Material Variability and Repetitive Member Allowable Property Adjustments in Forest Products Engineering

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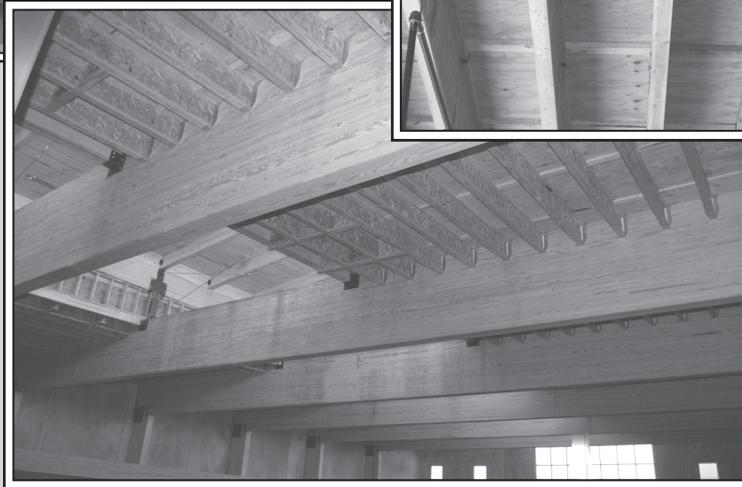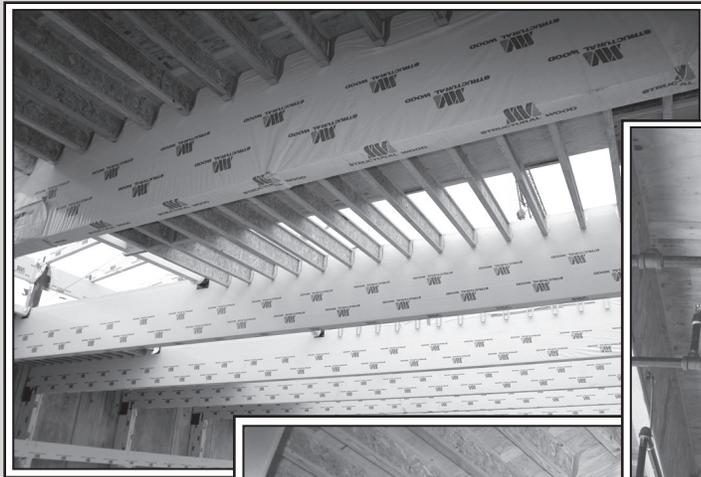
Abstract

It has been argued that repetitive member allowable property adjustments should be larger for high-variability material than for low-variability material. We report analytic calculations and simulations that suggest that the order of such adjustments should be *reversed*. That is, given the manner in which allowable properties are currently calculated, as the coefficient of variation of the strength distribution of individual elements increases, the upward repetitive member adjustments (if any) of assemblies constructed from these elements should decrease.

Keywords: assembly, reliability, redundancy, allowable property

Contents

Material Variability and Repetitive Member Allowable Property Adjustments

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1   Introduction

In existing standards (e.g., ASTM D 245, D 1990, 2007a,b), upward adjustments in allowable properties are permitted for repetitive member assemblies of solid sawn wood. In the pre-2004 D 5055 standard (ASTM, 2003), this upward adjustment was larger for high-coefficient-of-variation (COV) material than for low-COV material. In section X1.4.1 of the pre-2004 D 5055 standard a justification of the differing adjustments was given:

> The allowable bending strength increases for repetitive member use were derived taking into consideration the coefficient of variation (COV) of the stiffness of various flange materials. The original theory justifying this type of increase seems to be based on the relative stiffness of the members and positive correlation between bending strength and stiffness. Logic indicates that as stiffness COV decreases so would the load sharing. That is, as stiffness COV tends to zero, lack of differential deflection eliminates load transfer.

For pragmatic reasons, the differing adjustments were removed in the 2004 revision of D 5055 (ASTM, 2007c). However, the intuitive basis for the differential adjustments is still found in section 5.1.3 of D 6555 (ASTM, 2007d):

> Load sharing tends to increase as member stiffness variability increases ...

Based on the analytical calculations and computer simulations reported in this paper, we conclude that the differing repetitive adjustments that have been based on this intuition (that is, upward adjustments that are larger for higher COV material) are flawed.

It is important to note that we are not arguing with the contention that "load sharing tends to increase as member stiffness variability increases." Instead we dispute the conclusion that is drawn from this intuition — that assemblies composed of higher variability material deserve larger upward adjustments in allowable properties. This conclusion is not valid as it makes the implicit assumption that high-variability assemblies and low-variability assemblies begin with the same probability of individual element failure. As we will see below, this assumption is false. Thus, higher variability assemblies begin with a handicap and any advantage due to "better load sharing" does not make up for this initial disadvantage. In fact, for the assemblies that we consider, the probability that an assembly fails when subjected to a load equal to its allowable bending strength increases as COV increases. Given this basis, larger repetitive member upward adjustments for larger member strength COVs lead to assemblies with larger probabilities of failure and are thus unjustified. This is the essence of our argument. We provide the analytical and simulation details in the remainder of this paper.

For the purposes of this paper, we define assembly failure as the failure of any one of the assembly's members. A first approximation to the probability, P_F, of assembly failure is given by

$$P_F \approx 1 - (1 - p_S)^k \tag{1}$$

where p_S is the probability that an individual element fails and k is the number of elements in the assembly. Result (1) treats the elements of an assembly as independent members that all see the same load. As we will see below, this approximation overestimates P_F if weaker members tend to be less stiff and thus tend to see less of the load. However, it is a starting point for our analysis and emphasizes the fundamental role played by p_S.

In Section 2 we obtain analytic formulas for p_S for normal and lognormal distributions. In Section 2 we also point out the central fact that, as currently calculated, *allowable properties are not associated with a fixed probability of failure*. That is, the probability that a member fails when subjected to a load equal to its allowable bending strength is smaller for members that come from distributions with smaller coefficients of variation.

In Section 3 we derive an expression for P_F in the case of perfectly correlated, normally distributed strengths and stiffnesses.

In Section 4 we discuss simulations of a particular hypothetical assembly. Our simulated assembly contains seven members, and we distribute loads in proportion to member stiffnesses. Our simulations produce estimates of P_F. In Section 4 we discuss the tables that we produced from these simulations and the fact that they indicate that, at loads equal to allowable bending strengths, the probability of failure increases with increasing COV.

In Section 5 we extend these simulations to cover the case in which our assemblies are subjected to loads that have been adjusted upwards in accordance with the recommendations of the pre-2004 ASTM D 5055. These simulations indicate that such a procedure leads to higher probabilities of failure for assemblies associated with higher member COVs.

We note that our simulations are based on normal and lognormal data (both "truncated" and untruncated), a simple model of seven member assemblies, and a particular definition of failure. Thus we do not claim that they hold in general, and we do not believe that our estimates of failure probabilities should be applied to any real world situations. However, these results certainly call into question the validity of adjusting high-variability materials more than low-variability materials. Such adjustments cannot be based on intuition. If permitted, they must be based on detailed theoretical calculations, simulations, or empirical evidence.

2   Probability of failure of a single member

2.1   Intuition

In Section 1 we asserted that the probability, p_S, that an individual element fails when subjected to a load equal to its allowable strength level is larger for high-COV materials than for low-COV materials. Because this fact is central to our argument and might be counterintuitive for some readers, we provide some additional explanation here.

In developing this intuition, we assume these strengths are normally distributed. However, in our simulations, we work with untruncated normal and lognormal distributions, and with "truncated" normal and lognormal distributions.

For a normal strength distribution, the probability, p_S, that a particular specimen fails at its allowable strength level is the probability that the strength of the specimen lies below the fifth percentile of the strength distribution divided by 2.1 (the safety and duration of load factor). For

a more variable material (of the same mean strength), the fifth percentile will be lower so the allowable strength level will be smaller. One might assume that this will imply that the strength of a specimen from a more variable material will be less likely to lie below its allowable strength level. However, the more variable material has a greater standard deviation, and it turns out that its allowable strength level is not as many standard deviations from its mean as is the allowable strength level of a less variable material. In particular, for a specimen from a normal distribution with mean μ and standard deviation σ, we have

$$\text{allowable strength level} = (\mu - 1.645 \times \sigma)/2.1$$

and the number of standard deviations that the allowable strength level lies beneath the mean strength is

$$
\begin{aligned}
(\mu - (\mu - 1.645 \times \sigma)/2.1)/\sigma &= (1.1 \times \mu + 1.645 \times \sigma)/(2.1 \times \sigma) \\
&= 1.1/(2.1 \times \text{COV}) + 1.645/2.1
\end{aligned}
$$

where $\text{COV} = \sigma/\mu$. Thus the larger the COV, the closer (as measured in standard deviations) the allowable strength level is to the mean strength and the higher the probability that a specimen's strength will fall below the allowable strength level. This is made precise in Table 1 and is illustrated in Figure 1. In Figure 1 both normal distributions have mean 1. The broad distribution has standard deviation 0.25, and the narrow distribution has standard deviation 0.05. The dotted vertical lines are at the allowable strength level and fifth percentile of the broad distribution. The solid vertical lines are at the allowable strength level and fifth percentile of the narrow distribution. Visually, it should be clear that the strength of a specimen from the broad distribution ($\text{COV} = 0.25$) is more likely to fall below its allowable strength level than is the strength for a specimen from the narrow distribution ($\text{COV} = 0.05$). In fact, as we can see from Table 1, the allowable strength level for the broad distribution lies only 2.88 standard deviations below the distribution's mean, and the probability that a specimen from this distribution fails at its allowable strength level is 2.0E-3, while the allowable strength level for the narrow distribution lies 11.3 standard deviations below the distribution's mean, and the probability that a specimen from the narrow distribution fails at its allowable strength level is 1.0E-29.

In Subsections 2.2 and 2.3, we provide detailed formulas for p_S for normal and lognormal distributions in situations in which aging has reduced the original strength of the specimens.

2.2 Normal distribution

Let μ be the mean of the population and σ the standard deviation. Let $\text{COV} \equiv \sigma/\mu$ (rather than $\sigma/\mu \times 100$ as it is sometimes expressed). Suppose that a member has been in service for time t and that the strength reduction factor appropriate for this length of service is $r(t)$. Then, at time t the probability that a single member fails at its allowable strength level is given by

$$
\begin{aligned}
p_S &= \text{Prob}[\text{N}(\mu, \sigma^2)/r(t) < (\mu - 1.645 \times \sigma)/2.1] && (2) \\
&= \text{Prob}[\text{N}(0, 1) < (r(t)(\mu - 1.645 \times \sigma)/2.1 - \mu)/\sigma] \\
&= \text{Prob}[\text{N}(0, 1) < (r(t)/2.1 - 1)\mu/\sigma - r(t) \times 1.645/2.1] \\
&= \text{Prob}[\text{N}(0, 1) < (r(t)/2.1 - 1)/\text{COV} - r(t) \times 1.645/2.1]
\end{aligned}
$$

Note that for $r(t) < 2.1$, p_S increases as COV increases.

2.3 Lognormal distribution

Let μ and σ denote the mean and standard deviation of the population *after* natural logs have been taken. Let COV be the standard deviation of the *original* population divided by the mean of the original population. It can be shown that for lognormal data, $\sigma = \sqrt{\ln(1 + \text{COV} \times \text{COV})}$. Let $r(t)$ be as above. Then, at time t the probability that a single member fails at its allowable strength level is given by

$$
\begin{aligned}
p_S &= \text{Prob}[\text{LN}(\mu, \sigma^2)/r(t) < \exp(\mu - 1.645\sigma)/2.1] \\
&= \text{Prob}[\text{N}(\mu, \sigma^2) < \mu - 1.645\sigma + \ln(r(t)) - \ln(2.1)] \\
&= \text{Prob}[\text{N}(0, 1) < -1.645 + (\ln(r(t)) - \ln(2.1))/\sigma] \\
&= \text{Prob}\left[\text{N}(0, 1) < -1.645 + (\ln(r(t)) - \ln(2.1))/\sqrt{\ln(1 + \text{COV} \times \text{COV})}\right]
\end{aligned}
\tag{3}
$$

Note that, again, for $r(t) < 2.1$, p_S increases as COV increases.

2.4 "Truncated" distributions

Because real populations of boards sometimes have strength distributions that are left truncated (e.g., "number 2 and better"), we expanded our simulations to include "truncated" distributions. We did this as follows:

For a given correlation between modulus of elasticity (MOE) and modulus of rupture (MOR), we generated 100,000 samples from the bivariate normal distribution (in the lognormal case, the bivariate normal distribution was the joint distribution of the MOE and ln(MOR) values). We then identified the 60,000 boards with the largest MOEs (the top 60% of the MOEs) and treated them as MSR (machine stress rated) number 2 and better. We treated the 40,000 of these 60,000 with the lowest MOEs (the bottom two-thirds of the number 2 and better) as number 2 boards. We then found the nonparametric estimate of the fifth percentile of the MORs of these 40,000 boards, divided this by 2.1, and took this as our allowable strength property. When we generated assemblies we only accepted boards with MOEs above the 40th percentile of the MOE distribution. Of course, because the correlation between MOE and MOR is not 1, this led to boards with MORs that were sometimes below the 40th percentile of the MOR distribution. This led to MOR distributions that were not truly fully truncated. (For an MOE/MOR correlation of 1, the MOR distribution would be fully truncated. For an MOE/MOR correlation of 0, the "truncated" MOR distribution would not differ from the original MOR distribution. Intermediate levels of correlation lead to intermediate levels of "truncation.")

In Figure 2, we plot histograms of simulated MORs of MSR number 2 and better boards when MOR and MOE have bivariate normal distributions with COVs equal to 0.25 and correlations (top to bottom) of 1.0, 0.9, 0.6, and 0.0. In this figure we also mark allowable strength levels by solid vertical lines. In Figure 3, we present the corresponding plots for COVs equal to 0.05. These figures suggest that for "truncated" distributions, a board is more still more likely (except in the case in which the correlation is 1.0) to have a strength value below the allowable property level for a high-COV distribution than for a low-COV distribution. We report simulations based on "truncated" distributions in Section 4.

4

3 P_F for normally distributed, perfectly correlated strength and stiffness

In Subsections 2.2 and 2.3 we performed simple calculations that demonstrated that in the normal and lognormal cases, the probability of failure of a single member that is subjected to a load equal to its allowable property increases as COV increases. In this section we perform simple calculations that establish that, for a special case (bivariate normal MOE/MOR distribution, perfect correlation between MOE and MOR), the probability that an assembly fails at an unadjusted property level increases with COV. These calculations are merely meant to be suggestive. The increase in probability of failure with COV that they suggest has, however, been confirmed by the simulations reported in the remainder of the paper.

We have

$$P_F = \text{Prob(one or more failures)} = 1 - \text{Prob(no failure of any member)}$$

There is no failure of any member if and only if

$$(\mu + \epsilon_i)/r(t) \geq \frac{\mu + \epsilon_i}{\mu + \epsilon_1 + \ldots + \mu + \epsilon_k} \times \frac{k \times (\mu - 1.645 \times \sigma)}{2.1} \tag{4}$$

holds for $i = 1, \ldots, k$. Here, k is the number of members in the assembly, μ is the mean of the strength distribution of the members, the ϵ's are normally distributed with mean zero and standard deviation σ, $(\mu + \epsilon_i)/r(t)$ is the strength of the ith member, $(\mu + \epsilon_i)/(\mu + \epsilon_1 + \ldots + \mu + \epsilon_k)$ is the fraction of the load on the assembly that the ith element sees (under the assumption that strength and stiffness are perfectly correlated), and $k \times (\mu - 1.645 \times \sigma)/2.1$ is the total load on the assembly.

The k inequalities of (4) hold if and only if

$$1 \geq \frac{r(t)}{(\mu + \bar{\epsilon})} \times \frac{\mu - 1.645 \times \sigma}{2.1}$$

where $\bar{\epsilon} = (\epsilon_1 + \ldots + \epsilon_k)/k$. Thus,

$$
\begin{aligned}
\text{Prob(no failure of any member)} &= \text{Prob}\left(\mu + \bar{\epsilon} \geq \mu \times r(t) \times (1 - 1.645 \times \text{COV})/2.1\right) \tag{5} \\
&= \text{Prob}\left(\text{N}(\mu, \mu^2 \text{COV}^2/k) \geq \mu \times r(t) \times (1 - 1.645 \times \text{COV})/2.1\right) \\
&= \text{Prob}\left(\text{N}(0, \text{COV}^2/k) \geq r(t) \times (1 - 1.645 \times \text{COV})/2.1 - 1\right) \\
&= \text{Prob}\left(\text{N}(0,1) \geq \frac{\sqrt{k}}{\text{COV}} \times \left(r(t) \times \frac{1 - 1.645 \times \text{COV}}{2.1} - 1\right)\right) \\
&= \text{Prob}\left(\text{N}(0,1) \geq \sqrt{k} \times \left(\frac{1}{\text{COV}}\left(\frac{r(t)}{2.1} - 1\right) - \frac{r(t) \times 1.645}{2.1}\right)\right)
\end{aligned}
$$

and

$$\text{Prob(failure of at least one member)} \tag{6}$$

$$= \text{Prob}\left(\text{N}(0,1) < \sqrt{k} \times \left(\frac{1}{\text{COV}}\left(\frac{r(t)}{2.1} - 1\right) - \frac{r(t) \times 1.645}{2.1}\right)\right)$$

Provided that $r(t) < 2.1$, this probability will increase as COV increases.

4   Probability of failure of an assembly at an unadjusted allowable strength level

For the purposes of this paper, we consider a simple model of seven member assemblies. No claim is made that this is a highly realistic model or that the results can be extrapolated to arbitrary assemblies. However, the results from our simulations do suggest that assemblies of higher COV material do not deserve higher upward repetitive member adjustments.

4.1   The normal distribution version of the model

1. Member strengths are drawn from a normal distribution with mean μ, standard deviation σ, and coefficient of variation σ/μ.

2. Member strengths are reduced by a factor of $r(t)$. In this paper we consider $r(t) = 2.1$, 2.0, 1.9, 1.8, 1.7, 1.6, 1.5, and 1. The mean 50 year reduction factor due to duration of load is believed to be 1.6 (American Forest & Paper Association, 1997). We considered the other values in our simulations to establish the trends in P_F as a function of COV as the material ages. See Figures 4 through 7.

3. The assemblies contain seven members.

4. Each assembly is subjected to a load equal to seven times the allowable strength level of an individual element — $7(\mu - 1.645\sigma)/2.1$.

5. Because the assembly elements are connected by a diaphragm, they each bend the same distance, so loads are proportional to MOEs. In particular

$$\text{load}_i = (7(\mu - 1.645\sigma)/2.1) \times \text{MOE}_i/(\text{MOE}_1 + \ldots + \text{MOE}_7)$$

6. The correlation between strength and MOE is 0.5 or 0.7.

7. The failure of the assembly occurs when any member fails.

The FORTRAN code for this simulation can be obtained at

```
http://www1.fpl.fs.fed.us/repm.3.f
```

The results from this simulation depend on $r(t)$. Results for $r(t) = 2.1, \ldots, 1.5$ and $r(t) = 1$ are provided in Table 2. Table 2 contains the COV value, the corresponding probability of failure of a single member, p_S, the approximation to P_F of equation (1), and simulation estimates of P_F for $\rho = 0.5$ and $\rho = 0.7$ that take into account load sharing.

Here are the relevant facts to note from Table 2. (In Figure 4 we plot the log of the simulation estimate of P_F versus COV for $\rho = 0.7$ and $r(t) = 2.1, 2.0, \ldots, 1.5, 1.0$.)

1. As we would expect, due to load sharing, the simulation estimates of P_F are always lower than the weakest link approximation to P_F given by equation (1).

2. Also, as we would expect, P_F decreases as the correlation between MOE and MOR increases (as load sharing becomes more effective).

3. Except in the $r(t) = 2.1$ case, the simulation estimate of P_F increases with COV so we would expect that any upward adjustments of allowable strength level should be smaller for larger COVs.

6

4.2 Lognormal distribution version of the model

1. Member strengths are drawn from a $\mathrm{LN}(\mu, \sigma^2)$ distribution.

2. Member strengths are reduced by a factor of $r(t)$.

3. The assemblies contain seven members.

4. Each assembly is subjected to the load $7\exp(\mu - 1.645\sigma)/2.1$.

5. Because the assembly elements are connected by a diaphragm, they each bend the same distance, so loads are proportional to MOEs. In particular

$$\mathrm{load}_i = (7\exp(\mu - 1.645\sigma)/2.1) \times \mathrm{MOE}_i/(\mathrm{MOE}_1 + \ldots + \mathrm{MOE}_7)$$

6. The correlation between ln(strength) and MOE is 0.5 or 0.7.

7. The failure of the assembly occurs when any member fails.

The FORTRAN code for this simulation can be obtained at

```
http://www1.fpl.fs.fed.us/repm.3.ln.f
```

The results from this simulation are reported in Table 3. Table 3 is supplemented by Figure 5. The conclusions that can be drawn from Table 3 are essentially the same as those that can be drawn in the normal distribution case. The results do not support the assumption that higher COVs justify larger upward repetitive member adjustments in conditions that we see in practice.

4.3 "Truncated" normal distribution version of the model

1. Member strengths are drawn from a "truncated" (see Subsection 2.4) normal distribution.

2. Member strengths are reduced by a factor of $r(t)$.

3. The assemblies contain seven members.

4. Each assembly is subjected to the load $7 \times$ (5th percentile of number 2 strengths)/2.1.

5. Because the assembly elements are connected by a diaphragm, they each bend the same distance, so loads are proportional to MOEs. In particular

$$\mathrm{load}_i = (\mathrm{assembly\ load}) \times \mathrm{MOE}_i/(\mathrm{MOE}_1 + \ldots + \mathrm{MOE}_7)$$

6. The correlation between strength and MOE is 0.5 or 0.7.

7. The failure of the assembly occurs when any member fails.

The FORTRAN code for this simulation can be obtained at

```
http://www1.fpl.fs.fed.us/repm.4.f
```

The results from this simulation are reported in Table 4. The conclusions that can be drawn from Table 4 are essentially the same as those that can be drawn in the untruncated normal distribution case. The results do not support the assumption that higher COVs justify larger upward repetitive member adjustments in conditions that we see in practice.

4.4 "Truncated" lognormal distribution version of the model

1. Member strengths are drawn from a "truncated" (see Subsection 2.4) lognormal distribution.

2. Member strengths are reduced by a factor of $r(t)$.

3. The assemblies contain seven members.

4. Each assembly is subjected to the load $7 \times$ (5th percentile of number 2 strengths)/2.1.

5. Because the assembly elements are connected by a diaphragm, they each bend the same distance, so loads are proportional to MOEs. In particular

$$\text{load}_i = (\text{assembly load}) \times \text{MOE}_i/(\text{MOE}_1 + \ldots + \text{MOE}_7)$$

6. The correlation between ln(strength) and MOE is 0.5 or 0.7.

7. The failure of the assembly occurs when any member fails.

The FORTRAN code for this simulation can be obtained at

```
http://www1.fpl.fs.fed.us/repm.4.ln.f
```

The results from this simulation are reported in Table 5. The conclusions that can be drawn from Table 5 are essentially the same as those that can be drawn in the untruncated lognormal case. The results do not support the assumption that higher COVs justify larger upward repetitive member adjustments in conditions that we see in practice.

5 Probability of failure of an assembly at an adjusted allowable strength level

The fact that higher COV material is associated with a higher probability of failure at an unadjusted allowable strength level does not necessarily imply that it does not deserve a higher upward adjustment than a lower COV material. *A priori*, it is conceivable that the probability of failure of a lower COV material might increase more rapidly as load is increased than the probability of failure for a higher COV material. To check this in a practical situation we ran our model under the three conditions discussed in the pre-2004 ASTM D 5055. In particular the authors of that standard permitted a 15% upward adjustment for material with a 25% COV, a 7% upward adjustment for material with an 11% COV, and a 4% upward adjustment for material with a 7% COV. Our results for the two untruncated strength distributions (normal and lognormal) are presented in Tables 6 and 7. These tables are supplemented by Figures 6 and 7, which plot P_F versus COV for the normal and lognormal cases. Our results for the two "truncated" strength distributions are presented in Tables 8 and 9. The tables do not support the larger upward adjustments for larger COV material that were specified in the standard. Such adjustments lead to probabilities of failure that are often much larger for the high COV material than for the low COV material.

The FORTRAN code for these simulations can be obtained at

```
http://www1.fpl.fs.fed.us/1574.3.f
```

```
http://www1.fpl.fs.fed.us/1574.3.ln.f
```

8

```
http://www1.fpl.fs.fed.us/1574.4.f

http://www1.fpl.fs.fed.us/1574.4.ln.f
```

It is possible that the particular adjustments specified in pre-2004 ASTM D 5055 are inappropriate, but that upward adjustments should still be larger for larger COVs. The appropriate adjustments would certainly depend on the correct model, the correct distribution, the target failure probability, and $r(t)$. For the purposes of illustration, we consider $r(t) = 1.8$ and choose as our target failure probability the probability associated with a COV equal to 0.11 and an upward adjustment of 1.07. In Table 10 we report simulation results that indicate that the corresponding upward adjustment for a COV equal to 0.25 is smaller than 1.07, and the corresponding upward adjustment for a COV equal to 0.07 is larger than 1.07. In this case at least, the code had it backwards. Of course, the simulation model might be poor, but we believe that our work does call into question the validity of higher upward adjustments for higher COVs.

The FORTRAN code for these last simulations can be obtained at

```
http://www1.fpl.fs.fed.us/adj.3.f, and

http://www1.fpl.fs.fed.us/adj.3.ln.f

http://www1.fpl.fs.fed.us/adj.4.f, and

http://www1.fpl.fs.fed.us/adj.4.ln.f
```

6   An objection

A reader who is wedded to the idea that repetitive member adjustments should be larger for materials with a larger COV might make the following argument:

Suppose that individual elements have normally distributed bending moduli with mean μ and variance σ^2. Further suppose that the bending modulus of an assembly is the average of the bending moduli of its elements. Then the fifth percentile of the modulus distribution of an element is

$$\mu - 1.645 \times \sigma = \mu(1 - 1.645 \times \text{COV})$$

and the fifth percentile of the modulus distribution of an assembly of n elements is

$$\mu - 1.645 \times \sigma/\sqrt{n} = \mu(1 - 1.645 \times \text{COV}/\sqrt{n})$$

Thus the ratio of the fifth percentile of the modulus distribution of the assembly to the fifth percentile of the modulus distribution of an element is

$$(1 - 1.645 \times \text{COV}/\sqrt{n})/(1 - 1.645 \times \text{COV})$$

and a little calculus demonstrates that this ratio increases as COV increases. Thus, the argument goes, the repetitive member "adjustment" from the modulus of an element to the modulus of an assembly increases with COV.

There are two problems with this argument. The first and possibly more minor problem is that the modulus of an assembly is not in general the average of the moduli of its elements. In fact, if load transfer to stronger elements is limited (if the correlation between MOE and MOR is not high), the modulus of the assembly can be closer to the modulus of its weakest element

than to the average modulus of its elements. The second and more fundamental problem with this argument is that allowable properties are not fifth percentiles. Instead they are currently calculated as fifth percentiles divided by 2.1. Thus, since equivalent assembly designs and repetitive member adjustments should lead to equivalent probabilities of failure under equal loads, we must approach the problem as in the current paper, and as we saw in Subsection 2.1 and its sequel, the division by 2.1 leads to individual element probabilities of failure that are higher for more variable material. This in turn leads to assemblies that have higher probabilities of failure for more variable material, and thus we are not justified in awarding such assemblies higher repetitive member adjustments.

7   A second objection

It can be argued that the only age-related strength reduction that is relevant in our tables is $r(t) = 2.1$, and for this reduction, individual element probabilities of failure are 0.05 for all COVs, and assembly probabilities of failure do go down (initially) as COVs increase. One response to this argument is that we do not actually expect 2.1 reductions due to duration of load. Instead the mean 50-year reduction is believed to be 1.6 (American Forest & Paper Association, 1997). The 2.1 is achieved by multiplying by an additional "factor of safety" of 1.3. A second response is that we do not see failures with the frequency that would be predicted if we really had reductions on the order of 2.1. A third response is that even if we really believed that lifetime reductions in strength were on the order of 2.1, design engineers typically apply an additional 1.3 factor of safety (that does not take into account differences in COV) to the allowable property. In this case, even if we assume a 2.1 reduction in strength due to duration of load, we have (here we use the normal distribution for purposes of illustration)

$$
\begin{aligned}
p_S &= \text{Prob}[N(\mu, \sigma^2)/2.1 < ((\mu - 1.645 \times \sigma)/2.1)/1.3] \\
&= \text{Prob}[N(\mu, \sigma^2)(1.3/2.1) < (\mu - 1.645 \times \sigma)/2.1] \\
&\approx \text{Prob}[N(\mu, \sigma^2)/1.6 < (\mu - 1.645 \times \sigma)/2.1]
\end{aligned}
$$

so the appropriate sections of our tables are those in which $r(t) = 1.6$. In these sections of the tables, P_F increases as COV increases.

8   The wrong lesson

In questioning the belief that higher COV material deserves a higher upward repetitive member upward adjustment, we are *not* advancing an argument for upward adjustments in low-COV material that are higher than allowed under current standards. As we noted earlier, our simulations are based on a simple model of an assembly, and although they are sufficient to cast doubt on the belief that higher upward adjustments are justified for higher COV material, they are not sufficient to identify proper upward adjustments (if any).

9   Summary

We have demonstrated that, for at least one hypothetical assembly, from a reliability[1] standpoint, upward repetitive member adjustments that are larger for more variable material are not justified. On the contrary, our simulations suggest that given the manner in which individual element

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[1] In the simulations reported here, we assumed that the load was fixed at the allowable property. More complete simulations would replace the fixed load with a load distribution whose 99th (for example) percentile was set equal to the allowable property.

allowable strength levels are currently calculated, as the coefficient of variation of the strength distribution of individual elements increases, the upward repetitive member adjustments (if any) of assemblies constructed from these elements should decrease.

These results stem from the fact that under current standards, individual element allowable strength levels associated with higher COV material correspond to larger probabilities of assembly failure. This, in turn, follows from the fact that allowable strength levels are not as many standard deviations below mean strength for a high COV material as for a low COV material. Thus, the probability, p_S, that an individual element fails when subjected to a load equal to its allowable strength level is larger for members from strength distributions with larger COVs. This leads to a larger P_F, the probability of at least one member failure in an assembly.

Our results should not be used to argue for a particular upward adjustment in the standards. Improved analytical models and additional empirical testing would be required to establish appropriate (if any) upward repetitive member adjustments.

References

American Forest & Paper Association. 1997. Commentary. National Design Specification for Wood Construction. Washington, DC: American Forest & Paper Association.

ASTM. 2003. Standard specification for establishing and monitoring structural capacities of pre-fabricated wood I-Joists. D 5055-03. West Conshohocken, PA: ASTM International.

ASTM. 2007a. Standard practice for establishing structural grades and related allowable properties for visually graded lumber. D 245-06. West Conshohocken, PA: ASTM International.

ASTM. 2007b. Standard practice for establishing allowable properties for visually graded dimension lumber from in-grade tests of full-size specimens. D 1990-00. West Conshohocken, PA: ASTM International.

ASTM. 2007c. Standard specification for establishing and monitoring structural capacities of prefabricated wood I-Joists. D 5055-05. West Conshohocken, PA: ASTM International.

ASTM. 2007d. Standard guide for evaluating system effects in repetitive-member wood assemblies. D 6555-03. West Conshohocken, PA: ASTM International.

COV	Fifth percentile	Allowable strength	Standard deviations below mean	p_S
0.05	0.918	0.437	11.26	0.104E-28
0.10	0.836	0.398	6.02	0.864E-09
0.15	0.753	0.359	4.28	0.954E-05
0.20	0.671	0.320	3.40	0.334E-03
0.25	0.589	0.280	2.88	0.200E-02

Table 1: For a normal population of mean 1, and a series of coefficients of variation, this table provides the fifth percentile of the distribution, the associated allowable strength level (fifth percentile divided by 2.1), the number of standard deviations that the allowable strength level lies below the mean of the distribution, and the probability, p_s that a member of the normal population will have a strength value that falls below the allowable strength level.

$r(t)$	COV	p_S	Weakest link[1]	Load sharing[2] $\rho = 0.5$	$\rho = 0.7$
	0.05	0.500E−01	0.302E+00	0.271E+00	0.119E+00
	0.10	0.500E−01	0.302E+00	0.247E+00	0.108E+00
	0.15	0.500E−01	0.302E+00	0.228E+00	0.101E+00
2.1	0.20	0.500E−01	0.302E+00	0.214E+00	0.995E−01
	0.25	0.500E−01	0.302E+00	0.204E+00	0.102E+00
	0.30	0.500E−01	0.302E+00	0.199E+00	0.110E+00
	0.35	0.500E−01	0.302E+00	0.201E+00	0.124E+00
	0.40	0.500E−01	0.302E+00	0.208E+00	0.145E+00
	0.05	0.588E−02	0.405E−01	0.289E−01	0.438E−02
	0.10	0.205E−01	0.135E+00	0.941E−01	0.271E−01
	0.15	0.298E−01	0.191E+00	0.127E+00	0.454E−01
2.0	0.20	0.356E−01	0.224E+00	0.144E+00	0.593E−01
	0.25	0.394E−01	0.246E+00	0.156E+00	0.729E−01
	0.30	0.422E−01	0.261E+00	0.165E+00	0.887E−01
	0.35	0.443E−01	0.272E+00	0.177E+00	0.109E+00
	0.40	0.459E−01	0.280E+00	0.191E+00	0.135E+00
	0.05	0.346E−03	0.242E−02	0.110E−02	0.470E−04
	0.10	0.733E−02	0.502E−01	0.278E−01	0.492E−02
	0.15	0.169E−01	0.112E+00	0.642E−01	0.185E−01
1.9	0.20	0.247E−01	0.161E+00	0.933E−01	0.346E−01
	0.25	0.308E−01	0.197E+00	0.116E+00	0.514E−01
	0.30	0.355E−01	0.223E+00	0.135E+00	0.714E−01
	0.35	0.392E−01	0.244E+00	0.155E+00	0.948E−01
	0.40	0.421E−01	0.260E+00	0.177E+00	0.125E+00

Table 2: Failure probabilities of single members (p_S) and of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to load $7(\mu - 1.645\sigma)/2.1$, **normal** case.

[1]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[2]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

$r(t)$	COV	p_S	P_F		
				Load sharing[2]	
			Weakest link[1]	$\rho = 0.5$	$\rho = 0.7$
1.8	0.05	0.989E−05	0.693E−04	0.100E−04	0.000E+00
	0.10	0.227E−02	0.158E−01	0.656E−02	0.723E−03
	0.15	0.908E−02	0.618E−01	0.301E−01	0.675E−02
	0.20	0.168E−01	0.112E+00	0.580E−01	0.194E−01
	0.25	0.238E−01	0.155E+00	0.850E−01	0.361E−01
	0.30	0.296E−01	0.190E+00	0.111E+00	0.574E−01
	0.35	0.345E−01	0.218E+00	0.136E+00	0.843E−01
	0.40	0.386E−01	0.241E+00	0.163E+00	0.117E+00
1.7	0.05	0.119E−06	0.834E−06	0.000E+00	0.000E+00
	0.10	0.605E−03	0.423E−02	0.125E−02	0.730E−04
	0.15	0.464E−02	0.320E−01	0.129E−01	0.217E−02
	0.20	0.112E−01	0.757E−01	0.350E−01	0.104E−01
	0.25	0.181E−01	0.120E+00	0.617E−01	0.251E−01
	0.30	0.246E−01	0.160E+00	0.897E−01	0.466E−01
	0.35	0.303E−01	0.194E+00	0.119E+00	0.742E−01
	0.40	0.353E−01	0.222E+00	0.150E+00	0.108E+00
1.6	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.139E−03	0.975E−03	0.177E−03	0.700E−05
	0.15	0.225E−02	0.157E−01	0.511E−02	0.750E−03
	0.20	0.727E−02	0.498E−01	0.202E−01	0.562E−02
	0.25	0.137E−01	0.921E−01	0.445E−01	0.174E−01
	0.30	0.203E−01	0.134E+00	0.724E−01	0.376E−01
	0.35	0.266E−01	0.172E+00	0.105E+00	0.652E−01
	0.40	0.323E−01	0.205E+00	0.138E+00	0.101E+00

Table 2 continued: Failure probabilities of single members (p_S) and of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to load $7(\mu - 1.645\sigma)/2.1$, **normal** case.

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[1]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[2]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

$r(t)$	COV	p_S	P_F		
				Load sharing[2]	
			Weakest link[1]	$\rho = 0.5$	$\rho = 0.7$
1.5	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.277E−04	0.194E−03	0.200E−04	0.000E+00
	0.15	0.104E−02	0.723E−02	0.182E−02	0.207E−03
	0.20	0.461E−02	0.318E−01	0.118E−01	0.290E−02
	0.25	0.102E−01	0.694E−01	0.312E−01	0.122E−01
	0.30	0.167E−01	0.111E+00	0.587E−01	0.303E−01
	0.35	0.232E−01	0.152E+00	0.911E−01	0.578E−01
	0.40	0.294E−01	0.189E+00	0.127E+00	0.941E−01
1.0	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.954E−05	0.668E−04	0.110E−04	0.000E+00
	0.20	0.334E−03	0.234E−02	0.534E−03	0.143E−03
	0.25	0.200E−02	0.139E−01	0.528E−02	0.223E−02
	0.30	0.571E−02	0.393E−01	0.196E−01	0.116E−01
	0.35	0.113E−01	0.765E−01	0.470E−01	0.332E−01
	0.40	0.182E−01	0.121E+00	0.854E−01	0.692E−01

Table 2 continued: Failure probabilities of single members (p_S) and of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to load $7(\mu - 1.645\sigma)/2.1$, **normal** case.

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[1]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[2]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

$r(t)$	COV	p_S	P_F Weakest link[1]	Load sharing[2] $\rho = 0.5$	Load sharing[2] $\rho = 0.7$
2.1	0.05	0.500E−01	0.302E+00	0.287E+00	0.127E+00
	0.10	0.500E−01	0.302E+00	0.277E+00	0.121E+00
	0.15	0.500E−01	0.302E+00	0.267E+00	0.117E+00
	0.20	0.500E−01	0.302E+00	0.260E+00	0.114E+00
	0.25	0.500E−01	0.302E+00	0.255E+00	0.112E+00
	0.30	0.500E−01	0.302E+00	0.250E+00	0.110E+00
	0.35	0.500E−01	0.302E+00	0.246E+00	0.109E+00
	0.40	0.500E−01	0.302E+00	0.242E+00	0.107E+00
2.0	0.05	0.438E−02	0.302E−01	0.272E−01	0.393E−02
	0.10	0.164E−01	0.109E+00	0.970E−01	0.253E−01
	0.15	0.243E−01	0.158E+00	0.136E+00	0.428E−01
	0.20	0.293E−01	0.188E+00	0.157E+00	0.544E−01
	0.25	0.327E−01	0.207E+00	0.171E+00	0.624E−01
	0.30	0.351E−01	0.221E+00	0.179E+00	0.676E−01
	0.35	0.368E−01	0.231E+00	0.185E+00	0.722E−01
	0.40	0.382E−01	0.239E+00	0.189E+00	0.751E−01
1.9	0.05	0.132E−03	0.925E−03	0.741E−03	0.140E−04
	0.10	0.404E−02	0.280E−01	0.235E−01	0.326E−02
	0.15	0.103E−01	0.698E−01	0.576E−01	0.123E−01
	0.20	0.158E−01	0.105E+00	0.851E−01	0.226E−01
	0.25	0.201E−01	0.133E+00	0.106E+00	0.318E−01
	0.30	0.235E−01	0.153E+00	0.122E+00	0.394E−01
	0.35	0.262E−01	0.170E+00	0.133E+00	0.456E−01
	0.40	0.284E−01	0.183E+00	0.142E+00	0.502E−01

Table 3: Failure probabilities of single members (p_S) and of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to load $7\exp(\mu - 1.645\sigma)/2.1$, **lognormal** case.

[1]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[2]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

$r(t)$	COV	p_S	P_F		
				Load sharing[2]	
			Weakest link[1]	$\rho = 0.5$	$\rho = 0.7$
	0.05	0.113E−05	0.793E−05	0.900E−05	0.000E+00
	0.10	0.711E−03	0.496E−02	0.380E−02	0.261E−03
	0.15	0.370E−02	0.256E−01	0.202E−01	0.275E−02
1.8	0.20	0.769E−02	0.526E−01	0.415E−01	0.802E−02
	0.25	0.116E−01	0.782E−01	0.607E−01	0.144E−01
	0.30	0.150E−01	0.100E+00	0.779E−01	0.209E−01
	0.35	0.179E−01	0.119E+00	0.920E−01	0.268E−01
	0.40	0.204E−01	0.134E+00	0.103E+00	0.321E−01
	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.839E−04	0.587E−03	0.436E−03	0.110E−04
	0.15	0.110E−02	0.768E−02	0.563E−02	0.443E−03
1.7	0.20	0.334E−02	0.232E−01	0.176E−01	0.245E−02
	0.25	0.615E−02	0.423E−01	0.320E−01	0.584E−02
	0.30	0.902E−02	0.615E−01	0.460E−01	0.102E−01
	0.35	0.117E−01	0.791E−01	0.597E−01	0.148E−01
	0.40	0.141E−01	0.949E−01	0.709E−01	0.193E−01
	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.620E−05	0.434E−04	0.340E−04	0.000E+00
	0.15	0.262E−03	0.183E−02	0.123E−02	0.470E−04
1.6	0.20	0.127E−02	0.887E−02	0.639E−02	0.594E−03
	0.25	0.299E−02	0.207E−01	0.151E−01	0.209E−02
	0.30	0.507E−02	0.349E−01	0.254E−01	0.451E−02
	0.35	0.724E−02	0.496E−01	0.365E−01	0.754E−02
	0.40	0.937E−02	0.637E−01	0.471E−01	0.109E−01

Table 3 continued: Failure probabilities of single members (p_S) and of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to load $7\exp(\mu - 1.645\sigma)/2.1$, **lognormal** case.

[1]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[2]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

| $r(t)$ | COV | p_S | | P_F | |
| | | | | Load sharing[2] | |
			Weakest link[1]	$\rho = 0.5$	$\rho = 0.7$
1.5	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.238E−06	0.167E−05	0.000E+00	0.000E+00
	0.15	0.480E−04	0.336E−03	0.217E−03	0.000E+00
	0.20	0.413E−03	0.289E−02	0.195E−02	0.124E−03
	0.25	0.130E−02	0.906E−02	0.622E−02	0.614E−03
	0.30	0.263E−02	0.182E−01	0.130E−01	0.175E−02
	0.35	0.421E−02	0.291E−01	0.211E−01	0.357E−02
	0.40	0.589E−02	0.405E−01	0.293E−01	0.579E−02
1.0	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.20	0.596E−07	0.417E−06	0.000E+00	0.000E+00
	0.25	0.161E−05	0.113E−04	0.500E−05	0.000E+00
	0.30	0.151E−04	0.106E−03	0.470E−04	0.100E−05
	0.35	0.647E−04	0.453E−03	0.277E−03	0.700E−05
	0.40	0.178E−03	0.125E−02	0.776E−03	0.440E−04

Table 3 continued: Failure probabilities of single members (p_S) and of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to load $7\exp(\mu - 1.645\sigma)/2.1$, **lognormal** case.

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[1]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[2]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

		$\rho = 0.5$			$\rho = 0.7$		
			P_F				P_F	
$r(t)$	COV	p_S[1]	Weakest link[2]	Load sharing[3]	p_S[1]	Weakest link[2]	Load sharing[3]
	0.05	0.337E−01	0.213E+00	0.205E+00	0.330E−01	0.209E+00	0.138E+00
	0.10	0.339E−01	0.214E+00	0.194E+00	0.330E−01	0.209E+00	0.133E+00
	0.15	0.338E−01	0.214E+00	0.185E+00	0.330E−01	0.209E+00	0.130E+00
2.1	0.20	0.337E−01	0.213E+00	0.180E+00	0.330E−01	0.210E+00	0.128E+00
	0.25	0.336E−01	0.213E+00	0.177E+00	0.330E−01	0.210E+00	0.130E+00
	0.30	0.338E−01	0.214E+00	0.177E+00	0.330E−01	0.210E+00	0.132E+00
	0.35	0.338E−01	0.214E+00	0.177E+00	0.329E−01	0.209E+00	0.135E+00
	0.40	0.337E−01	0.213E+00	0.179E+00	0.330E−01	0.209E+00	0.139E+00
	0.05	0.247E−02	0.171E−01	0.155E−01	0.134E−02	0.934E−02	0.392E−02
	0.10	0.112E−01	0.758E−01	0.643E−01	0.855E−02	0.583E−01	0.294E−01
	0.15	0.175E−01	0.116E+00	0.953E−01	0.146E−01	0.977E−01	0.523E−01
2.0	0.20	0.216E−01	0.142E+00	0.115E+00	0.188E−01	0.125E+00	0.694E−01
	0.25	0.247E−01	0.161E+00	0.130E+00	0.218E−01	0.143E+00	0.830E−01
	0.30	0.266E−01	0.172E+00	0.140E+00	0.240E−01	0.156E+00	0.944E−01
	0.35	0.283E−01	0.182E+00	0.150E+00	0.257E−01	0.167E+00	0.104E+00
	0.40	0.295E−01	0.189E+00	0.158E+00	0.271E−01	0.175E+00	0.114E+00
	0.05	0.713E−04	0.499E−03	0.425E−03	0.123E−04	0.859E−04	0.220E−04
	0.10	0.309E−02	0.214E−01	0.166E−01	0.163E−02	0.113E−01	0.452E−02
	0.15	0.850E−02	0.580E−01	0.451E−01	0.578E−02	0.397E−01	0.183E−01
1.9	0.20	0.134E−01	0.903E−01	0.700E−01	0.101E−01	0.689E−01	0.352E−01
	0.25	0.175E−01	0.116E+00	0.915E−01	0.140E−01	0.939E−01	0.514E−01
	0.30	0.208E−01	0.137E+00	0.109E+00	0.172E−01	0.114E+00	0.664E−01
	0.35	0.235E−01	0.153E+00	0.125E+00	0.198E−01	0.130E+00	0.800E−01
	0.40	0.258E−01	0.167E+00	0.139E+00	0.220E−01	0.144E+00	0.928E−01

Table 4: Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor $r(t)$, the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, **"truncated normal"** case.

[1] Monte Carlo result based on 7,000,000 simulated members.

[2] Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[3] Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

		$\rho = 0.5$			$\rho = 0.7$		
			P_F			P_F	
$r(t)$	COV	p_S[1]	Weakest link[2]	Load sharing[3]	p_S[1]	Weakest link[2]	Load sharing[3]
	0.05	0.114E−05	0.793E−05	0.300E−05	0.000E+00	0.000E+00	0.000E+00
	0.10	0.688E−03	0.481E−02	0.340E−02	0.232E−03	0.162E−02	0.487E−03
	0.15	0.375E−02	0.259E−01	0.190E−01	0.201E−02	0.140E−01	0.575E−02
1.8	0.20	0.807E−02	0.551E−01	0.409E−01	0.516E−02	0.356E−01	0.166E−01
	0.25	0.124E−01	0.835E−01	0.638E−01	0.864E−02	0.589E−01	0.307E−01
	0.30	0.162E−01	0.108E+00	0.845E−01	0.121E−01	0.816E−01	0.458E−01
	0.35	0.195E−01	0.129E+00	0.104E+00	0.150E−01	0.101E+00	0.606E−01
	0.40	0.223E−01	0.146E+00	0.121E+00	0.177E−01	0.117E+00	0.746E−01
	0.05	0.000E+00	0.000E+00	0.100E−05	0.000E+00	0.000E+00	0.000E+00
	0.10	0.125E−03	0.878E−03	0.575E−03	0.250E−04	0.175E−03	0.370E−04
	0.15	0.162E−02	0.113E−01	0.771E−02	0.639E−03	0.447E−02	0.161E−02
1.7	0.20	0.466E−02	0.322E−01	0.232E−01	0.250E−02	0.173E−01	0.735E−02
	0.25	0.851E−02	0.581E−01	0.434E−01	0.523E−02	0.361E−01	0.177E−01
	0.30	0.123E−01	0.832E−01	0.646E−01	0.834E−02	0.570E−01	0.312E−01
	0.35	0.160E−01	0.107E+00	0.855E−01	0.113E−01	0.766E−01	0.454E−01
	0.40	0.193E−01	0.127E+00	0.105E+00	0.142E−01	0.955E−01	0.608E−01
	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.211E−04	0.148E−03	0.740E−04	0.129E−05	0.918E−05	0.200E−05
	0.15	0.611E−03	0.427E−02	0.271E−02	0.177E−03	0.124E−02	0.370E−03
1.6	0.20	0.263E−02	0.183E−01	0.126E−01	0.113E−02	0.787E−02	0.315E−02
	0.25	0.577E−02	0.397E−01	0.290E−01	0.304E−02	0.211E−01	0.992E−02
	0.30	0.943E−02	0.641E−01	0.490E−01	0.562E−02	0.387E−01	0.204E−01
	0.35	0.130E−01	0.876E−01	0.697E−01	0.845E−02	0.577E−01	0.338E−01
	0.40	0.166E−01	0.111E+00	0.915E−01	0.113E−01	0.766E−01	0.483E−01

Table 4 continued: Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor $r(t)$, the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, **"truncated normal"** case.

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[1]Monte Carlo result based on 7,000,000 simulated members.

[2]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[3]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

$r(t)$	COV	$\rho = 0.5$			$\rho = 0.7$		
			P_F			P_F	
		p_S[1]	Weakest link[2]	Load sharing[3]	p_S[1]	Weakest link[2]	Load sharing[3]
1.5	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.243E−05	0.171E−04	0.100E−04	0.143E−06	0.834E−06	0.000E+00
	0.15	0.213E−03	0.149E−02	0.896E−03	0.436E−04	0.305E−03	0.790E−04
	0.20	0.142E−02	0.991E−02	0.647E−02	0.479E−03	0.335E−02	0.127E−02
	0.25	0.380E−02	0.263E−01	0.188E−01	0.170E−02	0.118E−01	0.528E−02
	0.30	0.701E−02	0.481E−01	0.364E−01	0.368E−02	0.254E−01	0.133E−01
	0.35	0.106E−01	0.722E−01	0.575E−01	0.621E−02	0.427E−01	0.248E−01
	0.40	0.142E−01	0.953E−01	0.785E−01	0.890E−02	0.606E−01	0.382E−01
1.0	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.571E−06	0.417E−05	0.300E−05	0.000E+00	0.000E+00	0.000E+00
	0.20	0.414E−04	0.290E−03	0.161E−03	0.243E−05	0.171E−04	0.300E−05
	0.25	0.386E−03	0.270E−02	0.185E−02	0.639E−04	0.447E−03	0.196E−03
	0.30	0.145E−02	0.101E−01	0.760E−02	0.356E−03	0.249E−02	0.131E−02
	0.35	0.343E−02	0.238E−01	0.191E−01	0.112E−02	0.783E−02	0.477E−02
	0.40	0.633E−02	0.435E−01	0.367E−01	0.246E−02	0.171E−01	0.114E−01

Table 4 continued: Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor $r(t)$, the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, **"truncated normal"** case.

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[1] Monte Carlo result based on 7,000,000 simulated members.

[2] Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[3] Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are smaller than the weakest link estimates.

| $r(t)$ | COV | $\rho = 0.5$ | | | $\rho = 0.7$ | | |
| | | | P_F | | | P_F | |
		p_S[1]	Weakest link[2]	Load sharing[3]	p_S[1]	Weakest link[2]	Load sharing[3]
2.1	0.05	0.342E−01	0.216E+00	0.213E+00	0.330E−01	0.209E+00	0.139E+00
	0.10	0.343E−01	0.217E+00	0.204E+00	0.330E−01	0.209E+00	0.133E+00
	0.15	0.345E−01	0.218E+00	0.198E+00	0.330E−01	0.209E+00	0.129E+00
	0.20	0.342E−01	0.216E+00	0.191E+00	0.330E−01	0.210E+00	0.125E+00
	0.25	0.342E−01	0.216E+00	0.187E+00	0.330E−01	0.210E+00	0.122E+00
	0.30	0.343E−01	0.217E+00	0.182E+00	0.330E−01	0.210E+00	0.120E+00
	0.35	0.342E−01	0.216E+00	0.178E+00	0.329E−01	0.209E+00	0.118E+00
	0.40	0.342E−01	0.216E+00	0.176E+00	0.330E−01	0.209E+00	0.117E+00
2.0	0.05	0.179E−02	0.125E−01	0.131E−01	0.978E−03	0.682E−02	0.301E−02
	0.10	0.905E−02	0.616E−01	0.580E−01	0.682E−02	0.468E−01	0.239E−01
	0.15	0.145E−01	0.972E−01	0.868E−01	0.119E−01	0.805E−01	0.425E−01
	0.20	0.182E−01	0.120E+00	0.104E+00	0.156E−01	0.104E+00	0.549E−01
	0.25	0.207E−01	0.136E+00	0.114E+00	0.182E−01	0.121E+00	0.638E−01
	0.30	0.225E−01	0.147E+00	0.122E+00	0.201E−01	0.133E+00	0.699E−01
	0.35	0.239E−01	0.156E+00	0.126E+00	0.216E−01	0.142E+00	0.735E−01
	0.40	0.248E−01	0.161E+00	0.129E+00	0.227E−01	0.149E+00	0.776E−01
1.9	0.05	0.251E−04	0.176E−03	0.212E−03	0.329E−05	0.229E−04	0.700E−05
	0.10	0.165E−02	0.115E−01	0.110E−01	0.835E−03	0.583E−02	0.246E−02
	0.15	0.516E−02	0.356E−01	0.311E−01	0.343E−02	0.238E−01	0.105E−01
	0.20	0.858E−02	0.585E−01	0.498E−01	0.642E−02	0.441E−01	0.205E−01
	0.25	0.115E−01	0.779E−01	0.646E−01	0.913E−02	0.622E−01	0.295E−01
	0.30	0.140E−01	0.937E−01	0.752E−01	0.114E−01	0.773E−01	0.372E−01
	0.35	0.159E−01	0.106E+00	0.833E−01	0.133E−01	0.895E−01	0.434E−01
	0.40	0.175E−01	0.116E+00	0.903E−01	0.149E−01	0.100E+00	0.486E−01

Table 5: Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor $r(t)$, the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, **"truncated lognormal"** case.

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[1] Monte Carlo result based on 7,000,000 simulated members.

[2] Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[3] Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are generally smaller than the weakest link estimates.

| $r(t)$ | COV | $\rho = 0.5$ | | | $\rho = 0.7$ | | |
| | | | P_F | | | P_F | |
		p_S[1]	Weakest link[2]	Load sharing[3]	p_S[1]	Weakest link[2]	Load sharing[3]
1.8	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.188E−03	0.132E−02	0.133E−02	0.567E−04	0.397E−03	0.125E−03
	0.15	0.150E−02	0.104E−01	0.920E−02	0.748E−03	0.523E−02	0.197E−02
	0.20	0.365E−02	0.252E−01	0.208E−01	0.223E−02	0.155E−01	0.630E−02
	0.25	0.595E−02	0.409E−01	0.328E−01	0.407E−02	0.281E−01	0.118E−01
	0.30	0.811E−02	0.554E−01	0.438E−01	0.599E−02	0.412E−01	0.180E−01
	0.35	0.101E−01	0.686E−01	0.530E−01	0.771E−02	0.527E−01	0.231E−01
	0.40	0.117E−01	0.793E−01	0.607E−01	0.926E−02	0.631E−01	0.282E−01
1.7	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.147E−04	0.103E−03	0.910E−04	0.229E−05	0.159E−04	0.300E−05
	0.15	0.330E−03	0.230E−02	0.203E−02	0.118E−03	0.824E−03	0.255E−03
	0.20	0.135E−02	0.940E−02	0.767E−02	0.632E−03	0.442E−02	0.156E−02
	0.25	0.275E−02	0.191E−01	0.151E−01	0.157E−02	0.109E−01	0.408E−02
	0.30	0.437E−02	0.302E−01	0.235E−01	0.285E−02	0.197E−01	0.782E−02
	0.35	0.599E−02	0.412E−01	0.313E−01	0.416E−02	0.287E−01	0.118E−01
	0.40	0.759E−02	0.519E−01	0.387E−01	0.549E−02	0.378E−01	0.157E−01
1.6	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.286E−06	0.209E−05	0.700E−05	0.000E+00	0.000E+00	0.000E+00
	0.15	0.584E−04	0.409E−03	0.388E−03	0.123E−04	0.859E−04	0.260E−04
	0.20	0.418E−03	0.292E−02	0.226E−02	0.149E−03	0.104E−02	0.274E−03
	0.25	0.113E−02	0.788E−02	0.618E−02	0.521E−03	0.364E−02	0.116E−02
	0.30	0.217E−02	0.151E−01	0.113E−01	0.120E−02	0.834E−02	0.289E−02
	0.35	0.341E−02	0.236E−01	0.175E−01	0.204E−02	0.142E−01	0.527E−02
	0.40	0.461E−02	0.318E−01	0.231E−01	0.297E−02	0.206E−01	0.782E−02

Table 5 continued: Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor $r(t)$, the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, **"truncated lognormal"** case.

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[1]Monte Carlo result based on 7,000,000 simulated members.

[2]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[3]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are generally smaller than the weakest link estimates.

| $r(t)$ | COV | $\rho = 0.5$ | | | $\rho = 0.7$ | | |
| | | | P_F | | | P_F | |
		p_S[1]	Weakest link[2]	Load sharing[3]	p_S[1]	Weakest link[2]	Load sharing[3]
1.5	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.857E−05	0.601E−04	0.310E−04	0.857E−06	0.584E−05	0.100E−05
	0.20	0.101E−03	0.705E−03	0.538E−03	0.259E−04	0.181E−03	0.530E−04
	0.25	0.428E−03	0.300E−02	0.224E−02	0.136E−03	0.953E−03	0.278E−03
	0.30	0.100E−02	0.700E−02	0.494E−02	0.443E−03	0.310E−02	0.941E−03
	0.35	0.175E−02	0.122E−01	0.893E−02	0.903E−03	0.630E−02	0.206E−02
	0.40	0.264E−02	0.183E−01	0.131E−01	0.149E−02	0.104E−01	0.351E−02
1.0	0.05	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.10	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.15	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.20	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.25	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	0.30	0.571E−06	0.417E−05	0.800E−05	0.000E+00	0.000E+00	0.000E+00
	0.35	0.114E−04	0.801E−04	0.410E−04	0.143E−05	0.100E−04	0.300E−05
	0.40	0.343E−04	0.240E−03	0.165E−03	0.514E−05	0.359E−04	0.120E−04

Table 5 continued: Failure probabilities of single members (p_S) and seven member assemblies (P_F) when the strengths of the single members have been reduced by the factor $r(t)$, the single members are subjected to a load equal to their allowable property level, and the assemblies are subjected to a load equal to seven times the allowable property level for a single member, **"truncated lognormal"** case.

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[1]Monte Carlo result based on 7,000,000 simulated members.

[2]Weakest link — $P_F = 1 - (1 - p_S)^7$. This assumes that all members see the same load.

[3]Load sharing — Monte Carlo result based on 1,000,000 simulated assemblies. Due to load sharing, these simulation estimates of P_F are generally smaller than the weakest link estimates.

$r(t)$	COV	P_F[1]	
		$\rho = 0.5$	$\rho = 0.7$
2.1	0.07	0.584E+00	0.390E+00
	0.11	0.580E+00	0.390E+00
	0.25	0.417E+00	0.258E+00
2.0	0.07	0.203E+00	0.786E−01
	0.11	0.323E+00	0.160E+00
	0.25	0.330E+00	0.189E+00
1.9	0.07	0.381E−01	0.699E−02
	0.11	0.139E+00	0.484E−01
	0.25	0.253E+00	0.134E+00
1.8	0.07	0.409E−02	0.312E−03
	0.11	0.479E−01	0.109E−01
	0.25	0.188E+00	0.923E−01
1.7	0.07	0.264E−03	0.900E−05
	0.11	0.130E−01	0.189E−02
	0.25	0.137E+00	0.627E−01
1.6	0.07	0.120E−04	0.000E+00
	0.11	0.291E−02	0.264E−03
	0.25	0.964E−01	0.419E−01
1.5	0.07	0.100E−05	0.000E+00
	0.11	0.516E−03	0.220E−04
	0.25	0.675E−01	0.277E−01
1.0	0.07	0.000E+00	0.000E+00
	0.11	0.000E+00	0.000E+00
	0.25	0.887E−02	0.354E−02

Table 6: Failure probabilities of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to loads adjusted in accordance with pre-2004 ASTM D 5055, **normal** case.

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[1]Monte Carlo result based on 1,000,000 simulated assemblies. The simulation takes into account bot¡h load sharing and the upward adjusted loads.

$r(t)$	COV	P_F[1]	
		$\rho = 0.5$	$\rho = 0.7$
	0.07	0.628E+00	0.433E+00
2.1	0.11	0.653E+00	0.462E+00
	0.25	0.594E+00	0.399E+00
	0.07	0.218E+00	0.839E−01
2.0	0.11	0.369E+00	0.188E+00
	0.25	0.465E+00	0.273E+00
	0.07	0.364E−01	0.616E−02
1.9	0.11	0.151E+00	0.494E−01
	0.25	0.339E+00	0.170E+00
	0.07	0.298E−02	0.175E−03
1.8	0.11	0.436E−01	0.800E−02
	0.25	0.228E+00	0.939E−01
	0.07	0.940E−04	0.100E−05
1.7	0.11	0.858E−02	0.775E−03
	0.25	0.140E+00	0.470E−01
	0.07	0.200E−05	0.000E+00
1.6	0.11	0.115E−02	0.470E−04
	0.25	0.770E−01	0.203E−01
	0.07	0.000E+00	0.000E+00
1.5	0.11	0.980E−04	0.100E−05
	0.25	0.378E−01	0.743E−02
	0.07	0.000E+00	0.000E+00
1.0	0.11	0.000E+00	0.000E+00
	0.25	0.102E−03	0.300E−05

Table 7: Failure probabilities of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to loads adjusted in accordance with pre-2004 ASTM D 5055, **lognormal** case.

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[1]Monte Carlo result based on 1,000,000 simulated assemblies. The simulation takes into account both load sharing and the upward adjusted loads.

$r(t)$	COV	P_F[1] $\rho = 0.5$	$\rho = 0.7$
2.1	0.07	0.518E+00	0.487E+00
	0.11	0.526E+00	0.508E+00
	0.25	0.400E+00	0.399E+00
2.0	0.07	0.146E+00	0.912E−01
	0.11	0.262E+00	0.206E+00
	0.25	0.306E+00	0.279E+00
1.9	0.07	0.213E−01	0.660E−02
	0.11	0.998E−01	0.554E−01
	0.25	0.224E+00	0.183E+00
1.8	0.07	0.182E−02	0.218E−03
	0.11	0.295E−01	0.107E−01
	0.25	0.159E+00	0.114E+00
1.7	0.07	0.890E−04	0.100E−05
	0.11	0.691E−02	0.147E−02
	0.25	0.109E+00	0.678E−01
1.6	0.07	0.200E−05	0.000E+00
	0.11	0.144E−02	0.155E−03
	0.25	0.725E−01	0.381E−01
1.5	0.07	0.000E+00	0.000E+00
	0.11	0.227E−03	0.110E−04
	0.25	0.471E−01	0.205E−01
1.0	0.07	0.000E+00	0.000E+00
	0.11	0.000E+00	0.000E+00
	0.25	0.381E−02	0.555E−03

Table 8: Failure probabilities of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to loads adjusted in accordance with pre-2004 ASTM D 5055, **"truncated normal"** case.

[1]Monte Carlo result based on 1,000,000 simulated assemblies. The simulation takes into account both load sharing and the upward adjusted loads.

$r(t)$	COV	P_F[1]	
		$\rho = 0.5$	$\rho = 0.7$
2.1	0.07	0.547E+00	0.507E+00
	0.11	0.577E+00	0.544E+00
	0.25	0.518E+00	0.483E+00
2.0	0.07	0.147E+00	0.888E−01
	0.11	0.281E+00	0.215E+00
	0.25	0.381E+00	0.327E+00
1.9	0.07	0.173E−01	0.480E−02
	0.11	0.938E−01	0.493E−01
	0.25	0.256E+00	0.195E+00
1.8	0.07	0.921E−03	0.870E−04
	0.11	0.213E−01	0.661E−02
	0.25	0.156E+00	0.102E+00
1.7	0.07	0.200E−04	0.000E+00
	0.11	0.316E−02	0.503E−03
	0.25	0.851E−01	0.458E−01
1.6	0.07	0.000E+00	0.000E+00
	0.11	0.300E−03	0.170E−04
	0.25	0.412E−01	0.176E−01
1.5	0.07	0.000E+00	0.000E+00
	0.11	0.210E−04	0.000E+00
	0.25	0.176E−01	0.557E−02
1.0	0.07	0.000E+00	0.000E+00
	0.11	0.000E+00	0.000E+00
	0.25	0.900E−05	0.000E+00

Table 9: Failure probabilities of seven member assemblies (P_F) when strengths of single members have been reduced by the factor $r(t)$, and the assemblies are subjected to loads adjusted in accordance with pre-2004 ASTM D 5055, **"truncated lognormal"** case.

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[1]Monte Carlo result based on 1,000,000 simulated assemblies. The simulation takes into account both load sharing and the upward adjusted loads.

Distribution	COV	P_F[3]	
		$\rho = 0.5$	$\rho = 0.7$
Normal	0.07	0.138E−01	0.167E−02
	0.11	0.478E−01	0.110E−01
	0.25	0.126E+00	0.568E−01
"Truncated normal"	0.07	0.710E−02	0.137E−02
	0.11	0.300E−01	0.106E−01
	0.25	0.100E+00	0.590E−01
Lognormal	0.07	0.121E−01	0.111E−02
	0.11	0.430E−01	0.802E−02
	0.25	0.121E+00	0.383E−01
"Truncated lognormal"	0.07	0.472E−02	0.793E−03
	0.11	0.214E−01	0.661E−02
	0.25	0.727E−01	0.369E−01

Table 10: Failure probabilities, P_F, of seven member assemblies subjected to allowable strength levels adjusted upward by a factor of 1.07, $r(t) = 1.8$ (strengths of single members have been reduced by a factor of 1.8).

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[3]Monte Carlo result based on 1,000,000 simulated assemblies.
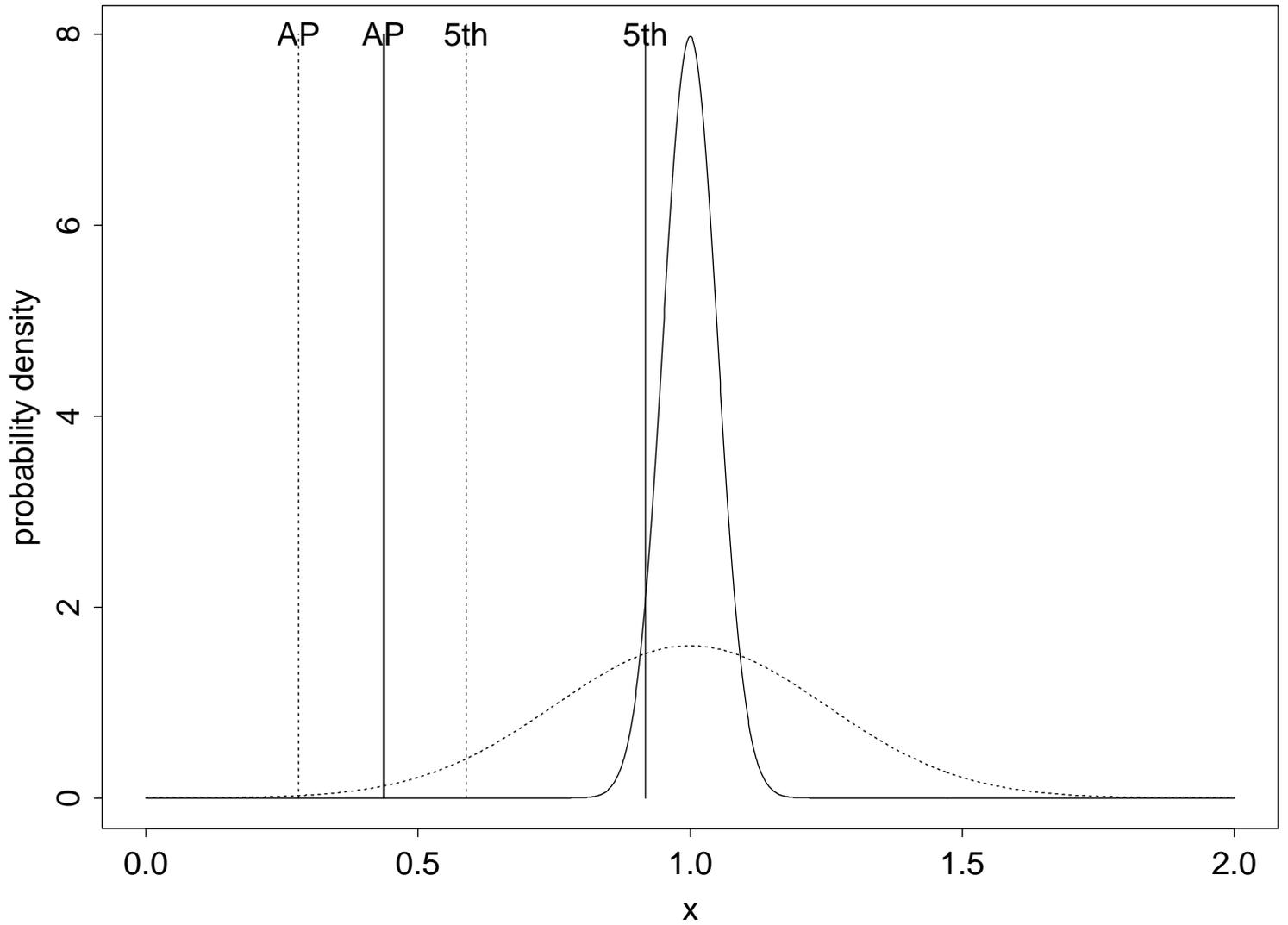
Figure 1: Both normal distributions have mean 1. The broad distribution has standard deviation 0.25 and the narrow distribution has standard deviation 0.05. The dotted vertical lines are at the allowable property level (AP) and fifth percentile (5th) of the broad distribution. The solid vertical lines are at the allowable property level (AP) and fifth percentile (5th) of the narrow distribution.
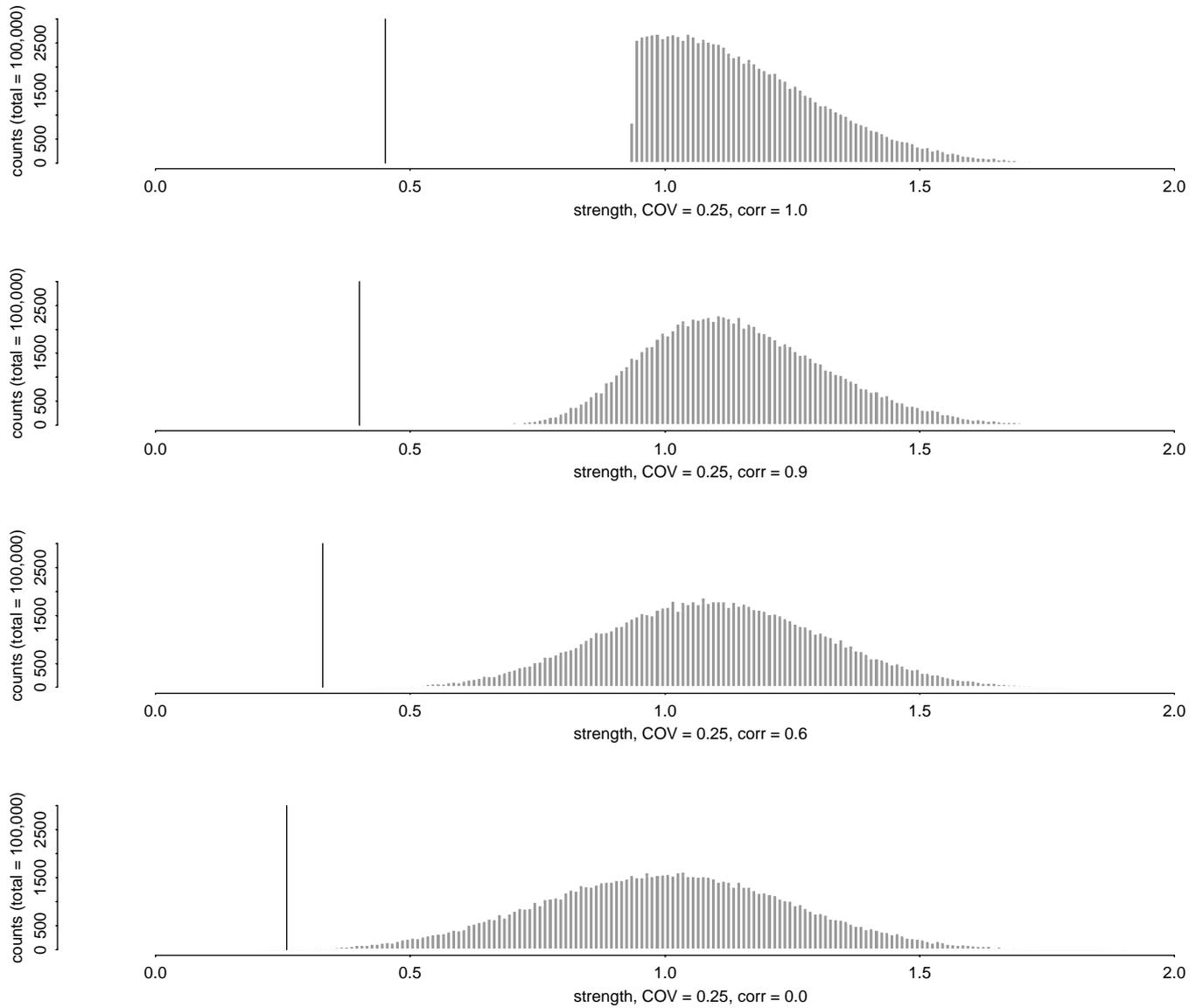
Figure 2: "Truncated normal" distributions. The coefficient of variation is 0.25. The correlations between MOE and MOR are (from top to bottom) 1.0, 0.9, 0.6, 0.0. The vertical lines are at the allowable property levels.
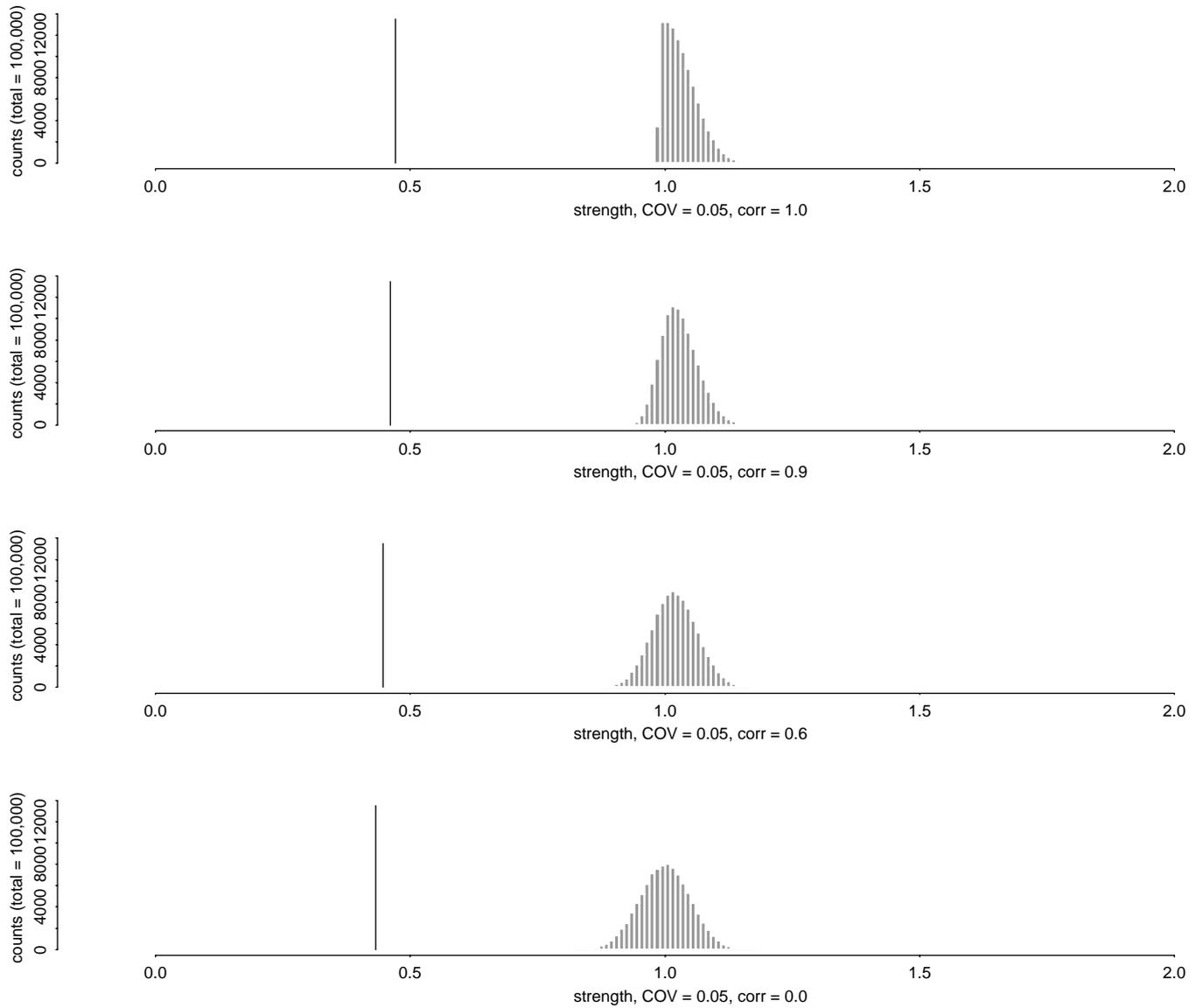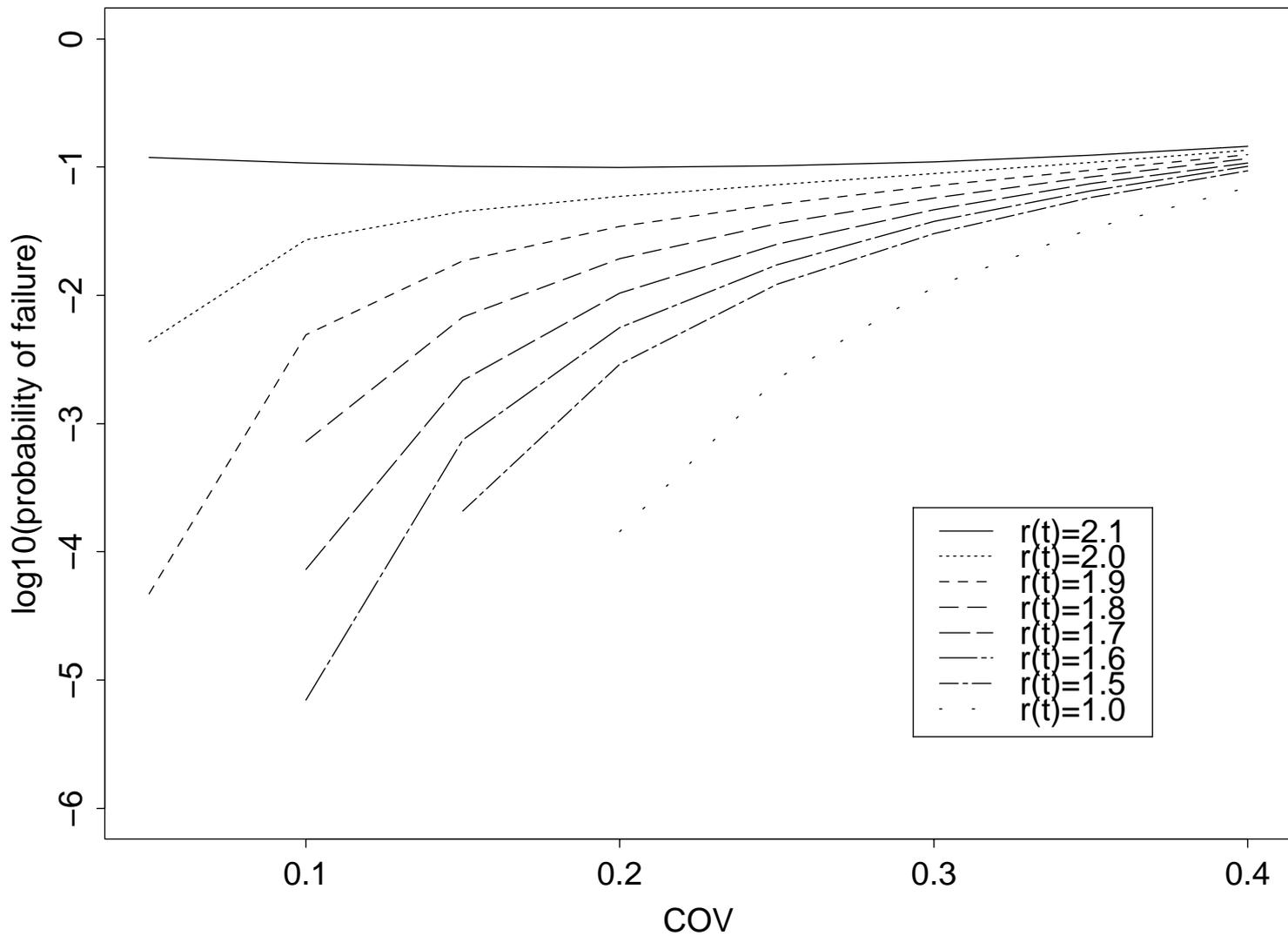
Figure 3: "Truncated normal" distributions. The coefficient of variation is
0.05. The correlations between MOE and MOR are (from top to bottom)
1.0, 0.9, 0.6, 0.0. The vertical lines are at the allowable property levels.

Figure 4: Normal case. Here, for $\rho = 0.7$, we have plotted the logs (base 10) of the simulation P_F data of Table 2.
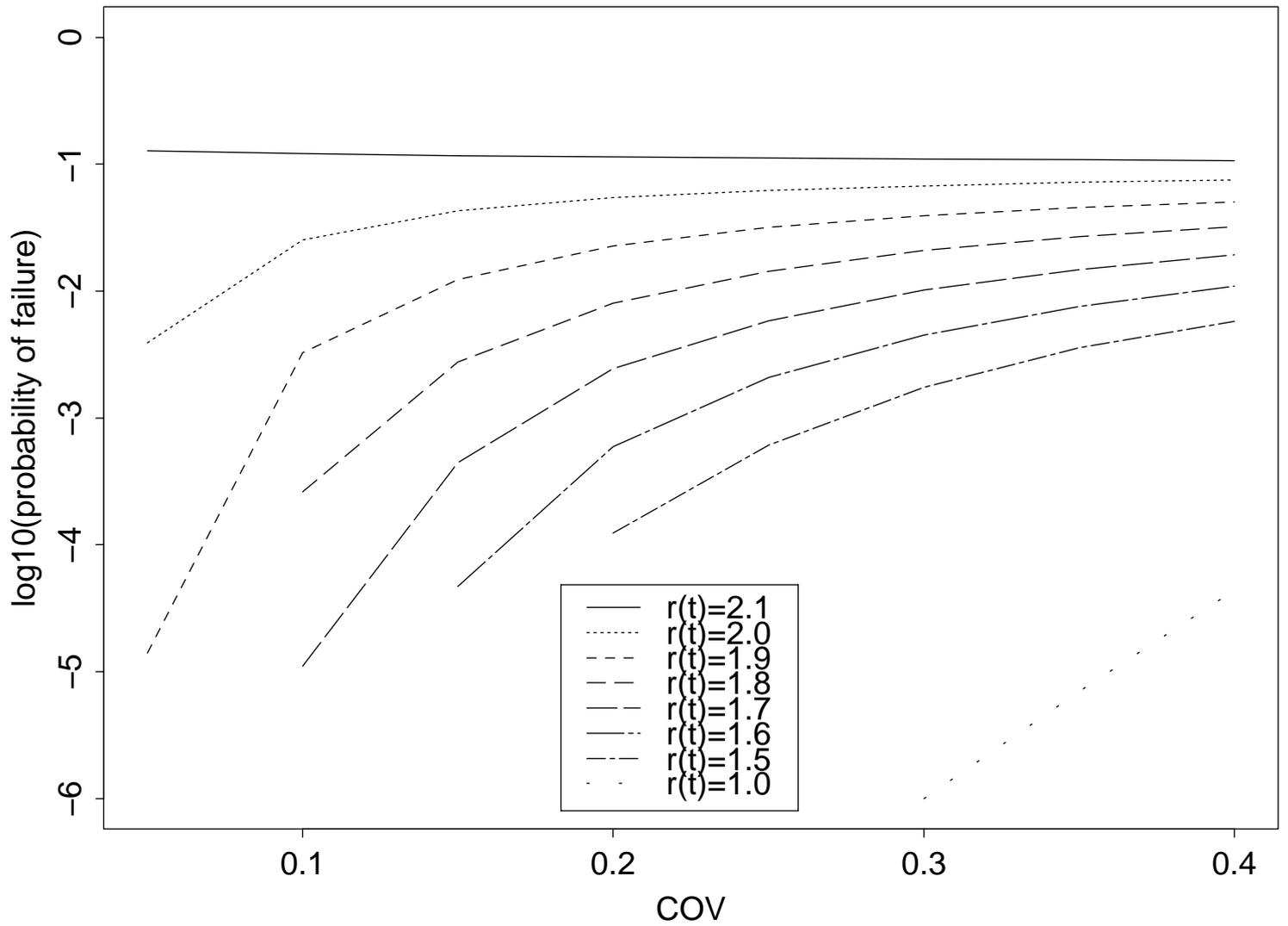
Figure 5: Lognormal case. Here, for $\rho = 0.7$, we have plotted the logs (base 10) of the simulation P_F data of Table 3.
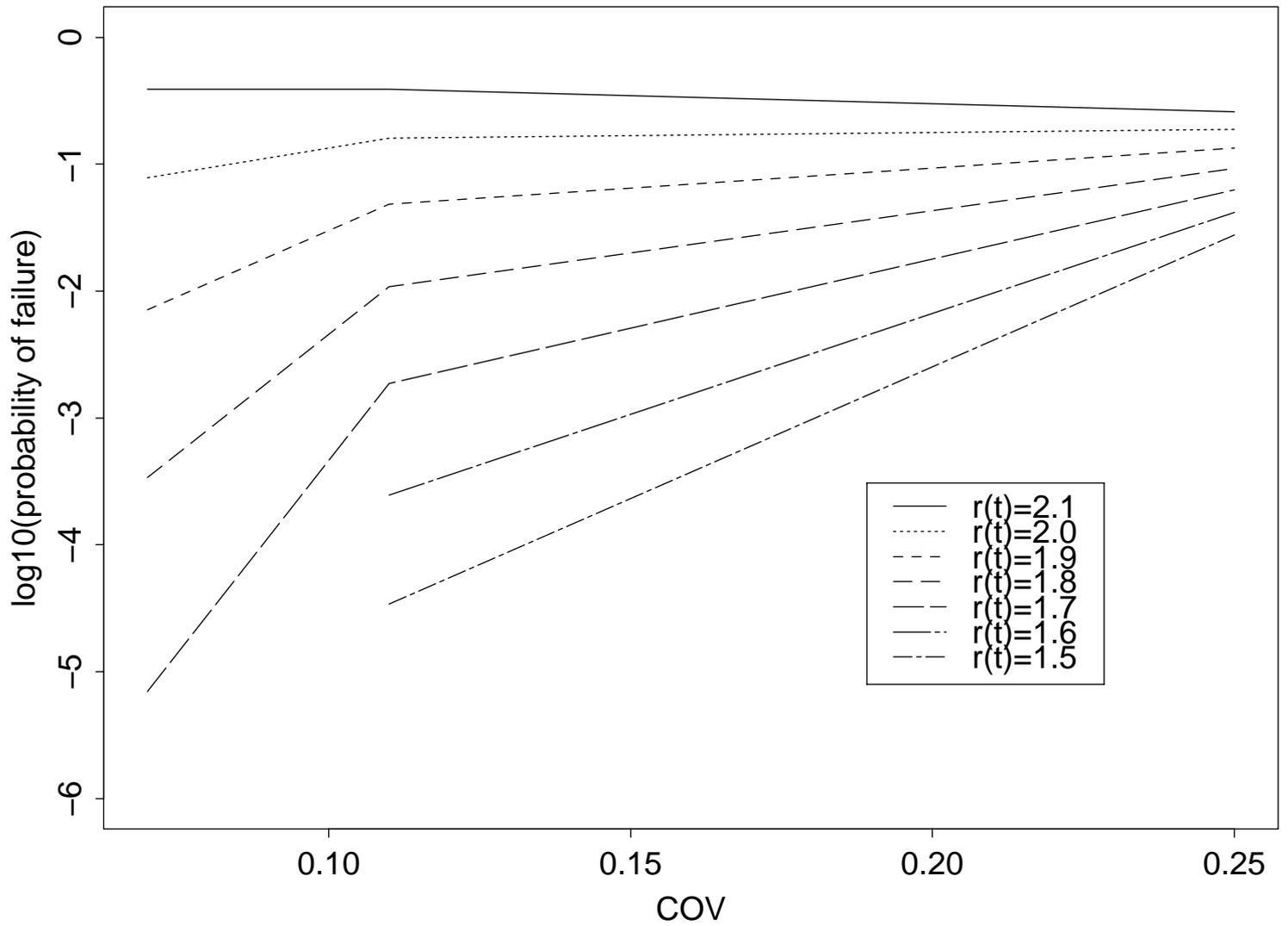
Figure 6: Normal case, adjusted design loads. Here, for $\rho = 0.7$, we have plotted the logs (base 10) of the simulation P_F data of Table 6.
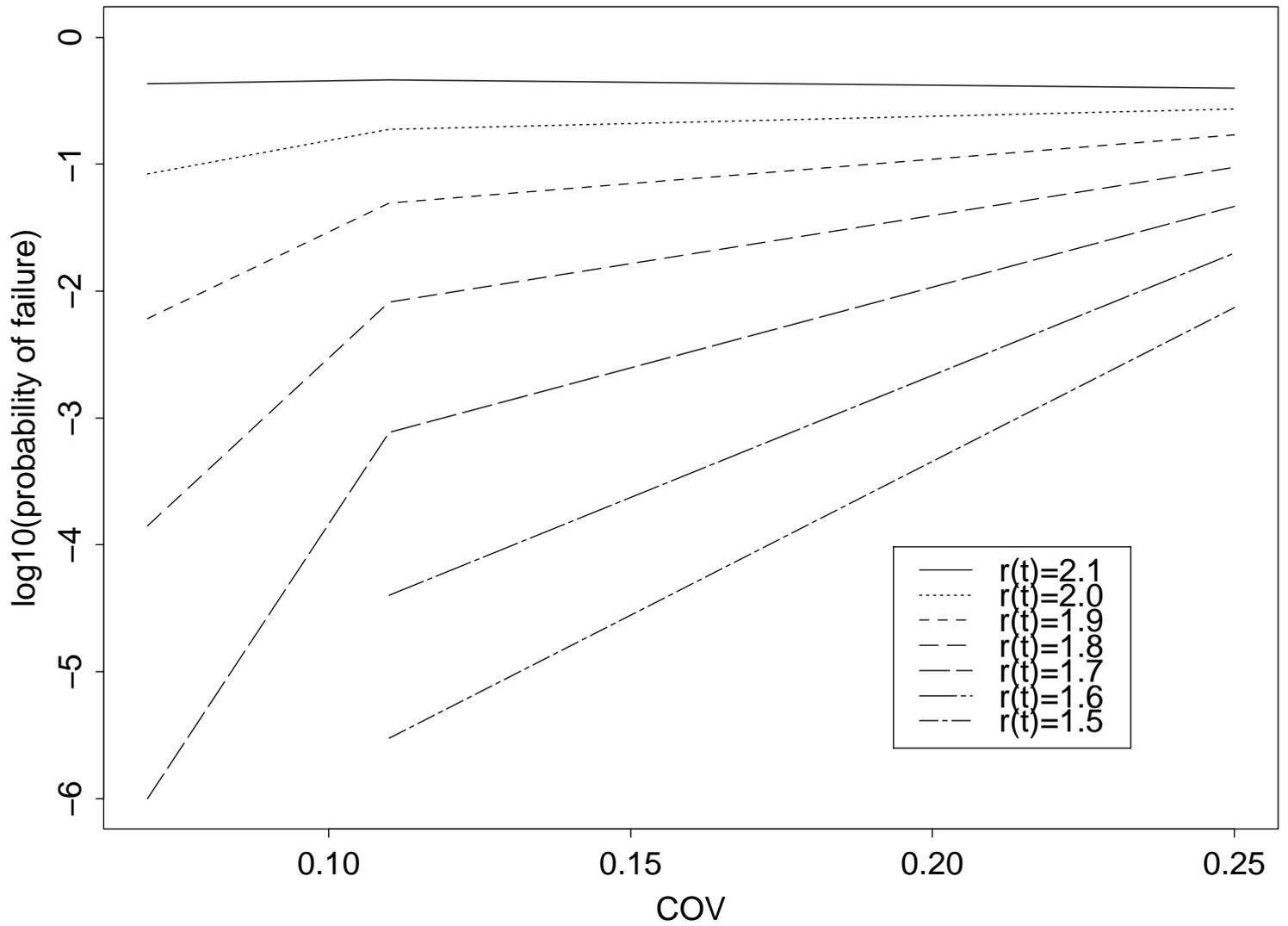
Figure 7: Lognormal case, adjusted design loads. Here, for $\rho = 0.7$, we have plotted the logs (base 10) of the simulation P_F data of Table 7.