Efficient dissemination of the orbit predictions in real-time

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Background and motivation

The real-time GNSS applications are of increasing interest. The Precise Point Positioning technique (Zumberge, 1997) applied in real-time requires dissemination of the precise satellite orbits and clocks with high efficiency and unique reproducibility.
Because of the rapid variation of the satellite clock corrections, the prediction could not be provided for a period longer than a few tens of minutes. The clock corrections should be thus better estimated in real-time.
On contrary, the orbit dynamical models are, in most cases, good sufficient even for a long-arc (few days) representation with a few centimeters accuracy in the position today. An excellent orbit prediction can be thus achieved in most cases, but during a satellite maintenance (which probably cannot be more improved by the determination in real-time). Also the accuracy in orbit prediction is more quickly degraded for the old-type 'Block-IIA' satellites during eclipsing periods (Dousa, 2008).
The IGS ultra-rapid product (Dow 2005) is respected as a primary in the quality and robustness. Up to seven analysis centers with individual approaches contribute to the IGS combination. Though the IGS ultra-rapid orbits are updated every six hours today, the product can already be declared as suitable for the real-time PPP support - at least, in the 'regional' PPP solution, which is not affected by the orbit errors as much as the 'global' PPP.
We are reviewing the possibility of dissemmination of the IGS predicted orbits for real-time applications.

Situation

Because of the overlaps between consequent IGS ultra-rapid products and because of its sampled position distribution (15 min epochs in SP3), a unique reproducibility of the orbits can not be guaranteed by the simple means of an interpolation technique. Figure 1 shows the interpolation of the IGS final and IGS ultra-rapid orbits from standard 15 min sampling rate with respect to high-rate IGS final orbit sampling rate, both in SP3 format. We can suppose providing the unique orbits via a continual real-time stream of SP3-like high-rate sampling of the satellite positions, e.g. such as provided by BKG in the EUREF-IP project (Weber, 2007). If 1-Hz sampling rate is applied, the subcentimeter orbit reproducibility will be guaranteed, but the dissemination in real-time would be highly inefficient (a single satellite orbit would be represented by 3*3600 ephemeris in one hour interval). In opposite, significantly reduced sampling will not provide sufficient reproducibility during the interpolation. A problem can also occur due to the interpolation at the end of a sample on contrary to the interpolation for central epoch.

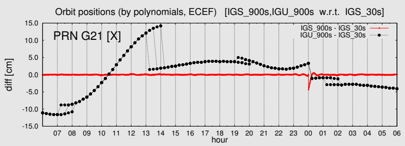

Fig. 1: IGS final and IGS ultra-rapid orbit 15 min sampling rate interpolated and compared to IGS final high-rate sampling rate.

We propose a simple functional orbit representation for dissemination of the predicted orbit for real-time.

Polynomial orbit representation

The precise orbits are originally determined by 6+ parameters of the dynamic/stochastic model in an inertial system. On the contrary, the PPP applications are designed only to reproduce the precise orbits by keeping them fixed. Usually, a simple method of ephemeris interpolation component by component in the terrestrial system is applied because it is a simple and efficient method and thus the transformation into the intertial system is avoided (no need for the Earth's orientation parameters, EOP).
An interpolation by the polynomials in the form

$$P(t) = A_0 + A_1.t^1 + A_2.t^2 + A_3.t^3 + \ldots + A_N.t^N$$

where P is X, Y, Z orbit position component, t is time, was demonstrated e.g. by Schenewerk (2003). The best performance, in terms of accuracy, was achieved with 10 degree polynomials while sufficient accuracy was achieved already using the degree 8 for interpolating 15 min ephemeris. Schenewerk additionally tested the trigonometric polynomials, which gave the sufficient results even with a lower degree, but only for the extrapolations in the inertial system. The disadvantages are the needs for EOP parameters and a higher computational burden.
A standard interpolation method (as described by Schenewerk) usually estimates strightly interpolated values instead of the coefficients of interpolating polynomials. It is well known that the coefficients are, in general, determined less accurately than their values (Press 1992).
Nevertheless, we are interested in the coefficients for the unique and efficient orbit representation (and dissemination) in real-time. Thus, we need to answer the following questions:

- Is the estimation of the polynomial coefficients acurate and stable ?
- Which period is reliable for a single batch representation ?
- Which polynomial degree is sufficient for a given period representation ?
- Should be the orbit representation be disseminated via real-time stream or via batch files ?

The polynomial orbit representation over a hour validity interval could reduce the huge amount of orbit positions disseminated in 1-Hz stream (10800 values) to about 30 values/hour/satellite.

Polynomials interpolation versus approximation

We applied two methods for estimating the polynomial coefficients. Firstly, we applied interpolating polynomials as defined by Lagrange formula (Press, 1992). Secondly, we approximated the polynomial function by fitting redundant data samples.
In both cases, we have evaluated orbit reproducibility using the estimated polynomials for different polynomial degrees, validity intervals and fitting the different data interval if applicable.
We have prepared the reference SP3 files with 'high-rate' (30s) orbit positions estimated in Bernese GPS Software (Dach 2007) using standard orbit model in inertial system for the IGS final and ultra-rapid product.
We then estimated the polynomial coefficients for the original IGS SP3 product (900s) referenced in every half hour. We compared the satellite positions calculated from different polynomials with those from the 'high-rate' SP3 files.
The comparison was provided every five minutes over the interval of ±90min from the reference epoch in the mid of every hour. Figs 2 and 3 displays all the maximal differences due to this interpolation for the IGS final and ultra-rapid orbits. The RMSs for all the differences were usually smaller by the factor of 2-3 (for degree <7) and by the factor of 20 (for degree >7). To achieve 1cm orbit position reproducibility for the one hour validity interval, the polynomials of degree 8 are still satisfactory.

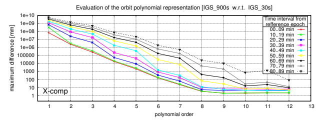

Fig. 2: Reproducibility of IGS final orbits via different degree of the interpolating polynomials and using different validity interval

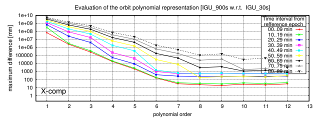

Fig. 3: Reproducibility of IGS ultra-rapid orbits via different degree of the interpolating polynomials and using different validity interval

The same test provided for the IGS ultra-rapid orbits (Fig 3) gaved slightly worse results due to the discontinuities in the vicinity of the IGS ultra-rapid product updates (compare to Figs 4 and 5).
Excluding the 15 min interval for every new IGS ultra-rapid product (4/day), the results were close to those for the IGS finals. The polynomial degree 8 is thus sufficient up to 1 hour validity and degree 10 up to 2 hour validity interval for subcentimeter orbit reproducibility. However, hourly update seems to be optimal because, as soon as new orbit product is available, new positions should replace the previously predicted values, except for the last one in order to protect the position continuity when applying the interpolating method (red line in Fig 5). These tests are valid for 15 min interpolated samples only.

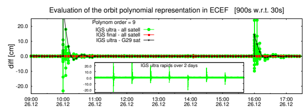

Fig. 4: The exceeding residuals due to inconsistencies of the reference and interpolated orbits when updated

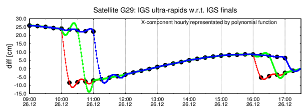

Fig. 5: Possible approaches to the IGU polynomial update for the one hour validity interval

Stability

When the reliable polynomial degree was selected, both methods proved stable enough in the long-term test. It can be roughly confirmed in a cummulative plot, Fig 6, (especially for degree <9), which displays the individual coefficient contribution to the interpolated values. The figure also demonstrates the real coefficient ranges (the contribution divided by $(T - T0)^i$, where i is degree). To efficiently encode the coefficients for the distribution, the significant digits can also be derived from the figure.

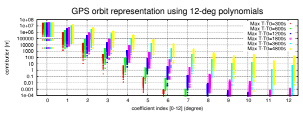

Fig. 6: Summary plot of the polynomial coefficient contributions. Coefficient ranges and significant digits can be easily derived.

Continuity for position and velocity ?

From the definition, the position continuity is guaranteed only in case of the interpolating polynomials, Fig 7. The velocities are not smooth implicitly in polynomial representation at the epoch of updating the IGS ultra-rapid product. However, there is no urgent request for such smoothness - the need for the estimation of the relativity correction due to the orbit excentricity is approx. 0.18 m/s in the radial component. This is easily attained even with the polynomial interpolation (Fig 7).

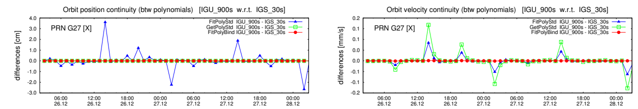

Fig. 7: Continuity for position (left) and velocity (right) for three different methods

However, by approximating the polynomial function, we have additionally implemented the option of constraining the continuity up to the function's N-derivative, with respect to the previous polynomials. Figure 8 shows examples of the standard fitting and for the fitting applied together with constraining the position and velocity to the estimated ones using the previous polynomials. Finally, the optimal interval for fitting was tested, and Fig 9 demonstrates possible instabilities when the fitting interval is too long for the applied polynomial degree.

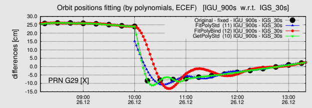

Fig. 8: Interpolating and approximating by fitting polynomials. Fitting is possibly constrained for the continuity in position and velocity

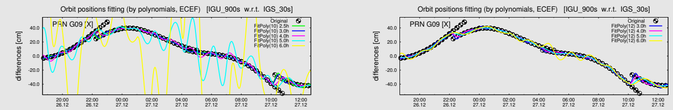

Fig. 9: Aproximating polynomials over different period, degree and constrained for position and velocity continuity

Product distribution

Contrary to the precise clocks, the polynomial interpolation coefficients for predicted orbits can be provided via real-time data stream as well as via the files updated from ftp server. While ftp approach is safe for transport, the streaming content can not be simply checked for its completeness. Every user starts his own reading session. Obviously, a single set of the polynomial coefficients should be resent repeatedly within the stream. An upcoming polynomials should be provided in advance too. We assume that streaming approach could be preferred, because real-time capable software already uses the streaming when getting the observed data and the precise clocks. Providing the polynomial representation of the differences, with respect to the broadcast orbits, can be considered too.

Prototype implementation

Using the GPSTk library (Harris, 2007), we have developed a software capable of providing the continuous orbit polynomials (COP) stream. The software was used in the evaluation described above. It enables reading the incoming SP3 files for the defined mask and concatenated broadcast orbit files too. It estimates the polynomials using a selected method and polynomial degree and retains them until the precise product is ready for the comparison (e.g. IGS rapid orbits). The software saves the hourly polynomials files, or it streams the polynomials in real-time to one or more (redundant) NTRIP casters.
The differences from the backward routine comparison are saved in log-files, archived into GOP database and visualized on the web (ftp://www.pecny.cz → GNSS → GPS-orbits → IGU real-time), see Fig. 10.

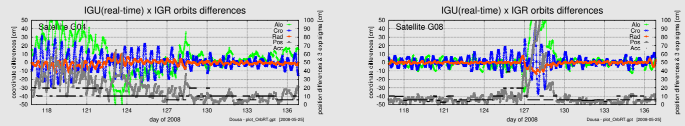

Fig. 10: Plots of differences of the 'real-time' IGS ultra-rapid orbits operationally compared to IGS rapid orbits

The prototype stream is running at ntrip.pecny.cz caster at 'COPS0' and 'COPS1' mounting points providing GPS and GPS+GLONASS orbits in polynomials, respectively. A preliminary ASCII format consists of two blocks of the polynomial sets coupled for two consecutive validity intervals (hours), Fig 11. Each block contains the frames for each GNSS satellite: satellite ID, validity interval, reference epoch, SP3 accuracy code, broadcast healthy code, subframes for X,Y,Z orbit components and predicted clocks. The component coefficient sets are preceeded by the component id, count of the coefficients, the degree of freedom and functional fitting RMS if applicable. The predicted clocks (IGS ultra-rapid), completed by the broadcast clocks for the missing values, are ready to serve as a priori values for any real-time clock generator. The GPS predicted orbits are used from the IGS ultra-rapids and the GLONASS predicted orbits from the CODE ultra-rapid solutions (Hugentobler et al., 2005).

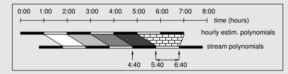

Fig. 11: Coupled streaming hourly polynomials

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