

Using Mathematics in Forestry¹

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Most jobs require good mathematics skills, and forestry is no exception. The exercises in this handout will show you some of the ways that foresters use mathematics in their work.

Foresters usually deal with large land areas. They need to know what kind of trees grow on their area, how many of them there are, and how big they are. Measuring every tree on a National Forest covering one and half million acres and containing over 500 million trees would not only be time consuming and expensive, but probably impossible! For those reasons, foresters only measure part of the forest, a process called sampling. They assume that the measured (sampled) part represents the whole area. Data collected from the sampled part is expanded to provide information for the unsampled portion.

Suppose we need to know how much usable timber is present in a 100-acre ponderosa pine stand. The entire stand (all 100 acres) is our population, but we can't measure every tree because it would take too much time. So the most efficient way to inventory the ponderosa pine timber is to sample it, which we'll do by establishing plots. Statistics indicates that 20 plots of $\frac{1}{4}$ -acre each will provide reliable information about the ponderosa pine timber. How can we figure out the plot size? By using some mathematical formulas, of course! Previous sampling experience shows that circular plots are easier to use than square ones. To lay out a plot in the shape of a circle, we must know its radius, which we'll figure out using this formula:

$$\text{Area} = \text{Pi} \times \text{Radius}^2$$

[Note: if you don't know what pi is, here is a short description. Pi is the number you get when you divide the distance around a circle (its circumference) by the distance through its middle (the diameter). The distance around every circle is "about" three times the distance across it. But it's the "about" part that creates the puzzle of pi. Mathematicians call pi an irrational number because when you divide circumference by diameter, the answer comes out in decimals that go on forever without any apparent pattern. Pi begins as 3.14159265, but it never ends. In 1999, a Japanese scientist used a supercomputer to calculate pi to about 206 billion digits, and it still goes on from there. All those digits aren't really necessary to use pi, of course – using only the first ten decimals, you can measure the earth's circumference to within a fraction of an inch. Sometimes, pi is shown in textbooks or in formulas using this Greek symbol: π]

The total area of an acre is 43,560 square feet. Since we want to use a $\frac{1}{4}$ -acre plot, its area is $43,560 \div 4$ or 10,890 square feet. We now have enough information to use the formula above to figure out our plot radius.

¹ This paper was prepared in 1990 for Mr. Kevin Steinmetz's sixth grade class at Humbolt Elementary School in John Day, Oregon. It was revised several times since then for use with other classes.

$$\text{Area} = \text{Pi} \times \text{Radius}^2$$

$$10,890 \text{ square feet} = 3.1416 \times \text{Radius}^2$$

$$10,890 \text{ square feet} \div 3.1416 = \text{Radius}^2$$

$$3,466.39 \text{ square feet} = \text{Radius}^2$$

$$\sqrt{3,466.39} \text{ square feet} = \text{Radius}$$

$$58.88 \text{ feet} = \text{Radius}$$

Each ¼-acre plot will be a circle with a radius of 58.88 feet. We'll use a wooden stake as the plot center, and mark the plot boundaries by measuring out a radius in several directions from the stake. Great! We've just used our knowledge about the geometry of a circle to design plots which will help us answer our inventory question about the ponderosa pine timber.

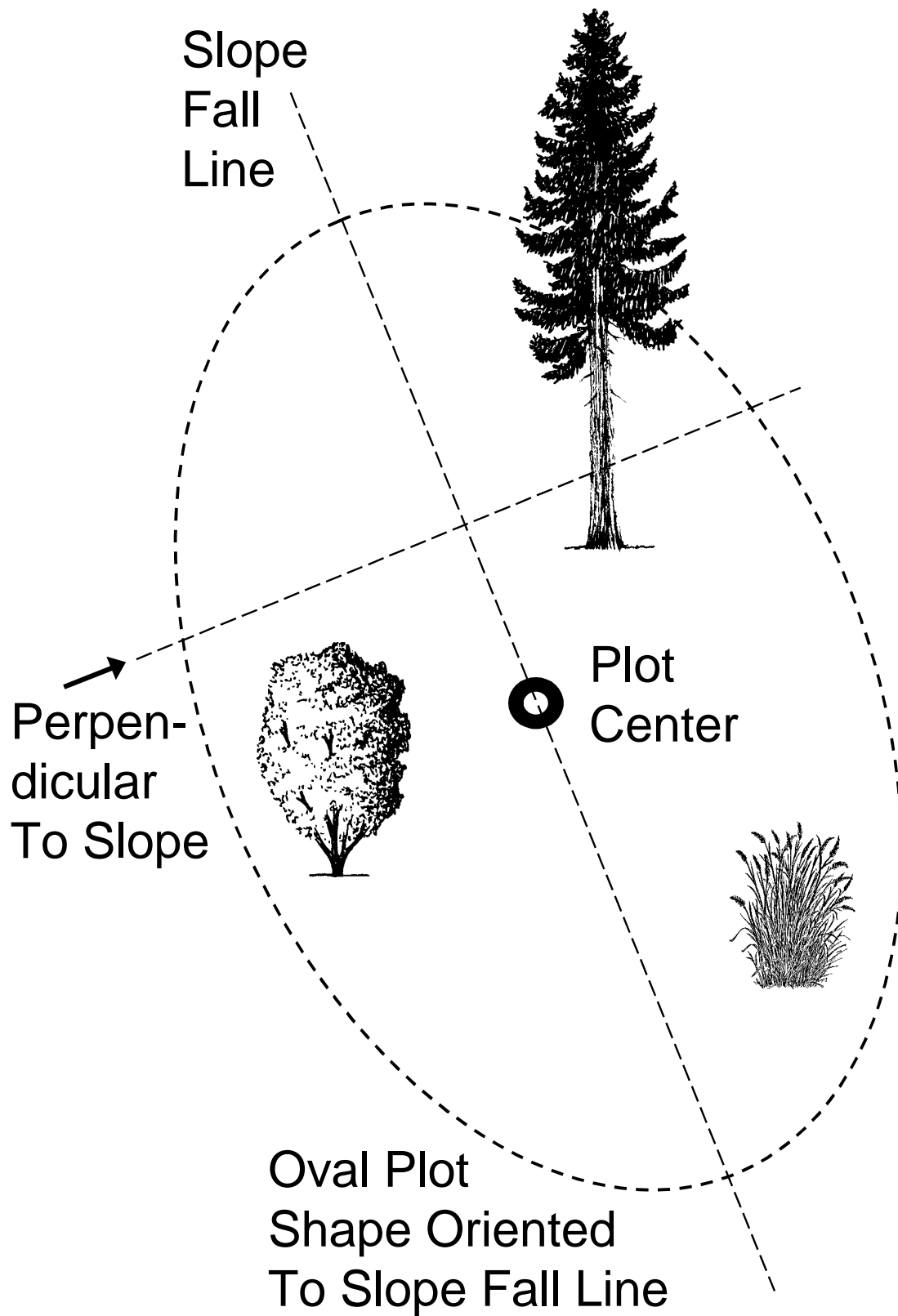
Exercise 1: If our circular plot has a radius of 58.88 feet, what is its diameter? [Do you remember the relationship between a circle's radius and its diameter?]

Diameter is: _____ feet.

Exercise 2: What would be the area (in square feet) and the radius (in feet) for circular plots of the following sizes? [Don't forget that 1 acre contains 43,560 square feet.]

Plot Size (Acres)	Plot Area (Square Feet)	Plot Radius (Feet)
1/4	10,890	58.88
1/10	_____	_____
1/100	_____	_____
1/250	_____	_____
1/300	_____	_____
1/500	_____	_____
1/1000	_____	_____

For plots on flat ground, we can use the radius just as it was calculated above. The problems start when you are working on a hill because the plot's radius then varies, depending on the steepness of the hill. By projecting a plot's radius on a steep hillside, we see that it becomes oval in shape, not circular:



On sloping ground, plots have an oval shape with their long axis parallel to the slope (called the "slope fall line"). Note that a line perpendicular to the slope forms a "right angle" with the slope fall line.

How do we adjust the radius of a plot that occurs on sloping ground? Well, you could figure out the adjusted radius using trigonometry (secants), but most foresters just carry around something called a slope correction table. Here is part of the slope correction table for slopes ranging up to 61 percent:

Table 1: Slope correction factors

Slope Percent	Slope Correction Factor
0 – 9	1.00
10 – 17	1.01
18 – 22	1.02
23 – 26	1.03
27 – 30	1.04
31 – 33	1.05
34 – 36	1.06
37 – 39	1.07
40 – 42	1.08
43 – 44	1.09
45 – 47	1.10
48 – 49	1.11
50 – 51	1.12
52 – 53	1.13
54 – 55	1.14
56 – 57	1.15
58 – 59	1.16
60 – 61	1.17

Now, let's use table 1's slope correction factors to figure out if some trees near our plot edge are "in" (inside the plot) or "out" (outside of the plot radius). These trees near the plot edge are referred to as "borderline" trees. Here is a way to measure borderline trees:

1. Use an instrument called a clinometer to measure the slope percent from the center (side) of a borderline tree to the plot center (let's say that it is 30 percent).
2. Find a slope correction factor in table 1 for the slope percent you just measured (the correction factor is 1.04 for 30 percent).
3. Multiply the correction factor by the plot radius. This is called the corrected radius. For our ¼-acre plots, the result is: 58.88 feet × 1.04 = 61.24 feet.
4. Measure the slope distance from the center (side) of the tree to the plot center. If the measured distance is less than the corrected radius (61.24 feet in this example), then the tree is in; if the measured distance is more, the tree is out.

Exercise 3: Are the following trees on a ¼-acre plot in or out? Record the slope correction factor (from table 1) and corrected plot radius for each tree too.

Tree	Slope Percent From Tree to Plot Center	Slope Correction Factor	Corrected Plot Radius (Feet)	Distance From Tree to Plot Center (Feet)	In or Out?
1	20	_____	_____	59.1	_____
2	25	_____	_____	58.6	_____
3	30	1.04	61.24	62.9	Out
4	35	_____	_____	62.4	_____
5	45	_____	_____	63.7	_____
6	50	_____	_____	59.9	_____
7	60	_____	_____	60.3	_____

MEASURING CIRCUMFERENCE AND DIAMETER

Now that we know how to figure out which trees are in or out of the plot, we need to learn how to measure the size of trees that are “in” on the plot. First, we need to find out how big around each tree is. There are two main ways that we can describe the size of round objects like tree stems:

- ◆ We can measure their circumference, which is the distance around the outside of the trunk, or
- ◆ We can measure the circumference and convert it to diameter, which is the distance through the middle of a tree’s trunk.

Foresters use special measuring tapes that show a tree’s circumference on one side, and its equivalent diameter on the other side. How is that done? Actually, it’s easy to do because circumference and diameter are closely related:

$$\text{Circumference} = \text{Pi} \times \text{Diameter}$$

$$\text{Diameter} = \text{Circumference} \div \text{Pi}$$

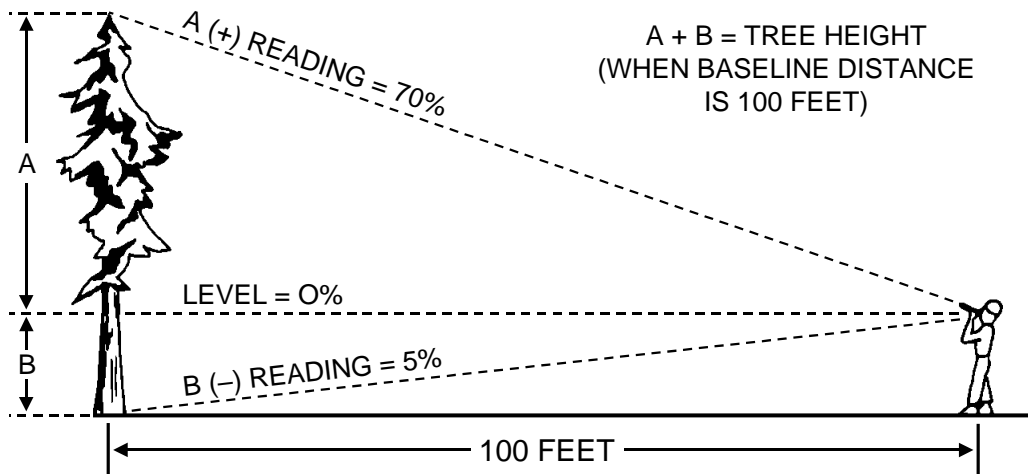
[Have you noticed yet that most mathematical formulas pertaining to circles use pi?]

Exercise 4: What are the diameters or circumferences of the following trees? (You might be interested to know that these are actual measurements for some of the biggest trees found in the Blue Mountains of northeastern Oregon.)

Species	Circumference (Inches)	Diameter (Inches)
Subalpine Fir	132	_____
Western Juniper	151	_____
Lodgepole Pine	_____	34.7
Ponderosa Pine	242	_____
Engelmann Spruce	_____	63.3
Grand Fir	218	_____
Quaking Aspen	129	_____
Douglas-fir	_____	65.9

MEASURING TREE HEIGHTS

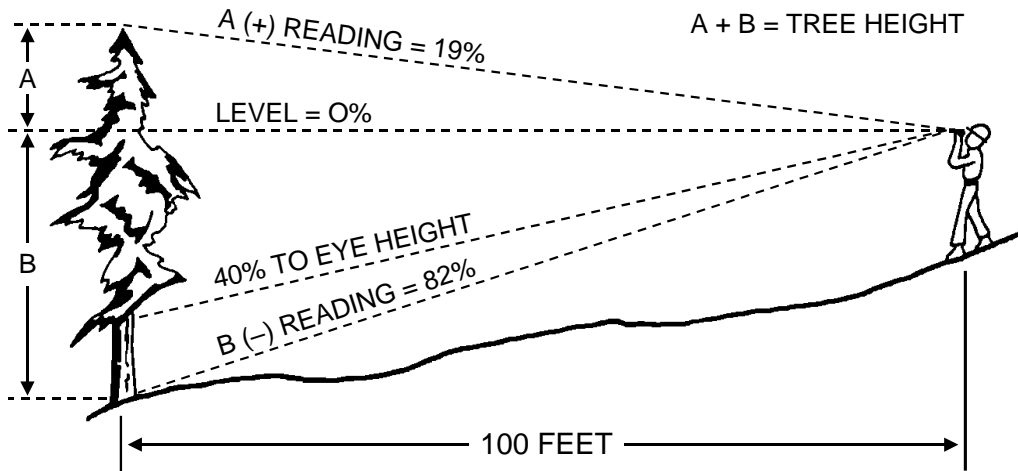
After measuring a tree’s diameter or circumference, we then need to find out how tall it is. This can be tricky until you get the hang of it, especially when working on hillsides or sloping ground. Foresters measure heights using an instrument called a clinometer (short for an “incline meter”). Clinometers are based on percent, which means that they are designed to be used at a distance of 100 feet from the tree. Let’s look at an example on flat ground:



The A (+) readings on a clinometer mean that you are looking up; the B (-) readings mean that you are looking down. A level (not up or down) reading would be 0 on a clinometer. In our example above, the A (70) reading was looking up at the top of the tree;

the B (5) reading was looking down at its base (where the trunk meets the ground). To get the tree's total height, you add the top reading (70) and the bottom reading (5) together: 75 feet tall for our example tree.

Now, let's look at a situation where measuring tree height is a little more complicated. When using a clinometer on sloping ground (hillsides), you must apply a slope correction factor just like we did when checking whether borderline trees were in or out of the plot area. Here's a height measurement example for sloping ground:



Here is the process you'd use to measure this tree's height:

1. Use the clinometer to measure the slope percent from you to your "eye height" on the tree trunk (this sighting is a line that's parallel to the ground surface). In our example, the slope percent is 40.
2. Find a slope correction factor in table 1 for the slope percent you just measured (1.08 is the slope correction factor for 40 percent).

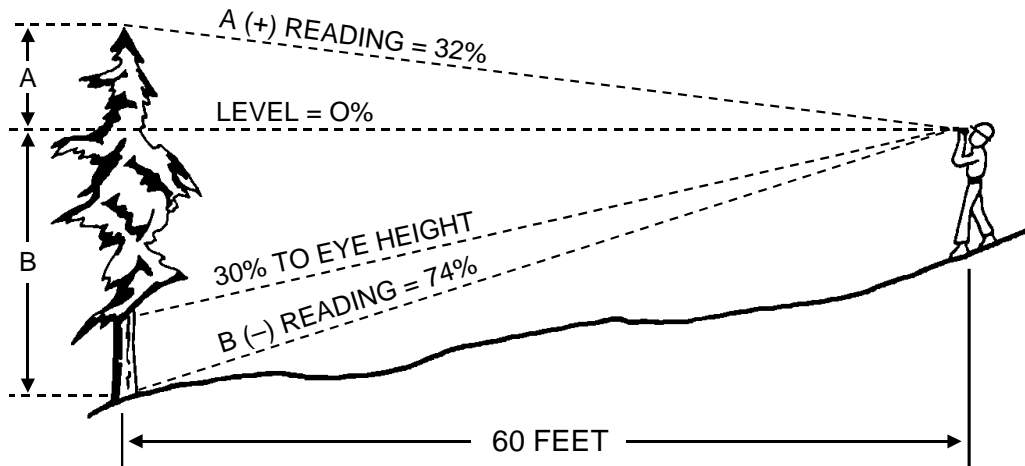
[Note: many clinometers show slope correction factors on the same scale as the slope percent values, which is handy because then you don't have to look them up in a table.]

3. Multiply the slope correction factor by the baseline distance. In our example with a 100-foot baseline, the result is: $100 \text{ feet} \times 1.08 = 108 \text{ feet}$. This means that on a 40 percent slope, you must be 108 feet away from a tree to get the same answer as if you were 100 feet away on flat ground.
4. Back up to 108 feet from the tree before taking clinometer readings of its top and base.
5. Take clinometer readings of the tree's top and base, and add them together to come up with a total height. In our example, the result would be: $19 + 82 = 101 \text{ feet}$.

Exercise 5: What are the heights of the following trees?

Tree	Slope Percent to Tree	Slope Correction Factor	Corrected Baseline (Feet)	Clinometer Readings:		Total Height (Feet)
				Up (A)	Down (B)	
1	35	_____	_____	96	32	_____
2	40	1.08	108	19	82	101
3	45	_____	_____	43	8	_____
4	50	_____	_____	78	16	_____
5	55	_____	_____	91	27	_____
6	60	_____	_____	66	35	_____

Next we'll look at another situation where measuring tree height is more complicated than usual. It is often hard to measure heights from 100 feet away if thick brush or downed logs hide your view of the tree's base. In those cases, you'll need to measure height from someplace other than 100 feet away and adjust the clinometer results accordingly. Here is our example tree:



Here is what you'd do to measure this tree's height:

1. Use the clinometer to measure a slope percent to eye height on the tree (30 percent in this example).
2. Get a slope correction factor from table 1 (1.04 for 30 percent).
3. Multiply the slope correction factor by your baseline distance: $60 \text{ feet} \times 1.04 = 62.4$ feet.
4. Back up to 62.4 feet before taking clinometer readings of the tree's top and base.

- Take clinometer readings of the top and base, and add them together. In our example, the result is: $32 + 74 = 106$.

Does this mean that the tree is 106 feet tall? No, it does not! Do you remember that a clinometer is designed to be used with a 100-foot baseline (see page 3)? Any time that you are not 100 feet away from the tree (slope-corrected distance), the clinometer readings must be adjusted to account for a different baseline distance. In our example, the slope-corrected baseline distance was only 60 feet, not 100.

- Calculate a baseline adjustment factor by dividing your baseline distance by 100: $60 \text{ feet} \div 100 = 0.6$.
- Multiply the sum of the clinometer readings (see step 5 above) by the adjustment factor to finally get a total height for your tree: $106 \times 0.6 = 63.6$ feet.

Exercise 6: What are the heights of the following trees?

Tree	Baseline Distance (Feet)	Baseline Adjustment Factor	Clinometer Readings:		Total Height (Feet)
			Up (A)	Down (B)	
1	40	_____	89	47	_____
2	50	_____	73	26	_____
3	60	.6	32	74	63.6
4	65	_____	68	29	_____
5	75	_____	52	11	_____
6	90	_____	64	26	_____

Now, let's see if we can pull some of this information together. We began sampling the 100-acre ponderosa pine stand last week and three plots are done so far. Here is the data at this point:

PLOT 1 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	32.6	109	196
2	Ponderosa Pine	27.4	101	173
3	Grand Fir	18.9	88	93
4	Douglas-fir	29.4	106	204
5	Grand Fir	12.3	72	79
6	Quaking Aspen	8.6	66	58
7	Western Larch	14.2	91	87

PLOT 1 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
8	Ponderosa Pine	9.6	48	54
9	Ponderosa Pine	15.2	68	74
10	Western Larch	16.9	96	86
	Average	_____	_____	_____
	How many trees per acre?	_____		

PLOT 2 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	33.9	111	194
2	Ponderosa Pine	30.7	108	188
3	Douglas-fir	26.9	98	147
4	Ponderosa Pine	27.8	102	156
5	Douglas-fir	21.6	92	106
6	Ponderosa Pine	28.9	103	129
	Average	_____	_____	_____
	How many trees per acre?	_____		

PLOT 3 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	26.2	98	114
2	Grand Fir	22.9	95	102
3	Ponderosa Pine	36.8	109	149
4	Douglas-fir	42.9	113	196
5	Lodgepole Pine	12.6	74	82
6	Ponderosa Pine	28.4	100	121
7	Ponderosa Pine	22.9	91	106
8	Grand Fir	18.7	89	97
9	Douglas-fir	27.6	99	112
	Average	_____	_____	_____
	How many trees per acre?	_____		

Exercise 7: Here's what you need to do now:

1. Calculate an average diameter, height and age for each of the 3 plots, and record it in the tables.
2. Calculate the total number of trees, per acre, that each plot represents; record your answers in the tables.

How can you do that? Remember that each plot samples exactly $\frac{1}{4}$ of an acre. If you count the number of trees that are "in" on your plot, and then multiply by 4 to expand the sample to a whole acre, you will know how many trees per acre are represented by your sample.

3. Calculate an average for the 3 plots combined by filling in the table below:

Plot	Average Diameter (Inches)	Average Height (Feet)	Average Age (Years)	Trees Per Acre
1	_____	_____	_____	_____
2	_____	_____	_____	_____
3	_____	_____	_____	_____
Average	_____	_____	_____	_____

4. What proportion of the sample trees, on the 3 plots combined, were ponderosa pines (don't forget that we are interested in how much ponderosa pine timber we have on the 100-acre tract)? _____ percent.
5. Fill in the blanks in this summary of our results so far. "After completing three plots, we can now say that the sampled area has an average of _____ trees per acre, with an average diameter of _____ inches, an average height of _____ feet and an average age of _____ years. The proportion of ponderosa pines on the 3 plots was _____ percent."

Since this summary is for 3 plots only, the data will almost certainly change after all 20 samples are completed!

LET'S SUMMARIZE

What have you learned from this exercise?

- ◆ How to figure out the radius of a circular plot after being given its area;
- ◆ How to adjust a plot radius to account for sloping ground;
- ◆ How to measure the circumference and diameter of trees;
- ◆ How to use a clinometer to measure tree heights on either flat or sloping ground;
- ◆ That many mathematical formulas involving circles use pi, a special constant that we round off to 3.1416;