## HEIGHT MODERNIZATION LESSON PLAN

## How High Is That?

Theme
Height Modernization

## Links to Overview Essays and Resources Needed for Student Research

http://oceanservice.noaa.gov/topics/navops/heightmodernization/ http://www.ngs.noaa.gov/heightmod/index.shtml

Subject Area<br>Earth Science/Geography/Mathematics

## Grade Level

9-12

## Focus Question

What is "height modernization," and why is it important?

## Learning Objectives

- Students will describe the National Spatial Reference System, and discuss at least three practical uses for accurate spatial information.
- Students will discuss at least three examples in which vertical position data are used.
- Students will solve practical problems involving geographic positioning.


## Materials Needed

- Copies of "Clinometer Scale" and "Model Benchmark," one copy for each student group; protractors may be substituted for "Clinometer Scales";
- Drinking straw, one for each student group
- Fishing weight, about 15 grams; one for each student group
- Masking tape or duct tape, about 20 cm for each student group
- Tape measure, at least 25 feet long; one for each student group
- (Optional) Copies of "Geodetic Height Worksheet," one copy for each student group


## Audio/Visual Materials Needed

None

## Teaching Time

One or two 45-minute class periods, plus time for students to complete worksheet problems

## Seating Arrangement

Classroom style or groups of 2-3 students
Maximum Number of Students
30

## Key Words

Height datum
Benchmark
Geodesy
Latitude
Longitude
Total station

## Background Information

An airliner navigating from London's Heathrow Airport to Dulles International Airport near Washington, a backpacker trekking across the Cascades, and a sportfisherman trolling over an artificial reef near Charleston, SC share at least one common feature: they all make use of the National Spatial Reference System (NSRS). Developed and maintained by NOAA's National Geodetic Survey (NGS), the NSRS is a national coordinate system that provides the foundation for transportation, communication, mapping, construction, and a multitude of scientific and engineering activities. Key components of the NSRS include:

- A consistent coordinate system that defines latitude, longitude, height, scale, gravity, and orientation throughout the United States;
- A network of permanently marked reference points;
- A network of continuously operating reference stations (CORS) which provides up-to-the-minute information on movements of the Earth's surface; and
- A set of accurate models that describe dynamic geophysical processes that affect spatial measurements.

The NSRS uses latitude and longitude to describe the location of points on the Earth's surface. Since the surface of the Earth is not uniformly flat, the NSRS also includes a description of vertical positions. The fundamental reference point for latitude measurements is the equator. The latitude of a point on Earth is its angular distance north or south from the equator expressed in degrees and fractions of degrees (fractions are stated in minutes, which equal $1 / 60$ th of a degree, and seconds which equal $1 / 3600$ th of a degree). The North Pole and South Pole have latitudes of $90^{\circ}$ north and $90^{\circ}$ south (written $90^{\circ} \mathrm{N}$ and $90^{\circ} \mathrm{S}$ ) respectively. The fundamental reference point for longitude measurements is the Royal Observatory at Greenwich, England. The longitude of a point on Earth is its angular distance east or west from an imaginary line passing from the north pole, through the Royal Observatory, to the south pole. Like latitude, longitude is expressed in degrees and fractions of degrees. The fundamental reference for vertical positions is mean sea level (see the Geodesy Discovery Kit at http://oceanservice.noaa.gov/education/kits/geodesy/geo03_figure. html and http://oceanservice.noaa.gov/education/kits/geodesy/geo06_ vertdatum.html for more information and lesson plans on the NSRS and some of the ways in which it is used).

Reference points in the NSRS are called "survey marks" and are physically indicated on the Earth's surface by metal disks set into concrete casings or very stable structures. Survey marks often are mistakenly called "benchmarks," but benchmarks are only one type of survey mark. Information about these reference points is organized into sets of data called datums. For each reference point, the distance and direction of adjacent fixed points is measured. This means that fixed reference points can be used to determine the location of any other point included in the datum. Traditionally, triangulation (a method for finding position from the angles between reference points) and trigonometry were used to determine location relative to reference points within a datum. See Appendix A for an example of how to use triangulation to define a geographic position.

The vertical datum is a set of data about a collection of reference points with known heights either above or below mean sea level. "Benchmark" is the name for a survey mark in
the vertical datum. Traditionally, these vertical reference points are established by using a known elevation at one location to determine the elevation at another location (a method known as differential leveling). This technique has been almost completely replaced by the advanced technology of the Global Positioning System (GPS) (see http://oceanservice.noaa.gov/ education/kits/geodesy/geo06_vertdatum.html and http://oceanservice. noaa.gov/education/kits/geodesy/geo09_gps.html, respectively, for more about the vertical datum and GPS).

Because these traditional methods can be complex and timeconsuming (and consequently, expensive), there has been widespread interest in combining information within the NSRS with location information determined with the Global Positioning System (GPS). GPS is a navigation system using 24 Earth-orbiting satellites that transmit signals which can be used to calculate the position and velocity of stations receiving the signals. Because of a number of errors involved in receiving signals from multiple satellites, position data from the basic GPS system are only accurate within several meters. While this level of accuracy may be adequate for some purposes, it would not be adequate, for example, for engineers trying to match segments of a bridge being built from opposite sides of a river. Many of these errors can be corrected, however, using a technique known as Differential GPS (DGPS). This technique uses reference points whose position is accurately known (such as the reference points in the NSRS) to calculate correction factors for GPS signals. With DGPS, positions can be determined with an accuracy of $10-50 \mathrm{~cm}$. Further accuracy (as good as 1 cm or better) is possible, but generally requires more expensive equipment and hours of data collected at a single point.

In situations where very accurate measurements of height are needed, there is another problem with GPS. As mentioned above, the elevation of a specific location on Earth is usually referenced to global mean sea level (this is called "orthometric" height). Global mean sea level is defined by an imaginary three-dimensional shape called the geoid. If the geoid actually existed, points on its surface would correspond to global mean sea level. GPS height calculations are not referenced to the geoid, in part because the surface of the geoid
is constantly changing due to variations in Earth's gravity. Instead, GPS height estimates are referenced to another mathematically defined shape called a reference ellipsoid. The reference ellipsoid is an approximation of the geoid, so elevation measured by GPS is an approximation of orthometric height (visit http://www.oceanservice.noaa.gov/education/kits/geodesy/ geo03_figure.html for additional discussion of the geoid). Where accurate measurements of height are needed, the difference between an approximation based on the reference ellipsoid and "true" height based on the geoid can be critical.

In 1998, the U.S. Congress directed NGS to conduct a study to investigate the importance of "height modernization;" that is, improving estimates of vertical position within the NSRS. The study found that providing more accurate heights through height modernization will:

- Improve the safety of aircraft landing in low-visibility conditions;
- Alert safety planners to evacuation routes that are slowly sinking and susceptible to flooding;
- Provide ships with better estimates of under-keel and overhead clearance (which is important, for example, when ships pass beneath bridges) to avoid dangerous collisions;
- Identify flood-prone areas to guide new construction and reconstruction projects;
- Improve the efficiency of fertilizer and pesticide use to reduce costs and counter pollution from chemical runoff; and
- Improve the efficiency and reliability of water delivery systems.

GPS measurements can be made much more quickly and at less expense than traditional surveying methods, but their usefulness depends upon their accuracy. Height modernization involves improving height measurements of individual reference points within a datum, as well as improving the accuracy of height measurements by GPS.

In this lesson, students will use simulated survey markers and triangulation to determine the height of selected locations. Optionally, they may also use basic elements of trigonometry and geometry to solve problems related to height and geo-
graphic position. While not all of these problems make specific use of NSRS datums, they are good examples of how geodetic tools can be used in diverse ways.

## Learning Procedure

1. 

To prepare for this lesson:
a. Review information on height modernization at http:// oceanservice.noaa.gov/topics/navops/heightmodernization.
b. Make copies of "Model Benchmark" and "Clinometer Scale" one for each student group, preferably mounted on cardboard or foamcore. Alternatively, you can use protractors instead of the "Clinometer Scale." Write elevations on the benchmarks (it doesn't really matter what figures you choose; if each group has a benchmark with a different elevation, the correct answer for Step 5 will also be different for each group).

Note: Many survey marks do not have the elevation stamped onto the marker itself, because the elevation may change over time. Instead, the elevation is given on a data sheet that the National Geodetic Survey maintains for each of its survey marks (for more information, see http://www. geocaching.com/mark/). To simplify this lesson, the "Model Benchmark" is based on an older type of mark that was used by the U.S. Coast and Geodetic Survey (originally established by President Thomas Jefferson as the "Survey of the Coast," now known as the National Geodetic Survey and the Office of Coast Survey).
c. Identify the area that students will use for the height determination activity. This may be outdoors, such as a portion of a schoolyard, or indoors, such as a gymnasium or cafeteria. Identify specific locations for which students will determine height. Outdoor locations may include the tops of trees, basketball hoops, building features, etc. Indoor locations could be tops of windows, electrical fixtures, or sheets of paper that you tape into place. Identify enough locations so that there is a unique location for each student group. If you do not plan to have students create their own benchmarks, prepare these as directed in Step 4, below.
d. If you plan to have students work problems on the "Geodetic Height Student Worksheet," review the "Geodetic Height Teacher Worksheet," and make copies of the Worksheets for each student group.
2.

If your students are not familiar with the science of geodesy, you may want to have them read part or all of the Geodesy Discovery Kit at http://oceanservice.noaa.gov/education/kits/geodesy

## 3.

Briefly review the NSRS, emphasizing the use of survey marks as reference points with known latitude, longitude and elevation. Be sure students understand the concept of triangulation as a means of using such reference points to determine the latitude, longitude and elevation of another point. Students should realize that traditional triangulation requires a line-of-sight between points. Since this is not a requirement for determining position with the global positioning system, GPS technology can greatly simplify this process.

Discuss some of the ways in which the NSRS is important, such as:

- Identifying land areas that are sinking (a process called subsidence)
- Assessing the safety of coastal evacuation routes (routes in subsiding areas may be underwater during storm emergencies);
- Designing efficient water supply and drainage systems that depend upon elevation differences to maintain water flow by gravity;
- Designing agricultural areas to minimize runoff pollution;
- Improving navigation aids (such as runway lights and electronic guidance systems) to make aircraft approaches and landings safer; and
- Identifying areas that are susceptible to flooding.

Remind students that Earth's crust (the lithosphere) consists of about a dozen large plates of rock with an average thickness of about 5 km . Beneath these plates is a hot flowing mantle layer called the asthenosphere, which is several hundred kilometers thick. Heat within the asthenosphere creates convection cur-
rents, which cause the tectonic plates to move several centimeters per year relative to each other. This means that Earth's crust is in constant motion, and this can cause reference points to change position. In addition, Earth's gravity varies as well, causing changes in mean sea level. These changes can affect the accuracy of elevation estimates based on global mean sea level, and are the reason that height modernization is necessary.

## 4.

Tell students that they are going to use traditional survey methods to find the height of certain locations relative to a fixed reference point. If you want to have students create their own reference points, provide each group with a copy of a "Model Benchmark" and have them write a height and date near the center of the mark.

Have each student group make a clinometer, using a "Clinometer Scale," drinking straw, string, fishing weight, and tape. Tape the drinking straw to the straight edge of the "Clinometer Scale" or protractor. Tape one end of the string next to the vertex of the "Clinometer Scale" or protractor. Attach a fishing weight to the other end of the string. Review the procedure for estimating height using the clinometer (see Step 5).

Finished Clinometer

5.

Bring students to the activity area. Have each group place their benchmark near one of the locations to be surveyed. Alternatively, you may place the benchmarks yourself. Assign a location to each group, and have students estimate the height of their assigned location relative to the benchmark as follows:

## Estimating Height with a Clinometer


a. One student should hold the clinometer at eye-level and look through the drinking straw at the assigned location. Another student should measure and record the height of the first student's eye (E), and read and record the number on the scale indicated by the string. This is the angle of elevation $(\alpha)$ of the location. If clinometers have been constructed using protractors, students will have to subtract their reading from $90^{\circ}$ to obtain the angle of elevation.
b. Measure the distance from the first student to the base of the location being surveyed (D).
c. Calculate the height of the assigned location:
height $=$ benchmark elevation $+\mathrm{E}+\mathrm{D} \bullet(\tan \alpha)$
6.

Return to the classroom and discuss students' results. Students should realize that no matter how carefully they make their measurements, the accuracy of the final height estimate
depends upon the accuracy of the elevation stamped on the benchmark. If the mark has moved, the calculated elevation will be incorrect.
7.

If you want to have students solve additional problems, follow procedures described on "Geodetic Height Teacher Worksheet."

## The Bridge Connection

http://www.vims.edu/bridge/ - In the "Site Navigation" menu on the left, click on "Previous Data Tips," then "Geology," and scroll down to "May 2000 Coastal Erosion: Where's the Beach?" and conduct your own beach profile or access profile data from a Maryland beach and plot the changes over time for a graphic illustration of these processes.

## The Me Connection

Have students write a short essay describing ways in which the ability to precisely determine vertical geographic position could be of direct personal importance, or ways in which students might personally use some of the resources described in this lesson.

## Extensions

1. Visit http://oceanservice.noaa.gov/education/kits/geodesy for the National Ocean Service Geodesy Discovery Kit, with tutorials, activities and lesson plans about geodesy.
2. Visit http://www.geocaching.com/mark/ to find out more about geodetic benchmarks, and "benchmark hunting."
3. For more activities involving the Global Positioning System, see "I Know Where You Are!" (15 pages, pdf, 180 Kb; http://oceanservice.noaa.gov/education/kits/geodesy/lessons/ geodesy_where_you_are.pdf) and the National Ocean Service Discovery Classroom (http://oceanservice.noaa.gov/education/ classroom/20_globalpositioning.html).

## Resources

http://oceanservice.noaa.gov/topics/navops/heightmodernization National Ocean Service Height Modernization Web page
http://oceanservice.noaa.gov/education/kits/geodesy - "Geodesy Discovery Kit" from NOAA's National Ocean Service
http://www.ngs.noaa.gov/heightmod/ - National Geodetic Survey height modernization Web page
http://www.ngs.noaa.gov/ - Web page for the National Geodetic Survey, with links to information about benchmarks, geodetic datums, software, and other resources
http://education.usgs.gov/common/lessons/teaching_with_topographic_ maps.html - "Ideas for Teaching With Topographic Maps" from the U. S. Geological Survey

## National Science Education Standards

Content Standard A: Science as Inquiry

- Abilities necessary to do scientific inquiry
- Understandings about scientific inquiry

Content Standard E: Science and Technology

- Abilities of technological design
- Understandings about science and technology

Content Standard F: Science in Personal and Social Perspectives

- Natural and human-induced hazards
- Science and technology in local, national, and global challenges


## National Geography Standards

Standard 1: How to use maps and other geographic representations, tools, and technologies to acquire, process, and report information.

Standard 15: How physical systems affect human systems.
Links to AAAS "Oceans Map" (aka benchmarks)
5D/H3 - Human beings are part of the earth's ecosystems. Human activities can, deliberately or inadvertently, alter the equilibrium in ecosystems.

## height modernization teacher worksheet

## Geodetic Height

Provide each student group with a copy of the "Geodetic Height Student Worksheet." Be sure students understand the terms "plan view" and "elevation view" used in Problem 4. A "plan view" shows a scene or object as it would appear when viewed from above; the plan view of a typical building would show the roof. An "elevation view" shows a scene or object as it would appear when viewed from the side; the plan view of a typical building would show doors, windows, and exterior walls.

The geometric relationships between locations and objects is the "meat" of the worksheet problems, so it is desirable that students be challenged to work out these relationships, rather than supply formulas into which they simply have to "plug in" appropriate values. Depending upon the time available and students' proficiency with basic trigonometry, you may want to assign a smaller number of problems to each group. Alternatively, you may want to begin by working through one or two problems with the entire class.

When students have solved their assigned problems, have each student group present their solutions. The following solutions are one way to proceed; there may be others:

## Problem 1:

The benchmark is 5.27 ft above mean sea level and the height of the total station is 5.50 ft above the benchmark, so an additional elevation of 4.23 ft is needed above the level of the total station to achieve a vertical height of 15 ft above mean sea level. Since the side of the right triangle that is opposite angle $\theta$ in Figure 1 has a length of 4.23 ft , and the side adjacent to angle $\theta$ has a length of 50.27 ft ,

$$
\tan \theta=4.23 \div 50.27=0.08415
$$

So,

$$
\theta=\tan ^{-1}(0.08415)=4.810^{\circ}
$$

## Problem 2:

Figure 2a illustrates the setup for solving the Lookout Tower problems. $D$ is the distance in feet between the location of the Lookout Tower and the Ranger's position; $\mathrm{E}_{\mathrm{L}}$ is the elevation of the Lookout Tower; $\mathrm{E}_{\mathrm{R}}$ is the elevation of the Ranger's position; $\alpha$ is the elevation above ( $\alpha$ has a positive sign) or below ( $\alpha$ has a negative sign) the horizontal plane. From the WCRL Web site, the distance between the Lookout Tower and the Ranger's position in the three examples is
(a) $\mathrm{D}=75,600 \mathrm{ft}$
(b) $\mathrm{D}=33,900 \mathrm{ft}$
(c) $\mathrm{D}=52,300 \mathrm{ft}$

Figure 2a.


Solving for $\alpha$ :
$\alpha=\sin ^{-1}\left[\left(\mathrm{E}_{\mathrm{R}}-\mathrm{E}_{\mathrm{L}}\right) \div \mathrm{D}\right]$
So,
(a) $\alpha=-2.3^{\circ}$
(b) $\alpha=5.7^{\circ}$
(c) $\alpha=-2.4^{\circ}$

## Problem 3:

The conceptual setup for solving the microwave antenna problems is shown in Figure 3a. F is the Required Fresnel Zone Clearance; $\mathrm{D}_{\mathrm{AB}}$ is the distance in feet from Peak A to Peak $B ; D_{A C}$ is the distance in feet from Peak A to Peak C; $E_{A}$ is the elevation of Peak A; $E_{B}$ is the elevation of Peak $B ; D_{H}$ is the difference in height between the antennae on Peaks A and C; $E_{T}$ is the elevation of the antenna on Peak C; point $A$ is the top of Peak A; point C is the top of Peak C; $\mathrm{E}_{\mathrm{C}}$ is the elevation of Peak C.

Figure 3a.


First, find the angle $\alpha$ :

$$
\alpha=\tan ^{-1}\left[\left(\mathrm{E}_{\mathrm{B}}-\mathrm{E}_{\mathrm{A}}+\mathrm{F}\right) \div \mathrm{D}_{\mathrm{AB}}\right]
$$

This is the minimum angle of elevation needed to ensure that the signal from the antenna on Peak A has the required Fresnel Zone Clearance above Peak B.

Next, find $\mathrm{D}_{\mathrm{H}^{\prime}}$ the difference in height between the antenna on Peak A and the antenna on Peak C:

$$
\mathrm{D}_{\mathrm{H}}=(\tan \alpha) \cdot\left(\mathrm{D}_{\mathrm{AC}}\right)
$$

Finally, find the height (elevation) of the antenna on Peak C:

$$
\mathrm{E}_{\mathrm{T}}=\mathrm{D}_{\mathrm{H}}+\mathrm{E}_{\mathrm{A}}
$$

The solutions for the three examples are:
(a) $\alpha=4.5^{\circ} ; \mathrm{E}_{\mathrm{T}}=11,364 \mathrm{ft}$
(b) $\alpha=2.1^{\circ} ; \mathrm{E}_{\mathrm{T}}=11,725 \mathrm{ft}$
(c) $\alpha=7.4^{\circ} ; \mathrm{E}_{\mathrm{T}}=11,030 \mathrm{ft}$

In example (a), $E_{T}(11,364 \mathrm{ft})$ is greater than $E_{C}(11,280 \mathrm{ft})$, so a tower would be needed to raise the antenna to the required height on Peak C. In Example (b), $\mathrm{E}_{\mathrm{T}}$ is equal to $\mathrm{E}_{\mathrm{C}^{\prime}}$, so the antenna can be mounted at ground level on Peak C. In Example (c), $\mathrm{E}_{\mathrm{T}}(11,030 \mathrm{ft})$ is less than $\mathrm{E}_{\mathrm{C}}(12,000 \mathrm{ft})$. It might be possible to locate the antenna on the side of Peak C facing Peak A, but a solution is required that allows antennas to be located on top of the Peaks. To meet this requirement, angle $\alpha$ must be increased to raise the line-of-sight from the antenna on Peak A, so
$\alpha=\tan ^{-1}\left[\left(\mathrm{E}_{\mathrm{C}}-\mathrm{E}_{\mathrm{A}}\right) \div \mathrm{D}_{\mathrm{AC}}\right]=\tan ^{-1}[(12,000-2,800) \div(63,360)]$ $=8.3^{\circ}$

## Problem 4:

Because the total station is mounted on a tripod, the line-of-sight distance measured to the reflector $(\mathrm{C})$ is greater than the horizontal distance that would be measured if the total station were flat on the ground (Figure 4c). So the first step is to find the horizontal distance $(\mathrm{H})$ between the total station and the reflector.

From Figure 4a, it is evident that the vertical angle between the total station and the location of the reflector $(\alpha)$ is equal to angle $\beta$ (see Figure 4 c ). So, $\cos \beta=(\mathrm{H}) \div(\mathrm{C})$. Substituting values for $\beta$ and (C):

$$
\cos \left(9.942^{\circ}\right)=(\mathrm{H}) \div 30.46 \mathrm{~m}
$$

$$
(H)=(0.9850) \cdot 30.46 \mathrm{~m}=30.00 \mathrm{~m}
$$

Next, find the distance (D) between the athlete's throwing position and the position of the shot (Figure 4d).
Figure 4c. (Elevation)


Figure 4d. (Plan)

(1) Construct a line (A) at right angles to line H , passing through the athlete's position. This line divides line H into segments $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$.
(2) $\cos \theta=\mathrm{H}_{1} \div \mathrm{B}$, so $\mathrm{H}_{1}=\mathrm{B} \cdot \cos \theta$
(3) $\mathrm{H}_{1}+\mathrm{H}_{2}=\mathrm{H}$, so $\mathrm{H}_{1}=\mathrm{H}-\mathrm{H}_{2^{\prime}}$ and $\mathrm{H}_{2}=\mathrm{H}-\mathrm{H}_{1}$
(4) $\mathrm{B}^{2}=(\mathrm{A})^{2}+\left(\mathrm{H}_{1}\right)^{2}$ [Pythagorean theorem]
(5) $(\mathrm{A})^{2}=\mathrm{B}^{2}-\left(\mathrm{H}_{1}\right)^{2}$
(6) $\mathrm{D}^{2}=(\mathrm{A})^{2}+\left(\mathrm{H}_{2}\right)^{2}$
(7) Substituting (5): $\mathrm{D}^{2}=\mathrm{B}^{2}-\left(\mathrm{H}_{1}\right)^{2}+\left(\mathrm{H}_{2}\right)^{2}$
(8) Substituting (3): $\mathrm{D}^{2}=\mathrm{B}^{2}-\left(\mathrm{H}-\mathrm{H}_{2}\right)^{2}+\left(\mathrm{H}_{2}\right)^{2}$
(9) So, $\mathrm{D}^{2}=\mathrm{B}^{2}-\left[\mathrm{H}^{2}-2 \mathrm{H} \cdot \mathrm{H}_{2}+\left(\mathrm{H}_{2}\right)^{2}\right]+\left(\mathrm{H}_{2}\right)^{2}$
(10) $\mathrm{D}^{2}=\mathrm{B}^{2}-\mathrm{H}^{2}+2 \mathrm{H} \cdot \mathrm{H}_{2}-\left(\mathrm{H}_{2}\right)^{2}+\left(\mathrm{H}_{2}\right)^{2}$
(11) $\mathrm{D}^{2}=\mathrm{B}^{2}-\mathrm{H}^{2}+2 \mathrm{H} \cdot \mathrm{H}_{2}$
(12) Substituting (3): $\mathrm{D}^{2}=\mathrm{B}^{2}-\mathrm{H}^{2}+2 \mathrm{H} \bullet\left(\mathrm{H}-\mathrm{H}_{1}\right)$
(13) Substituting (2): $\mathrm{D}^{2}=\mathrm{B}^{2}-\mathrm{H}^{2}+2 \mathrm{H} \bullet(\mathrm{H}-\mathrm{B} \cdot \cos \theta)$
(14) So, $\left.\mathrm{D}^{2}=\mathrm{B}^{2}-\mathrm{H}^{2}+2 \mathrm{H}^{2}-2 \mathrm{H} \cdot \mathrm{B} \cdot \cos \theta\right)$
(15) So, $\left.\mathrm{D}^{2}=\mathrm{B}^{2}+\mathrm{H}^{2}-2 \mathrm{H} \cdot \mathrm{B} \cdot \cos \theta\right)$
(16) Substituting known values into (15):
$\mathrm{D}^{2}=(25.00)^{2}+(30.00)^{2}-2 \cdot\left[25.00 \cdot 30.00 \cdot \cos \left(53.13^{\circ}\right)\right]$
$\mathrm{D}^{2}=625+900-1500 \cdot 0.600$
$D^{2}=625$

D = 25 m (a new record!)

## HEIGHT MODERNIZATION WORKSHEET

## Geodetic Height

1. 

To reduce property damage caused by flooding, building codes in coastal areas often require that the lowest floor of newly constructed buildings be a specified height above mean sea level. Suppose a builder wants to construct a new home on the site diagrammed in Figure 1, and has hired a surveyor to determine the elevation of the site. Surveyors and others who need to make precise measurements of the location of various objects often use an instrument called a "total station." This instrument combines a digital version of a theodolite (an instrument for measuring horizontal and vertical angles) with an electronic distance measuring device (EDM). To survey the site in Figure 1, the surveyor locates a benchmark near the site, drives a wooden post into the ground at the proposed construction site, and sets up his total station to make the necessary measurements. Based on the data in Figure 1, what is the angle $\theta$ that will indicate the point on the post that corresponds to a vertical height of 15 feet above mean sea level?

Figure 1.

2.
"Fire lookouts-in a blaze of summer heat or covered with snow, what more romantic image of solitude is there? Second only to Smokey Bear, fire lookouts have become one of the U.S. Forest Service's most widely recognized icons. At one
time in the 1940s, there were close to 4,000 fire towers in National Forests across the United States. Now there are fewer than 900. Computerized lightning detection systems and air patrols have taken over much of the role of lookouts in detecting and locating wildfires. In severe fire seasons, some fire lookouts are still staffed, because they offer views not covered by other systems. But for the most part, they are a dying breed." (Jill A. Osborn, from the "Passport in Time Web site: http://www.passportintime.com/lookouts.html)

Suppose that a Fire Lookout receives a radio call from a Forest Ranger on the ground who has discovered an abandoned campfire that was not completely extinguished and is now too large for the ranger to control. The Ranger has a GPS receiver, and is able to report his latitude, longitude, and elevation. The Fire Lookout has internet access, and can use Web pages that calculate the distance between two geographic locations given the latitude and longitude of the sites (such as http:// www.wcrl.ars.usda.gov/cec/javallat-long.htm, or http://www2.nau.edu/ $\sim$ cvm/latlongdist. html ). The Lookout also has a tripod-mounted binocular telescope that can be elevated to anydesired angle above or below the horizontal plane (see Figure 2). In the following problems, use the WCRL Web site listed directly above and the data provided to find the angle at which the Lookout should set her telescope so that it will be directed toward the Ranger's position. In these problems, assume that the elevations of the Lookout and Ranger are both referenced to the ellipsoid (so they can be directly compared).

Figure 2.

a.
Lookout's Location: Deadman Lookout Tower, Arapaho-
Roosevelt National Forests; Latitude: $40^{\circ} 49^{\prime} 48^{\prime \prime} ;$
Longitude: $105^{\circ} 45^{\prime} 03^{\prime \prime}$; Elevation: $10,710 \mathrm{ft}$
Ranger's Location: Latitude: $41^{\circ} 00^{\prime} 23^{\prime \prime}$; Longitude: $105^{\circ} 39^{\prime}$
$48^{\prime \prime}$; Elevation: $7,632 \mathrm{ft}$
Distance between Lookout's Position and Ranger's Position

Angle to which telescope should be set $\qquad$
b.

Lookout's Location: Spruce Mountain Lookout, Medicine Bow National Forest; Latitude: 41 12' 11"; Longitude: 106 13' 01 "; Elevation: 10,003 ft
Ranger's Location: Latitude: $41^{\circ} 14^{\prime} 34^{\prime \prime}$; Longitude: $106^{\circ} 19^{\prime}$ 43"; Elevation: 13,359 ft

Distance between Lookout's Position and Ranger's Position

Angle to which telescope should be set $\qquad$
c.

Lookout's Location: Squaw Mountain Lookout, ArapahoRoosevelt National Forests; Latitude: $39^{\circ} 40^{\prime} 46^{\prime \prime}$; Longitude: $105^{\circ} 29^{\prime} 32^{\prime \prime}$; Elevation: 11,486 ft
Ranger's Location: Latitude: $39^{\circ} 45^{\prime} 03^{\prime \prime}$; Longitude: $105^{\circ} 39^{\prime}$ 14"; Elevation: 9,324 ft

Distance between Lookout's Position and Ranger's Position

Angle to which telescope should be set $\qquad$
3.

Modern digital microwave radio systems are becoming increasingly popular for telecommunications. Cellular telephone networks are one of the most familiar applications, but the same technology is used in many other circumstances where a wireless communications system is desirable (for
example, in remote areas where ordinary telephone service is unreliable, or when a company wants to be able to exchange large amounts of information between two locations). Microwave radio systems routinely operate at distance up to 80 km between stations (much greater distances are achievable under some conditions), but a fundamental requirement is that there is a clear path between the antennas (parabolic "dishes"). This path is often called line-of-sight, but this term is somewhat misleading because reliable communication requires that the path be larger than a simple line between the antennas. Since the electromagnetic signal from a transmitting antenna disperses as it moves away from source, objects close to the direct "line-of-sight" can block part of the signal. The clearance needed around the line-of-sight is known as the "Fresnel Zone" clearance (pronounced "fray-NEL"). The Fresnel Zone clearance decreases as the frequency of the microwave signal increases.

Suppose you are installing a microwave radio system to provide high-speed data transfer between remote instrument stations monitoring seismic activity in the western United States. These stations are typically located 20 to 40 miles apart, and some of them require communications between mountain peaks with different heights, separated by one or more mountains having an intermediate height (see Figure 3). In the following problems, use the data provided to calculate the angle to which the antenna on Peak A should be elevated to provide the necessary Fresnel Zone clearance at Peak B, and the height

required for an antenna on Peak $C$ to establish a line-of-sight signal path. Assume that the antenna on Peak A is at ground level. Antennas need to be located on the top of all peaks so additional antennas pointing in other directions can be installed to expand the system. In these problems, distances are given in statute miles ( $5,280 \mathrm{ft}$ ). The Required Fresnel Zone Clearance represents the minimum vertical distance below the line-of-sight needed for reliable communications.
a.

Elevation of Peak A $=4,300 \mathrm{ft}$; Elevation of Peak B $=7,600$ ft ; Elevation of Peak C $=11,280 \mathrm{ft}$
Distance from Peak A to Peak $\mathrm{B}=8 \mathrm{mi}$; Distance from Peak
A to Peak C $=17 \mathrm{mi}$
Required Fresnel Zone Clearance $=11.5 \mathrm{ft}$

Angle $\alpha$ of Antenna on Peak A = $\qquad$

Height of Antenna on Peak C = $\qquad$
b.

Elevation of Peak A $=3,400 \mathrm{ft}$; Elevation of Peak B $=8,000 \mathrm{ft}$;
Elevation of Peak C $=11,725 \mathrm{ft}$
Distance from Peak A to Peak B = 24 mi ; Distance from Peak A to Peak C $=43 \mathrm{mi}$
Required Fresnel Zone Clearance $=18 \mathrm{ft}$
Angle $\alpha$ of Antenna on Peak $\mathrm{A}=$ $\qquad$

Height of Antenna on Peak C = $\qquad$
c.

Elevation of Peak A $=2,800 \mathrm{ft}$; Elevation of Peak B $=6,200$ ft ; Elevation of Peak C $=12,000 \mathrm{ft}$; Distance from Peak A to Peak B $=5 \mathrm{mi}$; Distance from Peak A to Peak C $=12 \mathrm{mi}$
Required Fresnel Zone Clearance $=27 \mathrm{ft}$
Angle $\alpha$ of Antenna on Peak $\mathrm{A}=$ $\qquad$

Height of Antenna on Peak C = $\qquad$
4.

Sporting events are making increasing use of modern technology. One example is the use of a total station to measure the distance of thrown objects in contests such as the shotput. In the shotput, the shot is thrown from a concrete circle approximately 2.1 m in diameter. At the front of the circle, there is a 10 cm high stop board. Any part of the thrower's body can touch the inside of the stop board, but not the top. The shot must land within a throwing sector in front of the circle.

To measure the distance a shot is thrown, the total station is mounted on a tripod and the distance (B) between the total station and the area from which the athletes will throw the shot is measured. As soon as an athlete has made a throw, the position of the shot is marked with a reflector that the total station uses to measure distance. The total station is used to measure:

- The line-of-sight distance from the total station to the reflector (C);
- The horizontal angle $(\theta)$ between the line-of-sight from the total station to the stop board and the line-of-sight from the total station to the location of the reflector; and
- The vertical angle ( $\alpha$ ) between the total station and the location of the reflector (see Figures 4a and 4b).

Given the following information, calculate the distance (D) between the athlete's throwing position and position of the shot:

$$
\begin{aligned}
& B=25.00 \mathrm{~m} \\
& C=30.46 \mathrm{~m} \\
& \theta=53.13^{\circ} \\
& \alpha=9.942^{\circ}
\end{aligned}
$$

Figure 4a. (Elevation View)


Figure 4b. (Plan View)


Model Benchmark


Clinometer Scale


