	**Venus**	**Earth**	**Mars**
Diameter in km				
Circumference in km				
Mass in kg				
Density in kg/m³				
Gravity in m/s²				
Composition (rocky, frozen, gaseous, etc)				
Surface flyby description				

Name: Date: .

Lesson 2 Planet Data Sheet — Outer Planets
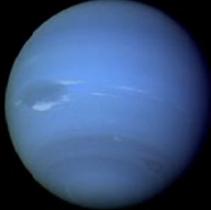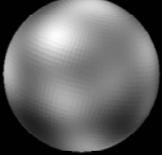
Planet	Jupiter	Saturn	Uranus	Neptune	Pluto
Diameter in km					
Circumference in km					
Mass in kg					
Density in kg/m^3					
Gravity in m/s^2					
Composition (rocky, frozen, gaseous, etc)					
Surface flyby description					

Name: _____ Date: _____ .

Lesson 2 Planet Data Sheet — Moons

Planet	The Moon	Titan	Io	Europa	Triton
Diameter in km					
Circumference in km					
Mass in kg					
Density in kg/m³					
Gravity in m/s²					
Composition (rocky, frozen, gaseous, etc)					
Surface flyby description					
Parent Planet					

Name: Date: .

Scale Model: Mass

Calculate the mass of each planet and moon, and then determine the number of cotton balls that will be needed for each planet (and moon) in the scale model.

Planet / Moon	Mass in kg x 10^{22}	Mass in scientific notation (kg)	Number of cotton balls based on Earth's mass
Mercury	33.0 x 10^{22} kg		
Venus	487 x 10^{22} kg		
Earth	597 x 10^{22} kg	5.97 x 10^{24} kg	1.0 cotton ball
Mars	64.2 x 10^{22} kg		
Jupiter	190,000 x 10^{22} kg		
Saturn	56,800 x 10^{22} kg		
Uranus	8,680 x 10^{22} kg		
Neptune	10,200 x 10^{22} kg		
Pluto	1.25 x 10^{22} kg		
Moon	7.35 x 10^{22} kg		
Titan	13.5 x 10^{22} kg		
Io	8.93 x 10^{22} kg		
Europa	4.80 x 10^{22} kg		
Triton	2.14 x 10^{22} kg		

Name: Date: .

Scale Model: Circumference

Calculate the scale circumference of each planet (and moon) based on Pluto's scale circumference being equal to 1 centimeter.

Planet / Moon	Diameter in km	Circumference in km	Scale circumference in cm
Mercury	4,878 km	15,329 km	
Venus	12,104 km	38,025 km	
Earth	12,755 km	40,075 km	
Mars	6,790 km	21,344 km	
Jupiter	142,796 km	449,179 km	
Saturn	120,660 km	378,675 km	
Uranus	51,118 km	160,592 km	
Neptune	49,528 km	155,597 km	
Pluto	2,300 km	7,232 km	1.0 cm
Moon	3,476 km	10,916 km	
Titan	5,150 km	16,171 km	
Io	3,630 km	11,398 km	
Europa	3,138 km	9,583 km	
Triton	2,700 km	8,748 km	

Name: Date: .

Scale Model: Observations

Quietly walk around the room and observe the scale model(s) that you and your classmates created. *Gently* use your hands to feel the differences in the models of the planets. Read the information on the cards.

What do you notice or wonder about:

  • the relationship between how the planets feel and their compositionf?

  • the relationship between how the planets feel and their mass?

  • the relationship between how the planets feel and their gravity?

What else do you notice or wonder? Write your observations and questions in the spaces below.

I Notice	**I Wonder**
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

Based on what you have observed and learned from the scale model and the information on the cards, which planets do you think would be best for humans to visit?   Why?

_____

_____

_____

Name: _____ Date: _____ .

Volume and Density

If you know the volume and mass of an object, then you can calculate its density. Follow the steps below to first calculate the volume of each planet and then to calculate each planet's density.

Note: *In this exercise, we will assume that each planet is a perfect sphere.*

Step 1: Refer to the Lesson 2 Planet Data Sheets to find a planet's diameter in km.

Step 2: Divide the diameter by 2 to find a planet's radius in km.

Step 3: Convert kilometers to meters using a unit ratio.

Step 4: Solve for the volume of a planet using the formula: $\text{volume} = \dfrac{4 \pi r^3}{3}$

Step 5: Solve for the density of a planet using the formula: $\text{density} = \dfrac{\text{mass in kg}}{\text{volume in m}^3}$

Planet (diameter)	Volume (m³)	Mass (kg x 10²²)	Density (kg/m³)
Mercury		33.0×10^{22} kg	
Venus		487×10^{22} kg	
Earth (12,755 km)	$\approx 1.08 \times 10^{21}$ m³	597×10^{22} kg	$\approx 5{,}528$ kg/m³
Mars		64.2×10^{22} kg	
Jupiter		$190{,}000 \times 10^{22}$ kg	
Saturn		$56{,}800 \times 10^{22}$ kg	
Uranus		$8{,}680 \times 10^{22}$ kg	
Neptune		$10{,}200 \times 10^{22}$ kg	
Pluto		1.25×10^{22} kg	

How do your answers for each planet's density compare to the density measurements you collected on your Lesson 2 Planet Data Sheets from *What's the Difference*?

Name: Date: .

Density: Large Planets vs. Small Planets

> **Density** *describes how tightly packed something is—how much matter (stuff) is in a certain volume.*

Compare the density of the nine planets by *listing them from greatest density to lowest density* based on the scale model. The first one (Earth) has been done for you.

most dense

Planet	Density kg/m³	Rocky or Gaseous	Mass kg x 10²²
1. Earth	5,515 kg/m³	rocky	597 kg x 10²²
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			

least dense

1. Which planets have *higher* densities?   ❑ rocky   ❑ gaseous
2. Which planets have *lower* densities?   ❑ rocky   ❑ gaseous

Circle the 4 planets with the *greatest* mass.

3. Which planets have *higher* densities?   ❑ more massive   ❑ less massive

4. Which planets have *lower* densities?   ❑ more massive   ❑ less massive

Name: Date: .

Characteristics: Large Planets vs. Small Planets

You have observed the planets in the scale model in terms of mass, volume, composition, and density. Use the chart below to organize general characteristics of large planets vs. small planets.

	Large Planets	Small Planets
Mass:	_____	_____
Circumference:	_____	_____
Composition:	_____	_____
Density:	_____	_____
Gravity:	_____	_____

Write your conclusions about the planets and their characteristics below.

Name: Date: .

Graphing Resource
Student Guide

Types of Graphs

There are several types of graphs that scientists and mathematicians use to analyze sets of numbers or data.

Bar graphs are often used to compare values.	**Pie graphs** are often used to compare percentages or parts of a whole.
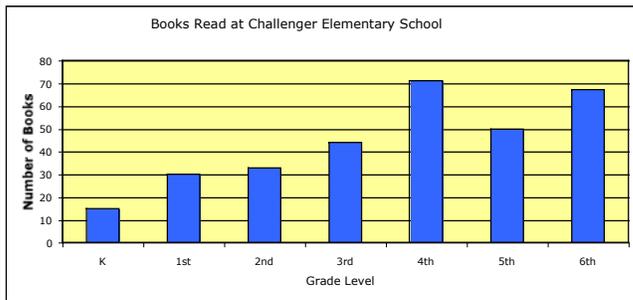
Books Read at Challenger Elementary School

% of Class That Likes the Color
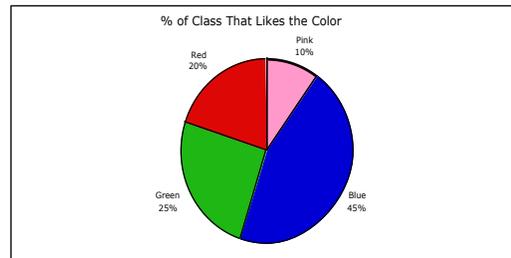
Line graphs are often used to show rates of change.

Tom's Test Scores
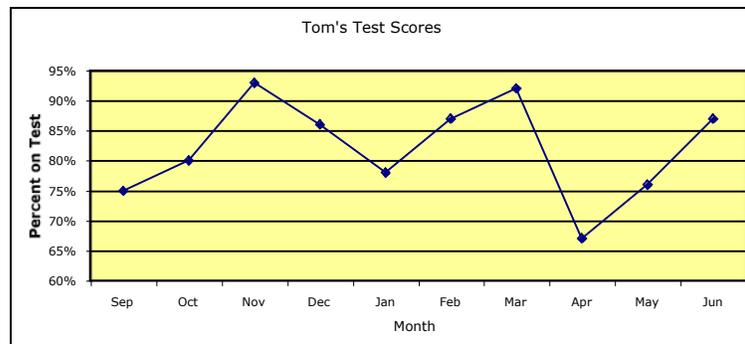
Name: Date: .

Before You Begin

When you are planning to graph data, you need to answer some questions before you begin.

    1. What type of graph will you use?

    2. What unit of measurement will you use?

    3. What scale will you use?

    4. What will be the minimum and maximum values on your graph?

    5. Will your graph start at 0?

Making Bar Graphs and Line Graphs

Every graph needs a **title** and **labels** on the horizontal "x" axis (side-to-side) and the vertical "y" axis (up and down).
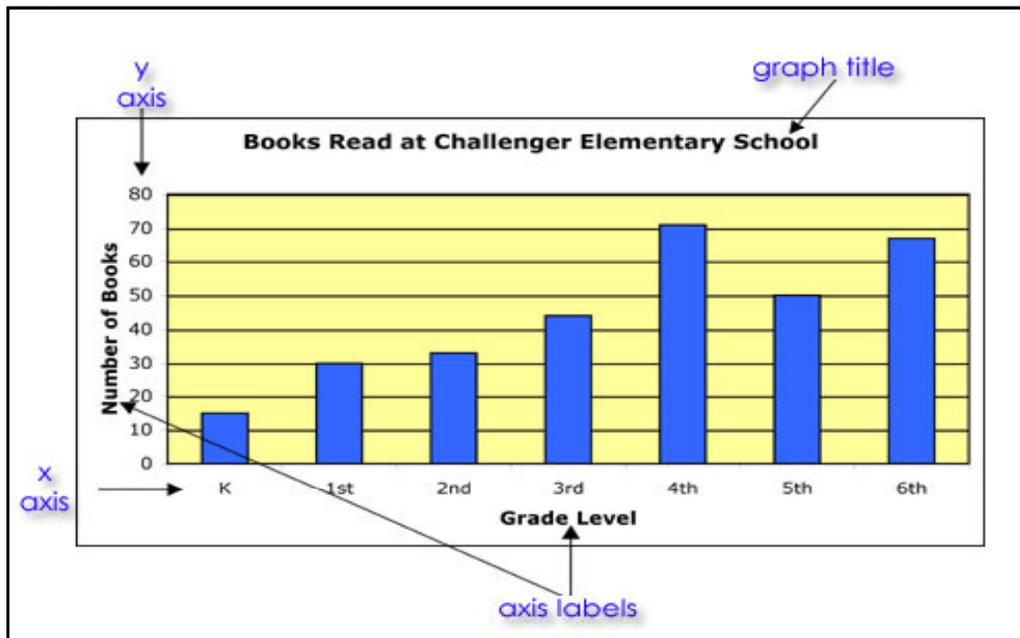
The **unit of measurement** you are using needs to be clearly shown (inches, kilograms, etc.). The unit for the bar graph above is "number of books" as is written in the vertical y-axis label.

You also must choose a **scale** for your vertical y-axis. The vertical scale on the bar graph above goes from 0 to 80 in increments of 10.

Name: Date: .

The scale is determined by the data you are graphing. To determine the scale, look at the largest and smallest numbers you will be graphing.

For this line graph, the vertical y-axis goes from 60% to 95% in increments of 5%.

The unit of measurement for this graph is "percent." (%)

Tom's Test Scores

Percent on Test

Month
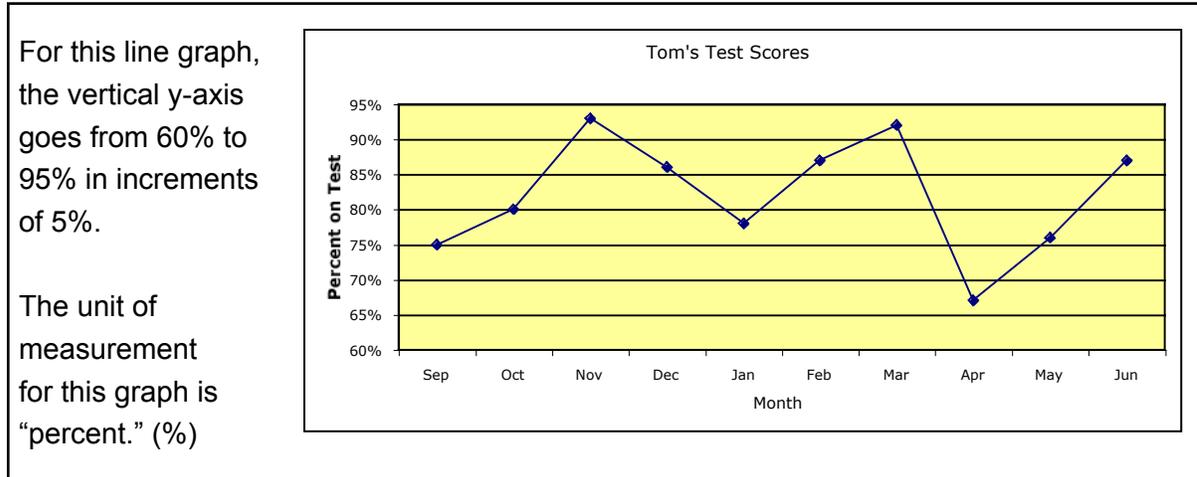
Making a Pie Graph

A pie graph is shown using a circle, which has 360 degrees. To make an accurate pie graph you will need a compass or a similar instrument to trace a circle and a protractor to measure angles in degrees.

Start by making a circle. You will then have to multiply your fractions or percents (in decimal format) by 360 degrees to find out how many degrees you will need in each wedge. For example:

Circle = 360º
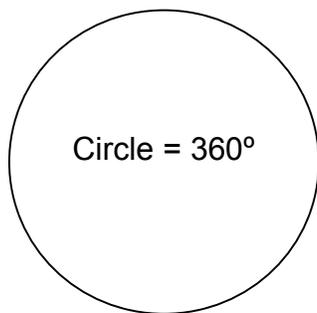
| Color	% of class that likes the color
Blue	45%
Green	25%
Red	20%
Pink	10%
Total:	**100%**

The sum of your fractions should total to 100%.

Name: _____   Date: _____   .

To find out how many degrees of the pie graph will represent the number of students in the class who like the color blue, you would multiply 360 degrees by 0.45. The result of your calculation is 162 degrees. To find out how many degrees of the pie graph will represent the number of students in the class who like the color green, you would multiply 360 degrees by 0.25. The result of your calculation is 90 degrees.

To mark off the blue portion of the pie graph, start by drawing a radius of the circle (a line segment from the center of the circle to the circle itself). Then use the protractor to measure an angle of 162 degrees and draw the corresponding radius. The green portion will have an angle measure of 90 degrees, the red portion will have an angle measure of 72 degrees, and the pink portion will have an angle measure of 36 degrees. The sum of these angles will have an angle measure of 360 degrees, the number of degrees in a circle.

Pink = 36°

Blue = 162°

Green = 90°

Red = 72°

Total = 360°

% of Class That Likes the Color

Pink 10%
Red 20%
Green 25%
Blue 45%
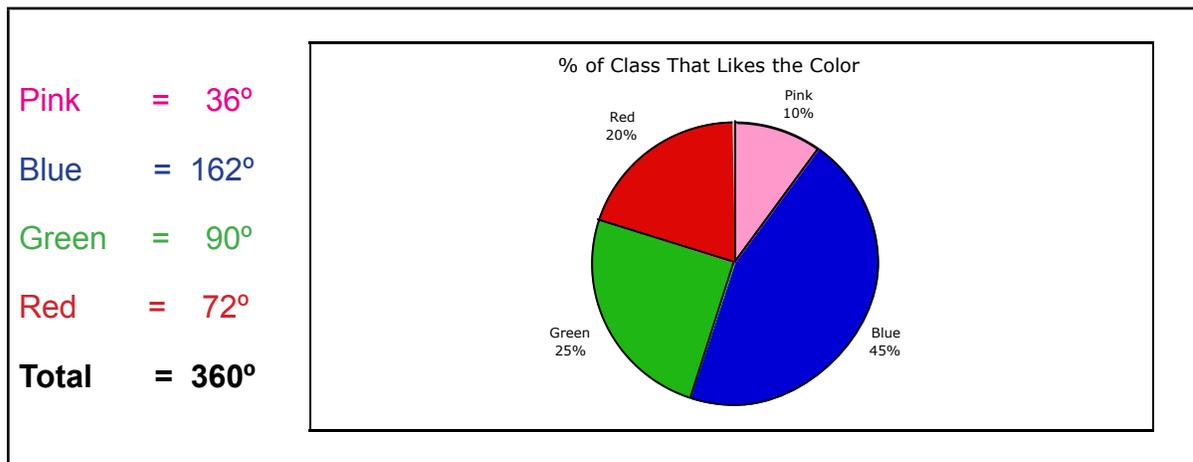
When the portions have been drawn into the circle, you then need to **color** each portion, label each portion with both the **category** and the **percent or fraction**, and give the graph an overall **title**.

Name: Date: .

Lesson 2 Extension Problems

| **Fractions, Percents, and Planets** |
| --- |

A pie chart is a way to represent fractions or percents of a whole. The following example calculates what percent of the total mass of the nine planets is contained in a given planet.

1. How is the mass of the solar system distributed amongst the nine planets? In other words, *what percent of the total mass of all nine planets is contained in each planet?* Be sure to include descriptions and pictures to explain how you solved the problem.

After finding solutions to the above problem, complete the following:

2. Which planets contain most of the mass of the nine planets?

3. If you made a pie chart showing the percent of mass for each of the nine planets, would you be able to represent the mass of the smaller planets?

4. Find the sum of the percent of mass for the *inner planets and Pluto*.

   Use that as one value and use the mass of *each of the gas giants* planets to create a pie chart to represent the distribution of mass in the solar system.
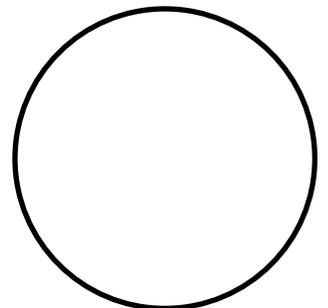
Name: Date: .

Additional Ratio and Proportion Problems

The following are problems that will require multiple steps to obtain a solution. You will need to measure lengths inside the classroom and apply what you know about scale, ratio, and proportion to solve them. You may choose the units you work with, as long as they are appropriate. Be sure to include descriptions and pictures to explain how you solved the problem.

1. An interesting relationship

Circles and spheres are very special geometric shapes. They have no corners. The formulas for the circumference and area of a circle, as well as for the surface area and volume of a sphere, are very different from the formulas for figures with straight edges like rectangles and squares. The following problem allows you to investigate an interesting relationship between two often-used measurements of any circle or any sphere.

a. Calculate the ratio of each planet's circumference to its diameter. (Refer to p.11) For example, the ratio for the planet Mercury would be as follows:

$$\frac{\text{Circumference of Mercury}}{\text{Diameter of Mercury}} = \frac{15{,}329 \text{ km}}{4{,}878 \text{ km}}$$

$$\approx \quad 3.14 \ (\text{or} \approx 22/7)$$

Calculate this ratio for each of the planets. Express your answers as both a fraction and as a decimal. What do you notice?

b. Measure the circumference of a great circle on a basketball. Measure the diameter of the basketball by holding a ruler against its widest point. Calculate the ratio of the circumference of the basketball to the diameter of the basketball. What do you notice? Try this for other spheres and their great circles. What is your result?

c. Look up the word "pi" on the Internet. Summarize what you learned about this very interesting number called pi (π).

Name: _____   Date: _____   .

2. A Bus to Pluto…

The mass of each planet is a measurement that is sometimes hard to conceptualize. It is helpful to compare the mass of a planet to objects that have a mass with which we are familiar. Here is an example.

a. Pluto is the least massive planet in the solar system. A public bus has a mass of approximately 14,515 kilograms. If the mass of Pluto is 125,000 kg followed by 17 zeros, estimate how many buses it would take to equal the mass of Pluto? Calculate the value (and then add 17 zeros to the end of the number!) How close was your estimate?

b. You calculated in this lesson that Pluto's mass is 0.002 of Earth's mass. Use this relationship to calculate how many buses it would take to equal the mass of Earth. (Be sure that your answer makes sense—Earth is more massive than Pluto.)

c. If Earth's mass is equal to the mass of the number of buses you calculated in part b, estimate how many buses it would take to equal the mass of Jupiter—the most massive planet in the solar system. (You calculated in this lesson that the mass of Jupiter is approximate 318 times the mass of Earth.) Calculate the number of buses that it would take to represent the mass of Jupiter. (Be sure your answer makes sense—Jupiter is more massive than Earth.) How close was your estimate?

Name: Date: .

3. I Gained HOW Much??!!

In this lesson, you learned that weight is affected by gravity. On another planet, an object would weigh less or more, depending on the gravity on that planet.

a. Weigh yourself here on Earth (Be careful! Think about what units you should use to record your weight and what units are on a typical scale. Remember: your mass will not change from planet to planet.)

b. For each of the other planets, use the percentage value of the other planet's gravity in terms of the Earth's gravity to calculate your weight on another planet. (For example, if Mercury has 0.4 of Earth's gravity, on Mercury you would weigh 40% of your weight on Earth.) Make a chart of what you would weigh on different planets in our solar system.

c. In addition to affecting your weight, the amount of gravity on a planet affects how high you can jump. Get a friend to measure how high you can jump standing in one spot (It might be helpful to try a few times and average your jumps.) What units of measurement will you use? This time you will use the inverse of the percentage value. To calculate the inverse, divide 1 by the decimal value of the gravity. For example, Mercury has only 40% of Earth's gravity. One divided by 0.4 is 2.5. You can jump 2.5 times higher on the surface of Mercury than you can on the surface of Earth! Make a chart showing how high you could jump on different planets in our solar system.