CORRELATING OFF-AXIS TENSION TESTS TO SHEAR MODULUS OF WOOD-BASED PANELS

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ABSTRACT: The weakness of existing relationships correlating off-axis modulus of elasticity E_q to shear modulus G_{12} for wood composite panels is demonstrated through presentation of extensive experimental data. A new relationship is proposed that performs better than existing equations found in the literature. This relationship can be manipulated to calculate the shear modulus G_{12} of orthotropic specimens such as wood-based panels solely from off-axis tension (or compression) tests. The proposed relationship reduces to classical theory for the isotropic case. In all cases tested, the newly proposed relationship is a better fit to the data than the traditional orthotropic elasticity equation.

INTRODUCTION

The shear strength and modulus of rigidity of composite wood materials are required for the rigorous design of many lumber-panel structural components such as trusses with gusset plates, box beams, folded plate roofs, roof or floor diaphragms, shear walls, and engineered wood products such as the webs of I-joists. Allowable shear strength values are readily available for the design of solid lumber and other engineered wood products such as glulam; however, the elastic shear modulus, or modulus of rigidity G_{12}, is often experimentally obtained for new and unique panel products. Yet, it is difficult to readily conduct pure shear tests that could directly calculate G_{12}.

Wood composite panel products, similar to those mentioned above, are traditionally modeled as orthotropic solids. In x-y space, the modulus of elasticity (MOE) may vary from its strongest value E_1 (x-axis aligned with the longitudinal grain direction) to E,, typically 90° away from the strong axis. The problems of complicated test setups and specimen size effects (*Annual* 1997) notwithstanding, the experimentally obtained shear modulus G_{12} should relate to the two principal MOE E_1 and E_2, the Poisson ratio v_{12}, and to the MOE at some angle **q** to the x-axis E_q by means of coordinate transformation of orthotropic elasticity relations (Jones 1975)

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \frac{\sin^4\theta}{E_2} - \frac{2v_{12}\cos^2\theta\sin^2\theta}{E_1} + \frac{\cos^2\theta\sin^2\theta}{G_{12}} \quad (1)$$

Such a relationship is more effective than the empirically derived Hankinson formula (Bodig and Jayne 1982)

$$E_\theta = \frac{E_1 E_2}{E_1\sin^2\theta + E_2\cos^2\theta} \quad (2)$$

in capturing the properties of panel products such as plywood, which can have a very small MOE at **q** = 45°, and large, approximately equal MOE at **q** = 0° (E_1) and **q** = 90° (E_2).

The original intention of this study was to investigate the use of (1) as a predictor of the shear modulus solely using uniaxial tests. These uniaxial tests establish E_1, E_2, and E,. Advantages of measurement of shear modulus by tension tests have been investigated by others (Rosen 1972; Ebrahimi and

Sliker 1981). A minor difficulty of using (1) to do this, was the need to experimentally obtain Poisson–ratios. The proposed method has eliminated this need.

The major difficulty in achieving the original objective was the somewhat surprising realization that (1) is ineffective in modeling E_q. This study has analyzed a historical set of data (Norris and McKinnon 1956) as well as newly gathered data on a variety of wood-based panel products (plywood, oriented strand board, particleboard), and has demonstrated that there is a lack of fit between (1) [or its alternate forms via (3) or (5)] and the experimental data. Standard methods were used to determine E (ASTM D 3500) from 0° to 90°, and also to determine G_{12} by means of a plate twisting test (ASTM D 3044).

Several researchers have investigated the off-axis MOE by means of traditional theory of elasticity, but without the empirical approach adopted herein. Bindzi and Samson (1995) recently have shown another format of (1) as follows:

$$E_q = \frac{E_1 E_2}{E_1\sin^2\mathbf{q}(\sin^2\mathbf{q} + a\cos^2\mathbf{q}) + E_2\cos^4\mathbf{q}} \quad (3)$$

where the nondimensional parameter a is given by

$$a = \frac{E_2}{G_{12}} - 2v_{12} \quad (4)$$

To complete the review of the literature, another equivalent formulation of (1) was presented by Liu and Ross (1998)

$$E_q = \frac{E_1 E_2}{E_2\cos^2\mathbf{q}(\cos^2\mathbf{q} + a\sin^2\mathbf{q}) + E_1\sin^4\mathbf{q}} \quad (5)$$

where the nondimensional parameter a is given by

$$a = \frac{E_1}{G_{12}} - 2v_{12} \quad (6)$$

It can be shown, however, that (3) and (5) are completely equivalent to (1) and are, consequently, equally ineffective in modeling composite wood panel MOE variations.

Given the poor performance of (1), the revised objective of this study became the demonstration of an improved equation relating E_1 E_2, G_{12}, and E_q. We have proposed such a relationship for modeling E_q, which reduces to the classical elasticity equation for the isotropic case ($E_1 = E_2$). This equation can be used in two ways. First, it can be used in conjunction with an experimentally determined E_1, E_2, and G_{12} to better predict E_q. Second, it can be rewritten to solve for G_{12} based on E_1, E_2, and E_q, at any angle **q**, each of which could be obtained from tension tests. Using the new relationship to extract G_{12} could be advantageous if the plate twisting test (ASTM D 3044) is not available.

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NEW RELATIONSHIP

The proposed relationship was formulated to appear similar to (1), yet without the Poisson ratio term. The Poisson ratio term was not included in this relationship to reduce the number of input variables. This simplification is reasonable as the Poisson ratio term in (1) can be one order of magnitude smaller than the others, because v_{12} is often approximately 0.1. Furthermore, this simplification eliminates the need to obtain transverse strains.

Then, the equation was modified to enforce agreement to experimentally obtained shear data by means of curve fitting. This was done by means of the calibrating term containing the nondimensional parameter A, where $A = E_2/E_1$. The result is as follows:

$$\frac{1}{E_q} = \frac{\cos^4 q}{E_1} + \frac{\sin^4 q}{E_2} + \frac{\cos^2 q \, \sin^2 q}{(A^{2A})G_{12}} \tag{7}$$

We have calibrated (7) to the ASTM D 3044 plate twisting data because that standard is based on well-accepted theoretical foundations (Hearmon and Adams 1952; Kuipers 1974). This empirical approach ensured that (7) could also be used to extract the shear modulus solely from tension test data. Having such a broad database to work with gave us confidence in our empirical approach. We have demonstrated that (7) performs better than (1) for a wide variety of wood-based panels.

The second application of (7) is to use it to solve for G_{12} based solely on E_q tests. Rewriting (7) gives the following results:

$$G_{12} = \frac{\cos^2 q \sin^2 q}{(A^{2A}) \cdot \left[\frac{1}{E_q} - \left(\frac{\cos^4 q}{E_1} + \frac{\sin^4 q}{E_2} \right) \right]} \tag{8}$$

However, if there is substantial variation in the experimental readings of E_q, then the resulting value calculation of G_{12} may have an unacceptably large variation (Seo et al. 1992). Actually, (8) is unstable at two particular angles, which are a function of the orthotropicity of the material. To minimize the effects of the variation of E_q in a systematic fashion, we have adopted the method proposed by Seo et al. (1992), who used this method to extract the shear modulus of orthotropic sheets of paper. Therefore

$$S = \frac{1}{G_{12}}, \quad (X1)_i = \frac{\cos^2 q_i \sin^2 q_i}{A^{2A}},$$

$$(X2)_i \quad \frac{\cos^4 q_i}{E_1} + \frac{\sin^4 q_i}{E_2} - \frac{1}{(E_q)i} \tag{9}$$

where $i = 1, 2, 3, , \ldots, n$ number of tests; and all other terms as previously defined. Define

$$F(X1_i, X2_i) = \sum_{i=1}^{''} (X1_i + X2_i)^2 \tag{10}$$

Minimizing for n trials

$$\frac{\partial F}{\partial S} = 0 \tag{11}$$

yields

$$2 \sum X1_i (SX1_i + X2_i) = 0 \tag{12}$$

Then

$$G_{xy} = \frac{1}{S} = \frac{-\sum (X1_i)^2}{\sum (X1_i X2_i)} \tag{13}$$

APPLICATION TO EXPERIMENTAL DATA

We conducted tests to determine E_q and G_{12} for plywood, oriented strand board, and particleboard. The value of E_q was found for $q = 0°$, $30°$, $45°$, $60°$, and $90°$ by means of tension tests conforming to ASTM D 3500. Four grades of plywood, three grades of oriented strand board, and four grades of particleboard were tested. Three replicates of these five angles (15 specimens) were tested for each of the 15 different panels for a total of 225 tension tests. Then, 60 plate twisting tests (4 on each panel) were performed on specimens taken from these panels in accordance with ASTM D 3044 to calculate the shear modulus G_{12}. These values will be referred to as G_{123044}. Table 1 summarizes the materials tested. Also examined were historical data (Norris and McKinnon 1956) on specially fabricated plywood panels. This data set contained 12 replicates of E_q tests at angles of $0 = 0°$, $15°$, $30°$, $45°$, $60°$, $75°$, and $90°$, resulting in 84 tension tests for each type of plywood. In all, there were 504 tension tests and 72 shear tests performed on these six types of specially fabricated plywood. Table 2 summarizes the historical data that we analyzed. Table 3 shows an example calculation of how we extracted the shear modulus solely from tension tests. The reason for presenting these calculations is to clarify the steps needed to extract the shear modulus from (8) by means of (9)–(13) and to show that it can easily be performed in a spreadsheet environment. Eqs. (1), (3), (5), and (7) were tested on all of these data in the following manner:

I. First the behavior of all four E, equations [(1), (3), (5), and (7)] was checked by putting E_1 ($E_{0°}$), E_2 ($E_{90°}$), and G_{123044}. Graphical representations of typical experimental data (solid symbols); (1), (3), and (5) (solid lines all coinciding); and (7) (dashed line with crosses) are shown in Figs. 1 and 2. Note that the agreement is the best for the newly proposed equation [(7)]. Also note that (1), (3), and (5) all require a value of Poisson's ratio. The following relationship for Poisson's ratio was used (Norris and McKinnon 1956):

$$v_{12} = \left[\frac{E_1}{E_2} + 1 \right] \frac{v_{TL}}{1.036} \tag{14}$$

TABLE 1. Materials Tested in This Study

| Panel type (1)	Grade (2)	Thickness (mm) (3)	Specimen names (4)	Number of shear specimens (5)	Number of tension specimens (6)
Plywood	N	12.7	N1, N2	8	30
	A-	12.7	A1, A2	8	30
	A-C	19.1	A3, A4	8	30
	C-C	12.7	C1, C2	8	30
Oriented strand board	Mill 1	12.7	O1	4	15
	Mill 2	12.7	O2	4	15
	Mill 3	15.7	O3	4	15
Particleboard	Mill 1	15.7	P1	4	15
	Mill 2	25.4	P2	4	15
	Mill 3	25.4	P3	4	15
	Mill 4	25.4	P4	4	15

TABLE 2. Materials from Norris and McKinnon Report (1956)

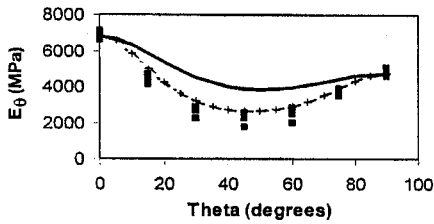| Panel type (1)	Number of piles (2)	Ply thickness (mm) (3)	Specimen names (4)	Number of shear specimens (5)	Number of tension specimens (6)
Plywood	3	1.59	T2	12	84
	5	1.59	T3	12	84
	7	1.59	T4	12	84
	5	0.79	T5	12	84
	7	0.79	T6	12	84
	9	0.79	T7	12	84

TABLE 3. Application of G12 Extraction to Data, Specimen T2

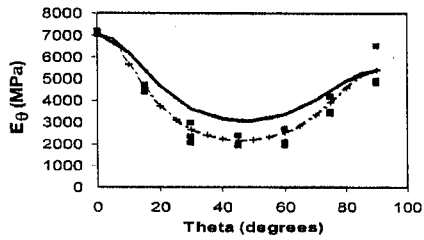| Theta (degrees) (1)	E_1 (MPa) (2)	E_2 (MPa) (3)	E_θ (MPa) (4)	$X1$ (5)	$X2$ (6)	$X1^2$ (7)	$X1X2$ (8)
15	10,090	4,629	4,978	0.12776	− 1.136E-04	0.01632	− 1.452E-05
15	10,090	4,629	5,647	0.12776	− 8.984845	0.01632	− 1.148E-05
15	10,090	4,629	4,923	0.12776	− 1.159E-04	0.01632	− 1.480E-05
30	10,090	4,629	2,489	0.38327	− 3.325E-04	0.14689	− 1.274E-04
30	10,090	4,629	2,606	0.38327	− 3.144E-04	0.14689	− 1.205E-04
30	10,090	4,629	2,668	0.38327	− 3.055E-04	0.14689	− 1.171E-04
45	10,090	4,629	1,889	0.51102	− 4.505E-04	0.26114	− 2.302E-04
45	10,090	4,629	1,800	0.51102	− 4.769E-04	0.26114	− 2.437E-04
45	10,090	4,629	1,765	0.51102	− 4.877E-04	0.26114	− 2.492E-04
60	10,090	4,629	2,186	0.38327	− 3.298E-04	0.14689	− 1.264E-04
60	10,090	4,629	2,200	0.38327	− 3.269E-04	0.14689	− 1.253E-04
60	10,090	4,629	2,213	0.38327	− 3.241E-04	0.14689	− 1.242E-04
75	10,090	4,629	3,620	0.12776	− 8.775E-05	0.01632	− 1.121E-05
75	10,090	4,629	3,303	0.12776	− 1.143E-04	0.01632	− 1.460E-05
75	10,090	4,629	3.420	0.12776	− 1.039E-04	0.01632	− 1.327E-05
Total						1.76272	− 1.06461E-05

Note: G calc = (−sum 1/sum 2) = 1,142 MPa.
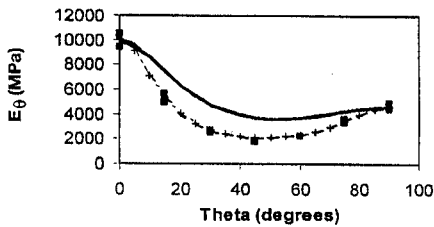
a.) T2 $G_{12\,eqn\,7}=1142$ MPa $G_{12\,3044}=1280$ MPa

b.) T3 $G_{12\,eqn\,7}=1056$ MPa $G_{12\,3044}=1429$ MPa

c.) T4 $G_{12\,eqn\,7}=886$ MPa $G_{12\,3044}=1005$ MPa

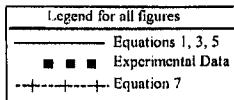Legend for all figures
Equations 1, 3, 5
■ ■ ■ Experimental Data
-+-+-+- Equation 7

FIG. 1. T2, T3, and T4 Specimens
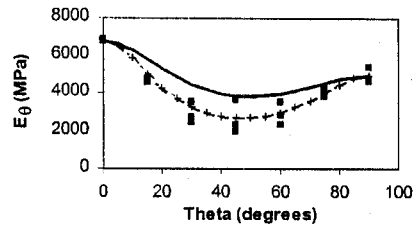
a.) T5 $G_{12\,eqn\,7}=1257$ MPa $G_{12\,3044}=1409$ MPa
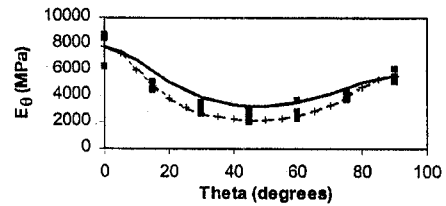
b.) T6 $G_{12\,eqn\,7}=1211$ MPa $G_{12\,3044}=1056$ MPa
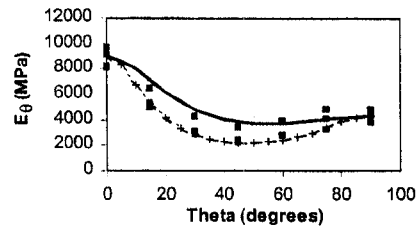
c.) T7 $G_{12\,eqn\,7}=1758$ MPa $G_{12\,3044}=1382$ MPa

Legend for all figures
Equations 1, 3, 5
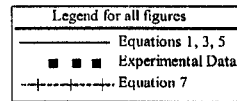■ ■ ■ Experimental Data
-+-+-+- Equation 7

FIG. 2. T5, T6, and T7 Specimens

where v_{TL}, = species-dependent material property. For example, for yellow poplar, v_{TL} is 0.019 ("The elastic" 1946).

2. The shear modulus calculations, based on the newly proposed (7) as well as the traditional orthotropic elasticity equation [(1)] are compared. In both cases, the error minimization scheme of (9)–(13) was applied. The value of shear modulus extracted from (1) will be designated as

$G_{12\,eqn\,1}$ and that from (7) will be designated as $G_{12\,eqn\,7}$ to differentiate them from the plate twisting test results called $G_{12\,3044}$. Fig. 3 summarizes these findings by showing $G_{12\,3044}$ versus the extracted values of $G_{12\,eqn\,1}$ and $G_{12\,eqn\,7}$. If the extracted values of G_{12} fell on a 45° line in this plot, then the extracted values would coincide perfectly with the independently obtained plate twisting values $G_{12\,3044}$. Clearly, only $G_{12\,eqn\,7}$ falls near the 45° line.

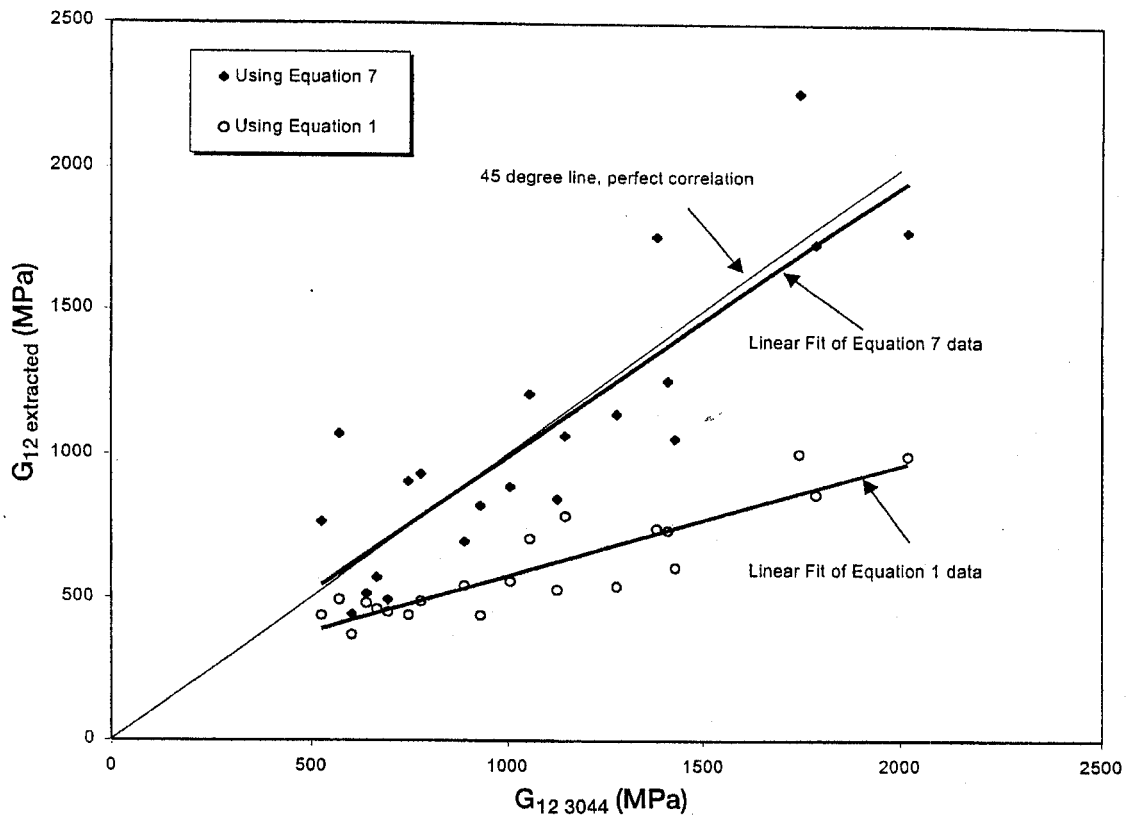FIG. 3. Comparing $G_{12\ extracted}$ to $G_{12\ 3044}$

TABLE 4. Comparison of Shear Modulus Values

Specimen (1)	$G_{12\ eqn\ 7}$ (MPa) (2)	$G_{12\ 3044}$ (MPa) (3)	Ratio $G_{12\ eqn\ 7}$ / $G_{12\ 3044}$ (4)	$G_{12\ eqn\ 1}$ (MPa) (5)	Ratio $G_{12\ eqn\ 1}$ / $G_{12\ 3044}$ (6)
A1	570	666	0.86	481	0.72
A2	441	603	0.73	381	0.63
A3	1,071	572	1.87	534	0.93
A4	763	526	1.45	458	0.87
C1	905	746	1.21	451	0.60
C2	492	693	0.71	464	0.67
N1	932	717	1.20	505	0.65
N2	514	640	0.80	500	0.78
T2	1,142	1,280	0.89	555	0.43
T3	1,056	1,429	0.74	633	0.44
T4	886	1,005	0.88	581	0.58
T5	1,257	1,409	0.89	777	0.55
T6	1,211	1,056	1.15	739	0.70
T7	1,758	1,382	1.27	771	0.56
O1	1,732	1,788	0.97	974	0.54
O2	2,263	1,745	1.30	1,254	0.72
O3	1,776	2,018	0.88	1,209	0.60
P1	819	929	0.88	567	0.61
P2	693	888	0.78	703	0.79
P3	1,067	1,147	0.93	1,017	0.89
P4	843	1,126	0.75	661	0.59
Average	—	—	1.01	—	0.66
Standard deviation	—	—	0.29I	—	0.136

DISCUSSION OF RESULTS

The proposed equation [(7)] is generally a better fit than (1) [or the equivalent (3) or (5)]. On some specimens, (1) performed equally as well as (7). A more substantial verification of the new equation is its ability to predict the experimental shear modulus $G_{12\ 3044}$ as well as match the E_q data. Table 4 summarizes these results. The average ratio of $G_{12\ eqn\ 7}$ /$G_{12\ 3044}$

= 1.01 is excellent, although the standard deviation is quite high at 0.291. Compare this, however, to G_{12} extracted from (1). Here we have better standard deviation (0.123) but the average ratio $G_{12\ eqn\ 1}$ /$G_{12\ 3044}$ = 0.60. This difference in shear modulus predictions is highlighted in Fig. 3. The scatter of the $G_{12\ eqn\ 1}$ data (circles in Fig. 3) is small, but the trendline is nowhere near the target line of 45°. Eq. (7) was calibrated such that the average $G_{12\ eqn\ 7}$ would be statistically close to $G_{12\ 3044}$ (here $G_{12\ eqn\ 7}$/$G_{12\ 3044}$ = 1.01). This means that individual values of G_{12} predicted by (7) will be higher than $G_{12\ 3044}$. It would be possible, however, to redefine (7) to produce a somewhat more conservative value of $G_{12\ eqn\ 7}$.

This method as presented in this paper requires off-axis data; thus it cannot be used if only parallel and perpendicular to grain test data (E_1 and E_2) are available. Currently, we are formulating empirical methods of estimating the shear modulus based solely on E_1 and E_2.

CONCLUSIONS

This study has improved the relationship governing the off-axis modulus of elasticity of orthotropic wood-based panels. The newly proposed relationship utilized the modulus of elasticity along the 1 and 2 axes, as well as the shear modulus, and it removed the need for a value of Poisson's ratio. This is an advantage over previous similar studies that required careful measurement of transverse strains. A useful application of the newly proposed equation is to solve it for the shear modulus. This allows for the calculation of shear modulus based solely on axial tests. This could be used to verify in-plane shear modulus data, or it could be used in place of the more cumbersome panel twisting test.

APPENDIX I. REFERENCES

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APPENDIX II.   NOTATION

The following symbols are used in this paper:

A = nondimensional parameter, ratio of E_1/E_2;

a = nondimensional parameter, function of E_i, G, and v_{12};

E_i = Young's modulus or modulus of elasticity, where $i = 1$, 2, or q; 1 being strong axis, 2 being weak axis, and q being some angle from strong axis;

$G_{\text{eqn } i}$ = shear modulus extracted from off-axis data, i being equation of interest;

G_{12} = shear modulus, along principal axes of panel;

$G_{12\ 3044}$ = shear modulus experimentally determined from plate twisting test ASTM D 3044; and

v_{12} = Poisson's ratio.