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Technical analysis of proposed sampling plans, their statistical equivalence, and comparison with sampling plans covered by existing requirements of 42 CFR Part 84, Subpart E, §84.41

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Technical Analysis of Proposed Sampling Plans

Outline

The purpose of this analysis

The purpose of this analysis of the proposed sampling plans is twofold:
1) To explain the changed philosophy and new sampling plans.
2) To help users in selecting or designing the new sampling plans.

The previous QC requirements:

The previous QC requirements, 42 CFR Part 84, Subpart E, § 84.41, involved AQL sampling schemes that were first developed by the U. S. Army in 1942. AQL focuses on the producer's need to accept good product (AQL="Acceptable Quality Level") rather than the consumer's need to reject off-grade product. The previous AQL based standards also required specific decision rules that did not provide producers relief from larger sample sizes than would be necessary with more recent developments in statistical sampling methodology.

This proposed rule:

The proposed rule allows the producer to use more recent developments in the application of statistical methodology to quality assurance. It shifts one's focus of attention more to protecting the consumer and addresses the relationship between the sampling inspection plan and statistical process control and capability.

Shift of focus to protecting the consumer:

The proposed rule does not require conformance to an AQL -- AQL is to protect the producer. Instead, it allows the use of ANSI/ASQC standard Q3 and/or Mil-Std-1916. These standards focus on the consumer's need that off-grade product lots be recognized and rejected -- in the unfortunate event that such product is made and submitted for inspection

AQL and RQL -- The difference between of producer &. consumer focused plans

Both AQL and RQL (RQL="Rejectable Quality Level) refer to the percentage of defective units in a manufactured lot. The old QC requirement focuses on the producer by specifying the sampling requirement in terms of AQL. The proposed rule focuses on the consumer by specifying the sampling requirement in terms of RQL. The difference between AQL and RQL standards is in their probability of accepting a lot whose percent defective equals the plan's named percentage. (AQL stated, or RQL stated.)

The following table the difference between defining 1.25% (for example) as AQL or RQL.

AQL or RQL: Then two ways to define a sampling plan's "stated percentage".

The true lot percent defective:	Producer-focused Plan:	Consumer-focused Plan:
1.25% (for example)	1.25% is AQL, **Pa=0.95**	1.25% is RQL, **Pa=0.05**
For reference: decision rule	n=208, Ac=5	n=240, Ac=0

Pa=Probability of acceptance, n=items sampled, Ac=maximum defectives allowed for accepting the lot.

Comment: See "LQ (Limiting Quality)" on page 3 -- LQ is somewhat similar to RQL.

Comment: This preamble will use the terminology AQL.95 and RQL.05:
"AQL.95" means AQL with Pa=0.95
"RQL.05" means RQL with Pa=0.05

With producer-focused plans (previous QC requirements): AQL=1.25% defective is a producer's protection level in the sense that a lot at the stated AQL has a high probability of acceptance and therefore a
low probability of rejection.

With consumer-focused plans (proposed rule): RQL=1.25% defective is a consumer's protection level in the sense that a lot at the stated RQL has a low probability of acceptance and therefore a
high probability of rejection.

In spite of the focus on protecting the consumer, we will see that this proposed rule provides a path to significantly reduce cost for the producer in many cases by reducing the number of units inspected.

Reward high quality processes:

The proposed rule will reward the producer for focusing on process improvement and control. It accomplishes this by allowing the producer to match standard plans with other plans that offer progressively favorable rewards in terms of time and effort and economy -- in return for progressively higher quality.

Key Characteristics of the Proposed Rule:

This proposed rule specifies three alternative standard methods. They are not identical but they are all three within the range of good QA practices.

1) Three alternative standard sampling plan methods (See the table below):
(1) Mil-Std-1916 (1996), (2) ANSI/ASQC Standard Q3-1988, and (3) minimum acceptable Cpk

2) Four classifications of quality characteristics (See the table below):
Critical, Major-A, Major-B, and Minor

3) The following table relates these defect classifications to "LQ" and "VL" of standards Q3 and 1916:

	Critical Defects	Major A Defects	Major B Defects	Minor Defects
Standard Q3 -- LQ	0.5%	1.25%	3.15%	8.0%
Standard 1916 -- VL	VI	V	IV	III

Other sampling methods are allowed if they can be shown to statistically match the protections and probabilities at the consumer's point of a standard plan.

LQ (Limiting Quality) levels of Q3:
Standard Q3 uses "LQ" index levels in two ways: (1) To define defect classification levels, (2) To indicate the percent defective that will have only a small probability of acceptance. In Q3, the probability associated with LQ, though small, is not a fixed value.

Comment: This preamble will later compare sampling plans to each other with respect to their ability to detect off grade lots. We will use RQL.05 as a point of comparison rather than LQ because the plans in Q3 do not have fixed Pa at LQ. To use RQL.05 to compare plans does not change their LQ -- it simply uses a different "yardstick" for that purpose.

Technical Analysis of Proposed Sampling Plans

1) Military Standard 1916:

These are C=0 attribute plans and alternative variables plans *(SD Unknown, SD= Standard Deviation*) that use a "cookbook" lookup method that does not burden the (non-statistical) user with terms that require probabilistic interpretation. It is suited for inspectors whose goal is to be in "compliance" but not to get involved in statistical definitions or to understand what the plan is doing. Mil-Std-1916 uses switching rules. It encourages the use of statistical process control. It also specifies attribute sampling plans for continuous processes in addition to lot by lot plans.

2) ANSI/ASQC Standard Q3

This attribute-only standard assigns sampling plans based on 1) the LQ that users assign in classifying the quality characteristic and 2) the lot size (N).

Q3 does not contain switching rules -- as it applies to isolated lots.

3) Minimum acceptable Cpk:

The index number Cpk reflects the proportion of individual items in the lot/process for which the characteristic is within specifications. (See the Glossary for how Cpk is calculated.) This method specifies a minimum Cpk and is thus adaptable to taking measurements in the process for control as well as for accept reject decisions. Sample size is not specified as is typical with SPC -- where more frequent, but smaller, samples are taken in process for subgroups smaller than lots.

Comment: When lot accept/reject decisions are needed with the Cpk method, the sample size could be determined from the Mil-Std-1916 variables plans. This would use the relationship that, using K-values of 1916: Cpk=K/3.

Comment: Q3 is more statistically oriented than 1916 -- as 1916 does not refer to points on the oc curve. Tables 3a to 3d show sample sizes and k values and Cpk=k/3 for variables plans that match the oc curves of the LQ plans in Q3. (See the Glossary for "oc curve".)

Other methods

The proposed rule allows for other methods of obtaining decision rules -- as long as those rules match the probabilities at the consumer's point of the oc curve of one of the three specified references. Sequential sampling plans can meet this criterion. ISO standards exist for sequential plans. Refer to ISO 8422 "Sequential Sampling Plans for Attributes" and ISO 8423 "Sequential sampling Plans for Variables". Those ISO standards do not necessarily contain plans that match the RQL of the desired authorized plans. Therefore the Appendix of this Preamble includes parameters to construct sequential plans that match the specified Q3 and 1916 plans exactly.

Removing barriers to improvement:

This proposed rule frees the inspecting organization from the previous requirement of having to follow a specific decision rule. Modern practice provides a variety of equally valid decision rules -- as examples below will show. The revision allows the producer to match the sampling plan with a more advantageous, efficient, and effective one, yet maintain the consumer's protections, and is flexible in setting the AQL and the type of statistical decision rule.

By focusing on the consumer’s point, the revision encourages the use of the most efficient decision rule that provides the specified consumer’s protection. Thus the revision does not lock in a favored decision rule type and it allows for using the many current and future developments in sampling methodology.

Examples using this proposed rule -- eight alternative sampling plans for: Major-A, N=2,000

Application: A Major-A characteristic, lot size N=2,000. These examples illustrate alternative plans that could be used for this same application.

(1) Major A -- under the 3 year "grandfather clause", using Mil-Std-105/Z1.4.

Example: N=2,000, n=125, Ac=2 (N=lot size, n=sample size, Ac=acceptance number)
The revision allows users 3 years to continue using Mil-Std-105/Z1.4 for attributes but tightens Major-A characteristics from 1% to 0.65% AQL.
According to the AQL based Mil-Std-105 (and equivalent ASQC/ANSI Z1.4) inspection level II:
Current: AQL=1.00%, n=125, Ac=3, RQL.05=6.0859%
Grandfathered: AQL=0.65%, n=125, Ac=2, RQL.05=4.9508%.

(2) Major A -- using Military-Standard-1916 -- Attributes:

Example: For VL=V, N=2,000: n=256, Ac=0
Comment: Table I gives CL=B. Table II gives n=256, Ac=0.
Protections: AQL.05=0.0200%, RQL.05=1.1634%. (Based on binomial distribution, not in the standard)
Note: Symbols are defined in the glossary.

(3) Major A -- using Military-Standard-1916 -- Variables (SD unknown):

Example: For VL=V, N=2,000: n=49, k=2.79 (Cpk=0.93)
Comment: Table I gives CL=B. Table III gives n=49, k=2.79.
Protections: AQL.05=0.0467%, RQL.05=1.1595% (Based on normal distribution, not in the standard)
Note: The RQL.05 of STD 1916 variables plans match that of the attribute plans (see example 2), but AQL.05 is about twice that of the attribute plan.

(4) Major A -- using ANSI/ASQC Standard Q3 -- Attributes:

Example: For LQ=1.25%, N=2,000: n=200, Ac=0
Comment: Table A1 gives n=200, Ac=0
Protections: AQL.05=0.0256%, RQL.05=1.4867%. (Based on binomial distribution, not in the standard)

(5) Major A -- matching Q3 (4) with a Variables plan -- SD Unknown:

Example: Q3 is n=200, Ac=0: The matched plan reduces n to 16% of Q3: n=32, k=2.82
Comments: These n and k values are from Table #3b, row 8 in the appendix of this document.
Protections: AQL.05=0.0256%, RQL=1.4867%

(6) Major A -- matching Q3 (4) with a Variables plan -- SD known::

Example: Q3 is n=200, Ac=0: The matched plan reduces n to 3.5% of Q3: n=7, k=2.824
Comments: These n and k values are from Table #3b, row 8 in the appendix of this document.
Protections: AQL.05=0.0256%, RQL=1.4867%

(7) Major-A -- matching Q3 (4) with a Sequential Variables Sampling Plan.

Example: Q3 is n=200, Ac=0: The matched plan reduces the average sample number (ASN, see the glossary) at AQL to 1.65% of Q3.
Comment: See the discussion and example of variables sequential sampling plans starting on Page 8.

(8) Major-A -- matching Q3 (4) with a Sequential Attribute Sampling Plan.

Example: Q3 is n=1,250, Ac=10: For zero defective in the sample at n=340, the matched plan reduces n to 27.2% of Q3.

Comment: See the discussion and example of attribute sequential sampling plans starting on Page 10.

Increased flexibility:

The previous eight examples illustrate the increased flexibility that this proposed rule allows to the producer in designing a quality system:

1) Using the smaller sample size that goes with using the more advanced statistical techniques.

2) Pick a lower RQL to exceed the requirements of the proposed rule in protecting the consumer.

3) Increase the AQL in order to accommodate a process average fraction defective that is higher than the original AQL but less than RQL. Increasing AQL increases n when RQL remains the same.

4) A combination of the above.

Incentive for Statistical Process Control (SPC)

The flexibility that the proposed rule allows provides incentive to producers to benefit from results of having high process quality through the use of SPC and other process strategies. The better the process, the less inspection required for the final product -- until in the extreme case almost all reliance is on controls and data at early points in the process. Thus the rule provides incentive toward process control necessary to make high quality in the first place rather that to rely on acceptance sampling to detect lower quality from a less tuned process.

About Matched Plans (Table #3a, 3b, 3c, 3d)

The matched sampling plans in Table #3 illustrate the opportunities to be gained by allowing the matching of the various types of acceptance decision rules to the same oc curve. These gains would not be attained with the existing requirements because its standard enforces one strict procedure.

Savings from smaller sample size:

Compared to the Q3 Ac=0 attribute plans, for the LQ=1.25 example, we can reduce sample size by factors like 7, 30, 40, or 70 times. This reduction of sample size is far better than incremental improvement. It can represent important savings in terms of material, time, and schedule bottlenecks.

When testing is destructive:

The advantage of requiring a specified oc curve instead of a specified, and perhaps primitive, decision rule is especially strong when the testing is destructive of the units tested. Matching to more efficient plans allows sample testing with increased confidence in the quality of the remaining undestroyed units.

Example applied to destructive tests:

Appendix Table #3b shows that a lot containing 90 items must be 100% inspected to meet Standard Q3 requirement of LQ=1.25% for Major-A characteristics. But rather than test and destroy all 90 items we can use matched variables plans to test only 24 individuals units, or 6, or 3 -- and still meet the requirement for Major-A characteristics.

When Lots are small lots:

Appendix Table #3b shows that Standard Q3 rules-out sampling inspection for items with major-A characteristics on lot sizes up to 90 units. But by matching variables plans to the Q3 oc curve we can meet the Q3 probabilities by testing 6 or fewer sample units.

Assumptions and best practices with variables sampling plans:

Special consideration for roll goods

Sampling roll goods requires a special consideration that does not usually arise when piece-parts are sampled. Samples may be cut from a roll prior to cutting the roll into individual consumable units.

But the individual specification limits (ISLs) of variables sampling plans apply to the (consumable) individuals -- not some other size slice of the roll. When using variables sampling plans it is essential that the samples be individuals in size, shape, etc. The importance of this is that the value of the calculated SD of variables plans represents the variability between consumable individuals within a lot. The amount of variability between small cut pieces of a roll is almost always different from that between larger sections. So to assure statistical accuracy it is essential that each measurement represents a (consumable) individual.

Additionally, with roll goods, the crossweb and downweb positions influence the calculated SD -- and thus the decision-making ability of the sampling plan. The crossweb and downweb patterns should be addressed by the method of selecting samples. (See References, "Is Your Process Too Good for its Control Limits?")

ASTM test methods may not be compatible with ISLs

Manufacturers sometimes use ASTM (American Society for Testing Materials) test methods in testing characteristics of roll goods. These methods cannot be depended on to automatically honor this principle of linking the measurements to consumable individuals. Thus ASTM sample processing rules may have to be modified so that each data point represents an individual. Many ASTM test methods average individuals together in a way that is incompatible with the "individuals" concept.

Assumption of known SD:

To justify the use of sampling plans with SD-Known the manufacturer should have evidence that the process SD is in a state of statistical control. This can take the form of an S-chart or an R-chart on historical lots. The past performance of a process is not a guarantee of current and future performance. Therefore, each sample SD should be applied to a decision rule for SD or, with small samples, a decision rule for the range.

Assumption of Normal Distribution

1) ISL based variables sampling plans assume a normal distribution of the characteristic among the individuals.
2) SD-Known variables sampling plans can be used with products having characteristics with non-normal distributions but techniques must be used that adjust for the shape of the actual distribution.

Sequential plans as switching rules:

One can think of sequential sampling plans as switching rules. They change the sample size depending on the available evidence of quality -- as do standard switching rules as described in Mil-Std-1916. Sequential plans differ from the typical switching in that all of their evidence comes from the current sampled lot. With sequential plans you do not have to assume historical continuity of quality from lot to lot. That is an important advantage of sequential plans because off-grade lots usually occur when they are not expected.

Sequential Sampling and Statistical Process Control:

Variables sequential sampling plans provide, in some cases, a means to move the accept/reject decision further back in the process. Sequential decision rules can be designed so that the sample size per decision is no greater than the typically smaller sample sizes often used with control charts. In this case, data that is collected for the purpose of using control charts can also be used to make accept/reject decisions. Although the two techniques can use the same data, they will have different limits. The control limits of control charts are not the same as accept/reject decision limits, as the latter are based on AQL and RQL.

Variables Sequential sampling plans

Fixed-n variables sampling plans can be matched with variables sequential plans to reduce the sample size. All of the matched plans in tables 1-4 of this preamble are matched at the points AQL.05, RQL.05

Variables Sequential Example -- matching Q3:

An operator or technician carries out a variables sequential decision rule by the method shown on the following page (9) titled "Graphical and tabular variables decision rule". This specific variables sequential decision rule is for Major A matched to Q3, LQ=1.25%, N=2,000. It is based on a lower specification limit (ISL) and known SD. For reference, the decision rule of the Q3 plan is n=200, Ac=0.

This resulting sequential decision rule will provide a reduction in sample size to 1.65% of the Q3 plan. (This occurs when the lot is at AQL quality.) This is almost equivalent to eliminating acceptance sampling altogether – a goal of some advocates of doing all of the QA effort in-process.

The variables sequential decision rule in this example was constructed using the parameters in Appendix Table 4b line 8. The method for manual calculation of the variables decision rule is on page 31.

Graphical and tabular variables decision rule

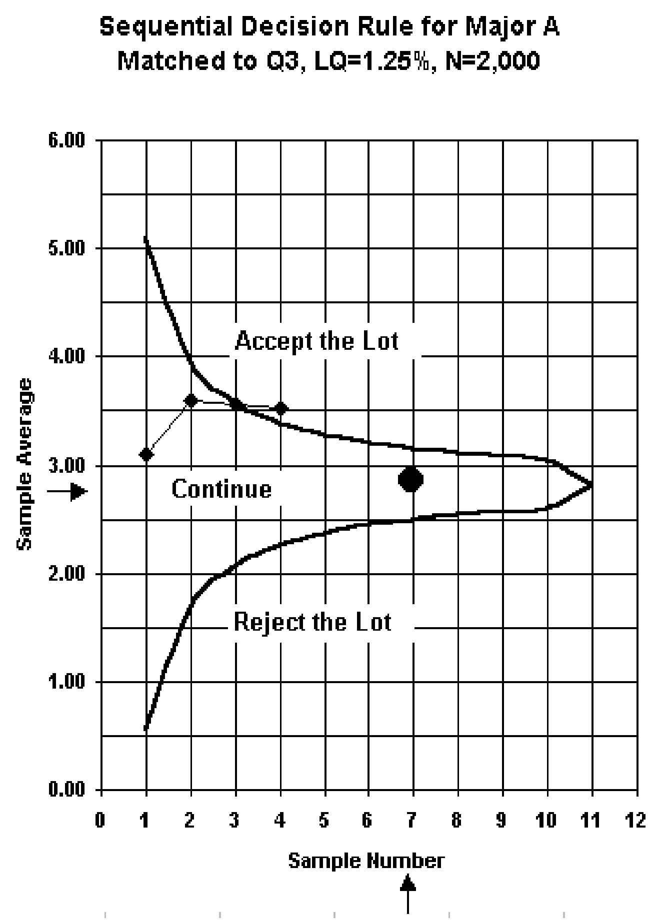
The following chart has the sequential data sheet on the left and sequential diagram on the right.

Data Sheet: The sequential decision rule consists of the varying Ac and Re in columns C and E of the data sheet. The data is coded to units of k = (X-LISL)/SD.

Diagram: On the diagram, the x-axis is sample number (n) and the y-axis is the cumulative sample average. The upper curve is a plot of (Ac vs. n) and the lower curve is a plot of (Re vs. n). The dot at n=7, Xbar=2.824 corresponds to the fixed-n plan.

Data Units: k = (X-LISL)/SD

n	Data	Re	Xbar	Ac
1	3.1	0.56	3.10	5.09
2	4.1	1.69	3.60	3.96
3	3.5	2.07	3.57	3.58
4	3.4	2.26	**3.53**	**3.39**
5		2.37	Accept	3.28
6		2.45		3.20
7		2.50		3.15
8		2.54		3.11
9		2.57		3.08
10		2.60		3.05
11		2.82		2.82



The technician executes the sequential plan by writing the data in column B. The cumulative average of the data goes in column D.

For this example, the technician would stop testing and reject the lot/batch if a cumulative average is less than the rejection number for that row. He/she would stop testing and accept the lot/batch if a cumulative average is greater than the acceptance number for that row.

This lot was accepted at n=4. This decision rule provides the same consumer's risk as the fixed-n attribute plan that it is matched to: n=200, C=0.

Attribute Sequential sampling plans

Fixed-n attribute sampling plans can be matched with attribute sequential plans to reduce the sample size. The reduction is not as dramatic as with variables sequential plans. Sequential attribute plans reduce n for all except C=0 plans.

Attribute Sequential Example -- matching Q3:

An operator or technician carries out an attribute sequential decision rule by the method starting on the following page (11) titled "Sequential Diagram of Attribute Decision Rule". This specific decision rule matches the Q3 plan for a Major-A characteristic: N=500,001, n=1250, Ac=10. (Appendix Table #3b)

Comment: This attribute example does not use the same Q3 plan (N=2,000, n=200,Ac=0) as the variables sequential example (page 8) because, as a general rule, the attribute sequential method cannot reduce the sample size for an Ac=0 plan. Therefore the example Q3 plan was chosen to have the greatest potential for sample size reduction.

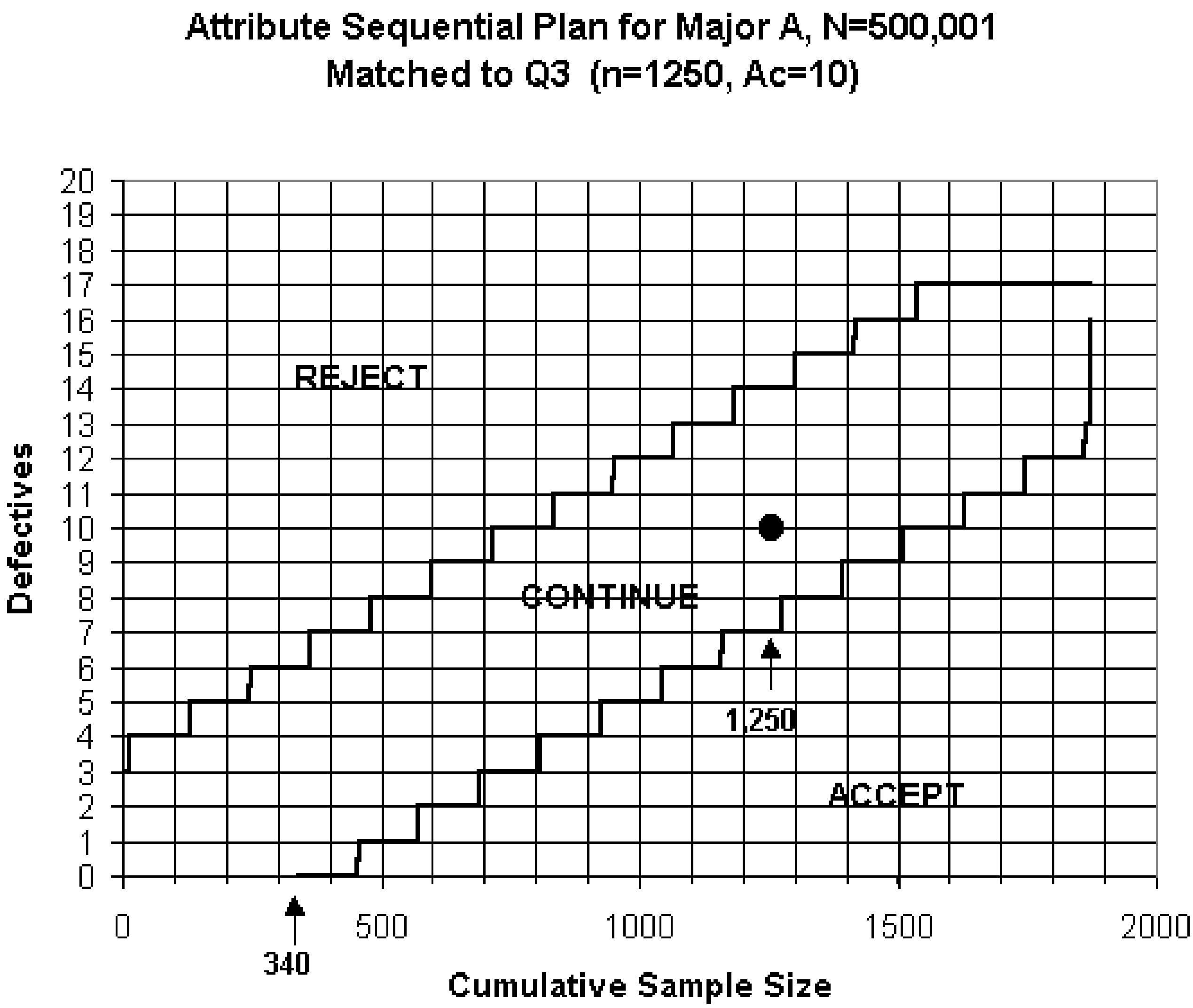
The resulting sequential decision rule will provide a reduction in sample size to as little as 27.2% of the Q3 plan. (This occurs when the quality is perfect or near perfect.)

The attribute sequential decision rule in this example was constructed using the parameters in Appendix Table #4b line 13. The method for manual calculation of the variables decision rule is on page 32.

Sequential Diagram of Attribute Decision Rule

The following diagram explains the sequential decision rule.

Diagram: On the diagram, the x-axis is the sample number (n) and the y-axis is the cumulative sample defectives. The upper staircase curve is a plot of (Ac vs. n) and the lower curve is a plot of (Re vs. n). The Ac and Re lines divide the diagram into three regions -- the reject zone, the "continue sampling" zone, and accept zone.
The dot at n=1,250, Ac=10 corresponds to the fixed-n plan.



Execution: The technician inspects the sample sequentially. The cumulative sample size (n) and cumulative defectives (d) are plotted on this diagram. At each sample, he/she compares the plotted point to the Ac and Re lines.

The execution process can be carried out without the diagram by using the equivalent numeric form of the decision rule, which is on the next page.

The acceptance line starts on the x-axis at defectives=0. The diagram shows that the technician can stop and accept the lot if 0 defective items occur by n=340. For reference, an arrow points to the n = 1,250 of the matching fixed-n sampling plan. Thus if the quality of the manufacturing process is good enough, the sample size will be n=340 instead of n=1,250, a reduction to 27.2% of the Q3 plan.

Sequential Table of Attribute Decision Rule

As an alternative to the diagram, the attribute sequential sampling plan can be executed with this table. The Ac and Re columns correspond to the Ac and Re lines of the previous diagram. The first two columns refer to the sample number on the x-axis.

Sample Size		Decision Rule	
FROM	TO	(AC)	(RE)
1	2	*	**
3	11	*	3
12	129	*	4
130	246	*	5
247	339	*	6
340	363	0	6
364	456	0	7
457	480	1	7
481	573	1	8
574	597	2	8
598	691	2	9
692	714	3	9
715	808	3	10
809	832	4	10
833	925	4	11
926	949	5	11
950	1,042	5	12
1,043	1,066	6	12
1,067	1,159	6	13
1,160	1,183	7	13
1,184	1,276	7	14
1,277	1,300	8	14
1,301	1,394	8	15
1,395	1,417	9	15
1,418	1,511	9	16
1,512	1,535	10	16
1,536	1,628	10	17
1,629	1,745	11	17
1,746	1,862	12	17
1,863	1,872	13	17
1,873	1,873	14	17
1,874	1,874	15	17
1,875	1,875	16	17

NOTES:
* = CANNOT ACCEPT
** = CANNOT REJECT

Execution: The technician increases the sample size either item by item or in groups as indicated in the table - until an accept/reject decision is made. The acceptance number (Ac) and the rejection number (Re) will determine the disposition of the lot.

Truncation: The table above shows that it is possible to take 1,875 samples to make a decision. The probability of that happening is very small, as the maximum average sample size (ASN, see the glossary) is 1000 for this example. That is still less than fixed-n=1,250.

Appendix

Glossary:

Ac, C

Ac and C are symbols for the acceptance number - the maximum number of defectives in the sample and still accept the lot.

AQL

The Acceptable Quality Level, AQL, is the lot fraction defective for which the lot has a high probability of acceptance. AQL is sometimes associated with 0.95 probability of acceptance. AQL and Alpha define the Producer's Point.

ASN

ASN (Average Sample Number) is useful to evaluate a sequential sampling plan. It is the average number of sample units inspected per lot in reaching decisions to accept or reject. The ASN curve is a plot of ASN versus true lot quality, p'.

Alpha Risk

The Producer's Risk, Alpha, is the risk of rejecting a good lot, that is, a lot that contains AQL fraction defective.

Beta Risk:

The Consumer's Risk, Beta, is the risk of accepting a rejectable lot, that is, a lot that contains RQL fraction defective. A typical value for the Beta risk is 0.05.

C, Ac

C and Ac are symbols for the acceptance number - the maximum number of defectives in the sample and still accept the lot.

Consumer's Point

Consumer focused sampling plans are indexed by the consumer's point on the oc curve. The consumer's point of an attribute sampling plan is defined by a rejectable percent defective (RQL or LQ) that will have a small probability of acceptance -- like Pa=0.05.

Cpk

An index reflecting the proportion of items for which a characteristic is within specification limits.

Cpk = the smaller of (Xbar-LISL)/(3*SD) and (UISL-Xbar)/(3*SD)

Fixed-n sampling plan

The simple size is specified, or fixed -- as opposed to a sequential sampling plan.

Individual Item

For roll goods, individual consumable items are cut from a larger web. Specification limits, or ISLs, apply to individual consumable items -- not statistics based on composite samples. The SD of variables sampling plans should be calculated from data representing individuals.

ISL

ISL is an acronym for Individual Specification Limit. ISLs defines when a measured individual item is defective on the high side (UISL), the low side (LISL), of both.

With measured data, ISLs are used to calculate a variables sampling plan's decision rules, calculate Cpk, and with rectification plans, to sort nonconforming units from rejected lots.

k

k is a standardized acceptance limit for a variables sampling plan for fraction nonconforming, k=3*Cpk.

LQ, RQL, LTPD:

The LQ (Limiting Quality) is the lot fraction defective for which the lot has a low probability of acceptance. ASQC/ANSI standard Q3 uses for levels of LQ to classify defects. Other names that have the same meaning in general as LQ are RQL and LTPD. (See AQL.)

Matched sampling plans:

In this preamble, a sampling plan is considered matched to another plan if they both have the same producer's and consumers' points at alpha=beta=0.05. That is , if the AQL and RQL of the two plans are the same.

N, n

n: Lower case n is the symbol for the sample size to be inspected.

N: Upper case N is the symbol for the lot size (Number of items in the lot.)

OC Curve

The oc curve (Operating Characteristic Curve) of a sampling plan is a graph of Pa versus p', where Pa = probability of acceptance, p' = true lot fraction defective of a lot. The oc curve tells you how the sampling plan will perform in making accept/reject decisions.

Pa

Probability of acceptance.

Producer's Point

Producer focused sampling plans are indexed by the producer's point on the oc curve. The producer's point of an attribute sampling plan is defined by an "acceptable" percent defective (see AQL) that will have a high probability of acceptance -- like Pa=0.95.

RQL, LQ, LTPD:

The Rejectable Quality Level, RQL, is the lot fraction defective for which the lot has a low probability of acceptance. RQL and Beta define the consumer's Point of the oc curve. Other names that have the same meaning in general as RQL are LQ and LTPD. (See AQL.)

Sequential Sampling Plan

A sequential sampling plan is a technique by which we build up our sample one item at a time or in small groups, and after inspecting each item, ask ourselves: "Can we be sure enough to accept or reject this batch on the information so far collected?"
Its value is in enabling reliable conclusions to be wrung from a minimum of data. This was deemed sufficient to require that it be classified "Restricted " within the meaning of the Espionage Act during the war of 1939-45.

SD

Standard Deviation

SD Known variables plans

Variables sampling plans have a "known SD" when the standard deviation is known in advance and has been determined to be in statistical control from lot to lot. This as concluded from using an S Chart. Past in-control conditions cannot demonstrate that the current lot has not changed. Therefore good practice is to test the current lot SD with a statistical F-test, or for small samples, a statistical range test. With the SD Known procedure the sample size is smaller than with the SD Unknown procedure.

SD Unknown variables plans

Variables sampling plans have an "unknown SD" when the standard deviation is not known prior to taking the sample. An estimate of the SD is then calculated from the sample and used in the accept/reject decision. These plans do not require that lots be in statistical control. With the SD Unknown procedure the sample size is larger than with the SD Known procedure.

Truncation Rule

Classical sequential sampling plans do not naturally truncate at a maximum sample size. Such plans need to be truncated by a truncation rule. The truncation rule use here is 1.5*Fixed-n.

Verification Level (VL)

In Mil-Std-1916, VL and lot size jointly determine the sample size. VL=VII requires the largest sample size, and n decreases as VL decreases to the lowest level, VL=I

Xbar

Xbar represents the sample average of variables data.

References:

Sequential Analysis of Statistical Data: Applications, (SRG Report 255)-- Prepared by the Statistical Research Group, Columbia University. Available: Amazon.com

From the Forward: Sequential analysis was devised by A. Wald in March 1943 for use in development work on military and naval equipment, in the analysis of combat experience, and in similar problems of war research. Its value in enabling reliable conclusions to be wrung from a minimum of data was deemed sufficient to require that it be classified Restricted within the meaning of the Espionage Act. The Army, the Navy, and the Office of Scientific Research and Development, however, introduced it into several thousand manufacturing establishments as a basis for acceptance inspection, and this resulted in a widespread demand for access to information on the subject. In response to representations from the War Production Board, the Army, and the Navy, the Restricted classification was therefore removed in May 1945.

Relevant Contents: How to apply sequential sampling for: Attributes, Double Dichotomies, Mean with known SD, (one-sided and two-sided), Standard deviation (one-sided)

Acceptance Sampling in Quality Control, Edward G. Schilling, Volume 42 of "STATISTICS: Textbooks and Monographs", Marcel Dekker, Inc., Available: Amazon.com

Relevant Contents:

Sampling by attributes -- single, double, multiple, sequential, and plans.
Sampling by variables -- fixed-n and sequential for process parameter, proportion nonconforming.
Sampling for reliability -- MTBF and failure rate.
Sampling of bulk material, narrow limit gauging, rectification schemes.
Acceptance control charts, cumulative sum charts.
Administration of acceptance sampling.

How to use Sequential Statistical Methods, Thomas P. McWilliams, The ASQC Basic References in Quality Control: Statistical Techniques, Volume 13, American Society for Quality Control, www.asq.org

Relevant Contents:

How to design sequential sampling plans
Sequential sampling plans for attributes (binomial, hypergeometric, and poisson)
Sequential sampling plans for variables (mean, process variability)
Sequential life testing
Sequential estimation

www.samplingplans.com:

Application tips for sampling plans.
Sampling plan discussion forum.
Software to develop sampling plans.

Title: Is Your Process Too Good for its Control Limits?

American Society For Quality (ASQ)

http://qic.asq.org/perl/search.pl?item=9583\

Statistical control charts adapted to specifically control downweb and crossweb variability of roll goods and injection molding.

Author: Janis, Stuart J.

Tables of Sampling Plan Properties

Description of tables

Table #1 -- Mil-Std-1916 Plans

Table #1 shows the relationship between the attribute and variables plans in Military Standard 1916.

Table #2a,b,c,d -- Mil-Std-1916 and Q3 compared

Table #2 compares the alignment of AQL, RQL, n, and Ac between standards 1916 and Q3 at each classification level.

Table #3a,b,c,d -- Variables plans that match Q3 Attribute Plans.

Table #3 lists the fixed-n variables plans that match the authorized attribute plans of Q3.

Table #4a,b,c,d -- Parameters to construct sequential sampling plans that match Q3 attribute plans.

Table #4 lists the sequential parameters HA, HR, and G for designing variables and attribute sequential plans that match Q3.

Table #1 -- Mil-Std-1916 Plans, Major-A (VL=V) -- Compare Attributes and Variables Plans

Purpose of Table #1: To study the relationship between the attribute and variables plans in 1916.
To actually design a plan, use table 3 and/or table 4

	Q3	1916	Mil-Std-1916 Attribute Plans				Exact Matching Variables Plans*		Mil-Std-1916 Variables Plans			
Row	Lot size to	CL	Attribute n	Attribute Ac	AQL.05	RQL.05	Variables n	Variables k	Variables n	Variables k	AQL.05	RQL.05
1	2-192	A	100%	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
2	193-1632	A	192	0	0.0267%	1.5482%	32	2.8103	44	2.69	0.0635%	1.5499%
3	1633-3072	B	256	0	0.0200%	1.1634%	35	2.9045	49	2.79	0.0467%	1.1596%
4	3073-5440	C	320	0	0.0160%	0.9318%	38	2.9756	54	2.86	0.0383%	0.9261%
5	5441-9216	D	384	0	0.0134%	0.7771%	41	3.0320	58	2.92	0.0318%	0.7665%
6	> 9216	E	512	0	0.0100%	0.5834%	45	3.1205	64	3.00	0.0249%	0.5905%

* Exact matching variables plans: These are the n and k of variables plans that would match the attribute plans exactly at AQL.05 and RQL.05. They are calculated independently of 1916.

What Table #1 shows:

(1) Table #1 shows that the RQLs of the variables plans of Mil-Std-1916 match quite closely the RQLs of the attribute plans. Thus from the standpoint of protecting the consumer from receiving lots with RQL quality, the two plans are equivalent.

(2) Table #1 shows that the AQL.05s of the variables plans of Mil-Std-1916 are more that twice that of the AQL.05s of the corresponding attribute plans. This explains why the sample size n of the 1916 variables plans are higher than the variables plans that match exactly.

(3) Not shown in table #1, but the VL=V variables plans match closely the oc curves of the VL=VI attribute plans. Possibly the designers of Mil-Std-1916 have moved the verification levels one level stricter for their variables plans.

Table #2a -- Critical Classification -- Mil-Std-1916 (VL=VI) and Q3 (LQ=0.5%)

Purpose of table #2a,b,c,d: To compare the alignment of AQL, RQL, n, Ac between standards 1916 and Q3 at each classification level.
To actually design a plan, use table 3 and/or table 4

		Mil-1916	Mil-Std-1916 Attribute Plans, VL=VI					ANSI/ASQC Q3 Attribute Plans,LQ=0.5			
Row	Lot size	Code Ltr	Attr. n	Attr. Ac	AQL.05	RQL.05		Attr. n	Attr. Ac	AQL.05	RQL.05
1	16-25	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
2	26-50	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
3	51-90	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
4	91-150	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
5	151-280	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
6	281-500	A	100%	N/A	N/A	N/A		280	0	0.0183%	1.0642%
7	501-512	A	512	0	0.0100%	0.5834%		380	0	0.0135%	0.7853%
	513-1,200	A	512	0	0.0100%	0.5834%		380	0	0.0135%	0.7853%
8	1,201-3,072	A	512	0	0.0100%	0.5834%		430	0	0.0119%	0.6943%
	3,073-3,200	B	640	0	0.0080%	0.4670%		430	0	0.0119%	0.6943%
9	3,201-5,440	B	640	0	0.0080%	0.4670%		450	0	0.0114%	0.6635%
	5,441-9,216	C	768	0	0.0067%	0.3893%		450	0	0.0114%	0.6635%
	9,217-10,000	D	1,024	0	0.0050%	0.2921%		450	0	0.0114%	0.6635%
10	10,001-17,408	D	1,024	0	0.0050%	0.2921%		500	0	0.0103%	0.5974%
	17,409-35,000	E	1,280	0	0.0040%	0.2338%		500	0	0.0103%	0.5974%
11	35,001-150,000	E	1,280	0	0.0040%	0.2338%		800	1	0.0444%	0.5916%
12	150,001-500,000	E	1,280	0	0.0040%	0.2338%		800	1	0.0444%	0.5916%
13	Above 500,000	E	1,280	0	0.0040%	0.2338%		1250	3	0.1094%	0.6191%

What Table #2a shows:

(1) For Critical characteristics, Mil-Std-1916 has higher sample sizes and lower RQLs than the corresponding Q3 plans.

(2) The Q3 plans in the table have higher RQLs than the nominal LQ designated by that standard. The reason for this is that Alpha=Beta=0.05 throughout.

Table #2b -- Major-A Classification -- Mil-Std-1916 (VL=V) and Q3 (LQ=1.25%)

Purpose of table #2a,b,c,d: To compare the alignment of AQL, RQL, n, Ac between standards 1916 and Q3 at each classification level.
To actually design a plan, use table 3 and/or table 4

	Q3	Mil-1916	Mil-Std-1916 Attribute Plans, VL=V					ANSI/ASQC Q3 Attribute Plans, LQ=1.25%			
Row	Lot size	Code Ltr.	Attr. n	Attr. Ac	AQL.05	RQL.05		Attr. n	Attr. Ac	AQL.05	RQL.05
1	16-25	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
2	26-50	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
3	51-90	A	100%	N/A	N/A	N/A		100%	0	N/A	N/A
4	91-150	A	100%	N/A	N/A	N/A		90	0	0.0570%	3.2738%
5	151-192	A	100%	N/A	N/A	N/A		130	0	0.0394%	2.2781%
	193-280	A	192	0	0.0267%	1.5482%		130	0	0.0394%	2.2781%
6	281-500	A	192	0	0.0267%	1.5482%		155	0	0.0331%	1.9142%
7	501-1,200	A	192	0	0.0267%	1.5482%		170	0	0.0302%	1.7468%
8	1,201-1,636	A	192	0	0.0267%	1.5482%		200	0	0.0256%	1.4867%
	1,637-3,072	B	256	0	0.0200%	1.1634%		200	0	0.0256%	1.4867%
	3,073-3,200	C	320	0	0.0161%	0.9318%		200	0	0.0256%	1.4867%
9	3,201-5,440	C	320	0	0.0161%	0.9318%		315	1	0.1129%	1.4971%
	5,441-9,216	D	384	0	0.0134%	0.7771%		315	1	0.1129%	1.4971%
	9,217-10,000	E	512	0	0.0100%	0.5834%		315	1	0.1129%	1.4971%
10	10,001-35,000	E	512	0	0.0100%	0.5834%		315	1	0.1129%	1.4971%
11	35,001-150,000	E	512	0	0.0100%	0.5834%		500	3	0.2737%	1.5434%
12	150,001-500,000	E	512	0	0.0100%	0.5834%		800	5	0.3271%	1.3096%
13	Above 500,000	E	512	0	0.0100%	0.5834%		1,250	10	0.4943%	1.3532%

What Table #2b shows:

(1) For Major-A characteristics, Mil-Std-1916 has higher sample sizes and lower RQLs than the corresponding Q3 plans.

(2) The Q3 plans in the table have higher RQLs than the nominal LQ designated by that standard. The reason for this is that Alpha=Beta=0.05 throughout.

Table #2c -- Major-B Classification -- Mil-Std-1916 (VL=IV) and Q3 (LQ=3.15%)

Purpose of table #2a,b,c,d: To compare the alignment of AQL, RQL, n, Ac between standards 1916 and Q3 at each classification level.
To actually design a plan, use table 3 and/or table 4

	Q3	Mil-1916	Mil-Std-1916 Attribute Plans, VL=VI					ANSI/ASQC Q3 Attribute Plans, LQ=3.15%			
Row	Lot size	Code Ltr.	Attr. n	Attr. Ac	AQL.05	RQL.05		Attr. n	Attr. Ac	AQL.05	RQL.05
1	16-25	A	100%	N/A	N/A	N/A		100%	N/A	N/A	N/A
2	26-50	A	100%	N/A	N/A	N/A		100%	N/A	N/A	N/A
3	51-80	A	100%	N/A	N/A	N/A		44	0	0.1165%	6.5819%
	81-90	A	80	0	0.0641%	3.6754%		44	0	0.1165%	6.5819%
4	91-150	A	80	0	0.0641%	3.6754%		55	0	0.0932%	5.3011%
5	151-280	A	80	0	0.0641%	3.6754%		65	0	0.0789%	4.5042%
6	281-500	A	80	0	0.0641%	3.6754%		80	0	0.0641%	3.6754%
7	501-960	A	80	0	0.0641%	3.6754%		125	1	0.2850%	3.7387%
	961-1,200	B	96	0	0.0354%	3.0724%		125	1	0.2850%	3.7387%
8	1,201-1,632	B	96	0	0.0354%	3.0724%		125	1	0.2850%	3.7387%
	1,633-3,072	C	128	0	0.0401%	2.3132%		125	1	0.2850%	3.7387%
	3,073-3,200	D	160	0	0.0321%	1.8549%		125	1	0.2850%	3.7387%
9	3,201-5,440	D	160	0	0.0321%	1.8549%		200	3	0.6859%	3.8310%
	5,440-10,000	E	192	0	0.0267%	1.5482%		200	3	0.6859%	3.8310%
10	10,001-35,000	E	192	0	0.0267%	1.5482%		315	5	0.8326%	3.3083%
11	35,000-150,000	E	192	0	0.0267%	1.5482%		500	10	1.2385%	3.3688%
12	150,000-500,000	E	192	0	0.0267%	1.5482%		800	18	1.5607%	3.3183%
13	Above 500,000	E	192	0	0.0267%	1.5482%		800	18	1.5607%	3.3183%

What Table #2c shows:

(1) For Critical Major-B, Mil-Std-1916 has, for small lots, higher sample sizes and lower RQLs than the corresponding Q3 plans. For larger lots, 1916 has smaller sample sizes. For N>500, Q3 has C>0

(2) The Q3 plans in the table have higher RQLs than the nominal LQ designated by that standard. The reason for this is that Alpha=Beta=0.05 throughout.

Table #2d -- Minor Classification -- Mil-Std-1916 (VL=III) and Q3 (LQ=8.0%)

Purpose of table #2a,b,c,d: To compare the alignment of AQL, RQL, n, Ac between standards 1916 and Q3 at each classification level.
To actually design a plan, use table 3 and/or table 4

	Q3	Mil-1916	Mil-Std-1916 Attribute Plans, VL=III					ANSI/ASQC Q3 Attribute Plans, LQ=8.0%			
Row	Lot size	Code Ltr.	Attr. n	Attr. Ac	AQL.05	RQL.05		Attr. n	Attr. Ac	AQL.05	RQL.05
1	16-25	A	100%	N/A	N/A	N/A		17	0	N/A	N/A
2	26-32	A	100%	N/A	N/A	N/A		22	0	0.2329%	12.7305%
	33-50	A	32	0	0.1602%	8.9368%		22	0	0.2329%	12.7305%
3	51-90	A	32	0	0.1602%	8.9368%		24	0	0.2135%	11.7346%
4	91-150	A	32	0	0.1602%	8.9368%		26	0	0.1971%	10.8830%
5	151-280	A	32	0	0.1602%	8.9368%		28	0	0.1830%	10.1466%
6	281-500	A	32	0	0.1602%	8.9368%		32	0	0.1602%	8.9368%
7	501-544	A	32	0	0.1602%	8.9368%		50	1	0.7153%	9.1398%
	545-960	B	40	0	0.1282%	7.2158%		50	1	0.7153%	9.1398%
	961-1,200	C	48	0	0.1068%	6.0503%		50	1	0.7153%	9.1398%
8	1,201-1,632	C	48	0	0.1068%	6.0503%		80	3	1.7255%	9.4075%
	1,233-3,072	D	64	0	0.0801%	4.5730%		80	3	1.7255%	9.4075%
	3,073-3,200	E	80	0	0.0641%	3.6754%		80	3	1.7255%	9.4075%
9	3,201-10,000	E	80	0	0.0641%	3.6754%		125	5	2.1106%	8.2260%
10	10,001-35,000	E	80	0	0.0641%	3.6754%		200	10	3.1146%	8.3335%
11	35,001-150,000	E	80	0	0.0641%	3.6754%		315	18	3.9854%	8.3562%
12	150,001-500,000	E	80	0	0.0641%	3.6754%		315	18	3.9854%	8.3562%
13	Above 500,000	E	80	0	0.0641%	3.6754%		315	18	3.9854%	8.3562%

What Table #2d shows:

(1) For Minor characteristics, Mil-Std-1916 has higher sample sizes and lower RQLs than the corresponding Q3 plans. For larger lots, 1916 has smaller sample size. Above N=500, Q3 has C>0

(2) The Q3 plans in the table have higher RQLs than the nominal LQ designated by that standard. The reason for this is that Alpha=Beta=0.05 throughout.

Table #3a -- Characteristics Classified as Critical -- Q3 (LQ=0.5%) attribute versus matched variables plans

Purpose of Table #3a,b,c,d: To list the fixed-n variables plans that match the authorized attribute plans of Q3.

	Section #1					Section #2			Section #3			Section #4		
	ANSI/ASQC Standard Q3 LQ=0.5%					**Matched Variables Fixed-n Plan SD Unknown**			**Matched Variables Fixed-n Plan SD Known**			**ASN of Matched Sequential Variables Plan, SD Known**		
Row	Lot size interval	n	Ac	AQL.05	RQL.05	n	k	Cpk units	n	k	Cpk units	At AQL	Maxi-mum	At RQL
1	16-25	100%	N/A	N/A	N/A	N/A	N/A	N/A	7	2.9332	0.98	3.4	5.6	3.4
2	26-50	100%	N/A	N/A	N/A	37	2.9332	0.98	7	2.9332	0.98	3.4	5.6	3.4
3	51-90	100%	N/A	N/A	N/A	37	2.9332	0.98	7	2.9332	0.98	3.4	5.6	3.4
4	91-150	100%	N/A	N/A	N/A	37	2.9332	0.98	7	2.9332	0.98	3.4	5.6	3.4
5	151-280	100%	N/A	N/A	N/A	37	2.9332	0.98	7	2.9332	0.98	3.4	5.6	3.4
6	281-500	280	0	0.0183%	1.0642%	37	2.9332	0.98	7	2.9332	0.98	3.4	5.5	3.4
7	501 - 1,200	380	0	0.0135%	0.7853%	41	3.0291	1.01	8	3.0291	1.01	3.9	6.4	3.9
8	1,201 - 3,200	430	0	0.0117%	0.6801%	43	3.0734	1.02	8	3.0734	1.02	3.9	6.4	3.9
9	3,201 - 10,000	450	0	0.0114%	0.6635%	43	3.0811	1.03	8	3.0811	1.03	3.9	6.4	3.9
10	10,001 - 35,000	500	0	0.0103%	0.5974%	45	3.1126	1.04	8	3.1126	1.04	3.9	6.4	3.9
11	35,001 - 150,000	800	1	0.0444%	0.5916%	88	2.9204	0.97	17	2.9204	0.97	8.3	13.6	8.3
12	150,001 - 500,000	800	1	0.0444%	0.5916%	88	2.9204	0.97	17	2.9204	0.97	8.3	13.6	8.3
13	Above 500,000	1250	3	0.1094%	0.6191%	167	2.7822	0.93	35	2.7822	0.93	17.1	28.0	17.1

* SD = Standard Deviation. ** ASN = Average Sample Number - for sequential plans, the average n to reach an accept/reject decision..

What table #3a shows:

Section #1 - These plans are from Q3 table A1, which requires 100% inspection for LQ=0.5, lot size (N) up to 280. Ac=0 up to N=10,000.

Section #2 - The fixed-n variables plans for SD Unknown (SD is calculated from the sample) reduce attribute n by a factor of **5-1/2**. By definition, the lot Cpk=k/3

Section #3 - The fixed-n variables plans for SD Known (SD is known from history) reduce attribute n by a factor of **25 times**.

Section #4 - For lots at AQL, the attribute n is reduced by a factor of **50-90 times**.

Table #3b -- Characteristics Classified as Major-A -- Q3 (LQ=1.25%) attribute versus matched variables plans

Purpose of Table #3a,b,c,d: To list the fixed-n variables plans that match the authorized attribute plans of Q3.

	Section #1					Section #2			Section #3			Section #4		
	ANSI/ASQC Standard Q3 LQ=1.25%					**Matched Variables Plan SD Unknown**			**Matched Variables Plan SD Known**			**ASN of Matched Sequential Variables Plan, SD Known**		
Row	Lot size interval	n	Ac	AQL.05	RQL.05	n	k	Cpk units	n	k	Cpk units	At AQL	Maxi-mum	At RQL
1	16-25	100%	N/A	N/A	N/A	N/A	N/A	N/A	6	2.5477	0.85	2.8	4.4	2.8
2	26-50	100%	N/A	N/A	N/A	24	2.5477	0.85	6	2.5477	0.85	2.8	4.4	2.8
3	51-90	100%	N/A	N/A	N/A	24	2.5477	0.85	6	2.5477	0.85	2.8	4.4	2.8
4	91-150	90	0	0.0570%	3.2738%	24	2.5477	0.85	6	2.5477	0.85	2.8	4.4	2.8
5	151-280	130	0	0.0394%	2.2781%	27	2.6782	0.89	6	2.6782	0.89	2.9	4.7	2.9
6	281-500	155	0	0.0331%	1.9142%	29	2.7383	0.91	7	2.7383	0.91	3.2	4.9	3.2
7	501-1,200	170	0	0.0302%	1.7468%	30	2.7695	0.92	7	2.7695	0.92	3.2	5.0	3.2
8	1,201 - 3,200	200	0	0.0256%	1.4867%	32	2.8240	0.94	7	2. 8240	0.94	3.3	5.1	3.3
9	3,201 - 10,000	315	1	0.1129%	1.4971%	62	2.6124	0.87	14	2. 6124	0.87	6.8	11.1	6.8
10	10,001 - 35,000	315	1	0.1129%	1.4971%	62	2.6124	0.87	14	2. 6124	0.87	6.8	11.1	6.8
11	35,001 - 150,000	500	3	0.2737%	1.5434%	115	2.4682	0.82	29	2. 4682	0.82	14.0	22.6	14.0
12	150,001 - 500,000	800	5	0.3271%	1.3096%	179	2.4713	0.82	44	2. 4713	0.82	21.5	35.2	21.5
13	Above 500,000	1,250	10	0.4943%	1.3532%	308	2.3952	0.80	80	2. 3952	0.80	39.0	63.6	39.0

* SD = Standard Deviation. ** ASN = Average Sample Number - for sequential plans, the average n to reach an accept/reject decision..

What table #3b shows:

Section #1 - These plans are from Q3 table A1, which requires 100% inspection for LQ=0.5, lot size (N) up to 280. Ac=0 up to N=10,000.

Section #2 - The fixed-n variables plans for SD Unknown (SD is calculated from the sample) reduce attribute n by a factor of **5-1/2**. By definition, the lot Cpk=k/3

Section #3 - The fixed-n variables plans for SD Known (SD is known from history) reduce attribute n by a factor of **25 times**.

Section #4 - For lots at AQL, the attribute n is reduced by a factor of **50-90 times**.

Table #3c -- Characteristics Classified as Major-B -- Q3 (LQ=3.15%) attribute versus matched variables plans

Purpose of Table #3a,b,c,d: To list the fixed-n variables plans that match the authorized attribute plans of Q3.

	Section #1					Section #2			Section #3			Section #4		
	ANSI/ASQC Standard Q3 LQ=3.15%					**Matched Variables Plan SD Unknown**			**Matched Variables Plan SD Known**			**ASN of Matched Sequential Variables Plan, SD Known**		
Row	Lot size interval	n	Ac	AQL.05	RQL.05	n	k	Cpk units	n	k	Cpk units	At AQL	Maxi-mum	At RQL
1	16-25	100%	N/A	N/A	N/A	17	2.2761	0.76	5	2.2761	0.76	2.3	3.7	2.3
2	26-50	100%	N/A	N/A	N/A	17	2.2761	0.76	5	2.2761	0.76	2.3	3.7	2.3
3	51-90	44	0	0.1165%	6.5819%	17	2.2761	0.76	5	2.2761	0.76	2.3	3.7	2.3
4	91-150	55	0	0.0932%	5.3011%	19	2.3637	0.79	5	2.3637	0.79	2.4	3.9	2.4
5	151-280	65	0	0.0789%	4.5042%	20	2.4275	0.81	6	2.4275	0.81	2.7	4.0	2.7
6	281-500	80	0	0.0641%	3.6754%	22	2.5048	0.83	6	2.5048	0.83	2.8	4.2	2.8
7	501-1,200	125	1	0.2850%	3.7387%	41	2.2732	0.76	12	2.2732	0.76	5.7	9.0	5.7
8	1,201 - 3,200	125	1	0.2850%	3.7387%	41	2.2732	0.76	12	2.2732	0.76	5.7	9.0	5.7
9	3,201 - 10,000	200	3	0.6859%	3.8310%	73	2.1176	0.71	23	2.1176	0.71	11.1	18.0	11.1
10	10,001 - 35,000	315	5	0.8326%	3.3083%	113	2.1158	0.71	35	2.1158	0.71	17.1	27.9	17.1
11	35,001 - 150,000	500	10	1.2385%	3.3688%	193	2.0371	0.68	63	2.0371	0.68	30.8	50.1	30.8
12	150,001 - 500,000	800	18	1.5607%	3.3183%	320	1.9951	0.67	107	1.9951	0.67	52.3	85.5	52.3
13	Above 500,000	800	18	1.5607%	3.3183%	320	1.9951	0.67	107	1.9951	0.67	52.3	85.5	52.3

* SD = Standard Deviation. ** ASN = Average Sample Number **-** for sequential plans, the average n to reach an accept/reject decision..

What table #3c shows:

Section #1 **-** These plans are from Q3 table A1, which requires 100% inspection for LQ=0.5, lot size (N) up to 280. Ac=0 up to N=10,000.

Section #2 **-** The fixed-n variables plans for SD Unknown (SD is calculated from the sample) reduce attribute n by a factor of **5-1/2**. By definition, the lot Cpk=k/3

Section #3 **-** The fixed-n variables plans for SD Known (SD is known from history) reduce attribute n by a factor of **25 times**.

Section #4 **-** For lots at AQL, the attribute n is reduced by a factor of **50-90 times**.

Table #3d -- Characteristics Classified as Minor -- Q3 (LQ=8.0%) attribute versus matched variables plans
Purpose of Table #3a,b,c,d: To list the fixed-n variables plans that match the authorized attribute plans of Q3.

	Section #1					Section #2			Section #3			Section #4		
	ANSI/ASQC Standard Q3 LQ=8.0%					**Matched Variables Plan SD Unknown**			**Matched Variables Plan SD Known**			**ASN of Matched Sequential Variables Plan, SD Known**		
Row	Lot size interval	n	Ac	AQL.05	RQL.05	n	k	Cpk units	n	k	Cpk units	At AQL	Maxi-mum	At RQL
1	16-25	17	0	0.3013%	16.1566%	10	1.8672	0.62	4	1.8672	0.62	1.8	2.8	1.8
2	26-50	22	0	0.2329%	12.7305%	12	1.9845	0.66	4	1.9845	0.66	1.9	3.0	1.9
3	51-90	24	0	0.2135%	11.7346%	12	2.0229	0.67	4	2.0229	0.67	1.9	3.1	1.9
4	91-150	26	0	0.1971%	10.8830%	13	2.0578	0.69	4	2.0578	0.69	2.0	3.2	2.0
5	151-280	28	0	0.1830%	10.1466%	13	2.0896	0.70	5	2.0896	0.70	2.2	3.3	2.2
6	281-500	32	0	0.1602%	8.9368%	14	2.1460	0.72	6	2.1460	0.72	2.2	3.4	2.2
7	501-1,200	50	1	0.7153%	9.1398%	25	2.8908	0.63	9	2.8908	0.63	4.3	6.9	4.3
8	1,201 - 3,200	80	3	1.7255%	9.4075%	42	1.7151	0.57	17	1.7151	0.57	8.3	13.6	8.3
9	3,201 - 10,000	125	5	2.1106%	8.2260%	65	1.7107	0.57	27	1.7107	0.57	13.1	21.1	13.1
10	10,001 - 35,000	200	10	3.1146%	8.3335%	109	1.6236	0.54	47	1.6236	0.54	23.0	37.4	23.0
11	35,001 - 150,000	315	18	3.9854%	8.3562%	176	1.5669	0.52	79	1.5669	0.52	38.6	63.0	38.6
12	150,001 - 500,000	315	18	3.9854%	8.3562%	176	1.5669	0.52	79	1.5669	0.52	38.6	63.0	38.6
13	Above 500,000	315	18	3.9854%	8.3562%	176	1.5669	0.52	79	1.5669	0.52	38.6	63.0	38.6

* SD = Standard Deviation. ** ASN = Average Sample Number - for sequential plans, the average n to reach an accept/reject decision..

What table #3d shows:

Section #1 - These plans are from Q3 table A1, which requires 100% inspection for LQ=0.5, lot size (N) up to 280. Ac=0 up to N=10,000.

Section #2 - The fixed-n variables plans for SD Unknown (SD is calculated from the sample) reduce attribute n by a factor of **5-1/2**. By definition, the lot Cpk=k/3

Section #3 - The fixed-n variables plans for SD Known (SD is known from history) reduce attribute n by a factor of **25 times**.

Section #4 - For lots at AQL, the attribute n is reduced by a factor of **50-90 times**.

Table #4a -- Attribute and Variables Sequential Plan Parameters for Characteristics Classified as Critical -- Q3 (LQ=0.5%)

Purpose of Table #4a,b,c,d: To list the sequential parameters HA, HR, and G for designing sequential plans that match Q3.

	Section #1					Section #2				Section #3			
	ANSI/ASQC Standard Q3 for LQ=0.5%					Attribute Sequential *				Variables Sequential, SD=Known			
Row	**Lot size interval**	**n**	**Ac**	**AQL.05**	**RQL.05**	**HA**	**HR**	**G**	**Trunk n*1.5**	**HA**	**HR**	**G**	**Trunk n*1.5**
1	16-25	100%	N/A	N/A	N/A	*	*	*	*	2.3358	-2.3358	2.9332	11
2	26-50	100%	N/A	N/A	N/A	*	*	*	*	2.3358	-2.3358	2.9332	11
3	51-90	100%	N/A	N/A	N/A	*	*	*	*	2.3358	-2.3358	2.9332	11
4	91-150	100%	N/A	N/A	N/A	*	*	*	*	2.3358	-2.3358	2.9332	11
5	151-280	100%	N/A	N/A	N/A	*	*	*	*	2.3358	-2.3358	2.9332	11
6	281-500	280	0	0.0183%	1.0642%	*	*	*	*	2.3358	-2.3358	2.9332	11
7	501-1,200	380	0	0.0135%	0.7853%	*	*	*	*	2.4000	-2.4000	3.0291	12
8	1,201 - 3,200	430	0	0.0117%	0.6801%	*	*	*	*	2.4303	-2.4303	3.0734	12
9	3,201 - 10,000	450	0	0.0114%	0.6635%	*	*	*	*	2.4347	-2.4347	3.0811	12
10	10,001 - 35,000	500	0	0.0103%	0.5974%	*	*	*	*	2.4581	-2.4581	3.1126	12
11	35,001 - 150,000	800	1	0.0444%	0.5916%	-1.1346	1.1346	0.002115	1200	3.5600	-3.5600	2.9204	26
12	150,001 - 500,000	800	1	0.0444%	0.5916%	-1.1346	1.1346	0.002115	1200	3.5600	-3.5600	2.9204	26
13	Above 500,000	1250	3	0.1094%	0.6191%	-1.6937	1.6937	0.002943	1875	5.2356	-5.2356	2.7822	53

* **Note**: A limitation of sequential attribute sampling plans is that they cannot reduce the sample size of C=0 plans.

What Table #4a shows:

For characteristics classified as critical, table #4a shows the sequential parameters that you need to construct a sequential decision rule to match the Q3 plan. The method for constructing these sequential attribute and sequential variables plans is described on pages 31 and 32.

These sequential parameters are listed in the table. They are described more completely on the pages where they are applied.

G = slope of a decision line.
HA = y-axis intercept of the acceptance line.
HR = y-axis intercept of the rejection line.
Trunk n*1.5 = the sample size at which you truncate the sequential plan.

Table #4b -- Attribute and Variables Sequential Plan Parameters for Characteristics Classified as Major-A -- Q3 (LQ=1.25%)

Purpose of Table #4a,b,c,d: To list the sequential parameters HA, HR, and G for designing sequential plans that match Q3.

	Section #1					Section #2				Section #3			
	ANSI/ASQC Standard Q3 for LQ=1.25%					Attribute Sequential				Variables Sequential			
Row	**Lot size interval**	**n**	**Ac**	**AQL.05**	**RQL.05**	**HA**	**HR**	**G**	**Trunk n*1.5**	**HA**	**HR**	**G**	**Trunk n*1.5**
1	16-25	100%	N/A	N/A	N/A	*	*	*	*	2.0860	-2.0860	2.5477	9
2	26-50	100%	N/A	N/A	N/A	*	*	*	*	2.0860	-2.0860	2.5477	9
3	51-90	100%	N/A	N/A	N/A	*	*	*	*	2.0860	-2.0860	2.5477	9
4	91-150	90	0	0.0570%	3.2738%	*	*	*	*	2.0860	-2.0860	2.5477	9
5	151-280	130	0	0.0394%	2.2781%	*	*	*	*	2.1689	-2.1689	2.6782	9
6	281-500	155	0	0.0331%	1.9142%	*	*	*	*	2.2088	-2.2088	2.7383	11
7	501-1,200	170	0	0.0302%	1.7468%	*	*	*	*	2.2294	-2.2294	2.7694	11
8	1,201 - 3,200	200	0	0.0256%	1.4867%	*	*	*	*	2.2636	-2.2636	2.8240	11
9	3,201 - 10,000	315	1	0.1129%	1.4971%	-1.1330	1.0330	0.005370	473	2.3340	-2.3340	2.6124	21
10	10,001 - 35,000	315	1	0.1129%	1.4971%	-1.1330	1.0330	0.005370	473	2.3340	-2.3340	2.6124	21
11	35,001 - 150,000	500	3	0.2737%	1.5434%	-1.6898	1.6898	0.007353	750	4.7571	-4.7571	2.4682	44
12	150,001 - 500,000	800	5	0.3271%	1.3096%	-2.1075	2.1075	0.007090	1,200	5.9370	-5.9370	2.4713	66
13	Above 500,000	1,250	10	0.4943%	1.3532%	-2.8988	2. 8988	0.008535	1,825	7.9752	-7.9752	2.3952	120

* **Note**: A limitation of sequential attribute sampling plans is that they cannot reduce the sample size of C=0 plans.

What Table #4b shows:

For characteristics classified as Major-A, table #4b shows the sequential parameters that you need to construct a sequential decision rule to match the Q3 plan. The method for constructing these sequential attribute and sequential variables plans is described on pages 31 and 32.

These sequential parameters are listed in the table. They are described more completely on the pages where they are applied.

G = slope of a decision line.
HA = y-axis intercept of the acceptance line.
HR = y-axis intercept of the rejection line.
Trunk n*1.5 = the sample size at which you truncate the sequential plan.

Table #4c -- Attribute and Variables Sequential Plan Parameters for Characteristics Classified as Major-B -- Q3 (LQ=3.15%)

Purpose of Table #4a,b,c,d: To list the sequential parameters HA, HR, and G for designing sequential plans that match Q3.

	Section #1					Section #2				Section #3			
	ANSI/ASQC Standard Q3 for LQ=3.15%					Attribute Sequential				Variables Sequential			
Row	**Lot size interval**	**n**	**Ac**	**AQL.05**	**RQL.05**	**HA**	**HR**	**G**	**Trunk n*1.5**	**HA**	**HR**	**G**	**Trunk n*1.5**
1	16-25	100%	N/A	N/A	N/A	*	*	*	*	1.9158	-1.9158	2.2761	8
2	26-50	100%	N/A	N/A	N/A	*	*	*	*	1.9158	-1.9158	2.2761	8
3	51-90	44	0	0.1165%	6.5819%	*	*	*	*	1.9158	-1.9158	2.2761	8
4	91-150	55	0	0.0932%	5.3011%	*	*	*	*	1.9698	-1.9698	2.3637	8
5	151-280	65	0	0.0789%	4.5042%	*	*	*	*	2.0099	-2.0099	2.4274	9
6	281-500	80	0	0.0641%	3.6754%	*	*	*	*	2.0586	-2.0586	2.5048	9
7	501-1,200	125	1	0.2850%	3.7387%	-1.1285	1.1285	0.01351	188	2.9962	-2.9962	2.2732	18
8	1,201 - 3,200	125	1	0.2850%	3.7387%	-1.1285	1.1285	0.01351	188	2.9962	-2.9962	2.2732	18
9	3,201 - 10,000	200	3	0.6859%	3.8310%	-1.6803	1.6803	0.01836	300	4.2432	-4.2432	2.1176	35
10	10,001 - 35,000	315	5	0.8326%	3.3083%	-2.0958	2.0958	0.01800	473	5.2862	-5.2862	2.1158	53
11	35,001 - 150,000	500	10	1.2385%	3.3688%	-2.8797	2.8797	0.02137	750	7.0812	-7.0812	2.0371	95
12	150,001 - 500,000	800	18	1.5607%	3.3183%	-3.8124	3.8124	0.02333	1,200	9.2479	-9.2479	1.9951	161
13	Above 500,000	800	18	1.5607%	3.3183%	-3.8124	3.8124	0.02333	1,200	9.2479	-9.2479	1.9951	161

* **Note**: A limitation of sequential attribute sampling plans is that they cannot reduce the sample size of C=0 plans.

What Table #4c shows:

For characteristics classified as Major-B, table #4c shows the sequential parameters that you need to construct a sequential decision rule to match the Q3 plan. The method for constructing these sequential attribute and sequential variables plans is described on pages 31 and 32..

These sequential parameters are listed in the table. They are described more completely on the pages where they are applied.

G = slope of a decision line.
HA = y-axis intercept of the acceptance line.
HR = y-axis intercept of the rejection line.
Trunk n*1.5 = the sample size at which you truncate the sequential plan.

Table #4d -- Attribute and Variables Sequential Plan Parameters for Characteristics Classified as Minor -- Q3 (LQ=8.0%)

Purpose of Table #4a,b,c,d: To list the sequential parameters HA, HR, and G for designing sequential plans that match Q3.

	Section #1					Section #2				Section #3			
	ANSI/ASQC Standard Q3 for LQ=8.0%					Attribute Sequential				Variables Sequential			
Row	**Lot size interval**	**n**	**Ac**	**AQL.05**	**RQL.05**	**HA**	**HR**	**G**	**Trunk n*1.5**	**HA**	**HR**	**G**	**Trunk n*1.5**
1	16-25	17	0	0.3013%	16.1566%	*	*	*	*	1.6746	-1.6746	1.8672	6
2	26-50	22	0	0.2329%	12.7305%	*	*	*	*	1.7417	-1.7417	1.9845	6
3	51-90	24	0	0.2135%	11.7346%	*	*	*	*	1.7640	-1.7640	2.0229	6
4	91-150	26	0	0.1971%	10.8830%	*	*	*	*	1.7845	-1.7845	2.0578	6
5	151-280	28	0	0.1830%	10.1466%	*	*	*	*	1.8033	-1.8033	2.0896	8
6	281-500	32	0	0.1602%	8.9368%	*	*	*	*	1.8371	-1.8371	2.1460	9
7	501-1,200	50	1	0.7153%	9.1398%	-1.1169	1.1169	0.03363	75	2.6353	-2.6353	1.8908	14
8	1,201 - 3,200	80	3	1.7255%	9.4075%	-1.6566	1.6566	0.04579	120	3.6898	-3.6898	1.7151	26
9	3,201 - 10,000	125	5	2.1106%	8.2260%	-2.0665	2.0665	0.04528	188	4.5907	-4.5907	1.7107	41
10	10,001 - 35,000	200	10	3.1146%	8.3335%	-2.8324	2.8324	0.05326	300	6.1186	-6.1186	1.6236	71
11	35,001 - 150,000	315	18	3.9854%	8.3562%	-3.7416	3.7416	0.05920	458	3.4939	-3.4939	2.7688	119
12	150,001 - 500,000	315	18	3.9854%	8.3562%	-3.7416	3.7416	0.05920	458	3.4939	-3.4939	2.7688	119
13	Above 500,000	315	18	3.9854%	8.3562%	-3.7416	3.7416	0.05920	458	3.4939	-3.4939	2.7688	119

* **Note**: A limitation of sequential attribute sampling plans is that they cannot reduce the sample size of C=0 plans.

What Table #4d shows:

For characteristics classified as Minor, table #4d shows the sequential parameters that you need to construct a sequential decision rule to match the Q3 plan. The method for constructing these sequential attribute and sequential variables plans is described on pages 31 and 32.

These sequential parameters are listed in the table. They are described more completely on the pages where they are applied.

G = slope of a decision line.
HA = y-axis intercept of the acceptance line.
HR = y-axis intercept of the rejection line.
Trunk n*1.5 = the sample size at which you truncate the sequential plan.

Calculations for Sequential Sampling Plans

Variables (Technical) -- Calculation of Ac and Re for variables sequential plans:

The plans in Table 1 of ISO8423 do not line up perfectly with the AQL.05 and RQL.05 of the ANSI/ASQC Q3 plans. Therefore here is a method of calculating this decision rule that uses the equations on page 62 of ISO 8423 to calculate the Ac and Re columns of the diagram.

The acceptance and rejection lines are parallel when Ac and Re are in units of cumulative measurements. The equations used here are for Ac and Re in units of the sample average. The equations of the two lines are:

Ac = SD*(G*n – HA)/n
Re = SD*(G*n + HR)/n

Where: Ac and Re are the acceptance and rejection numbers on the y-axis. Units = sample average.
SD is the known historical within-lot standard deviation.
(SD=1 here to make Ac and Re in units of k.)
G is the slope of the lines when the y-axis is the sum of the measurements - same for both lines.
n is the cumulative sample size - on the x-axis.
HA is the acceptance line's y-axis intercept - when the y-axis is the sum of the measurements.
HR is the rejection line's y-axis intercept - when the y-axis is the sum of the measurements.

Variables Example: A sequential plan to match n=200, Ac=0:
See Table #4b for these values.
G = 2.8240
HR = 2.2636
HA = 2.2636

The acceptance line:
Ac = 1*(2.8240*n – 2.2636)/n

Calculate a point on the acceptance line: (see the completed calculation, page 8)
Example for n=1, Ac = (2.8240*1 - 2.2636)/1 = 0.56
Example for n=2, Ac = (2.8240*2 - 2.2636)/2 = 1.69
Example for n=3, Ac = (2.8240*3 - 2.2636)/3 = 2.07

The rejection line:
Re = 1*(2.8240*n + 2.2636)/n

Calculate a point on the rejection line
Example for n=1, Re = (2.8240*1 + 2.2636)/1 = 5.09
Example for n=2, Re = (2.8240*2 + 2.2636)/2 = 3.96
Example for n=3, Re = (2.8240*3 + 2.2636)/3 = 3.58

These Ac and Re numbers appear with the complete variables sequential decision rule, page 9.

Attribute (Technical) -- Calculation of Ac and Re for attribute sequential plans

The plans in Table 1A of ISO8422 do not line up perfectly with the AQL.05 and RQL.05 of the ANSI/ASQC Q3 plans. Therefore here is a method of calculating this decision rule that uses the equations on page 13 of ISO 8422 to calculate the Ac and Re column.

The acceptance line and rejection line in the diagram look like staircases, but they are actually straight lines in which the y-axis numbers have been rounded to whole numbers of defectives. Before rounding, the acceptance and rejection lines are parallel. The equations of the two lines are:

Ac=G*n – HA
Re=G*n + HR

Where: Ac and Re are the acceptance and rejection numbers on the y-axis. Units = cumulative sample defectives.
G is the slope - same for both lines.
n is the cumulative sample size on the x-axis.
HA is the acceptance line's y-axis intercept.
HR is the rejection line's y-axis intercept.

Attribute Example: A sequential plan to match n=1250, Ac=10
See Table #4b for these values:
G = 0.008535
HR = 2.8988
HA = –2.8988

The acceptance line:

Ac=0.008535*n – 2.8988

Calculate a point on the acceptance line: (see the completed calculation, page 10)
Example for n=340, Ac = 0.008535*340 – 2.8988 = 0.0031 = 0 = Ac (rounded down)
Example for n=457, Ac = 0.008535*457 – 2.8988 = 1.0016 = 1 = Ac (rounded down)
Example for n=574, Ac = 0.008535*574 – 2.8988 = 2.0002 = 2 = Ac (rounded down)

The rejection line:

Re=0.008535*n + 2.8988

Calculate a point on the rejection line: (see the completed calculation, page 10)

Example for n=3, Re=0.008535*3 + 2.8988 = 2.9244 = 3 =Re (rounded up)
Example for n=12, Re=0.008535*12 + 2.8988 = 3.0012 =4 = Re (rounded up)
Example for n=130, Re=0.008535*130 + 2.8988 = 4.0084 =5 = Re (rounded up)

These Ac and Re numbers appear with the complete attribute sequential decision rule, page 12.