Joining Resource Sheds

We desire to (eventually) extend a lake's resource shed back into a contributing watershed, requiring the joining of resource sheds, estimated with different techniques, through a point (the mouth of a watershed). See Figure 1 where the superscripts denote Lake (L) and Watershed (W). The joint resource sheds for material departing in time interval i and arriving at the location of interest (in the lake) in time interval j, $V_{i,j}^{C}$, are

$$\begin{aligned} V_{i,j}^{C} &= V_{i,j}^{L}, & i > k_j \\ &= V_{i,j}^{L} \cup \bigcup_{m=i,k_j} V_{i,m}^{W} & i \le k_j \end{aligned} \tag{1}$$

where k_j is the last time interval for material leaving the watershed arriving at the location of interest in the lake in time interval j. Since $V_{i,k_j}^{W} = \varnothing \ \ \forall \ \ i > k_j$, we can write (1) simply as

$$V_{i,j}^{C} = V_{i,j}^{L} \cup \bigcup_{m=i,k_j} V_{i,m}^{W} \tag{2}$$

Summing (2),

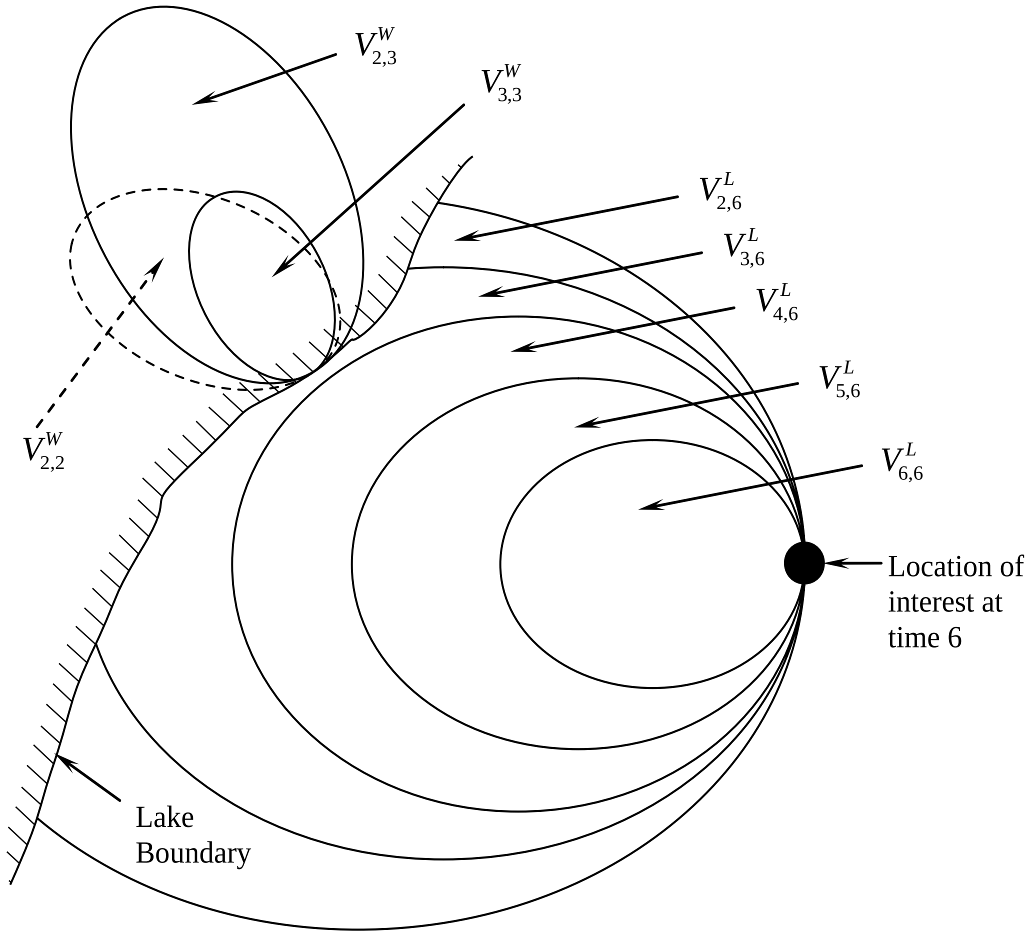

Figure 1. Example Lake Resource Shed Just Touching Watershed Mouth 3δ Ago.

$$\begin{aligned}\bigcup_{n=i,j} V_{n,j}^{C} &= \bigcup_{n=i,j} V_{n,j}^{L} \cup \bigcup_{n=i,k_j} \bigcup_{m=n,k_j} V_{n,m}^{W} \\ &= \bigcup_{n=i,j} V_{n,j}^{L} \cup \bigcup_{m=i,k_j} \bigcup_{n=i,m} V_{n,m}^{W}\end{aligned} \tag{3}$$

Then by **Error! Reference source not found.**, we can construct the resource shed with material departing during time intervals $i, ..., j$ and arriving during time interval j, $S_{i,j}^{C}$ (noting that $S_{i,k_j}^{W} = \varnothing \ \forall \ i > k_j$)

$$S_{i,j}^{C} = S_{i,j}^{L} \cup \bigcup_{m=i,k_j} S_{i,m}^{W} \tag{4}$$

Likewise, summing (4)

$$\bigcup_{n=i,j} S_{i,n}^{C} = \bigcup_{n=i,j} S_{i,n}^{L} \cup \bigcup_{n=i,j} \bigcup_{m=i,k_n} S_{i,m}^{W} \tag{5}$$

or

$$T_{i,j}^{C} = T_{i,j}^{L} \cup \bigcup_{n=i,j} \bigcup_{m=i,k_n} S_{i,m}^{W} \tag{6}$$

The joint resource shed distributions for material departing in time interval i and arriving in time interval j are

$$\begin{aligned} \overline{x}_{i,j,c}^{C} &= \overline{x}_{i,j,c}^{L}, & c = 1, ..., v_{i,j}^{C}/a, \quad i > k_j \\ &= \overline{x}_{i,j,c}^{L} + \sum_{m=i,k_j} \alpha_{m,j} \overline{x}_{i,m,c}^{W} & c = 1, ..., v_{i,j}^{C}/a, \quad i \le k_j \end{aligned} \tag{7}$$

where $\alpha_{m,j}$ (calculated with the lake circulation model) is the fraction of material leaving the watershed in time interval m that arrives at the location of interest in the lake in time interval j. (Since $\alpha_{m,j}$ is applied to $\overline{x}_{i,m,c}^{W}$, $c = 1, ..., v_{i,m}^{W}/a$, $i \le k_j$, the assumption is that watershed outflow is fully mixed.) Note that the watershed and lake do not overlap (are non-intersecting); therefore one of the two terms on the right side of (7) will be zero for each $c = 1, ..., v_{i,j}^{C}/a$. Because $\alpha_{m,j} = 0 \;\; \forall \;\; m > k_j$, we write simply

$$\overline{x}_{i,j,c}^{C} = \overline{x}_{i,j,c}^{L} + \sum_{m=i,j} \alpha_{m,j} \overline{x}_{i,m,c}^{W} \quad c = 1, ..., v_{i,j}^{C}/a \tag{8}$$

By summing (8) over consecutive time intervals $i, ..., j$, analogous to (3), one can demonstrate

$$\overline{z}_{i,j,c}^{C} = \overline{z}_{i,j,c}^{L} + \sum_{m=i,j} \alpha_{m,j} \overline{z}_{i,m,c}^{W} \quad c = 1, ..., s_{i,j}^{C}/a \tag{9}$$

Similarly, summing (9), analogous to (5), gives

$$\overline{w}_{i,j,c}^{C} = \overline{w}_{i,j,c}^{L} + \sum_{n=i,j} \sum_{m=i,n} \alpha_{m,n} \overline{z}_{i,m,c}^{W} \quad c = 1, ..., t_{i,j}^{C}/a \tag{10}$$

Finally, looking at fractions of material moved, by
Error! Reference source not found. and (8)

$$\begin{aligned} p_{i,j,c}^{C} = \frac{\overline{x}_{i,j,c}^{C}}{g_j^L} &= \frac{\overline{x}_{i,j,c}^{L}}{g_j^L} + \frac{1}{g_j^L} \sum_{m=i,j} \alpha_{m,j} \overline{x}_{i,m,c}^{W} \\ &= p_{i,j,c}^{L} + \sum_{m=i,j} \frac{\alpha_{m,j}}{g_j^L} g_m^W p_{i,m,c}^{W} \\ &= p_{i,j,c}^{L} + \sum_{m=i,j} \beta_{m,j} p_{i,m,c}^{W}, \qquad c = 1, ..., v_{i,j}^{C}/a \end{aligned} \tag{11}$$

where $\beta_{m,j} = \alpha_{m,j} g_m^W \big/ g_j^L$ which is the fraction of material arriving at the location of interest in the lake in time interval j that came from the watershed in time interval m. Likewise

$$q_{i,j,c}^C = q_{i,j,c}^L + \sum_{m=i,j} \beta_{m,j} q_{i,m,c}^W \qquad c = 1, \ldots, s_{i,j}^C / a \tag{12}$$

$$u_{i,j,c}^C = u_{i,j,c}^L + \sum_{n=i,j} \gamma_{i,j,n} \sum_{m=i,n} \beta_{m,n}\, q_{i,m,c}^W \qquad c = 1, \ldots, t_{i,j}^C / a \tag{13}$$

where $\gamma_{i,j,n} = g_n^L \Big/ \sum_{k=i,j} g_k^L$ which is the fraction of all material arriving at the location of interest in the lake during time intervals $i, \ldots, j$ that arrived in time interval n $\left(i \le n \le j\right)$.

Procedure: in actual practice, how we go about coordinating with lake researchers and a note on the assumptions involved in simply assigning a fraction (α_m) to couple the resource shed distributions between the watershed and the lake.

Example Applications

Note that the classical "travel-time" isochronal map for a watershed (Linsley et al. 1982 p280) is an example of our definitions. It is built, by assuming steady uniform rainfall (so that $Y(\tau, t) = Y(\tau + \Delta, t + \Delta)$ for all Δ), to estimate a watershed's time-area histogram, which is then used further to estimate the unit hydrograph for a watershed. For example, consider the mouth of a watershed at time 0 with constant outflow resulting from sustained spatially and temporally uniform precipitation over the entire watershed. If we determine the travel times from all locations in the watershed to the mouth and plot them, we have the classical hydrological travel time isochronal map shown on the left side of Figure 6 for an arbitrary watershed. Each isochrone represents $Y(\tau, 0)$; for example, the 3δ isochrone represents $Y(-3, 0)$. Then, resource sheds $\left(V_{i,0}\right)$ for water departing during the i th time interval $i = 0, -1, -2, \ldots$, and arriving at the watershed mouth during the 0th time interval are those areas with travel times within $\left[(-i-1)\delta, -i\delta\right)$; the shaded area in Figure 6 shows $V_{-3,0}$. By identifying similar resource sheds for other time intervals from the isochronal map, we can build the "time-area" histogram of classical hydrology, shown on the right side of Figure 2, which can be transformed into an estimate of a unit hydrograph. The shaded bar in the time-area histogram corresponds to the shaded resource shed in the isochronal map.

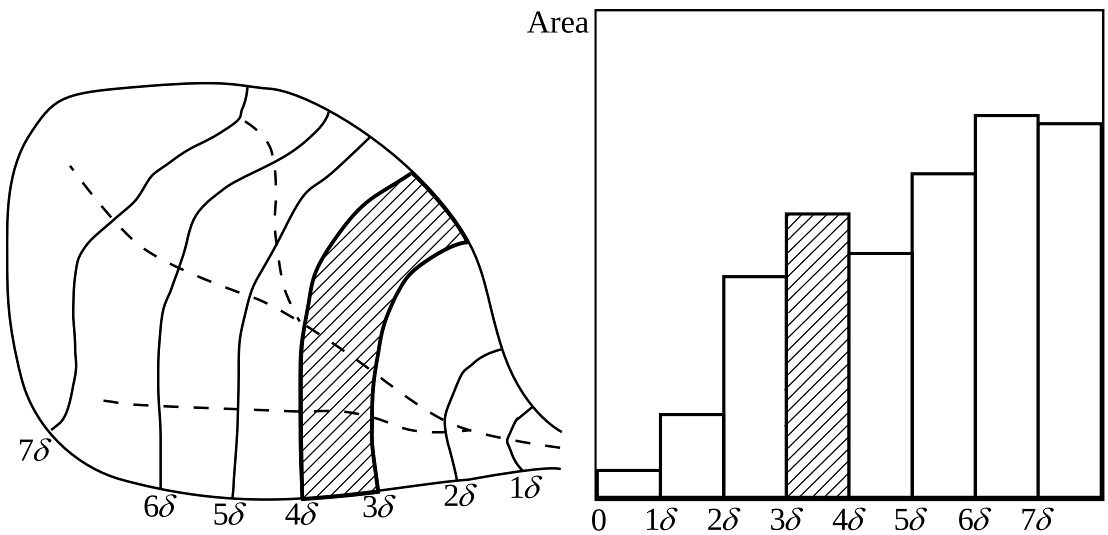

Figure 2. Isochronal Travel Time Map and Resultant Time-Area Histogram.