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Censoring Data for Resistance Factor Calculations in Load and Resistance Factor Design: A Preliminary Study

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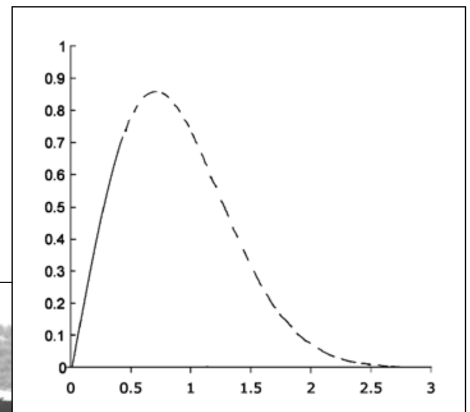

Abstract

Reliability estimates for the resistance distribution of wood product properties may be made from test data where all specimens are broken (full data sets) or by using data sets where information is obtained only from the weaker pieces in the distribution (censored data). Whereas considerable information exists on property estimation from full data sets, much less information is available on property estimation using censored data. To assess the need for a more rigorous study, a small simulation study was conducted to identify potential problems that could be associated with censoring effects on property estimates from an assumed Weibull distribution for use in reliability-based standards such as ASTM D 5457. Results suggest that reasonable estimates of property percentiles may be obtained when the censoring point is above the percentile needed. However, censoring also affects the estimate of the Weibull shape parameter, and therefore the coefficient of variation. This is important because the coefficient of variation is used to estimate the data confidence factor and the normalization factor also used in determining the data resistance factor. The simulation suggests that for a given sample size, the estimate of Weibull-shape parameter gets better as more of the distribution is included. Further studies are recommended to provide guidance on use of censored data with both the two- and three-parameter Wiebull distributions.

Keywords: Weibull distribution, reliability-based design, censored data

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Censoring Data for Resistance Factor Calculations in Load and Resistance Factor Design: A Preliminary Study

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Introduction

In 1984, the American Society of Civil Engineers (ASCE) Committee on Wood formed a Task Committee on Load and Resistance Factor Design (LRFD) for Engineered Wood Construction. The intent was to develop a reliability-based approach to safety and to put wood practices in line with those being used for concrete and steel. This new LRFD approach is described in AF&PA/ASCE 16-95 (1996). Calculation methods for the resistance part of the LRFD approach are in ASTM D 5457 (ASTM 2005). This ASTM standard applies only to individual wood elements, not to assemblies. The two-parameter Weibull distribution is the mandated basis of D 5457 calculations. The standard does not use full reliability methods but is loosely based on the approach presented in Thoft-Christensen and Baker (1982). This design procedure starts with fitting a two-parameter Weibull distribution to either a complete data set or a set of data representing the lower tail of the distribution. The fit can be made using either maximum likelihood methods or a regression-based estimation procedure. A distribution percentile estimate R_p is calculated from the fitted two-parameter Weibull distribution. For many properties, this will be a 5th percentile estimate. A coefficient of variation (CV) is calculated from the fitted shape parameter, α, using the formula

$$\mathrm{CV} \cong \alpha^{-0.92} \qquad (1)$$

From this value and the sample size, a data confidence factor Ω is calculated from a table in the standard. Generally this factor (≤1) goes up as sample size increases and down as the CV increases. Finally, a reliability-normalization factor K_R is also calculated from a table in the standard based on the CV and the mechanical property being considered. In general, this factor reaches its largest value at a CV of 12% and decreases as CV rises above 12%. The final LRFD resistance factor is a product of the three values:

$$R_n = (R_p)(\Omega)(K_R) \qquad (2)$$

The resulting value is used as the upper limit of the 50-year maximum load to which the element can be subjected. Fifty-year maximum loads are based on fitting various distributions to combinations of dead loads and wind, rain, snow, earthquake, and occupancy live loads. These distributions are codified in ASCE 7-88 (1990).

Because of the sensitivity of reliability calculations to distributional form and method of fit, the wood engineering community is interested in evaluating the effect of using a three- versus a two-parameter Weibull distribution in evaluating the effect of estimation method and the effect of using censored data sets versus full data sets. Simulations (such as Durrans and others 1998) have been published to address some of these issues. Further, the standard states, "Estimates of the distribution and its parameters give the most accurate reliability estimates when they represent a tail portion of the distribution rather than the full distribution" (ASTM 2005, Appendix X1.1.2). Whereas most strength properties for lumber are based on lower tail properties, the use of the standard is not limited to lower tail properties. Little information is currently available in the wood literature on the effect of data censoring on property estimates from the Weibull distribution. The objective of this study is to investigate the sensitivity of reference resistance to censoring procedures as applied in load and resistance factor design.

Methodology

This paper looks at the effect of censoring data when using three-parameter maximum likelihood estimates. This is relevant for beginning to look at ASTM D 5457, which can be interpreted to suggest that sampling lower tail data sets might be better for fitting distributions than complete data sets. Section 1.1.3 of the standard states, "By permitting tail fitting of the data, it [the standard] provides a way of fitting data in this important region that is superior to the full-distribution types." The standard also states, "For lower tail data sets, a minimum of 60 failed observations is required for sample sizes of n = 600 or less," and, "For sample sizes greater than 600, a minimum of the lowest 10% of the distribution is required" (ASTM 2005).

This statement implies that a censored data set can or should be used even when a smaller complete data set could be tested. Such an assumption is unusual because the amount of distribution represented by the censored data has no relationship to the overlap in load and resistance distributions. Using it to calculate a reliable LRFD value associated with a property needing a higher percentile than the 5th might lead to problems in some cases. The 10% figure apparently was chosen to make sure the 5th percentile is well bracketed.

We ran a small simulation to look at the effect of using censored samples on our ability to estimate percentiles from a three-parameter Weibull distribution using maximum likelihood estimates. We simulated data from known Weibull distributions using International Mathematical & Statistical Library (IMSL) routines and then used maximum likelihood estimation for a three-parameter Weibull distribution on the bottom 5%, 10%, 15%, … , 90%, 95%, and 100% of the data. Sample sizes for the complete data sets were 20, 60, 100, 200, and 500. For each fit of the data, we estimated a 5th, 30th, 50th, 70th, and 95th percentile. This was done for shape parameters of 2.0, 3.5, and 5.0; a scale parameter of 1; and a location parameter of 0.5 with 100 replications for each combination. These shape parameters cover the range of shape parameters found for major species tested in the U.S. in-grade program (Table 1) (Green and Evans 1987). We calculated the mean standard deviation and mean square error for each parameter and percentile. From the present study, we hope to get an idea of potential problems if a true reliability calculation were done when we look at the mean square error of the estimated 30th and 50th percentiles. Because the mean square error takes into account both bias and variability, it should give us the best feel for when our estimates will be close to the true values.

Results

Distribution Percentile Estimates

Table 2 shows the mean square error for percentile estimates for the right-censored three-parameter Weibull distributions studied. Trends show that extrapolated estimates for large samples can be poorer than estimates based on much smaller full distributions. This can be seen in the mean square value of our estimate of the 50th percentile, where a full sample of 60 specimens is smaller than 60 of 200, 50 of 500, and 75 of 500. We see that censored data with the censoring point above the percentile being estimated may be very good, as is shown in the 30th percentile estimates of 60 of 100 or 60 of 200 compared to 60 of 60. The results imply that the further we have to extrapolate, the poorer the estimate 50 of 500 and 75 of 500 estimates. If we assume that a poor estimate of a percentile means a poor estimate of distribution in the neighborhood of the percentile, any actual reliability calculations that involve an extrapolated percentile may be suspect. These observations offer a mixed view of what might happen under this ASTM standard. In small samples, it is unlikely that we would be extrapolating percentile estimates above the censoring point of the data and there should be no problem. Also, for many mechanical properties, the percentile of interest might well be a 5th percentile that the bottom 10% of the data would cover. However in section 6.3.1 of the standard, a 5th percentile is only an example. This implies that for large samples and situations where a higher percentile is needed, there might be problems with extrapolation. It also implies that other applications than this ASTM standard that use censored data from a Weibull distribution might have problems if they are extrapolating from data.

Coefficient of Variation

Censoring procedures can also affect estimates of the coefficient of variation (CV_W) for Weibull distributions. This is a more complex problem than the variation in percentile estimates because the coefficient of variation is used to calculate the factors Ω and K_R (Table 3 and 4).

Table 5 demonstrates how the mean square error of the estimated shape parameter varies with censoring strategies and shows that it can be higher than the mean square error obtained with full distributions. For approximately equal numbers of broken boards, the mean square error of the shape parameter increases as the percentage broken decreases. An estimate of the range in CV_W may be obtained by taking the square root of the mean square error. The resulting value incorporates bias and standard deviation of the estimate. As such, it can give an idea of how far from the actual true-shape parameter an estimate might be. It also ignores whether the bias creates a conservative estimate or a non-conservative estimate of R_n. Plus or minus this value would be like plus or minus a standard deviation from the mean value, which is a conservative estimate of what might occur. The expected ranges in CV_W can be quite large (Table 6). Because the bias is not 0, the actual range of values for CV will be different than this example depending on whether the bias is positive or negative. However, this range illustrates that the potential bias and variability of the maximum likelihood-shape parameter estimate might also be of concern.

For small sample sizes, the MLE of the shape parameter can be quite biased. Billmann and others (1972) looked at the two-parameter Weibull with either 25% or 50% of the largest observations censored for sample sizes $n = 40, 60, 80, 100, 120$. Their results showed that the greater the censoring, the greater the bias. If we define the bias as the mean of our estimated shape parameters minus the true shape parameter, we see the bias in the estimated shape parameters (Table 7). Positive biases represent cases in which we overestimated the shape parameter. This would cause us to underestimate the coefficient of variability. An error in this direction is therefore not conservative. The variability in these numbers can also reflect the small sample size in the simulation when there is a large variability to results. The small number of simulations used for each case should be increased in future studies to see if it clarifies the picture. When we use all the data, the bias gets smaller as the sample size gets larger (Table 8). A similar pattern occurs for the mean square error, as should be expected.

Conclusions

Although this is a very small study and uses a three- instead of the two-parameter Weibull, trends in the results indicate that a need exists for a more thorough investigation of the effects of censoring on percentile estimates from an assumed Weibull distribution. From the current results, we conclude the following:

1. Percentile estimates below the censoring point were estimated fairly well.
2. It appears that extrapolating to percentiles above the censoring point should be avoided. In most cases, extrapolation produced poorer estimates than a much smaller sample that had a censoring point above the percentile to be estimated.
3. We need to further evaluate the effect of censoring strategies on property estimates for both the two- and three-parameter Weibull distributions.
4. The coefficient of variation of the assumed Weibull distribution can also be affected by censoring strategies. Because the coefficient of variation affects both the data confidence and normalization factors, effects of censoring on the resistence factor are more complex than those on percentile estimates. Our results suggest that for a given sample size, the estimate of the shape parameter gets better as more of the distribution is included.
5. Further study is needed to evaluate the effect of data censoring on estimation of the shape parameter and estimated coefficient of variation for both the two- and three-parameter Weibull distributions.

Literature Cited

AF&PA/ASCE. 1996. Standard for load and resistance factor design (LRFD) for engineered wood construction. Washington, D.C.: American Wood Council of the American Forest & Paper Association. 16–95.

ASCE. 1990. Minimum design loads for buildings and other structures. Reston, VA: American Society of Civil Engineers. 7–88.

ASTM. 2005. Standard specification for computing the reference resistance of wood-based materials and structural connections for load and resistance factor design. ASTM D 5457-04a. Annual Book of Standards, Volume 04.10. West Conshohoken, PA: American Society for Testing and Materials.

Billmann, B.R.; Antle, C.E.; Bain, L.L. 1972. Statistical inference from censored Weibull samples. Alexandria, VA: Technometrics 14:831–840.

Durrans, S.R.; Triche, M.H.; Suddarth, S.K. 1998. Estimation of lower tail quantiles of Weibull probability distributions for lumber strength. Forest Products Journal 48(1):96–101.

Green, D.W. and Evans, J.W. 1987. Mechanical properties of visually graded lumber. Volumes 1–7. Springfield, VA: National Technical Information Service. PB-88-159-371.

Thoft-Christensen, P.; Baker, M.J. 1982. Structural reliability theory and its applications. New York, NY: Springer-Verlag.

Table 1. Shape parameter for nominal 2 by 4 lumber at 12% moisture content (Green and Evans 1987)[a]

Grade	Species group	MOR		UTS		UCS	
		2P	3P	2P	3P	2P	3P
Sel. Str.	Douglas Fir–Larch	4.69	4.18	3.06	2.40	5.77	3.18
	Hem–Fir	4.42	3.82	3.33	2.24	5.41	2.16
	Southern Pine	4.47	3.77	3.29	2.58	5.84	5.12
No. 2	Douglas Fir–Larch	2.89	2.37	2.15	1.54	4.10	2.75
	Hem–Fir	2.74	2.00	2.48	1.85	4.46	2.41
	Southern Pine	2.61	2.09	2.12	1.32	3.94	2.12

[a] Two- (2P) and three-parameter (3P) Weibull-shape parameter (α) for modulus of rupture (MOR), ultimate tensile stress parallel to the grain (UTS), and ultimate compression stress parallel to the grain (UCS) of Select Structural (Sel. Str.) and No. 2 grade dimension lumber.

Table 2. Mean square errors of estimated 30th and 50th percentiles from right-censored three-parameter Weibull distributions

Specimens broken (no.)	Specimens broken (%)	Shape 2.0		Shape 3.5		Shape 5.0	
		30th	50th	30th	50th	30th	50th
60 0f 60	100	0.0557	0.0613	0.0423	0.0404	0.0300	0.0272
60 of 100	60	0.0510	0.0547	0.0409	0.0382	0.0303	0.0280
60 of 200	30	0.0342	0.0742	0.0262	0.0473	0.0220	0.0386
75 of 500	15	0.0569	0.1032	0.0402	0.0879	0.0332	0.0678
50 of 500	10	0.1032	0.2524	0.0779	0.1600	0.0573	0.1189

Table 3. Data confidence factor Ω on $R_{0.05}$ for two-parameter Weibull distribution with 75% confidence [a] (table 1 of ASTM D 5454-04a, 2005)

CV_W	Sample size, *n*									
	30	40	50	60	100	200	500	1000	2000	5000
0.10	0.95	0.95	0.96	0.96	0.97	0.98	0.99	0.99	0.99	1.00
0.15	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	0.99	0.99
0.2	0.89	0.91	0.92	0.93	0.94	0.96	0.98	0.98	0.99	0.99
0.25	0.87	0.88	0.90	0.91	0.93	0.95	0.97	0.98	0.98	0.99
0.30	0.84	0.86	0.88	0.89	0.92	0.94	0.96	0.97	0.98	0.99
0.35	0.81	0.84	0.86	0.87	0.90	0.93	0.96	0.97	0.98	0.99
0.40	0.79	0.81	0.84	0.85	0.89	0.92	0.95	0.96	0.97	0.98
0.45	0.76	0.79	0.82	0.85	0.87	0.91	0.94	0.96	0.97	0.98
0.50	0.73	0.77	0.80	0.81	0.86	0.90	0.94	0.95	0.97	0.98

[a] Interpolation is permitted. For CV_W values below 0.10, the values for 0.10 shall be used.

Table 4. Fifth-percentile-based reliability normalization factors, K_R (table 3 of ASTM D 5454–04a, 2005)

CV_W (%)	K_R Compression and bending	Bending	Tension parallel	Shear (2.1 basis)	Shear (SCL, 3.15 basis)	Shear (I-Joist, 2.37 basis)
10	1.303	1.248	1.326	1.414	0.943	1.253
11	1.307	1.252	1.330	1.419	0.946	1.257
12	1.308	1.253	1.331	1.420	0.947	1.258
13	1.306	1.251	1.329	1.418	0.945	1.256
14	1.299	1.244	1.322	1.410	0.940	1.249
15	1.289	1.235	1.312	1.400	0.933	1.240
16	1.279	1.225	1.302	1.388	0.926	1.230
17	1.265	1.212	1.288	1.374	0.916	1.217
18	1.252	1.199	1.274	1.359	0.906	1.204
19	1.237	1.185	1.259	1.343	0.895	1.190
20	1.219	1.168	1.241	1.324	0.882	1.173
21	1.204	1.153	1.225	1.307	0.871	1.158
22	1.186	1.136	1.207	1.287	0.858	1.141
23	1.169	1.120	1.190	1.269	0.846	1.125
24	1.152	1.104	1.173	1.251	0.834	1.109
25	1.135	1.087	1.155	1.232	0.821	1.092
26	1.118	1.071	1.138	1.214	0.809	1.076
27	1.105	1.059	1.125	1.200	0.800	1.063
28	1.084	1.038	1.103	1.176	0.784	1.042
29	1.066	1.021	1.085	1.157	0.771	1.025
30	1.049	1.005	1.068	1.139	0.759	1.009

Table 5. Mean square errors for estimated shape parameters from right-censored three-parameter Weibull distributions

Specimens broken (no.)	Specimens broken (%)	True shape parameter 2.0	3.5	5.0
60 of 60	100	0.3506	0.8678	1.6425
60 of 100	60	0.4142	1.3292	1.6852
60 of 200	30	0.4431	1.4163	2.4448
75 of 500	15	0.5029	1.7580	2.5467
50 of 500	10	0.6066	2.2284	3.0601

Table 6. Effect of censoring on the variation in the shape parameter for the three-parameter Weibull distribution.

True shape parameter	Specimens broken	Range of shape parameter [a]		Coefficient of variation [b]	
		Low	High	Low	High
2.0	60 of 60	1.41	2.59	0.42	0.73
	60 of 100	1.36	2.64	0.41	0.78
	60 of 200	1.33	2.67	0.41	0.77
	75 of 500	1.29	2.71	0.40	0.79
	50 of 500	1.22	2.78	0.39	0.83
3.5	60 of 60	2.57	4.43	0.25	0.42
	60 of 100	2.35	4.65	0.24	0.46
	60 of 200	2.31	4.69	0.24	0.46
	75 of 500	2.17	4.83	0.24	0.49
	50 of 500	2.01	4.99	0.23	0.53
5.0	60 of 60	3.72	6.28	0.18	0.30
	60 of 100	3.70	6.30	0.18	0.30
	60 of 200	3.44	6.56	0.18	0.32
	75 of 500	3.40	6.60	0.18	0.32
	50 of 500	3.25	6.75	0.17	0.34

[a] $\pm$ one square root of the mean square error.
[b] $CV \cong \alpha^{-0.92}$; for $\alpha = 2.0$, CV = 0.53; for $\alpha = 3.5$, CV = 0.32; and for $\alpha = 5.0$, CV = 0.23.

Table 7. Bias for estimated shape parameters from right-censored three-parameter Weibull distributions

Specimens broken (no.)	True shape parameter		
	2.0	3.5	5.0
60 of 60	– 0.1040	– 0.1105	– 0.2319
60 of 100	– 0.1484	0.1027	– 0.5682
60 of 200	– 0.0488	– 0.0102	0.1988
75 of 500	– 0.1034	0.4053	0.4205
50 of 500	– 0.634	0.3144	0.1759

Table 8. Bias for estimated shape parameters from complete samples from three-parameter Weibull distributions

Specimens broken (no.)	True shape parameter		
	2.0	2.0	2.0
20 of 20	– 0.2864	– 0.2451	– 0.8857
60 of 60	– 0.1040	– 0.1105	– 0.2319
100 of 100	– 0.0617	0.0821	– 0.1492
200 of 200	– 0.0374	– 0.390	– 0.0451
500 of 500	– 0.0189	0.0087	0.0154