On the Retrieval of Effective Radius with Cloud Radars

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Abstract.

In-situ sampling of cloud droplets by aircraft in Oklahoma in 1997 , SHEBA – [1]FIREACE in 1998 and from a collection of droplet spectra measured from various locations around the world are used to evaluate the potential for a ground-based remote-sensing technique for retrieving profiles of cloud droplet effective radius. The technique uses vertically-pointing measurements from a high-sensitivity millimeter-wavelength radar to obtain height-resolved estimates of the effective radius in clouds.

Although most meteorological radars lack the sensitivity to detect small cloud droplets, millimeter-wavelength Acloud@ radars provide opportunities for remotely monitoring the properties of non-precipitating clouds. These high-sensitivity radars can reveal detailed reflectivity and velocity structure of most clouds within several kilometers range. In order to put these reflectivity ad velocity measurements into usable microphysical quantities, relationships between the measured quantities and the desired quantities must be developed. This can be done through theoretical analysis, modeling, or empirical measurements. Then the problem is determining the uncertainty of each procedure in order to know which to use.

A number of procedures have been developed recently to estimate the microphysical features of clouds from millimeter-wave radar observations. In this article we restrict our attention to liquid-water clouds; retrievals for ice clouds are described in other studies (e.g., *Matrosov* [1997]). Approaches for retrieving cloud radar reflectivity and ice and liquid water content were suggested by Liao and Sassen, 1994, which was expanded on and validated by Sassen et al, 1999 . Other retrieval for stratocumulus cloud properties using solar radiation, microwave radiometer and millimeter cloud radar were developed by Mace and Sassen 2000. Retrievals for marine boundary layer

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clouds were done by Dong et al., 1997 and Frisch et al., 1995. *Gossard et al.* [1997] approached the problem by using radar measurements of the full spectrum of measured Doppler vertical velocities with deconvolution adjustments for the effects of atmospheric turbulence. Further work using spectra has been done by Babb et al.,1999. In this study, we use in-situ comparisons with the effective radius retrieval of Frisch et al., 1995 along with the use of radar reflectivity alone for determining the effective radius.

Methods

One method for determining the modal radius from cloud radar and microwave radiometer measurements was developed in Frisch et al, 1995. This method used a log-normal model of the droplet distribution to relate the modal radius to radar reflectivity and liquid water path. They assumed a value for the droplet spread and that the droplet concentration was constant with height. The validity of these assumptions were noted by Davidson 1984 who observed that in stratus clouds the number of droplets per unit volume is almost constant with height. He based this observation from works of Slingo et al.1982a, Slingo et al., 1982b, and Nichols 1987. The log-normal distribution was used to represent the droplet distribution rather than the gamma distribution since it was computationally more convenient and was a good approximation for droplet distributions (Borovikov 1961, Levin 1961, Atlas et al. 1989, White et al. 1991) and has been used to characterize cloud droplets (Davidson et al., 1984). The log-normal cloud droplet distribution is written as

$$n = \frac{N}{\sqrt{2p}\, r\, s_x} \exp\left(\frac{(x - x_0)^2}{2 s_x^2}\right) \ldots.(1)$$

where $x=\ln(r)$ and $x_0=\ln(r_0)$ and σ_x is the logarithmic spread of the distribution.

The moments of the distribution are

$$< r^k >= r_0^k \exp\left(\frac{k^2}{2} s_x^2\right) \ldots(2).$$

The effective radius is defined as the third moment over the second moment, so the effective radius is related to the modal radius by

$$r_e = r_0 \exp - \frac{5}{2} s_x^2 ....(3).$$

The radar reflectivity factor is

$$Z = 2^6 N < r^6 >= 2^6 N r_e^6 \exp 18 s_x^2 .....(4)$$

Solving for r_e in equation (4) gives

$$r_e = \frac{1}{2}\left(\frac{Z}{N}\right)^{1/6} \exp - 0.5 s_x^2 ....(5)$$

Fox and Illingworth (1997) noted a relationship between r_e and the reflectivity factor Z from aircraft measurements of marine stratus. From (5), we can see that if we have and estimate of the droplet concentration and the droplet spread, r_e can be retrieved from Z. If microwave radiometer measurements are available for estimating the integrated liquid water, then constraining N and σ_x to be constant with height, we can use the method of Frisch et al.,1995, and solve for the effective radius.

$$r_e(h) = \frac{Z^{1/6}(h)}{2Q^{1/3}}\left(\frac{p r}{6}\right)^{1/3}\left(\sum_{i=1}^{i=m} Z^{1/2}(h_i)\Delta h\right)^{1/3} \exp(-2 s_x^2)......(6)$$

where h_i is height in the cloud, i=1 and i=m represent the radar range gate at the cloud base and cloud top respectively, Δh is the radar range gate thickness and Q is the microwave radiometer derived integrated liquid water through the depth of the cloud. This additional measurement eliminates the need to know N, however, we still need an estimate of σ_x. In order to evaluate the error introduced by our assumptions and the measurement errors, we can use equations (5) and (6) to determine the errors in both of these retrievals. The relative error in the first retrieval (equation 5) is

$$e = \frac{\Delta r_e}{r_e} = \pm\sqrt{\left(\frac{\Delta N}{6N}\right)^2 + \left(s_x \Delta s_x\right)^2 + \left(\frac{\Delta Z}{6Z}\right)^2} ....(7)$$

Evaluating the error in the second retrieval (equation 6) is a little more complicated. The error can be written as

$$\frac{\Delta r_e}{r_e} = \sqrt{\left(\frac{\Delta Z}{6Z}\right)^2 + \left(\frac{\Delta\left(\int_{h1}^{h2} Z(h)^{1/2}\,dh\right)^{1/3}}{\left(\int_{h1}^{h2} Z(h)^{1/2}\,dh\right)^{1/3}}\right)^2 + \left(4s_x \Delta s_x\right)^2 + \left(\frac{\Delta Q}{3Q}\right)^2} \text{.........(8)}$$

Equation (8) shows that even though there is no longer an error in N to contend with, we have increased the contribution of the droplet spectral spread error by a factor of 4 and have added some error due to the measurement of the integrated liquid water. There is an added complication due to the second term under the radical involving the height integral of the reflectivity factor. In order to access the error, we can approximate equation 8 by assuming the second term is negligible. This is a good approximation if the error in Z is either random, constant or a combination on these and the cloud thickness is several range gates thick. With this term eliminated, equation (8) becomes

$$\frac{\Delta r_e}{r_e} \approx \sqrt{\left(\frac{\Delta Z}{6Z}\right)^2 + \left(4s_x \Delta s_x\right)^2 + \left(\frac{\Delta Q}{3Q}\right)^2} \text{.........(9)}$$

Measurements

In order to determine the range of values in the parameters needed for the two retrievals, we used two sets of measurements, one for the marine strato-cumulus clouds and the one for the continental strato-cumulus clouds. The first set of measurements was taken during an April, 1997 Intensive Observation Period (IOP) at the Atmospheric Radiation Measurement(ARM) Southern Great Plains(SGP) site near Ponca City, Oklahoma. The second set were taken from instrumented aircraft in April-July, 1998 during the FIRE-ACE program in the arctic. The droplet size distributions were measured in both experiments with a Forward Scattering Spectrometer Probe (FSSP).

For the IOP at the SGP observations, a FSSP was installed on the University of North Dakota Citation. For the FIRE-ACE measurements, FSSP measurements were made with the University of Washington (UW) Convair 580 and the National Center for Atmospheric Research (NCAR) C-130. In order to get good statistics, the in- situ data were only considered when the liquid partical concentrations were greater than 10 cm^{-3}. The FSSP droplet spectra were used to calculate the effective radius radar reflectivity, the droplet concentration, and the logarithmic spread of the radii distribution.

The first retrieval (equation 5) is based on the assumption that we know approximately what the droplet concentration is for marine and continental stratus clouds. These measurements gave us the mean and standard deviation of the quantities that were necessary to help evaluate this effective radius retrieval. For example, Frisch et al., 1995 used a value of 0.35 for σ_x . The ARM IOP had a value σ_x of 0.32 ± 0.09 . The droplet concentration varied from a low of 25 to a maximum of about 400 with a few measurements of much higher concentrations.. The average was 212 cm^{-3} with a standard deviation of 107 cm^{-3}. We used about 5000 1- second spectra in these calculations. For the FIRE-ACE data, the average $\sigma_x = 0.34 \pm 0.09$, the range for N was from 10 to 400 cm^{-3} with a mean of 98 ± 74 cm^{-3} . Here we had about 45,000 1-second droplet spectra available. The estimate of the reflectivity error is more complicated. If we assume that at a reflectivity factor of -30 dBZ, and that we can measure dBZ to ±1dBZ, then its contribution to the error in r_e will be about 0.08. From equation 6, using the a value of N=200 for the continental stratus and N=100 for the marine stratus values with the standard deviation of 0.09, the error would be about 15% for this effective radius retrieval. Of course, this assumes that our approximation of a log-normal droplet distribution is a reasonable approximation although we did not attempt to separate out any multi-modal spectra. For the second retrieval, using the same values for Z , σ_x and their standard deviations plus the assumption that we can measure the liquid water to 20% , gives about a 16% error.

Retrieval comparisons with in-situ FSSP measurements

We used the measured droplet size distributions to compute the radar reflectivity and the effective radius size and plotted the effective radius vs the calculated radar reflectivity. These results are shown in figure 1a for the FIRE –ACE arctic measurements, and in figure 1b for the SGP IOP measurements with the curves representing equation 5 for different values of N with a value of σ_x=0.32. In figure 1a, we can see that the measurements from FIRE-ACE show that most of the droplets fall between 10 to 200 cm^{-3} . Figure 1b shows similar results from the SGP site. Here the droplet concentrations are higher with most values between 100 to 400 cm^{-3}.

Comparisons with in situ-measurements

During the April, 1997 IOP at the SGP site, we had an aircraft instrumented with an FSSP and a 2DP. We compared radar and radar-radiometer retrieved effective radius with the in-situ FSSP measurements of effective radius. We also used the 2DP to tell the number of events where we had particles that were large enough to cause large errors in our radar reflectivity measurements. Because of the height error in the aircraft and the sharp vertical gradients in the radar reflectivity measurements of the clouds, we had to adjust the aircraft height explained in Frisch et al, 2000. We set an arbitrary horizontal circle of 1.5 km around the radar for our comparisons. If the aircraft was within this circle, then we would do a comparison between the FSSP and radar-microwave radiometer retrieval. These comparisons were made on April 9,1997 from 15:33 to 17:31 Local time.

Figure 3 shows a time series plot of the aircraft FSSP calculated reflectivity factor along with the radar measured reflectivity factor for measurements within a 1.5 km horizontal distance from the radar. The measurements track very well until about 16.4 hours UT when the radar reflectivity becomes much lower relative to the FSSP reflectivity calculated reflectivity. This is the time when the cloud was rapidly dissipating and probably becoming less horizontally homogenous and not suitable for a comparison. Figure 4 shows the radar-radiometer effective radius retrieval compare with the FSSP for the times before 16.4 UT, and figure 5 a similar comparison for the radar reflectivity effective radius retrieval comparison with the FSSP. In both cases we used σ_x =0.32 and for the second retrieval N=212 cm^{-3}. These correspond to an average of measurements taken over several days during the IOP (Frisch, et al., 1998)

As another check on the method using equation (5), we used data from Pinnick et al., 1985. This is a set of cloud droplet data taken from several sites around the world and contains cloud droplet data from various types of clouds. Figure 5 shows a plot of the reflectivity vs the effective radius calculated from these droplet spectra. Again, we can see that there is a good correspondence in this relationship. We used a droplet concentration of 200 drops cm^{-3} and σ_x of 0.35 in these retrieval calculations.

A potential problem with either of these retrievals is that occasional large droplets occur in the cloud at low reflectivities. We examined 2dp measurements for large particles and found about 20 events when large particles were present. The total aircraft flight time was about 90 minutes, and the sampling rate was 1 sample per second. During this time, there were over 5000 samples, so the 20 or so events appear to be negligible for the continental stratus case.

Discussion and Conclusion

We have shown an analysis of 2 methods for determining the effective radius. The first method uses only the reflectivity factor, the second is based on a method of Frisch et al, 1990 which uses the reflectivity factor and a measurement on the integrated liquid water. In both methods, an estimate of the logarithmic spread of the cloud droplets is required, however, large changes in this spread contribute small changes in the effective radius retrieval using the reflectivity alone. In the technique using the radar alone, an estimate of the droplet concentration is required, although large changes in the concentration give small changes in the effective radius.

An error analysis based on in-situ measurements of both marine and continental stratus clouds show that the effective radius retrieval accuracy should be on the order of 15%, however, from our comparisons, the reflectivity alone technique appears significantly superior. This may be due do larger errors in the microwave radiometer retrieval.

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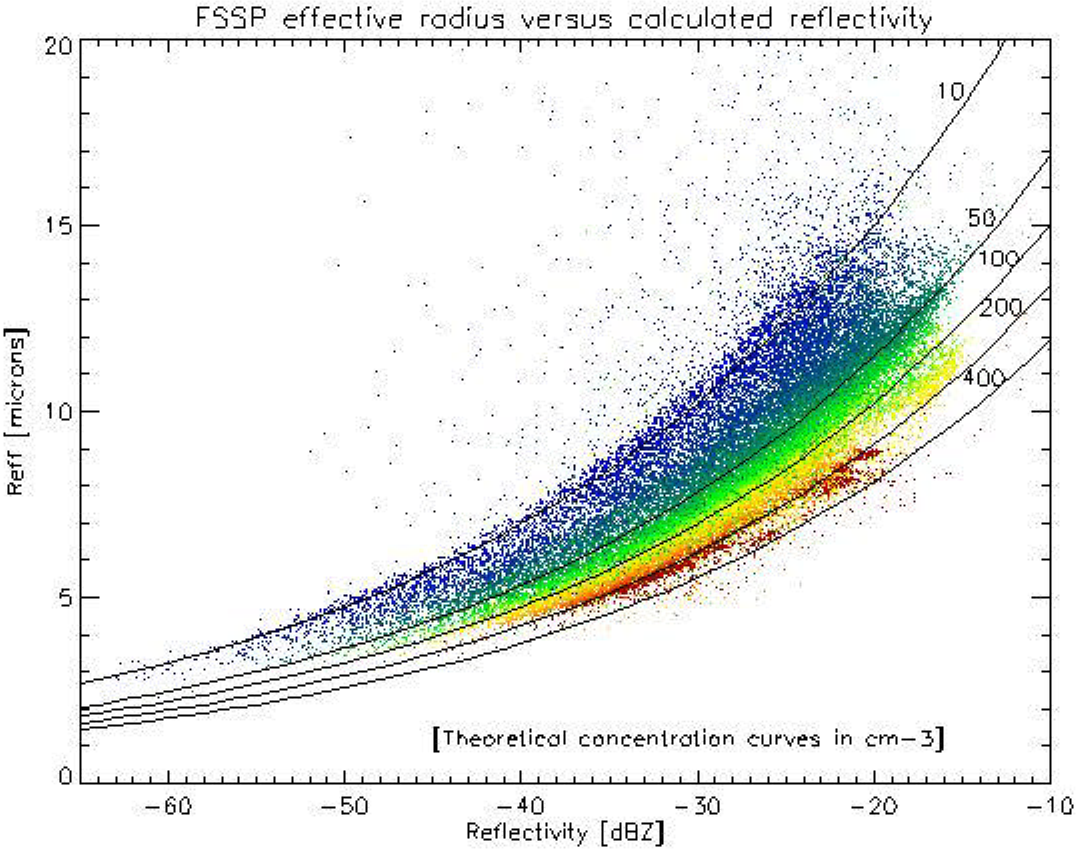

Figure 1a. FSSP derived effective radius vs FSSP derived reflectivity for the FIRE-ACE data. Color scale indicates droplet concentration range from 10 (blue) to about 400 (red).

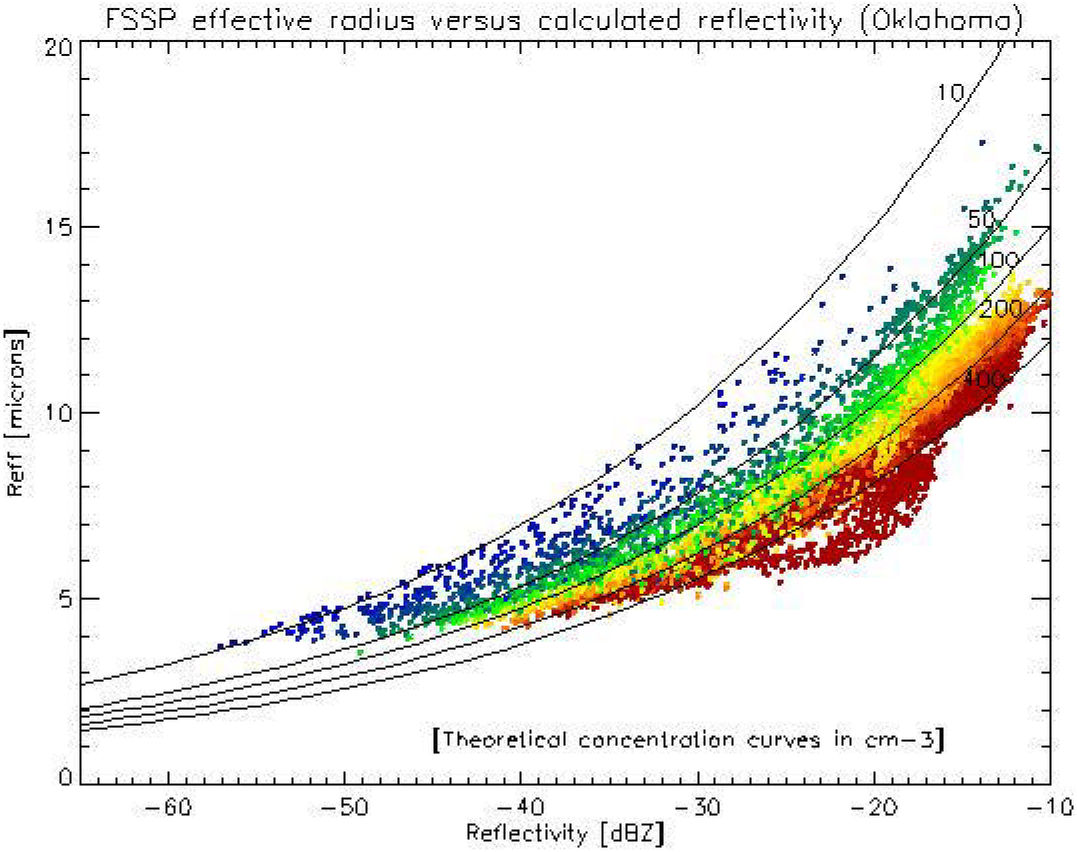

Figure 1b. FSSP derived effective radius vs FSSP derived reflectivity for the SGP-IOP data. Color scale indicates droplet concentration range from 10 (blue) to about 400 (red).

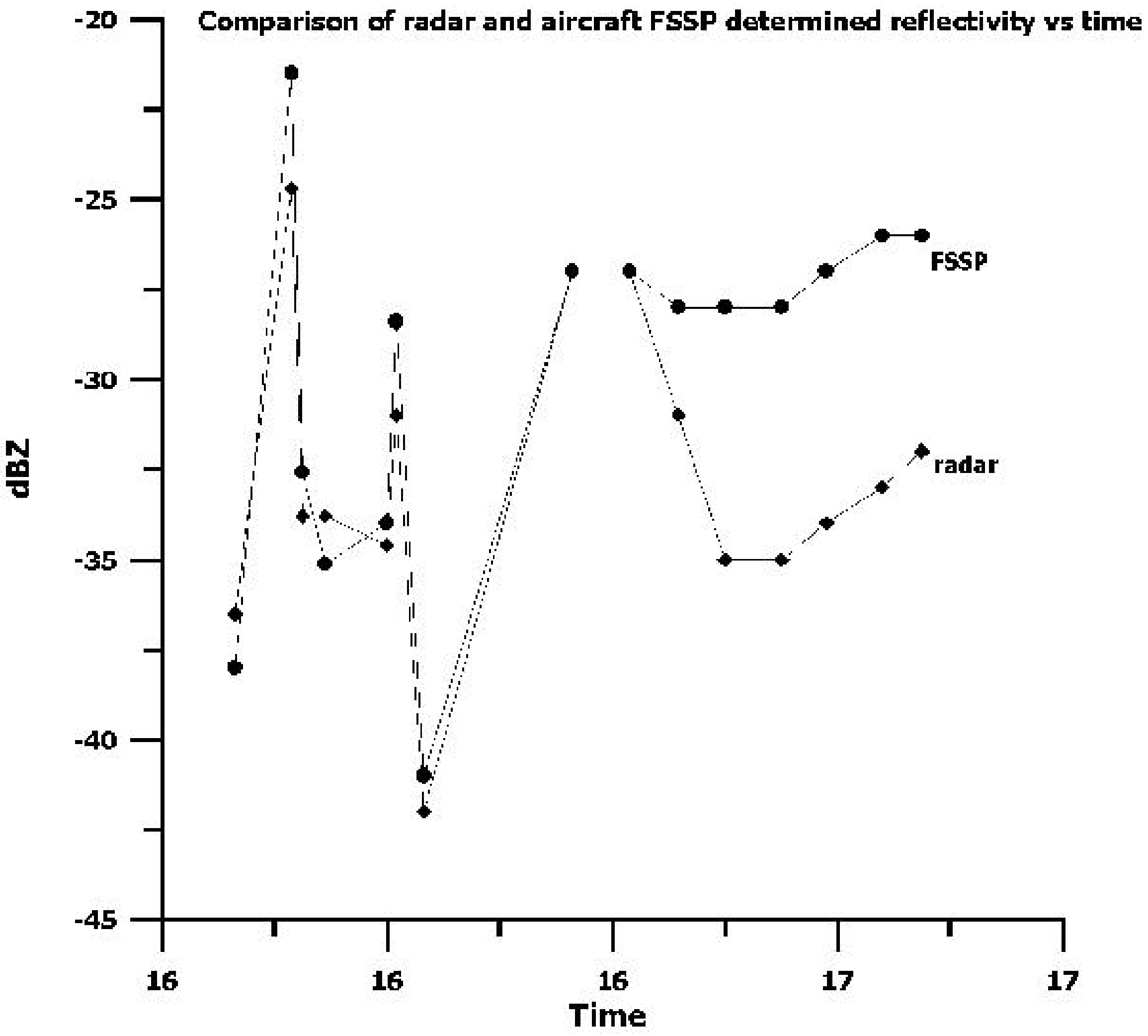

Figure 2. Comparison of radar and aircraft FSSP reflectivities vs time.

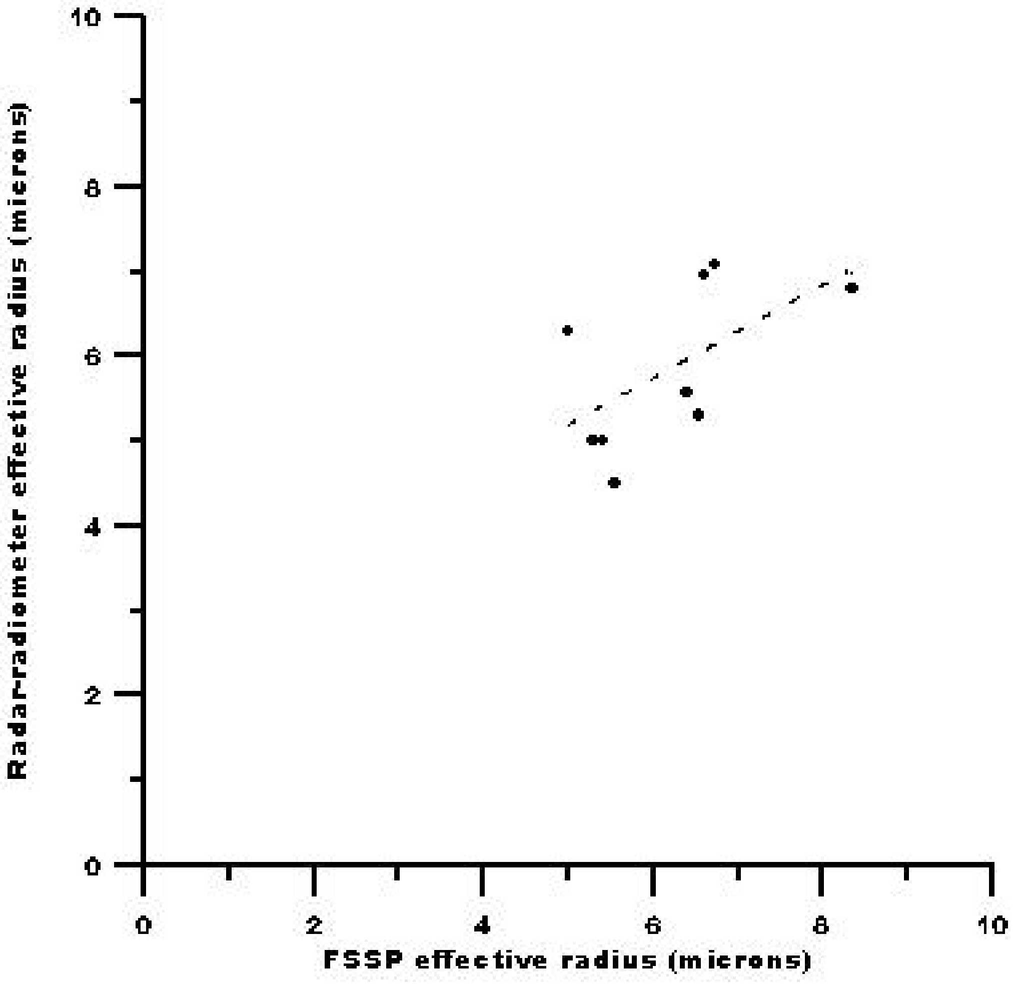

Figure 3. Comparisons of the aircraft FSSP derived effective radius with radar-radiometer technique.

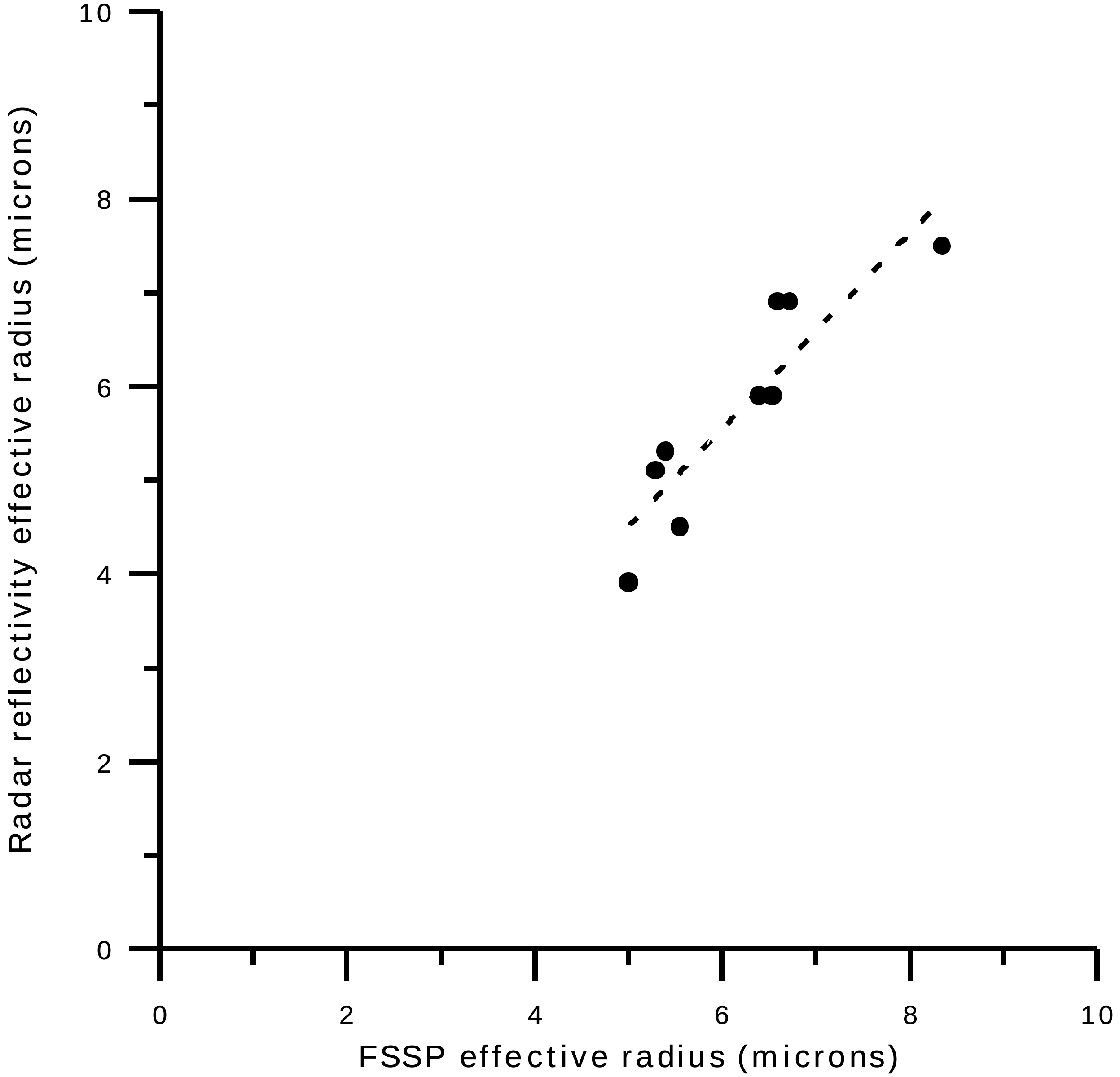

Figure 4. Comparisons of the aircraft FSSP derived effective radius with radar reflectivity technique.

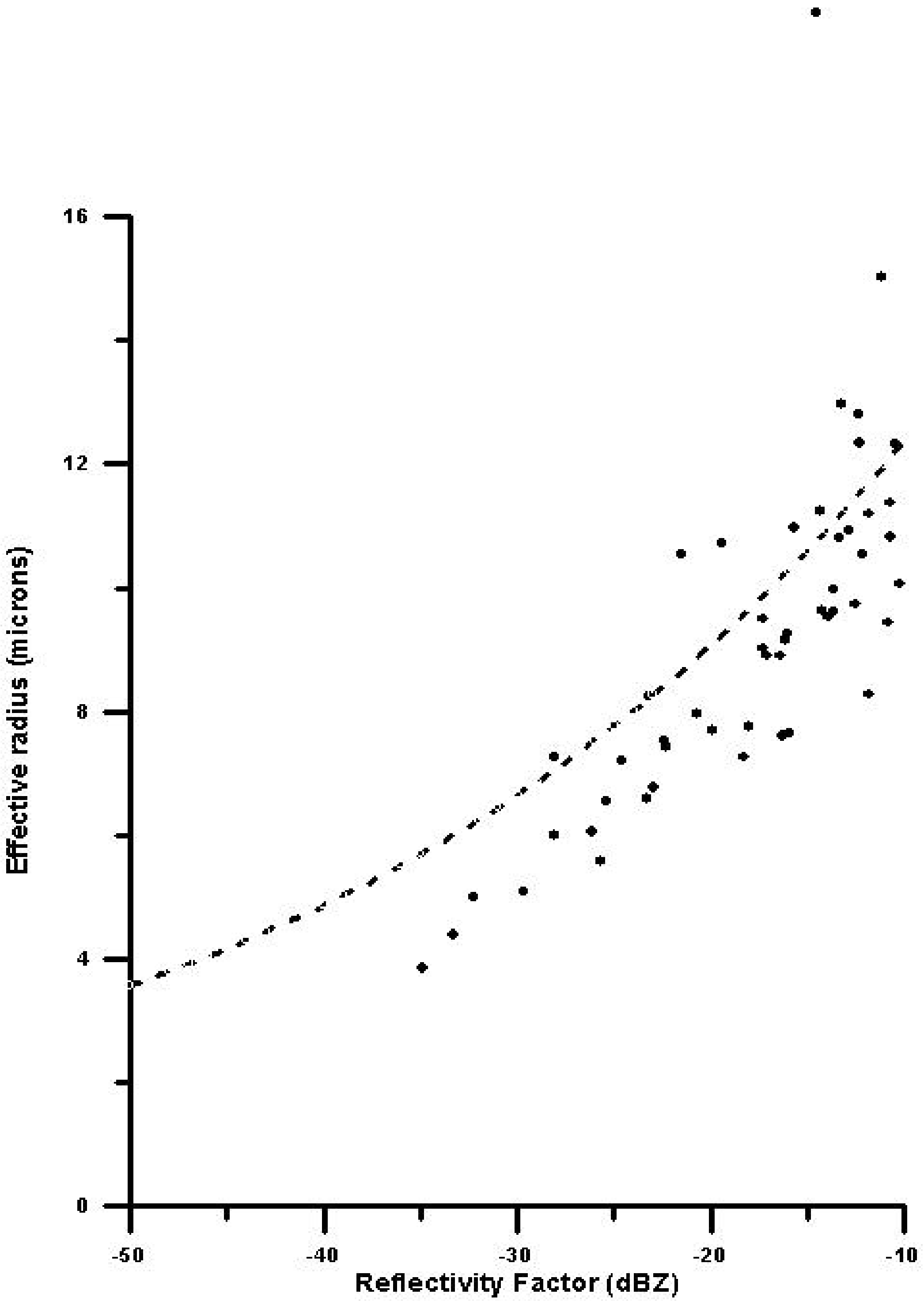

Figure 5. Effective radius vs dBZ from the data of Pinnick et al., 1985. Dashed line is equation 5 with N of 200 and σ_x of 0.35.