A MODEL DESCRIBING GROWTH AND DEVELOPMENT OF LONGLEAF PINE PLANTATIONS: CONSEQUENCES OF OBSERVED STAND STRUCTURES ON STRUCTURE OF THE MODEL

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Abstract—As longleaf pine (*Pinus palustris* Mill.) may currently represent as little as 1/30th of its former acreage, restoration within its former range in the southern coastal plain is active. Although the focus of these new plantings is aimed at ecosystem restoration, knowledge of the growth and development of longleaf plantations is essential to allow land managers to evaluate different management options. Stand development in longleaf plantations differs from development of plantations of other southern pines. Longleaf seedlings exist in a grass-stage for a varying period, and longleaf saplings and poles can often exist in an intermediate or suppressed crown class for long periods. Other southern pines do not exhibit this behavior. The consequence of these characteristics is that smooth, unimodal diameter distributions are inappropriate for characterizing longleaf pine stands. We will use alternative methods to describe the diameter distributions of longleaf pine. Depending upon viewpoint, the proposed model structure could be called a nonparametric diameter distribution model, or a diameter class model where a uniform distribution is not employed within a class. The model can also be implemented as an individual tree model, if the user desires. A neural net approach has proved promising for initially allocating trees to diameter classes for unthinned stands. A whole-stand basal area prediction equation ensures consistency between these components.

INTRODUCTION

Longleaf pine stands were once a major component of the southern coastal plain from North Carolina to Texas. Currently, longleaf pine may represent as little as 1/30th of its acreage in pre-colonial times (Franklin 1997). An aggressive planting program has developed to restore the longleaf ecosystem within its former range. Although the focus of that work is aimed at ecosystem restoration, knowledge about the growth and development of longleaf plantations is essential for sound management. Longleaf is well-suited for lower-intensity management, particularly longer rotation ages. Also, longleaf pine is less susceptible to most insect and disease problems than other southern pines (Boyer 1990).

Longleaf pine provides higher-value products, such as poles and pilings, more frequently than the more-abundant loblolly pine (*Pinus taeda* L.), and also has a higher specific gravity. Finally, longleaf pine is desirable because a forest of large, old, widely-spaced trees with a grassy understory is "parklike" and visually attractive to visitors.

Many of the older (30 years or older) plantations of longleaf pine arose in a restoration context that is different than the current situation. Often, longleaf plantations were established in cutover areas that had been repeatedly grazed and burned. The current context of restoration is afforesting agricultural fields or converting cutover stands formerly dominated by loblolly pine or mixed pine and hardwoods.

The silvics of longleaf pine distinguish it from other species in the U.S. (Boyer 1990). Three characteristics affect the stand structure of longleaf stands, and hence the structure of a model to describe longleaf plantations. First, longleaf seedlings exist in a "grass stage" for a varying period. The grass stage is a condition where the terminal bud is at or near ground level, and the needles appear similar to a bunchgrass. Although current management practices can often achieve active height growth of most seedlings in the second growing season, individual seedlings may reside in the grass stage for five or more years. Second, although longleaf is an intolerant species, saplings and poles can often exist in an intermediate or suppressed crown class for long periods. Other southern pines do not exhibit this behavior. Suppressed trees rarely respond to release, although trees with live crown ratios of 30 percent or more in the intermediate crown class do respond (Boyer 1990). Third, prescribed fire is an intrinsic part of longleaf pine management, although current practices restrict fire from plantations of other pine species. Interval between fires is often between 2 and 5 years. Prescribed fire ensures that mortality, though rare, will occur throughout the life of a stand, and will restrict ingrowth of volunteer hardwoods and loblolly pine.

There has been little growth and yield modeling done for plantation-grown longleaf pine. A relatively recent model for natural longleaf stands has been provided by Somers and Farrar (1991). The only existing model for plantation-grown

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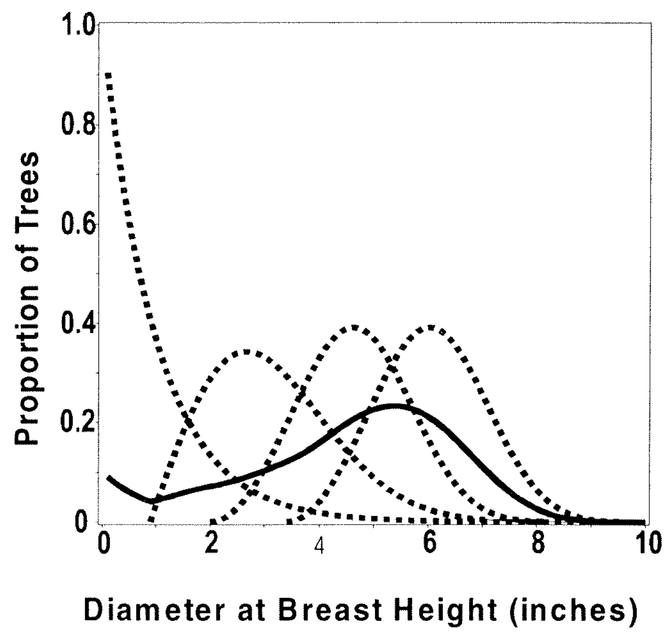

Figure 1—A theoretical diameter distribution represented as a mixture of four populations defined by length of time in the grass stage. The dotted lines represent the four populations, and the solid line is the mixture of these four populations. The proportion of the total is 0.1, 0.2, 0.3, and 0.4 for the four distributions proceeding from the distribution with the smallest mean to the distribution with the largest mean.for natural longleaf stands has been provided by Somers and Farrar (1991). The only existing model for plantation-grown longleaf is restricted to unthinned stands (Lohrey and Bailey 1977). Lohrey and Bailey's work is based on a part of the data available to us; most of those plots have been measured several additional times.

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A Theoretical Example

As individual seedlings reside in the grass stage for differing lengths of time, a diameter distribution for longleaf pine can be considered to be a mixture of distributions. The theoretical example in figure 1 suggests a mixture of four distributions: 40 percent of the trees resided in the grass stage for one year, 30 percent for two years, 20 percent for three or four years, and 10 percent for more than four years. Although the diameter distribution for each cohort is smooth, the pooled diameter distribution for the stand is not unimodal. Most commonly-used diameter distribution functions do poorly for stands that have a long, or heavy, left-hand tail, and are incapable of describing multimodality.

A Brief Primer on Alternative Model Structure

Growth and yield models are typically classified into convenient discrete classes. Although such simplistic polychotomies ignore functional similarities among models (see Goelz [in press] for a novel synthesis of modeling structures), we will describe model forms as discrete entities in this brief listing. One distinction is whether growth of individual trees is projected, and whole stand growth is defined by the aggregation of the individual trees, or whether whole-stand variables are directly predicted. When whole stand variables are predicted, the model might disaggregate whole stand growth into a diameter distribution. Intermediate among these are the size class models that project the growth of trees from one size class to another (typically 1 or 2 inch wide dbh classes). Potentially, there are intermediate structures between these classes (Goelz [in press]).

OUR APPROACH TO MODEL STRUCTURE

We believe that model structure should be determined by the needs of the eventual users of the model, the idiosyncracies of the biology of the system to be modeled, and the data available for estimation of the model. For example, if all trees were of uniform value per unit volume, a whole stand type model would be appropriate. On the other hand, if value of the trees varied with species, size, and tree grade (and if these variables were not highly correlated), then an individual tree model might be suggested. If diameter tends to exhibit relatively smooth unimodal distributions, a diameter distribution model might be suggested; if diameter distributions tend to be irregular or multimodal, use of a parametric distribution function may be inappropriate.

Our Data

Our data are described in Goelz and Leduc [in press]. Over 250 plots are scattered from Texas to Alabama and each records over 20 years of stand dynamics. While technically arising following clearcutting natural stands, the areas were often repeatedly burned and grazed for many years before the plantations were planted. Thus, previous use for many of our plots was open-range grazing rather than forest or cropland. The oldest plantations in our database were last measured at age 65.

Example Diameter Distributions from our Data

We provide several diameter distributions from our plots in figure 2. The plots vary considerably. Some resemble the classical unimodal diameter distributions for even-aged stands (e.g. plot A). Others are very irregular, often being bi- or multi-modal (e.g. plot C, age 65). A distinct grass stage, or the vestige of trees that lingered in the grass stage, is evident in some of the graphs (e.g. plots B, C, D). In some cases, thinning encouraged bimodality as thinning was from below, but only merchantable (greater than 4 in. dbh) trees were removed. In other cases, thinning removed a long left-hand tail or subsidiary mode of the distribution. These example diameter distributions suggest that diameter distributions for longleaf plantations take various shapes, many of which do not comply with standard parametric distributions, and thus we will not use standard parametric distributions in our model for longleaf pine.

A Tentative Model Structure

The objectives for our model structure are to: (1) allow for varied diameter distributions, and potentially maintain those structures; (2) allow stand structure, rather than simply whole-stand variables, to influence growth projections; (3) allow relatively simple implementation (at least to the user); (4) allow the model to be invoked as an individual tree model, diameter distribution model, or diameter class model to facilitate use by different clientele; (5) be applicable to inventory data tallied by diameter classes; (6) make extrapolations reasonable by being conditioned by a whole-stand basal area prediction equation; (7) be tractable for investigating optimal stand management. Regarding the use in extrapolation, although our oldest data are from 65 year old plantations, rotation age for longleaf may be as long as 150 years for some managers. To achieve these objectives, we suggest the following structure:

(1) Initially allocate trees into fixed-width diameter classes.
(2) Generate a diameter distribution that is a quadratic polynomial within a diameter class, but is discontinuous at the limits of each diameter class. Thus the diameter distribution consists of a number of pieces.
(3) Adjust number of trees in each class using an individual tree mortality function.
(4) Adjust/recalculate the parameters of the quadratic polynomial to reflect the effects of mortality.
(5) Use an individual tree diameter growth equation to project the limits of the now varying-width diameter classes.
(6) Adjust the growth in tree basal area to be consistent with a whole-stand basal area growth equation.
(7) Adjust the parameters of the quadratic polyno mial using a simple transformation.
(8) Integrate (using appropriate limits of integra tion) the within-class diameter distributions to reconstitute a fixed-width diameter distribution.

These integrals are simple analytic integrals. The definite integrals will define movement ratios (or growth-index

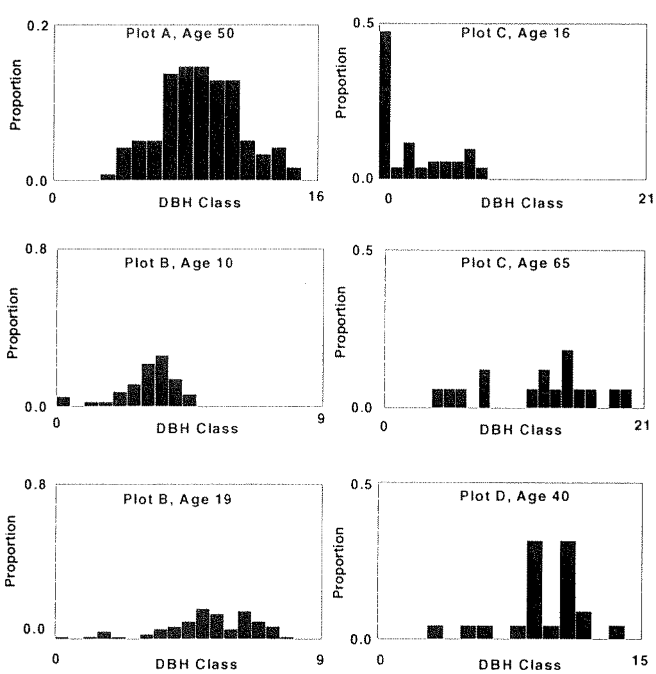

Figure 2—Six observed diameter distributions
variability among diameter distribution shapes within the data.

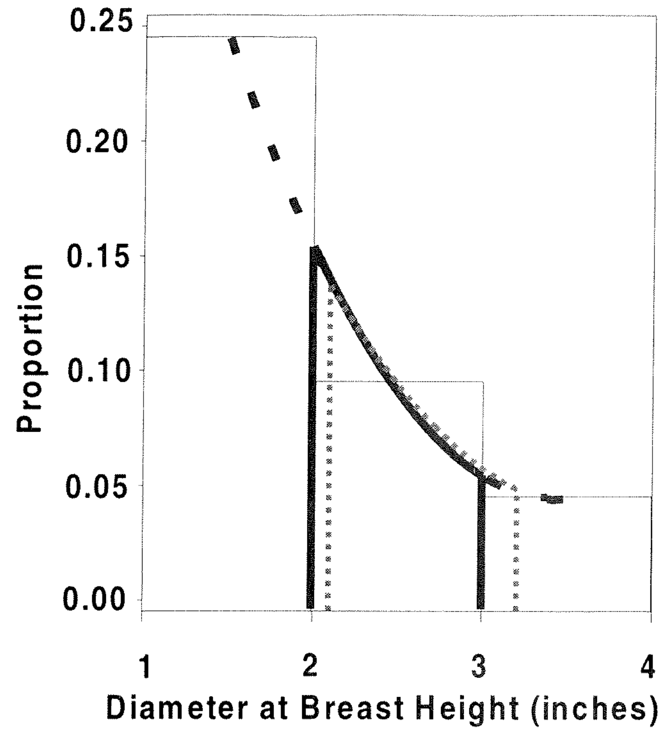

Figure 3—Example of projecting a diameter class into the future. The initial within-class distribution is a simple quadratic polynomial constrained to pass through the midpoints of the adjacent classes (solid lines) and integrate to the area of the histogram within the class. After projection (dotted line), the limits of the diameter class are changed, but the line still integrates to the same area.

ratios) in a diameter class (or stand table projection) model context.

Alternatively, the model could be implemented as an individual tree model, as the component parts are included in this model.

Figure 3 describes some of these steps. To avoid redundancy, we are starting with a diameter distribution that already reflects mortality. The quadratic polynomial ($Y = b_0 + b_1X + b_2X^2$; the math is easier if X is set equal to diameter minus the lower limit of the one-inch diameter class) is constrained to pass through the midpoint of the previous and succeeding diameter classes, and to integrate to the known probability within the diameter class. This is simple to ensure, as there are three parameters of the quadratic equation, and there are three pertinent known values, the proportion of trees in the subject, preceding, and succeeding diameter classes.

If we consider the lower limit of the diameter class to be 0, and the upper limit to be 1, and thus the midpoint of the previous diameter class to be –0.5 and the midpoint of the subsequent diameter class to be 1.5, we have three equations and three unknowns:

$$p_{i-1} = b_0 - \frac{b_1}{2} + \frac{b_2}{4} \quad [1]$$

$$p_{i+1} = b_0 + \frac{3b_1}{2} + \frac{9b_2}{4} \quad [2]$$

$$p_i = b_0 + \frac{b_1}{2} + \frac{b_2}{3} \quad [3]$$

where p_{i-1} is the proportion of trees in the previous diameter class, p_{i+1} is the proportion of trees in the succeeding diameter class, and p_i is the proportion of trees in the diameter class of interest. Equation [3] is the definite integral of the quadratic polynomial from 0 to 1. Equation [1] is obtained by setting Y of the quadratic polynomial equal to p_{i-1} and X equal to –0.5, and equation [2] is obtained by setting Y equal to p_{i+1} and X equal to 0.5. The parameters of the equation can be solved analytically to provide:

$$b_0 = \frac{9}{11} p_i - \frac{7}{44} p_{i+1} + \frac{15}{44} p_{i-1} \quad [4]$$

$$b_1 = \frac{12}{11} p_i + \frac{1}{22} p_{i+1} - \frac{23}{22} p_{i-1} \quad [5]$$

$$b_2 = -\frac{12}{11} p_i + \frac{6}{11} p_{i+1} + \frac{6}{11} p_{i-1} \quad [6]$$

In the case when the diameter class of interest is bounded by 0.0, then b_2 equals zero and equation [1] is not needed.

In that case, b_0 equals $\frac{3p_i - p_{i+1}}{2}$, and b_1 equals $p_{i+1} - p_i$ if one-inch-wide diameter classes are employed.

When future condition of the diameter class is projected, the limits of the diameter class are predicted with an individual tree diameter growth equation. As larger trees grow more than smaller trees, the width of the diameter class expands. In our example, we used a diameter growth equation that was constrained to be consistent with whole-stand basal area growth, however this constraint could be invoked later. The parameters of the new within-class distribution are obtained by a simple transformation and another solution need not be calculated.

For example, if x is a given diameter within a diameter class with x_0 as the lower limit and x_1 as the upper limit, then the initial distribution might be:

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)^2 \quad [7]$$

and after projecting future conditions of the limits of the diameter class (indicated by the additional subscript, $_2$):

$$f(x_2) = \frac{b_0}{(x_{12} - x_{02})} + \frac{b_1}{(x_{12} - x_{02})}\left(\frac{x - x_0}{x_{12} - x_{02}}\right) + \frac{b_2}{(x_{12} - x_{02})}\left(\frac{x - x_0}{x_{12} - x_{02}}\right)^2 \quad [8]$$

Equation [8] is a simple transformation to ensure integration to the same proportion for that diameter class. To recover a fixed-width diameter distribution, the transformed equation is integrated from the lower level of the projected diameter class to the upper level of the previous fixed-width diameter class (3 inches in the example given in figure 3). That obtains the trees that remained in the same diameter class. Then, the number that moved up into the next diameter class may be obtained by subtraction, or by integration from the upper limit of the previous fixed-width class to the upper limit of the variable-width class. The method is applicable to situations when all trees of a diameter class move one or two classes, or even when trees of an initial fixed-width diameter class are projected to occur in three or more of the fixed-width classes at the end of the projection period. Although this procedure may seem somewhat involved, all of the math can be directly calculated without resorting to numerically solving for the parameters.

Initial Conditions

Although the preceding model structure can project the growth of stands of varying structure, there is no provision for initial conditions when the model will be applied to a "bare ground" starting point. Leduc and others [in press] has applied neural networks to predicting diameter distributions for longleaf pine plantations. We will also apply neural nets to provide the initial diameter distribution for a stand. This module of the model will be applicable to ages of 5 to 20 years. Although Leduc and others. applied neural networks to a much broader range of ages, the technique is less suited for the projection of future conditions, given some initial conditions, as it would be difficult to ensure that illogical behavior was avoided (such as abrupt shifts of diameter distributions within relatively short time periods). We will condition the neural net predictions of trees per acre in each diameter class to be consistent with the whole-stand basal area prediction equation that will also be used in projection. Thus, the basal area prediction equation will link the initial condition and projection components of the model, and will provide consistency.

CONCLUSION

This structure secures all of the objectives stated previously while no standard methodology does so. It could be considered to be an integration of standard diameter distribution models (although with a nonparametric distribution) and individual tree forest models, as well as evocative of "enhancements" to standard size class models (e.g. Cao and Baldwin 1999; Nepal and Somers 1992; Pienaar and Harrison 1988), and the "limitless" diameter class model of Clutter and Jones (1980). Thus, it falls between classically-defined classes of models and incorporates an intermediate structure as discussed by Goelz [in press].

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