EFFECTS OF SHEAR COUPLING ON SHEAR PROPERTIES OF WOOD

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ABSTRACT

Under pure shear loading, an off-axis element of orthotropic material such as pure wood undergoes both shear and normal deformations. The ratio of the shear strain to a normal strain is defined as the shear coupling coefficient associated with the direction of the normal strain. The effects of shear coupling on shear properties of wood as predicted by the orthotropic elasticity theory were validated using our recently developed shear test fixture. The validation also serves to demonstrate that the shear test fixture possesses the capability to introduce a state of pure shear to the critical section of a specimen as required and that orthotropic elasticity theory should be used to describe the mechanical properties of pure wood.

Keywords: Orthotropic elasticity, shear coupling, shear modulus, shear strength, shear test.

INTRODUCTION

For a two-dimensional square element of orthotropic material such as clear wood with its grain oriented obliquely with respect to the edges, a pure shear loading applied to the edges will induce two normal strains perpendicular to the edges. The ratio of the shear strain to a normal strain is defined as the shear coupling coefficient associated with the axis of the normal strain (Tsai and Hahn 1980). Likewise, a normal loading applied to two opposite edges will induce a shear strain to distort the element. The ratio of the normal strain in the loading direction to the shear strain is defined as the normal coupling coefficient (Tsai and Hahn 1980). Some authors have made no differentiation between the two and have called the latter phenomenon shear coupling (due to normal loading) (Daniel and Ishai 1994; Pindera and Herakovich 1986), or just coupling effects (Pierron et al. 1998).

The theory of orthotropic elasticity has been fully developed (Jones 1975; Tsai and Hahn 1980; Daniel and Ishai 1994), and equations for the coefficients of normal coupling and shear coupling are well documented in it. In the experimental aspect, observation of the effects of normal coupling requires only a simple tension test with a specimen of a long strip of wood. When the grain is aligned with the loading direction, the specimen will elongate in the same direction and contract in the transverse direction due to Poisson's ratio. When the grain is oblique, the specimen will distort into a curved shape while being stretched due to the normal coupling coefficient (Pindera and Herakovich 1986). To observe the effects of shear coupling, we have used a recently developed shear test fixture (Liu et al. 1999), which is capable of determining the shear strength and shear modulus of clear and straight-grained wood specimens.

The current shear strength values of clear wood (Forest Products Laboratory 1999) were based on the shear block test (ASTM 1978), that actually does not give the shear strength because of high stress concentrations (Youngs 1957). The test cannot determine the shear modulus, nor can it be used to study the effect of grain slope, a parameter of special importance in the mechanics of orthotropic materi-

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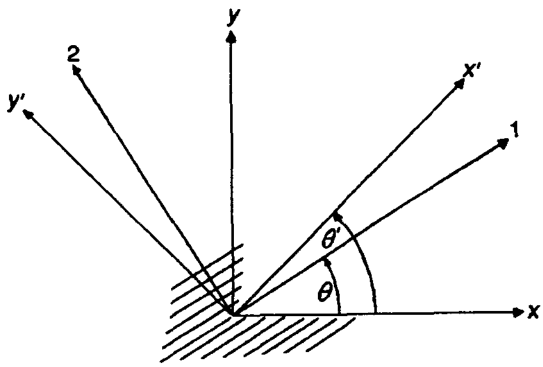

FIG. 1. Coordinate axes in principal material plane.

als. The current shear modulus values of clear wood were obtained using the method by March et al. (1942). The method is called the plate twist test (ASTM 1976). For solid wood, it is difficult to make plate specimens of practical size with uniform properties. Also, the plate twist test does not consider the effect of grain slope. These problems can all be solved using the shear test fixture recently developed by Liu et al. (1999).

In this paper, we present the results of a small test program on shear coupling and compare them against analytical predictions. The agreement of the comparison has demonstrated not only that the effects of shear coupling on shear properties of wood are as predicted by the orthotropic elasticity theory but, more significantly, that our shear test fixture is properly designed to make this comparison successful.

STRESS-STRAIN RELATIONS

Let the 1-2 coordinate system represent the principal material axes and the x-y coordinate system the geometrical or loading axes with angle θ from the *x* to the 1 axis as shown in Fig. 1. The stress-strain relations with respect to the x-y axes are expressed in the form

$$\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_s\end{bmatrix} = \begin{bmatrix}S_{xx} & S_{xy} & S_{xs}\\ S_{yx} & S_{yy} & S_{ys}\\ S_{sx} & S_{sy} & S_{ss}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_s\end{bmatrix} \quad (1)$$

where

$\epsilon_x, \epsilon_y, \gamma_s$ are axial, transverse, and shear strains, respectively,
$S_{xx}, S_{xy} \ldots$ are components of compliance matrix, and
$\sigma_x, \sigma_y, \tau_s$ are axial, transverse, and shear stresses, respectively.

Unlike the matrix for isotropic materials, the compliance matrix for orthotropic materials is fully populated. A normal stress will induce a normal strain in the same direction, another normal strain in the transverse direction, and a shear strain; a shear stress will induce a shear strain and two normal strains.

Components of the compliance matrix can be expressed in terms of the engineering constants (Daniel and Ishai 1994):

$$\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_s\end{bmatrix} = \begin{bmatrix}1/E_x & -\nu_{yx}/E_y & \eta_{sx}/G_{xy}\\ -\nu_{xy}/E_x & 1/E_y & \eta_{sy}/G_{xy}\\ \eta_{xs}/E_x & \eta_{ys}/E_y & 1/G_{xy}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_s\end{bmatrix} \quad (2)$$

where

E_x and E_y are Young's moduli along the *x* and *y* directions, respectively,
G_{xy} is shear modulus in the x-y plane,
ν_{xy}, ν_{yx} are Poisson's ratios (first subscript denotes loading direction and second subscript, strain direction), and
$\eta_{xs}, \eta_{ys}, \eta_{sx}, \eta_{sy}$ are normal and shear coupling coefficients; normal = first two coefficients and shear = last two (first subscript denotes loading or loading direction and second subscript, strain or strain direction).

From symmetry consideration of the compliance matrix, we obtain

$$\nu_{xy}/E_x = \nu_{yx}/E_y, \qquad \eta_{xs}/E_x = \eta_{sx}/G_{xy},$$
$$\eta_{ys}/E_y = \eta_{sy}/G_{xy} \quad (3)$$

Using the compliance transformation relations, we obtain the following equations for the transformed engineering constants (Daniel and Ishai 1994):

$$\frac{1}{E_x} = \frac{m^2}{E_1}(m^2 - n^2\nu_{12}) + \frac{n^2}{E_2}(n^2 - m^2\nu_{21}) + \frac{m^2n^2}{G_{12}}$$

$$\frac{1}{E_y} = \frac{n^2}{E_1}(n^2 - m^2\nu_{12}) + \frac{m^2}{E_2}(m^2 - n^2\nu_{21}) + \frac{m^2n^2}{G_{12}}$$

$$\frac{1}{G_{xy}} = \frac{4m^2n^2}{E_1}(1 + \nu_{12}) + \frac{4m^2n^2}{E_2}(1 + \nu_{21}) + \frac{(m^2 - n^2)^2}{G_{12}}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} = \frac{m^2}{E_1}(m^2\nu_{12} - n^2) + \frac{n^2}{E_2}(n^2\nu_{21} - m^2) + \frac{m^2n^2}{G_{12}}$$

$$\frac{\eta_{xs}}{E_x} = \frac{\eta_{sx}}{G_{xy}} = \frac{2mn}{E_1}(m^2 - n^2\nu_{12}) - \frac{2mn}{E_2}(n^2 - m^2\nu_{21}) + \frac{mn^3 - m^3n}{G_{12}}$$

$$\frac{\eta_{ys}}{E_y} = \frac{\eta_{sy}}{G_{xy}} = \frac{2mn}{E_1}(n^2 - m^2\nu_{12}) - \frac{2mn}{E_2}(m^2 - n^2\nu_{21}) + \frac{m^3n - mn^3}{G_{12}}$$

$$m = \cos\theta \qquad n = \sin\theta \tag{4}$$

where the basic engineering constants E_1, E_2, G_{12}, and ν_{12} are referred to the principal axes in Fig. 1. (Note: $\nu_{12}/E_1 = \nu_{21}/E_2$.)

SHEAR TEST FIXTURE

The newly developed shear test fixture (Liu et al. 1999) uses the Iosipescu specimen of the V-notched beam specimen. The purpose of the design was to introduce a pure shear state to the critical section of the specimen during testing. Other shear test fixtures (e.g., Walrath and Adams 1983; Conant and Odom 1995) obviously have the same purpose. The difference lies in the extent to which a design can achieve its goal.

The test method for shear properties of composite materials by Walrath and Adams (1983), later adopted by the American Society for Testing and Materials (ASTM 1993) as a standard, was found to cause specimen twisting, among other things, resulting in uneven stress distributions across the thickness of the specimen. In their effort to correct the shortcomings of the standard, Conant and Odom (1995) designed a series of prototype fixtures. They succeeded in eliminating the specimen twist problem, but they restricted the fixture halves to move only in the vertical direction during testing. Such restriction introduces normal stresses to the critical section of the specimen, adversely affecting the accuracy of the shear strength estimate, which should be a primary goal of any shear test fixture.

Figure 2 presents the shear test fixture by Liu et al. (1999). The right upper and left lower parts are the conventional antisymmetrical halves. The right lower and left upper parts are the two controlling blocks. Each block is connected to both halves of the fixture by two pairs of shafts that move on ball bushings. The left upper block is connected to the left lower part by two vertical shafts (Fig. 2A) and to the right upper part by two horizontal shafts (Fig. 2B; only the front shaft is partially visible). The right lower block is similarly connected to both halves of the fixture. Figure 3 shows the tips of the two horizontal shafts that connect the right lower block to the left lower Part.

Two heavy plastic doors, which can slide in the plane of Fig. 2, are used to mount and dismount the specimen. When closed, the doors also help locate the specimen in the horizontal direction of Fig. 3. Location of the specimen in this direction is also controlled through two knobs that are connected to steel components (Figs. 3 and 4). The specimen is centered in the fixture by two spring-controlled pegs that rest in the valleys of the V-notches when the specimen is properly located in the horizontal direction (Fig. 2). The load-

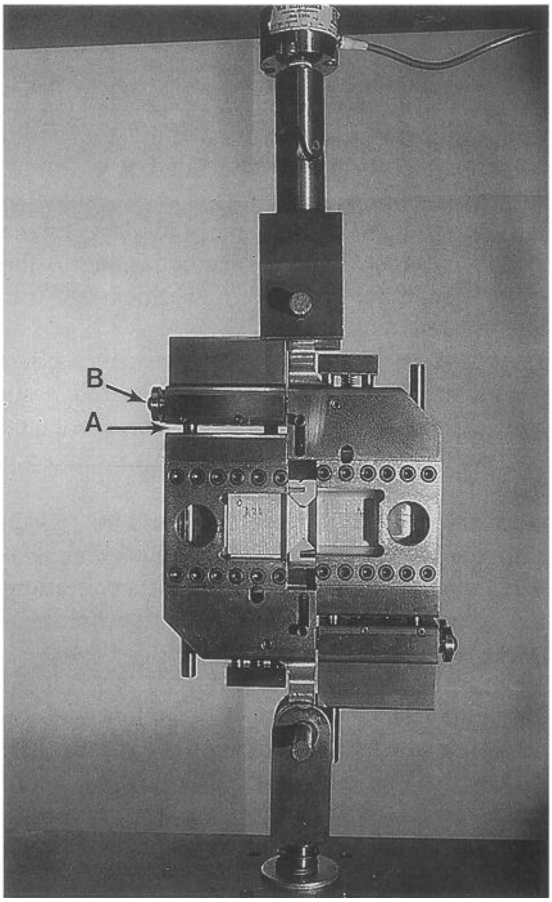

FIG. 2. Shear test fixture with specimen in place (Liu et al. 1999).

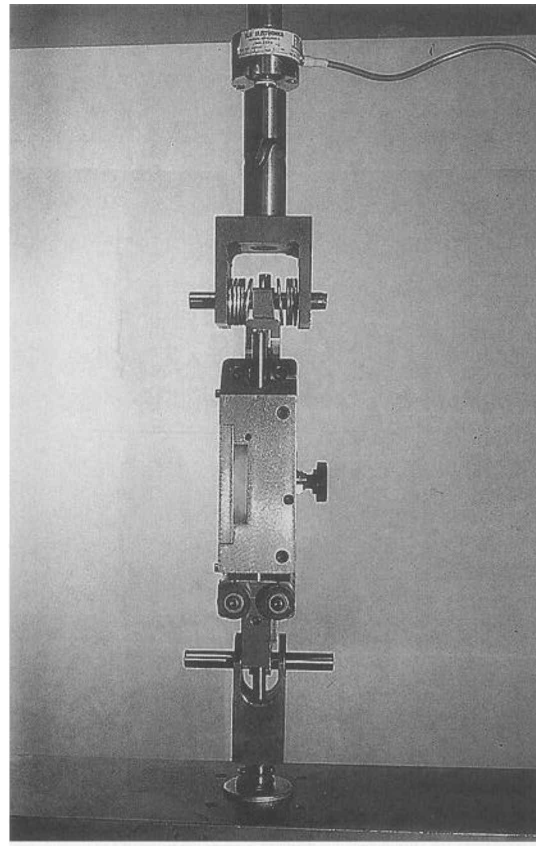

FIG. 3. Right side view of shear test fixture.

ing devices rest on the top of the right upper part and at the bottom of the left lower part and are controlled by screws (Fig. 2). These screws must be tightened before the test.

The deflection of the spring between the fixture and the lower frame of the test machine is calibrated to support half of the total fixture weight. At the start of a test, the initial load on the specimen can be adjusted to be close to zero. During testing, the lower frame of the test machine moves downward, applying a tensile load to the fixture, which is transmitted as shear loading on the critical section of the specimen.

At the end of the test, the lower frame of the test machine returns to its initial position. The screws controlling the loading devices and the knobs controlling the alignment of the specimen are released, the plastic doors are opened, and the specimen is removed from the fixture.

The test fixture operates like the Arcan shear test (Arcan et al. 1978) but uses the specimen geometry suggested by Iosipescu (1967) for efficient gripping and short turnaround time. The fixture has corrected the tendency to twist and misalign experienced by the Arcan test and several versions of the Iosipescu test when specimens are not of homogeneous materials. There was no restriction to prevent relative movement of the two fixture halves in the horizontal direction in the designs by Arcan and Iosipescu. Our shear test fixture has introduced none.

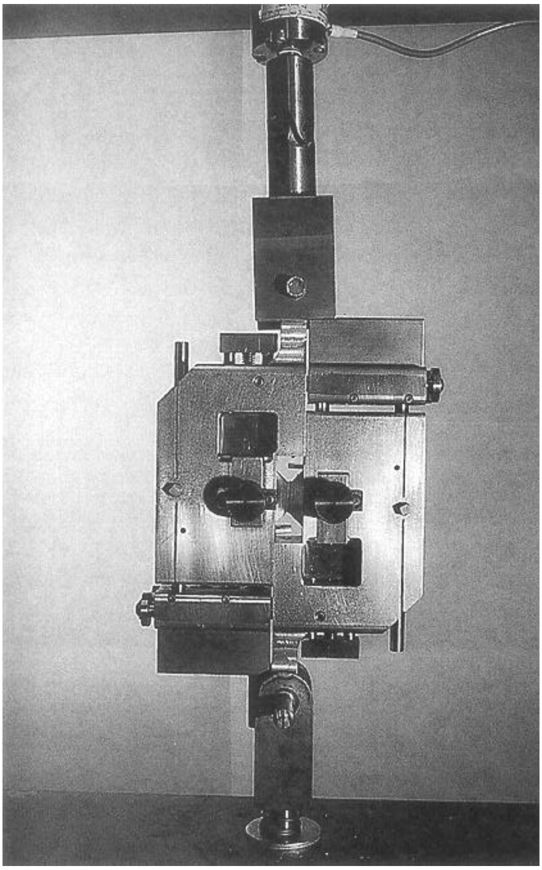

FIG. 4. Rear view of shear test fixture.

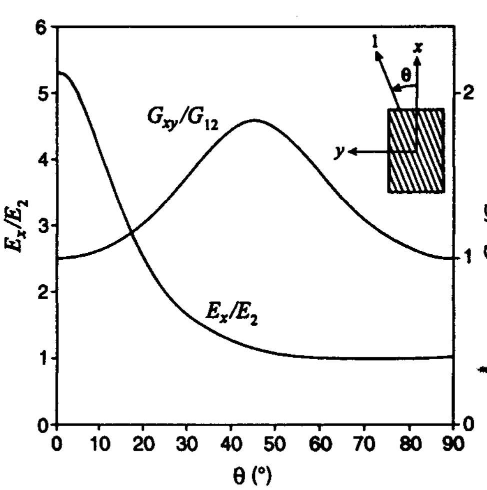

FIG. 5. Elasticity and shear moduli of Sitka spruce versus grain slope (E_1 = 11,800 MPa, E_2 = 2,216 MPa, G_{12} = 910 MPa, v_{12} = 0.37).

SPECIMENS AND TEST PROCEDURES

Specimen dimensions were as follows: total length, 95 mm; width, 38 mm; thickness, 12.7 mm; depth of 90° notch, 9.5 mm; and width of critical section between notches, 19 mm. The surface of the specimen was oriented in the longitudinal-radial plane; the slope of the grain was oriented at a specified angle to the critical section. The specimen thickness was in the tangential direction.

A board of Sitka spruce *(Picea sitchensis)* of unknown history but of the desired grain and growth ring orientations was selected from storage at the Forest Products Laboratory. The board was cut into five samples by slope of grain (0°, 30°, 45°, 60°, and 90°). Each sample consisted of 20 specimens. Before testing, the specimens were stored in a conditioning room at 23°C and 50% relative humidity for several weeks until stabilized.

All specimens were tested for shear strength. Tensile loading was applied with an Instron test machine, which pulled the left half of the fixture downward while the right half remained stationary. Crosshead speed was 1.27 mm/mm. Load and crosshead displacement data were recorded electronically.

RESULTS AND DISCUSSION

We intended to use the shear test fixture to make a comprehensive study of shear failure and grain slope relations. However, many specimens did not fail at the critical section as expected, making it impossible to conduct a meaningful statistical analysis. Nonetheless, all failures can be satisfactorily explained and the study has yielded conclusive results concerning shear coupling, which have also fulfilled our study objectives.

In a previous study (Liu and Ross 1998) with the same material processed as in the present study, the values of the basic engineering constants in Eq. (4) were found to be as follows: E_1 = 11,800 MPa, E_2 = 2,216 MPa, G_{12} = 910 MPa, and v_{12} = 0.37. With these values as input, we obtained the numerical results plotted in Figs. 5 and 6 from Eq.

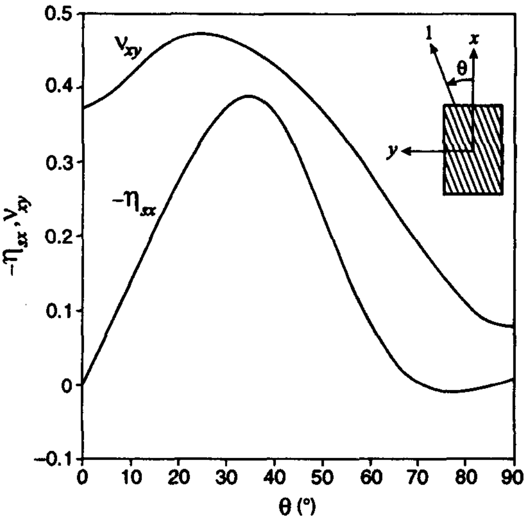

FIG. 6. Poisson's ratio and shear coupling coefficient of Sitka spruce versus grain slope.

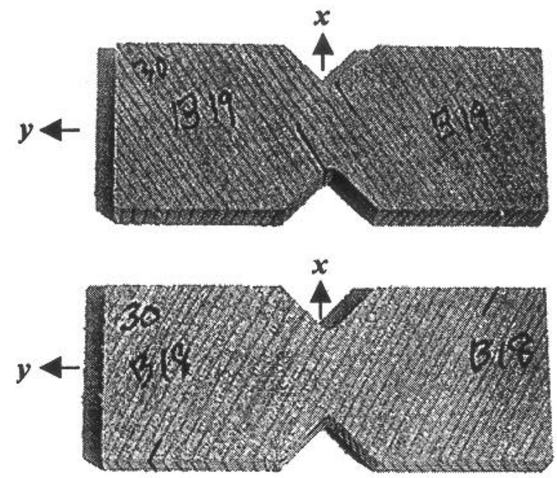

FIG. 7. Specimens with failure locations (top: θ = 30°, bottom: θ = –30°).

(4). From these figures, results for the other transformed engineering constants can readily be derived. These figures and equations facilitate the analysis of the test results.

Our test results can be demonstrated using the two specimens in Fig. 7. If the lower specimen is flipped over from right to left or from bottom to top, the two specimens are geometrically identical. As shown in the figure, the upper specimen has a grain slope of 30°, and the lower specimen has a grain slope of –30°, with respect to the critical section between the notches, which is on the x-axis. When tested in the shear test fixture, the upper specimen failed at the critical section, but the lower specimen did not. In the latter case, as the applied load is increased, the grain of the specimen was squeezed, and the specimen bulged out of plane at the critical section. The relative approach of the fixture halves in the horizontal direction was readily visible. The specimen finally failed at the upper right and lower left edges away from the critical section as a result of tensile stresses, as shown in Fig. 7. The same phenomenon was observed in all specimens with θ ≠ 0° or 90°.

This phenomenon can be explained from the stress-strain relations in Eq. (2), which gives

$$\epsilon_x = \eta_{sx}\tau_s/G_{xy} \qquad \epsilon_y = \eta_{sy}\tau_s/G_{xy}$$

$$\gamma_s = \tau_s/G_{xy} \tag{5}$$

for pure shear loading. When the left half of the fixture is lowered with respect to the right half, the shear at the critical section of the specimen τ_s is negative (Tsai and Hahn 1980). With θ = 30°, $\eta_{s\,x}/G_{xy} = -2.67E - 4MPa^{-1}$, as calculated from Eqs. (4). Therefore, ϵ_x is positive in Eq. (5). Likewise, we can show for θ = 30°, ϵ_y is also positive with $\eta_{s\,y}/G_{xy} = -5.04E - 5MPa^{-1}$. A positive strain corresponds to a positive stress. Therefore, the tensile stresses σ_x and σ_y due to the shear coupling coefficients η_{sx} and η_{sy} are acting together with the applied shear τ_s at the critical section as shown in Fig. 8a to cause the specimen to fail.

When θ = –30°, the signs for η_{sx} and $\eta_{s\,y}$ change from negative to positive, but their magnitudes remain the same. As a result of τ_s or τ_{xy}, σ_x and σ_y are both negative, as shown in Fig. 8b. The compressive stresses σ_x and σ_y are acting together with the applied shear τ_s at the critical section to delay the failure of the specimen. In this case, as the applied load in-

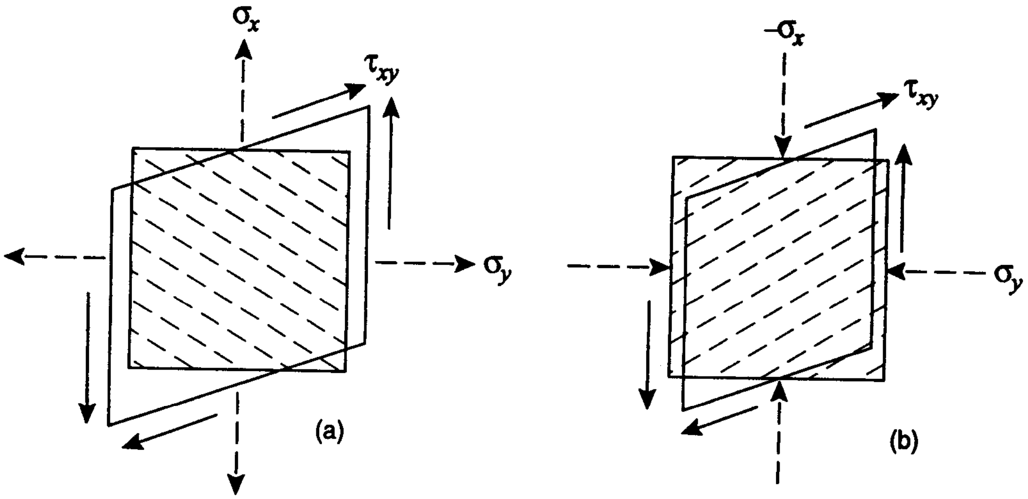

FIG. 8. Response of material elements with grain slope under pure shear loading: (a) + θ, (b) – θ.

creased, failure occurred elsewhere but not at the critical section.

These results show that when the grain slope changes from positive to negative, the induced normal stresses change from tensile to compressive due to a change in sign of the shear coupling coefficients. This results in increased strength of the material to resist the applied shear loading, which is in line with the Tsai-Wu strength criterion (Daniel and Ishai 1994). Although we were not able to show the magnitudes of the compressive stresses that cause failure together with the applied shear loading as described above, the trend of change in material shear resistance was evident.

These results also demonstrate that for any shear test fixture, the two halves of the fixture must be allowed to move horizontally while one is pulling away from the other in the vertical direction so that pure shear can occur at the critical section of the specimen. Even when $\theta = 0°$ where both η_{sx} and η_{sy}, are zero, the relative horizontal movement of the two parts cannot be avoided during testing unless the material of the specimen is extremely brittle and its shear strength is very low.

While the shear test fixture can only determine the shear strength at $\theta = 0°$, it can determine the shear modulus G_{xy} at any value of θ. Referring to Fig. 1, we obtain (Jones 1975)

$$\epsilon_{x'} = m'^2\epsilon_x + n'^2\epsilon_y + m'n'\gamma_s$$

$$\epsilon_{y'} = n'^2\epsilon_x + m'^2\epsilon_y - m'n'\gamma_s$$

$$m' = \cos\theta' \qquad n' = \sin\theta' \tag{6}$$

where θ' is the angle from the x to the X' axis. By setting θ' = 45°, it follows

$$\gamma_s = \epsilon_{x'} - \epsilon_{y'} \tag{7}$$

Equations (6) are derived from the laws of stress and strain transformation, which are independent of θ and any other material properties. Therefore, we can obtain γ_s using the 45° strain rosette or the shear gage by Ifju (1994) based on Eq. (7). With γ_s known for any value of θ in Fig. 1, the corresponding G_{xy} can be obtained. The data for G_{xy} in Fig. 5 were based on Eqs. (4) and validated experimentally using the shear gage (Liu and Ross 1998).

CONCLUSIONS

1. The shear coupling effects demonstrated in the present study clearly show that any improvements over the Arcan and the Iosipescu shear test fixtures must observe the

requirements that the fixture halves be antisymmetric and the relative movement in the horizontal direction be unrestricted during testing.

2. For orthotropic materials such as wood, the shear test fixture can determine the shear strength with respect to a principal material axis in a principal material plane. In any other directions in the same plane, shear coupling coefficients will introduce normal stresses to the critical section of the specimen together with the applied shear. However, in such situations, the shear modulus can still be determined using the shear test fixture.
3. Shear coupling is a direct consequence of grain slope, as demonstrated in the study presented here. In any mechanical design of structural wood members, the effect of grain slope on stress development should be taken into consideration.

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