An Empirical Evaluation of Value at Risk by Scenario Simulation

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Abstract: Scenario simulation was proposed by Jamshidian and Zhu (1997) as a method to separate computationally intensive portfolio revaluations from the simulation step in VaR by Monte Carlo. For multicurrency interest rate derivatives portfolios examined in this paper, the relative performance of scenario simulation is erratic when compared with standard Monte Carlo results. Although by design the discrete distributions used in scenario simulation converge to their continuous distributions, convergence appears to be slow, with irregular oscillations that depend on portfolio characteristics and the correlation structure of the risk factors. Periodic validation of scenario-simulated VaR results by cross-checking with other methods is advisable.

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One major obstacle to using Monte Carlo simulation for Value at Risk (VaR) calculations on large bank portfolios is the need to revalue a very large number of positions. Jamshidian and Zhu (1997) propose scenario simulation as a method to drastically reduce the computational burden. The key feature of this technique is the separation of revaluation and simulation. Scenario simulation samples a fixed set of precomputed "scenarios" using a Monte Carlo procedure. In contrast, as Jamshidian and Zhu (JZ) put it, standard Monte Carlo involves an "extortionate" number of portfolio revaluations.

While rapid advances in computing speed may eventually obviate the need for approximation techniques, such approximations are widely used at financial institutions today. It is common for such institutions to rely on different valuation models for computing VaR from those used in the "front-office" system for pricing and hedging. The former models generally have simple closed-form formulas, whereas the latter may use numerically intensive lattice, Monte Carlo, or finite difference solutions for prices and other outputs. Risk management practice is also moving toward more timely and frequent VaR reports, such as intra-day VaR. In addition, since January 1, 1998, the banking regulatory agencies have required capital to be charged against market-risk exposures in trading portfolios held by banks that meet certain criteria.[1] The charge is determined by the daily computation of VaR based on a bank's own internal model.

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[1]The risk-based capital regulations are contained in the *Federal Register*, September 6, 1996 (Volume 61, Number 174) [61 FR 47357 12 CFR Part 3, 208, 225, 325 - Joint Final Rule: Risk-Based Capital Standards: Market Risk]. The rule was issued jointly by the Office of the Comptroller of the Currency, the Board of Governors of the Federal Reserve System, and the Federal Deposit Insurance Corporation. Compliance has

The examples in Jamshidian and Zhu give some evidence that scenario simulation approximations are accurate. These examples include 10-year currency and interest rate swaps, a 5-year interest rate floor, and a 5×5 interest-rate receiver's swaption.[2] Because these instruments have long-term tenors, convexity effects for the options-based instruments are minor. VaR at the 97.5 and 99.0 percent levels are reported only for the swaps, whereas the floor and swaption examples are limited to mean, standard deviation, and extrema of the 30-day horizon value. The scenario simulated values are within 2 percent of the Monte Carlo values.

The purpose of this paper is twofold: first, to detail the steps involved in doing scenario simulation and to show its relationship with standard Monte Carlo and principal component simulation; and, second, to evaluate the relative performance of these three methods on several test portfolios. The precise meaning of "scenario" is defined below. The empirical focus of the paper is on LIBOR-based option portfolios, in which convexity effects are more pronounced than those found in JZ's long-dated instruments. JZ's examples are also based on LIBOR derivatives.

The results indicate that the relative performance of scenario simulation on nonlinear portfolios deteriorates compared with alternative approaches. Low dimensional discretizations of the risk factor inputs can give poor estimates of VaR for linear and nonlinear portfolios. The quality of the approximation depends on the extent of nonlinearity in a portfolio. Furthermore, the tests described below demonstrate that convergence of scenario simulation VaR results to benchmark values is slow as the

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been mandatory since January 1, 1998, for banks whose trading activity [gross sum of trading assets and liabilities on a worldwide consolidated basis] equals: 10 percent or more of total assets; or $1 billion or more. However, the banking regulators have discretion in deciding which banks must comply.

[2] The swaption is a 5-year option on a 5-year swap which comes into being if the option is exercised.

discretization gets finer. Although scenario simulation appears to be a useful alternative to other, more computationally intensive methods, periodic validation of scenario-simulated VaR results by cross-checking with other methods is advisable.

Alternative approaches to accelerating VaR by Monte Carlo have been proposed. Picoult (1997) develops an extension of a Taylor series approach that relies on "grids of factor sensitivities." In contrast to the local approximation of a Taylor series, factor sensitivities are the derivatives of the instrument value with respect to the risk factor evaluated along a discrete set of values of a risk factor. Before running a VaR, these sensitivities, including first, second, and higher order derivatives, including possibly cross derivatives, are computed and stored, and then subsequently, in the VaR simulation, changes in portfolio value are calculated by interpolation based on the stored factor sensitivities. The user decides how many and what type of terms to include in the approximation. Frye (1997) proposes a conservative approximation to VaR that is predicated on a discrete scenario analysis rather than a Monte Carlo. He defines principal component-based scenarios using a small set of large, prespecified shocks to the risk factors, such as 2.33 standard deviations for a 99th percentile VaR. The greatest loss that results in the process of revaluing a portfolio to these shocks is recorded as the VaR. Frye (1998) suggests a Monte Carlo approach based on a stored multidimensional grid of portfolio values as a function of a multidimensional grid of shocks to principal components. Simulation proceeds by revaluing the portfolio by linearly interpolating along the precomputed grid of portfolio values with respect to Monte Carlo draws for the risk factors.

The final paper related to JZ is Reimers and Zerbs (1998), who use simple, multi-currency portfolios of fixed-rate government bonds to assess the impact of the principal component stratification technique proposed by JZ. This method is described in detail in the following sections. They conclude that the relative difference in VaR based on the full covariance matrix versus the stratified principal component covariance matrix is on the order of one percent.

Sections 1, 2, and 3 review standard "brute-force" Monte Carlo, principal component Monte Carlo, and scenario simulation, respectively. Most of the exposition focuses on scenario simulation. Section 4 discusses the construction of test portfolios of LIBOR-derivatives, and section 5 compares the simulation results on these portfolios for all three methods. Section 6 gives concluding observations.

1. Brute-Force Monte Carlo

Monte Carlo methods are widely used in empirical finance and asset pricing.[3] The basic algorithm for VaR is:

1. Mark portfolio to market on initial date.
2. Generate simulated changes in risk factors based on estimated covariance matrix.
3. Revalue portfolio using simulated changes.
4. Iterate revaluations a large number of times.
5. Sort changes in portfolio value by size.
6. Select the desired percentile of the changes as the VaR.

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[3] There are many good sources on Monte Carlo methods applied to finance. Hull's (2000) text gives a thorough overview of the general procedure for valuing derivatives. Chapter 7 of the *RiskMetrics—Technical Document* [J.P. Morgan/Reuters, December 1996] covers Monte Carlo as applied to VaR. This chapter does not discuss principal component data reduction.

A frequently insurmountable roadblock to using this algorithm on large portfolios is that each Monte Carlo draw for the risk factors requires revaluation of all securities in the portfolio. A second-order Taylor series (delta-gamma) approximation for the portfolio value can be used to avoid the burden of full revaluation. However, such approximations may perform poorly in VaR applications for portfolios that contain free-standing or embedded out-of-the-money options, which would be missed by a local approximation.

2. Principal Component VaR

Principal component (PC) VaR reduces the computational burden by compressing the number of risk factors through the use of a reduced set of principal components. Fewer random numbers need to be drawn; however, the principal components must be inverted back into the original number of risk factors in order to revalue the portfolio. The VaR algorithm is the same as that for brute-force Monte Carlo, except for the addition of the computation of the principal components and the inversion process.[4]

By construction, principal components are uncorrelated. The principal components for the test portfolios are derived from the covariance matrix of the monthly log changes in a set of "key" interest rates along the yield curve. A complete discussion of the data construction appears below. Let the interest rate risk factors be given by the $n \times 1$ vector $\mathbf{RF}$ and evolve through one discrete time step as

(1) $$\mathbf{RF}_1 = \mathbf{RF}_0 \exp(\mathbf{u}),$$

where the shocks $\mathbf{u} \sim N(0,Q)$.

[4] Recent references on principal component VaR are Singh (1997) and Kreinin et al. (1998). Singh's examples use a Taylor series approximation to the pricing function rather than full revaluation.

The covariance matrix Q is factored into a diagonal matrix of eigenvalues Λ and an orthogonal matrix of eigenvectors $E = [e_1 \cdots e_n]$:

(2) $\Lambda = EQE'$.

Both the eigenvalue and eigenvector matrices are truncated to $m < n$ columns in order to include only a subset of the largest eigenvalues and their associated eigenvectors. The shock vector $\mathbf{u}$ is generated by a linear combination of standard normal shocks $h \sim N(0, I_m)$:

(3) $$\mathbf{u} = \underset{n\times m}{E'} \underset{m\times m}{\sqrt{\Lambda}} \underset{m\times 1}{h},$$

where the principal components are $\sqrt{\Lambda}h$. PC simulation consequently entails using a covariance matrix approximation for the full $n \times n$ covariance matrix.

In LIBOR derivative portfolios examined below, four principal component risk factors are retained from the original eight key-rate risk factors per market. These account for over 97 percent of the total variance of the original data (as measured in the standard way by the ratio of the sum of the eigenvalues of the four components to the sum of all the eigenvalues).

3. Scenario Simulation

The mechanics of scenario simulation are quite different from the other types of Monte Carlo applied to VaR because of the separation of the revaluation stage from the simulation stage.

The core step in scenario simulation is the approximation of the multivariate normal distribution by the binomial distribution. The joint occurrence of particular discrete states of the risk factors constitutes a "scenario." Such discrete approximations are conventional in statistics, but their incorporation into a simulation analysis, and particularly the "stratified" discretization discussed below, make the JZ approach a novel way to calculate VaR.

Principal component analysis is treated as a necessary adjunct to scenario simulation because the number of scenarios becomes huge even for a relatively small number of risk factors. JZ note that with 12 key rates in a yield curve model and an assumption that each can take three possible values, the total number of scenarios is $3^{12} = 531{,}441$. With five states for each variable, the total number of scenarios explodes to more than 200 million. The use of principal component analysis can dramatically reduce the number of scenarios with relatively little sacrifice of accuracy because interest rate movements tend to be highly correlated. Even with principal components standing in for market factors, the number of scenarios for portfolios involving term structures in more than just a few currencies can number in the trillions. Further assumptions must be made on the structure of the covariance matrix of the risk factors to pare down the problem into one of manageable proportions. The large number of scenarios requires a Monte Carlo procedure to compute VaR, instead of a direct calculation based on the full set of scenarios and their associated probabilities of occurrence. These procedures are explained below, reproducing JZ's equations (15)–(17) and adding some further explanatory detail.

There are assumed to be $m+1$ states, ordered from 0 to m, with probabilities determined in the conventional way for the binomial:

(4) $$\text{Probability}(i) = 2^{-m} \cdot \frac{m!}{i!(m-1)!}, \quad i = 0, \cdots, m .$$

These probabilities are not explicitly calculated or used in scenario simulation. They enter the VaR calculation in the construction of the "lookup table" for scenarios. The essence of scenario simulation is the discretization of continuously distributed random variables that represent the principal components (linear transformations of the risk factors). For example, if each principal component can be in five possible states, the total number of states is $5^3 = 125$. These can be enumerated and the portfolio can be revalued for each discrete shock. The key step is the mapping of a continuously distributed shock into a discrete shock. Although normality is assumed, the method can be applied to other distributions. The relationship between the normal and the binomial is given by

(5) $$\frac{1}{\sqrt{2p}} \int_{a_i}^{a_{i+1}} \exp\left(-\frac{x^2}{2}\right) dx = \frac{2^{-m} m!}{i!(m-i)!}, \quad i = 0, \cdots, m .$$

Starting from $a_0 = -\infty$ for the initial lower limit of integration, the next boundary point a_1 that equates the area of the normal density to the binomial probability for the 0th binomial state is determined numerically by any standard root-solving procedure. The remaining upper limits of integration a_{i+1}, defining areas under the normal density curve in the interval $[a_i, a_{i+1}]$, are then found successively, state by state. For five states, these values are:

(6) $$a_1 = -1.53412;\ a_2 = -0.48878;\ a_3 = 0.48878;\ a_4 = 1.53412 .$$

A standard normal variate *z* drawn from a random number generator can then be mapped into a discrete state by testing where it falls relative to the break points:

(7) $$B^{(m)}(z) = i \quad \text{if } a_i \le z < a_{i+1},$$

where $i \in \{0,\ldots,m\}$. This mapping comes into play in two places: in the construction of the lookup table of scenarios and in the simulation of scenarios.

Lookup Table Construction

The discrete variable $B^{(m)}(z)$, which represents a particular state, can be normalized as

(8) $$b^{(m)}(z) = (2B^{(m)}(z) - m)/\sqrt{m},$$

that is, the transformed random variable has mean 0 and variance 1 (the mean of the binomial distribution is $m/2$ and the variance is $m/4$). This variable's dependence on z is not explicitly used in the lookup table construction. Rather, a set of shocks $b^{(m)}(z)$ is defined based on the set of integers that the discrete variable B assumes. For 5 states $(m = 4)$, the variable $B^{(m)}(z)$ takes values $\{0,1,2,3,4\}$ and this translates into normalized shocks $b^{(m)}(z)$ with possible values $\{-2,-1,0,1,2\}$.

The $b^{(m)}(z)$ values are the discrete values of z that are the primary ingredients for constructing the lookup table. A matrix of scenario indexes is computed that exhausts all possible state combinations. These discrete state values $i \in \{0,\ldots,m\}$ for each principal component are translated into the corresponding $b^{(m)}(z)$, which in turn are plugged in as simulated principal components. These in turn are inverted into realizations of the risk factors that are used to revalue each instrument in each scenario. The resulting valuations are stored for later use during the simulation. The following table illustrates the mapping process using 5 states and a total of 125 possible scenarios.

Table 1
Mapping of Discrete States into
Shocks and Changes in Portfolio Value
5-State Discretization of 3 Principal Components

Discrete State Index			⇒	$b^{(m)}(z)$			Invert into Risk Factors	Change in Portfolio Value	Row
0	0	0	⇒	-2.0	-2.0	-2.0	⇒	-0.075	1
1	0	0	⇒	-1.0	-2.0	-2.0	⇒	-0.008	2
2	0	0	⇒	0.0	-2.0	-2.0	⇒	0.061	3
3	0	0	⇒	1.0	-2.0	-2.0	⇒	0.132	4
4	0	0	⇒	2.0	-2.0	-2.0	⇒	0.205	5
0	1	0	⇒	-2.0	-1.0	-2.0	⇒	-0.093	6
1	1	0	⇒	-1.0	-1.0	-2.0	⇒	-0.025	7
⋮	⋮	⋮		⋮	⋮	⋮		⋮	⋮
2	3	4	⇒	0.0	1.0	2.0	⇒	-0.048	118
3	3	4	⇒	1.0	1.0	2.0	⇒	0.021	119
4	3	4	⇒	2.0	1.0	2.0	⇒	0.092	120
0	4	4	⇒	-2.0	2.0	2.0	⇒	-0.198	121
1	4	4	⇒	-1.0	2.0	2.0	⇒	-0.132	122
2	4	4	⇒	0.0	2.0	2.0	⇒	-0.065	123
3	4	4	⇒	1.0	2.0	2.0	⇒	0.004	124
4	4	4	⇒	2.0	2.0	2.0	⇒	0.075	125

Simulation

Each $B^{(m)}(z)$ will substitute for a principal component in the simulation. A set of k such discrete state indexes defines a particular sampled scenario:

(9) $$B = B^{(m)} = (\mathbf{B}^{(m)}(z_1), \ldots, \mathbf{B}^{(m)}(z_k)),$$

where the z_i are independently drawn standard normal variates. (The principal components are assumed to be independent.) As noted above, the first four principal components per market are used in the examples of interest rate derivative portfolios. All possible scenarios are enumerated in the lookup table.

The discretization of any draw from a "continuous" random number generator on a computer is assigned to an element of the discrete vector of shocks.[5] The probability of the occurrence of any particular draw for $B^{(m)}(z)$ (given by equation (4)) is implicitly captured by the process of sorting the draws of the continuous variable z into discrete states as represented by equations (5) and (7). For independent principal components, the joint probability of a scenario—a combination of discrete variables represented by equation (9)—is the product of probabilities of each individual discrete variable. (The general case of correlated discrete variables is discussed subsequently.) In other words, the simulation samples with replacement from the scenarios in the lookup table. The frequency of the sampling of any given scenario is in proportion to its discrete probability.

A synopsis of the scenario simulation algorithm is:

1. Construct the lookup table by enumerating all possible scenarios involving the predefined states for each principal component or risk factor.
2. Compute $b^{(m)}(z)$ for each principal component or risk factor in a given scenario.
3. Compute the corresponding risk factors for each scenario (see equations (13) and (14) below).
4. Revalue each instrument in the portfolio using the risk factors for each scenario. Compute the change in portfolio value from its initial value. Assign an index number (the row number in the table) to the change in portfolio value based on a given scenario.
5. Simulate by drawing independent normal random variates z and map them into the states $B^{(m)}(z)$.
6. Scan the lookup table for the corresponding scenario and return the precomputed change in portfolio value and store it.

[5] Of course, any computer-based random number generator is also discrete, given the finite number of integers that can be represented as a 32-bit integer, based on current system constraints. Still, typical linear congruential generators can create at least 2 billion unique random numbers, and many can crank out vastly more than this (at the cost of being slower in execution) before they start to repeat their cycle. See Dwyer (1995).

7. Repeat simulation steps (5) and (6) a large number of times.
8. Sort the vector of simulated changes in portfolio value by size and evaluate the desired percentile value for the VaR.

In contrast with standard Monte Carlo or principal component Monte Carlo, the number of portfolio revaluations is independent of the number of simulation iterations under scenario simulation. Although the discretization limits the sampling from a given risk factor's distribution, the joint probabilities across risk factors can be very low. In the example given here, the principal components are independent. For example, in the 5 state example, while the smallest probability for one variable is $1/16 = 0.0625$, the smallest joint probability is $(1/16)^3 = 0.0002$. Nevertheless, for portfolios with sufficiently nonlinear payoffs, the failure to sample far enough into the tails may result in material inaccuracies of the scenario simulation approximation. The same may be true of extreme positions in digital options or other options positions that create spikes that the discretization misses.

Stratification of Principal Components

JZ propose a stratification of principal components to handle multimarket, multi-currency portfolios. Principal components are computed for a given market, say, for the term structure in a given currency, and consequently individual markets retain a distinct identity in the simulation. Although mutually uncorrelated for interest rates in a given currency, the principal components have correlations with principal components of interest rates in other currencies, as well as with other risk factors, such as foreign exchange rates.

However, even with the reduction in dimension gained through the use of principal components, there still can be trillions of scenarios to reckon with in multicurrency, multimarket portfolios. The number of dimensions for the scenario simulation could easily explode out of hand, wiping out its advantage over conventional Monte Carlo. For example, the total number of scenarios in the multicountry portfolios examined below is 3.4×10^{12} for the 7-state discretization and 4.7×10^{15} for the 11-state discretization.[6] The solution to the dimensionality problem is simply to use Monte Carlo to simulate the discrete joint distribution of the risk factors. Scenarios need to be tabulated only within each currency block.

Dealing with dependent discrete joint random variables is straightforward. Let Q_s be the covariance matrix based on the stratified decomposition of risk factors into principal components. Generating correlated standard random normal variates follows the usual procedure for Monte Carlo. A factorization of the covariance matrix Q_s is used to create a vector of correlated shocks X from the vector z of uncorrelated standard normal variates, such that

(10) $$X = (X_1, \ldots, X_n) \sim N(0, Q_s).$$

The discrete scenarios (9) use X in place of z:

(11) $$B = B^{(m)} = (\mathbf{B}^{(m)}(X_1), \ldots, \mathbf{B}^{(m)}(X_n)) .$$

Otherwise, scenario simulation proceeds exactly as given in the synopsis above. JZ formally prove the convergence of the multinomial approximation given by (11) to the continuously distributed random vector (10).

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[6] This is computed by

$\left(\text{\# states per \$ yield curve factor}\right)\times\left(\left(\text{\# states per foreign yield curve factor}\right)\times\left(\text{\# states per exchange rate}\right)\right)^3$.

As JZ note, premultiplying the B matrix derived from uncorrelated normal variates by the Cholesky factor of Q_s to induce the desired covariance structure would scramble the stratification. To avoid this loss of information, the Cholesky factor is applied to create the X vector in (10) before discretization.

4. Test Portfolios

Four kinds of multicurrency, LIBOR-derivatives portfolios are constructed to compare standard Monte Carlo, principal component Monte Carlo, and scenario simulation. Each type of portfolio was designed to contain one type of instrument: swaps, caps/floors, caplets/floorlets, and swaptions. Such derivatives represent an important share of most large banks' trading portfolios, and typically trading books are organized into subportfolios by instrument type.

Data

All portfolios involve swap or option positions on four currencies: the U.S. dollar, German mark, Swiss franc, and British pound. The exchange rate and interest rate data were obtained from the Bloomberg History Tool and are end-of-day Wednesday observations. The data sample period runs from December 1, 1993 to November 25, 1998, totaling 261 observations per time series. The use of Wednesday observations minimized the number of missing observations in the data set. There were eight missing Wednesday observations across all countries for interest rates or FX. These dates were backfilled using observations from the day before.

Spot LIBOR at maturities of 3, 6, 9, and 12 months and swap rates at maturities of 2, 5, 7, and 10 years were used to derive forward LIBOR curves. These maturities constitute the key rate maturities. Standard Granger causality tests, reported in the appendix, indicate that daily spot swap rates Granger-cause daily spot LIBOR rates, but not the converse. To mitigate potential distortion to measured correlations of daily changes in rates, weekly changes in risk factors, and hence a weekly holding period for the VaR, was employed in the simulations.[7]

All derivatives considered in this study have quarterly resets. Linear interpolation was used to fill out points along the yield curve: 8 key rates were expanded into 40 rates at quarterly maturity intervals out to 10 years.[8] Spot swap rates with maturity less than 2 years were derived from the spot LIBOR rates. In turn, daily 3-month forward LIBOR curves in each currency were derived from the spot swap rate curve.[9]

The forward LIBOR curve was used to construct a discount bond price curve and corresponding yield curve. Yields are more compactly represented in a principal component decomposition and are less contaminated by noise than are forward rates. Yields are the variables that get simulated in the VaR calculations. Bond prices are needed as inputs into the Black model for pricing interest-rate derivatives.

Covariance Matrix

The five-year data sample is subdivided into five subsamples, each spanning one year starting with the first Wednesday in December. This choice was motivated by the Basle Committee for Banking Supervision's internal models approach for setting regulatory

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[7] The general issue of data nonsynchroneity is discussed in *RiskMetrics* (RM Data Sets.pdf), section 8.5. Strips of Eurodollar futures rates for intermediate tenors are often used in constructing LIBOR curves. See Overdahl et al. (1997).

[8] Linear interpolation of a small set of key rates is a problematic practice because the resulting forward curve takes a saw-tooth shape. A difference in liquidity between the interbank deposit market (the spot LIBOR rates) and the swap market may also contribute to a jump in the derived forward LIBOR curve where the two maturity segments join. See Wang (1994).

[9] Formulas and conventions for LIBOR-based derivatives are described in Rebonato (1998) and Musiela and Rutkowski (1997).

capital for trading portfolios.[10] (The U.S. banking regulations for using VaR for capital determination, which derive from the Basle framework, stipulate that no less than one year's worth of daily data be incorporated into the covariance matrix and that a given yield curve incorporate no fewer than six "segments" of the curve "to capture differences in volatility and less than perfect correlation of rates along the yield curve."[11])

The full covariance matrix consists of interest rate and foreign exchange rate blocks of risk factors: the yield curve key rates from each country and their exchange rates, giving a total of 31 factors. A stratified principal component decomposition of each yield curve, retaining the four largest factors, reduced the dimension of the covariance matrix to 19×19. The first four principal components captured at least 97 percent of the total variation in the interest rate data in each currency. Tables 2–6 show the stratified correlation matrix for 1995–1998, respectively, where correlations for the first three interest rate PCs are displayed to conserve space. The currency labels are USD, DEM, CHF, and GBP, denoting the U.S., Germany, Switzerland, and U.K, respectively. The transformation of the original covariance matrix into the stratified matrix is similar to the standard decomposition of a covariance matrix in (2). The off-diagonal blocks, such as the covariance of DEM principal components with USD principal components, are computed by pre- and post-multiplying the original covariance matrix block by the DEM and USD blocks of eigenvector matrices, respectively. The main diagonal of own-country interest rate principal component blocks are identity matrices since principal components are orthonormal. However, the off-diagonal blocks show a pattern of correlations between countries that exhibits a degree of stability from year to year.

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[10] Basle Committee on Banking Supervision, "Amendment to the Capital Accord to Incorporate Market Risk," January 1996.

The tables indicate correlations greater than 0.20 in absolute value in dark shading. The cross-correlations of PCs in the off-diagonal blocks tend to show positive correlation of corresponding PCs—for example, first USD PC with first DEM PC. The results are variable from year to year, with 1998 having the greatest correlations and 1997 the weakest.

Exchange rate–interest rate principal component correlations, though strong at times (such as the first Swiss interest rate PC versus FX rates in 1995), are much more erratic. However, the FX block has high cross-rate FX correlation every year.

Portfolio Construction

An arbitrary set of positions in each instrument is assumed for each currency. The basic portfolios are designed to investigate nonlinear payoffs, the classic inverted "U" of a negative gamma exposure. As in any Monte Carlo exercise, the results do not necessarily generalize. They are valid for the particular portfolios being examined. However, qualitatively similar results were obtained for other test portfolios, which are not reported here to conserve space.

The portfolios consist of identical positions in derivatives in each currency that have identical U.S. dollar notional value ($10,000) at the initial date. The four types of portfolios are:

1. 10-year pay-fixed/receive floating **swaps** at current market rates.
2. 6-year **caps** and **floors**. At-the-money long cap and long floor (that is, strikes are set equal to the current forward LIBOR curve). Short out-of-the-money caps and floors with strikes set equal to $L_T \exp(\pm 3s_T)$, where T is the reset date of a caplet, L_T is forward LIBOR for time T, and s_T is the volatility of forward LIBOR for a weekly holding period. The notional value of the out-of-the-money positions is twice that of the at-the-money positions.

[11] See footnote 1.

3. 3-month **caplets** and **floorlets**. This portfolio is the first leg of (2), but with out-of-the-money strikes spaced at $\pm 2s_T$.
4. 6-month **swaptions** on forward-start 9-year pay-fixed swaps. The portfolio has the same structure as (2) and (3). The strike for the long at-the-money payor and receiver swaptions is the current forward swap rate for a 9-year swap. The strikes for the short out-of-the-money payor and receiver swaptions is $S_T \exp(\pm 3s_T)$, where S_T is the forward swap rate and s_T is its volatility.[12]

The swap portfolio is taken as the base case because swaps were used in JZ's examples. The caps/floors portfolio is an intermediate-term options portfolio. The caplets/floorlets portfolio is a short-dated options portfolio. The swaptions portfolio is a shorted-dated options portfolio on a long-maturity reference rate.

These portfolios involve all currencies and identical positions. The inherent diversification of a multicurrency portfolio will ameliorate extreme outcomes, although key interest rate PC cross correlations tend to be (weakly) positive, propagating shocks in the same direction across portfolios. Because the FX–interest rate correlations are weaker than the interest rate block cross correlations, the FX shocks mainly register as noise in the results.

Figure 1 shows the payoff profiles as of the initial date and exchange rates to shocks ranging ± 3 standard deviations (measured weekly) of the underlying forward LIBOR, swap, or forward swap rates; that is, the current underlying L_T is varied from $L_T \exp(-3s_T)$ to $L_T \exp(+3s_T)$. The first panel of Figure 1 illustrates the approximately linear response (except for a very slight convexity effect) of the individual swaps in each currency in the portfolio. All have zero value at the current market rate. The remaining figures exhibit negative convexity to different degrees. These figures simply trace out the pricing function as the underlying rate varies. The VaR results will assess the portfolio performance as simulated interest rate and currency shocks hit these portfolios over the weekly holding period. A volatility shock could also readily be included as an additional risk factor in each market, but the focus of this study is on "price" risk, as in JZ.

Risk Factor Simulation

Following the examples in JZ, risk factors are simulated as lognormal processes. Incorporating other continuous distributions for the risk factors is readily done using the fractile-to-fractile mapping described in Hull and White (1998).

For the crude Monte Carlo simulation, the vector of risk factors **RF** at the one-week horizon is generated by

(12) $$\mathbf{RF} = \mathbf{RF}_0 \exp(\mathbf{u}),$$

where the vector of shocks $\mathbf{u} \sim N(0,Q)$ and is 35×1. The risk factors are the eight key rates along the yield curve for each country and three foreign exchange rates. For the stratified principal component simulation, the vector is given by

(13) $$\mathbf{RF} = \mathbf{RF}_0 \exp(\tilde{\mathbf{u}}),$$

where

$$\tilde{\mathbf{u}} = \underset{8\times4}{E'_{USD}} \underset{4\times4}{\sqrt{\Lambda_{USD}}} \underset{4\times1}{h_{USD}} \Big| E'_{DEM} \sqrt{\Lambda_{DEM}} h_{DEM} \Big| E'_{CHF} \sqrt{\Lambda_{CHF}} h_{CHF} \Big| E'_{GBP} \sqrt{\Lambda_{GBP}} h_{GBP} \Big| \underset{3\times1}{\mathbf{u}_{FX}} ,$$

$\mathbf{x}_1 | \mathbf{x}_2$ denotes stacked vectors, and E_N is the truncated eigenvector matrix for country N that retains eigenvectors corresponding to the four largest eigenvalues of that country's yield curve. The interest rate principal components and FX rate shocks are

$$\sqrt{\Lambda_{USD}}\, h_{USD} \,|\cdots|\, \sqrt{\Lambda_{GBP}}\, h_{GBP} \,|\, \mathbf{u}_{FX} \sim N(0, Q^s),$$

[12] A receiver swaption is a call on a swap; that is, the right to receive fixed and pay floating. A payor swaption is a put on a swap; that is, the right to pay fixed and receive floating.

where Q^s is the 19×19 stratified covariance matrix. The main diagonal of the stratified covariance matrix contains the (stratified) eigenvalues of the principal components and its last three elements are the variances of the log changes in exchange rates.

Scenario simulation tabulates risk factors and corresponding changes in portfolio value in a lookup table. The risk factor equation is equivalent to (13), except that the vector of shocks $\tilde{\mathbf{u}}$ is replaced by

$$(14) \qquad \hat{\mathbf{u}} = \underset{8\times4}{E'_{USD}} \underset{4\times4}{\sqrt{\Lambda_{USD}}} \underset{4\times1}{B_{USD}} \Big| E'_{DEM} \sqrt{\Lambda_{DEM}} B_{DEM} \Big| E'_{CHF} \sqrt{\Lambda_{CHF}} B_{CHF} \Big| E'_{GBP} \sqrt{\Lambda_{GBP}} B_{GBP} \Big| \underset{3\times1}{B_{FX}} ,$$

where B_N is a vector of standardized discrete shocks to country N's yield curve that have been stored as a particular scenario, as in the example displayed in Table 1, and B_{FX} contains standardized discrete shocks to the exchange rates. Separate lookup tables for interest rate scenarios and corresponding changes in portfolio values are constructed for each currency. FX rate changes are also tabulated in separate lookup tables. As discussed above, Monte Carlo simulation is used to sample from the discretized joint distribution of the risk factors. The correlated normal random variables X defined by (10) drives the sampling from both interest rate and exchange rate scenarios, which translate the discretized multicurrency portfolio value changes back into dollars.

Portfolio Valuation

The VaR evaluated in this study represents a ten-day exposure, measured from the first business day of December for each year in the sample. Each portfolio is priced at the initial date and fully revalued ten days later. On each draw, the simulated 8 yield risk-factor values per currency are interpolated to a set of 40 yields, from 3 months to 10 years at quarterly maturity intervals. The corresponding discount price curve is computed, which provides the discount factors for all options-based instruments and from which is derived spot and forward swap rates (for swaps and swaptions) or the forward LIBOR curve (for caps and caplets).

Caps, floors, and swaptions are valued by the standard one-factor Black model. Although once regarded as an inconsistent, ad hoc application of the Black-Scholes model to interest rate derivatives, in recent years academic research has established that the model is fully consistent and arbitrage-free when applied to single instrument classes, such as caps or swaptions.[13]

In the examples below, a VaR run consists of 1,000 draws for Monte Carlo risk factor vector (12) or principal components risk factor vector (13), and 10,000 draws for scenario simulation vector (14), which drives the sampling from the joint discrete distribution for the portfolio value changes. Although 1,000 draws is relatively small and inaccurate for crude Monte Carlo (that is, without variance reduction techniques), this number is of the same order of magnitude as the number of iterations that large banks use for Monte Carlo and historical VaR simulation. Scenario simulation can be iterated to much higher numbers for the same total CPU time as Monte Carlo; 10,000 iterations was arbitrarily chosen. All of these runs are repeated 20 times and the means and standard errors of the resulting VaRs are reported in Table 8.

Two discretization choices for four principal components were used for scenario simulation: a coarse discretization of $7\times5\times3\times3$, yielding 315 distinct scenarios for a

[13] See Rebonato (1998) or Musiela and Rutkowski (1997). The Black model for interest rate derivatives can be derived as a single-factor, lognormal case of the Brace-Gatarek-Musiela (1995) model. The Black model is inconsistent across instruments, the most notable case being the model's simultaneous assumption that forward LIBOR is lognormally distributed in valuing caps while forward swap rates are lognormally distributed in valuing swaptions. Nevertheless, this discrepancy is negligible compared to other sources of error in VaR calculations. Furthermore, most practitioners and some academics disregard the inconsistency in pricing and hedging applications (see Jamshidian (1997) and Derman (1996)).

single country's yield curve, and a fine discretization of $11\times7\times5\times5$, giving 1925 scenarios. The fine discretization is intended to sample deeper into the tails of the risk factor distributions to approximate more accurately the convexity of the option portfolios. Foreign exchange rate distributions were discretized into 7 states for both high and low-density interest rate discretizations.

The results in the next section are based on crude Monte Carlo, principal component Monte Carlo, and scenario simulation runs. The VaR outcome for each of the methods is repeated 20 times to determine empirical distributions of the estimates for each method.

5. Test Portfolio VaR Results.

The simulation results are sensitive to time period and to type of portfolio. Chart 1 summarizes the output for the basic portfolios for the full four-currency portfolios. The panels give a relative comparison of 99th percentile VaRs, based on averages of 20 simulation runs. Within the chart, each panel gives individual yearly results, for each type of instrument portfolio (caps, caplets, swaps, and swaptions). The bars on the left show the percentage deviation of the principal component VaR in relation to the Monte Carlo VaR. The central bars indicate the comparison of the $7\times5\times3\times3$ scenario simulation (SS7) with principal components taken as the benchmark, and the bars on the right represent the comparison of the $11\times7\times5\times5$ scenario simulation (SS11) with principal components.

At the 99th percentile level, the PC/MC comparison for swaps is in good agreement—off by less than 1.5 percent, except for 1997 where the discrepancy is an underestimate by PC of 7.5 percent. The PC/MC match is good for the nonlinear instruments, except for caplets in 1996 and swaptions, for which a 5 percent gap exits in most years. Evidently, the approximation error due to excluding higher order PCs as risk factors has a disparate impact on the VaR outcomes, particularly for nonlinear instruments. The PC results are taken as the benchmark for scenario simulation rather than the Monte Carlo results in order to keep errors arising from discretization distinct from those stemming from PC approximation of the risk factors.[14]

The deviations in Chart 1 tend to be in the same direction from year to year for each instrument portfolio, with scenario simulation understating the VaR in relation to PC simulation. These gaps can be substantial. The 1995 SS7 underestimates the VaR by over 20 percent and the SS7 swaption errors are consistently greater than 10 percent. The discrepancy narrows only slightly in the SS11 results. In the case of the caps, the deviation reverses sign and equals or exceeds 10 percent in 1994, 1997, and 1998. Table 7 gives the output in tabular form along with the corresponding 95th percentile results, while Table 8 shows the dollar values of the VaRs and corresponding standard deviations, as well as the standard error for the mean of the 20 VaR estimates. Generally, the differences in outcomes across simulation runs, particularly for scenario simulation, are statistically significant.

These results contrast with JZ's, who find in their single currency swap examples that scenario simulation VaRs (at the 97.5 and 99 percent confidence levels) differ by no more than 2 percent from the Monte Carlo results. In their Table 6, JZ give an example of

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[14] Furthermore, one can argue that Monte Carlo is not the appropriate benchmark against which to judge the other results because Monte Carlo places too few restrictions on the way in which key rates can evolve through time. The covariance matrix only weakly constrains the way shocks hit the yield curve. Monte Carlo allows improbable movements in yields of different maturities in relation to one another, such as the 6-month and 2-year key rates rising sharply as the 1-year key rate falls. On the other hand, shocking principal components greatly limits the possible configuration of relative rate movements. However, low-order representations of the term structure, such as by two or three components, limit the possible movements and shapes too much. See Rebonato (1998), chapter 3. Another consideration is that the

a multicurrency interest rate swap portfolio, but they only report results for one discretization that is sampled at different iteration levels and make no comparison to Monte Carlo.

Accounting for Erratic Results

The variation in scenario simulation results in relation to the PC benchmarks appears to be analogous to well-known behavior of binomial option pricing models—namely, as the density of the approximating lattice decreases, the binomial model value oscillates more widely around the true analytic price. A similar phenomenon may account for the scattered values of the scenario simulation VaR around the PC benchmark. Simply increasing the number of nodes by using a moderate number of time steps remedies the oscillation problem for standard options. In contrast, the computational burden of scenario simulation rapidly becomes excessive as the discretization of each state becomes finer.

The size of the deviations induced by coarse discretizations of the risk factors and their variation with finer discretizations is examined systematically in another simulation exercise. The interest-rate derivatives portfolios above are too complex to use in a computationally intensive convergence analysis. Instead, simplified linear and nonlinear portfolios are used. The linear portfolio consists of two assets, with one unit of each. Each unit is valued initially at par. Their prices are correlated and are lognormally distributed. The VaR of the portfolio is determined by scenario simulation, with each price (risk factor) distribution discretized into an equal number of odd-numbered states ranging from 5 to 63. The nonlinear portfolio is similar to the interest rate caplet portfolio

excluded higher order PCs contain most of the errors in the data. Monte Carlo includes all sources of variation.

above, with out-of-the-money options spaced at two standard deviations from the at-the-money price; instead of four currencies there are two prices. The constituent options are priced by Black-Scholes as before.

Chart 2 displays the percentage deviation of the scenario simulation VaRs for the linear portfolio, at both the 95th and 99th percentile levels, from the VaR computed by the standard covariance method. The upper panel shows the outcome for zero correlation between the two risk factors; the lower the results for a correlation of 0.5. Each 95th and 99th percentile VaR pair for a given discretization level was generated based on one million scenario simulation draws. Virtually the same plot is reproduced if the simulations are repeated—the deviations do not predominately represent random "sampling variation." The pattern varies with instrument portfolio and correlation assumption.

Qualitatively, the two plots for the linear portfolio are similar: the most striking aspect of these graphs is that the deviations oscillate in an irregular pattern as the discretization becomes finer. Although the fluctuations diminish as the number of states in a simulation run increases, they are still sizable—convergence is slow. For the coarse discretization cases that would typically be used for scenario simulation, the deviations can swing 15 to 20 percentage points from one discretization level to the next. Although accurate results are possible with a coarse discretization, the problem is knowing which size partition achieves an accurate approximation. Another feature of the plots is that throughout the range of discretization fineness, the 99th percentile scenario simulation VaRs appear to underestimate the corresponding Monte Carlo VaR more so than do the 95th percentile scenario simulations.

The volatility in the convergence results is most pronounced in the tails of the distribution of changes in portfolio value, the area that matters for VaR measurement. In contrast, the means and medians of these empirical distributions (considered by instrument or correlation assumption) show much less variation as the number of states per risk factor increases. For the linear portfolio, the standard deviation of the VaRs at the 99^{th} or 95^{th} percentile level, computed across all discretizations levels in Chart 2, is about 100 times greater than the standard deviation of the medians of the empirical distributions and 25 times greater than that for the means.

The pattern for the nonlinear portfolio shown in Chart 6 differs from the one in Chart 5 mainly in the substantial underestimate of the scenario simulation VaR in relation to the corresponding Monte Carlo VaR. The sizable understatement diminishes only at discretization levels—about 13 to 20 states per factor—that probably are impractical for scenario simulation on large portfolios. As in the linear case, there also appears to be a stronger tendency for the 99^{th} percentile VaRs to underestimate the actual 99^{th} percentile VaR compared to the 95^{th} percentile results.[15]

6. Conclusions

The chief difficulty with standard Monte Carlo and principal component simulation in VaR applications is the potentially intractable number of portfolio revaluations that must be made in the course of computing VaR for very large portfolios. Jamshidian and Zhu have suggested scenario simulation as a solution that permits the use of full revaluation.

[15] Non-parametric 99 percent confidence intervals on the Monte Carlo benchmarks are:

Correlation	99% VaR	95% VaR
0.0	(-0.85%, 0.63%)	(-0.64%, 0.50%)
0.5	(-0.98%, 0.67%)	(-0.71%, 0.58%)

This paper has clarified the mechanics of computing VaR by scenario simulation and has compared the VaR results from scenario simulation on several LIBOR-derivatives portfolios with those from standard Monte Carlo and principal component simulation.

For the multicurrency interest rate derivatives portfolios examined in this paper, the relative performance of scenario simulation was erratic. The outcomes for the nonlinear test portfolios demonstrate that scenario simulation using low- and moderate-dimensional discretizations can give “poor” estimates of VaR. Although the discrete distributions used in scenario simulation converge to their continuous distributions, convergence appears to be slow, with irregular oscillations that depend on portfolio characteristics and the correlation structure of the risk factors. It would therefore be prudent for any bank adopting this approach to test scenario-simulated VaR results periodically against results from standard Monte Carlo or principal component simulation.

The two-sided confidence intervals were computed using the method in Morokoff et al. (1998).

Appendix

Standard Granger causality tests were run on "adjacent" maturity spot LIBOR L_t and swap rates S_t to determine the degree of nonsynchroneity in the two data sources for the LIBOR term structure. The spot LIBOR maturity was 1 year and the swap rate tenor was 2 years, the breakpoint in the market data used to estimate the forward LIBOR curve. The following autoregression tests the null that coefficients $b_1 = b_2 = \cdots = b_p = 0$, that is, the swap rates do not Granger-cause spot LIBOR:

(A.1) $$L_t = c_1 + \sum_{j=1}^{p} a_j L_{t-j} + \sum_{j=1}^{p} b_j S_{t-j} .$$

Similarly, the null that spot LIBOR does not Granger-cause the swap rate is assessed using

(A.2) $$S_t = c_2 + \sum_{j=1}^{p} a_j S_{t-j} + \sum_{j=1}^{p} b_j L_{t-j} .$$

These autoregressions are run on both daily and weekly interest rate data. The null is tested based on the White heteroscedasticity-consistent covariance matrix estimator; hence a chi-square rather than an *F* statistic is reported.

Granger Causality Tests

		Daily		Weekly	
Variable tested:		LIBOR	Swap Rate	LIBOR	Swap Rate
USD	c^2	22.5	300.19	7.95	29.97
	p-value	0.001	0	0.242	0
DEM	c^2	7.08	244.39	10.23	11.86
	p-value	0.31	0	0.115	0.065
CHF	c^2	8.17	273.36	13.35	18.2
	p-value	0.226	0	0.038	0.006
GBP	c^2	13.39	312.78	10.53	14.6
	p-value	0.037	0	0.104	0.024

The table shows the results for 6 lags in the autoregressions. Qualitatively similar results were obtained for other values of *p*. The chi-square values overwhelmingly indicate that swap rates Granger-cause spot LIBOR in the daily data, with weaker reverse feedback from LIBOR to the swap rate for USD and GBP. Switching to a weekly periodicity greatly reduces the magnitudes of the chi-square statistics for the swap rates, although Granger-causation is still highly statistically significant for USD, CHF, and GBP.

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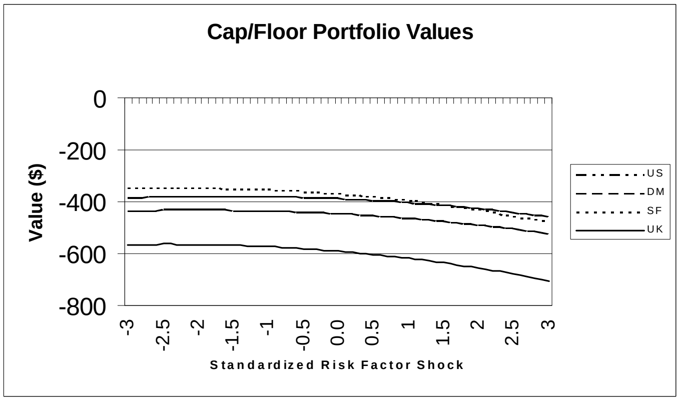
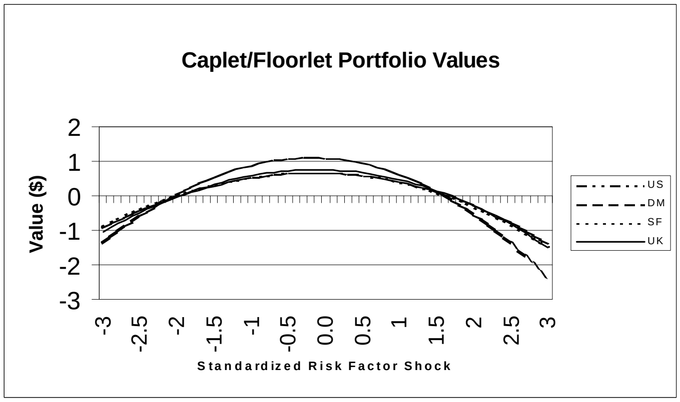
Figure 1


Cap/Floor Portfolio Values
Value ($)
0
-200
-400
-600
-800
-3
-2.5
-2
-1.5
-1
-0.5
0.0
0.5
1
1.5
2
2.5
3
Standardized Risk Factor Shock
US
DM
SF
UK


Caplet/Floorlet Portfolio Values
Value ($)
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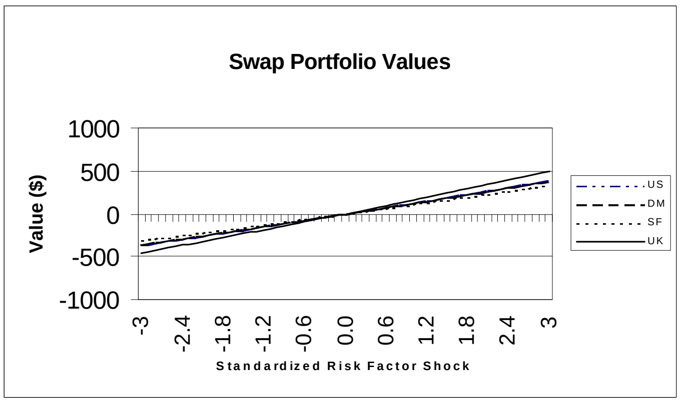
1
0
-1
-2
-3
-3
-2.5
-2
-1.5
-1
-0.5
0.0
0.5
1
1.5
2
2.5
3
Standardized Risk Factor Shock
US
DM
SF
UK


Swap Portfolio Values
Value ($)
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500
0
-500
-1000
-3
-2.4
-1.8
-1.2
-0.6
0.0
0.6
1.2
1.8
2.4
3
Standardized Risk Factor Shock
US
DM
SF
UK

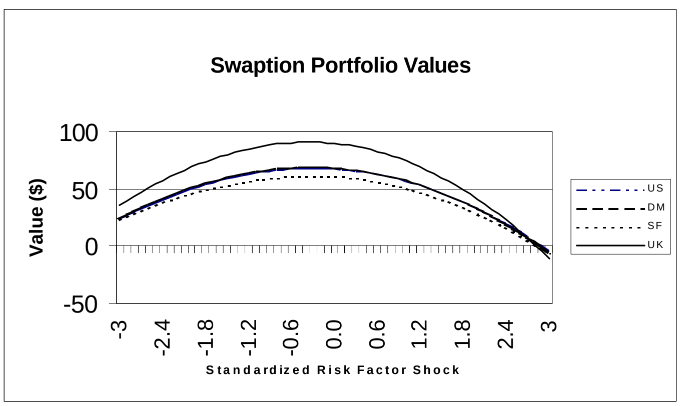

Swaption Portfolio Values
Value ($)
100
50
0
-50
-3
-2.4
-1.8
-1.2
-0.6
0.0
0.6
1.2
1.8
2.4
3
Standardized Risk Factor Shock
US
DM
SF
UK

Table 2
Correlation Matrix for 1994

		USD			DEM			CHF			GBP					
		PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	DEM	CHF	GPB
	PC1	1.0000	0.0000	0.0000												
USD	PC2	0.0000	1.0000	0.0000												
	PC3	0.0000	0.0000	1.0000												
	PC1	0.2433	-0.0357	0.1458	1.0000	0.0000	0.0000									
DEM	PC2	-0.2416	0.0137	0.0724	0.0000	1.0000	0.0000									
	PC3	0.1810	-0.0906	-0.1117	0.0000	0.0000	1.0000									
	PC1	0.2366	0.1240	0.2267	0.7006	-0.0345	-0.0837	1.0000	0.0000	0.0000						
CHF	PC2	-0.2309	0.0207	0.2085	-0.3727	0.3201	-0.0895	0.0000	1.0000	0.0000						
	PC3	0.0708	0.0886	-0.1887	0.0094	-0.5085	0.2588	0.0000	0.0000	1.0000						
	PC1	0.4194	-0.0908	-0.1216	0.5305	-0.3091	0.1053	0.4163	-0.4553	0.1207	1.0000	0.0000	0.0000			
GBP	PC2	-0.2337	0.0954	0.1328	-0.1965	0.4296	-0.1241	-0.1825	0.1844	-0.2631	0.0000	1.0000	0.0000			
	PC3	0.0438	-0.0724	-0.0660	-0.0469	0.0377	0.3783	-0.2672	-0.0510	-0.1307	0.0000	0.0000	1.0000			
	DEM	0.0314	-0.0725	0.0206	-0.1388	0.2672	0.1691	-0.0511	0.1776	-0.1031	-0.1736	0.0332	0.0499	1.0000	0.9239	0.7634
	CHF	0.1150	-0.0036	0.0726	-0.1197	0.2388	0.1932	-0.0426	0.1977	-0.1046	-0.1348	-0.0313	0.0040	0.9239	1.0000	0.7563
	GPB	0.1943	-0.1161	0.0324	-0.0663	0.2006	0.2157	0.1110	0.3074	-0.1020	-0.0765	-0.1612	-0.0256	0.7634	0.7563	1.0000

Orange or dark shading denotes correlation coefficients $\geq |0.20|$.

Table 3
Correlation Matrix for 1995

		USD			DEM			CHF			GBP					
		PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	DEM	CHF	GPB
	PC1	1.0000	0.0000	0.0000												
USD	PC2	0.0000	1.0000	0.0000												
	PC3	0.0000	0.0000	1.0000												
	PC1	0.2977	0.2511	0.1268	1.0000	0.0000	0.0000									
DEM	PC2	-0.4841	0.0152	0.0831	0.0000	1.0000	0.0000									
	PC3	0.0688	0.0644	0.1454	0.0000	0.0000	1.0000									
	PC1	0.0872	0.1761	0.0553	0.5861	0.1963	-0.0588	1.0000	0.0000	0.0000						
CHF	PC2	-0.3354	-0.0020	-0.0035	-0.3962	0.5769	0.0562	0.0000	1.0000	0.0000						
	PC3	-0.0124	0.1440	0.0728	-0.0835	0.0840	0.0833	0.0000	0.0000	1.0000						
	PC1	0.4785	0.1449	0.0788	0.4872	-0.5455	0.0555	0.1444	-0.4044	-0.1173	1.0000	0.0000	0.0000			
GBP	PC2	-0.0648	0.1061	0.0908	0.0392	0.4639	0.1713	0.1627	0.2105	0.1186	0.0000	1.0000	0.0000			
	PC3	0.2025	0.1449	0.0325	0.2762	-0.0923	0.2700	-0.0187	-0.3401	0.1400	0.0000	0.0000	1.0000			
	DEM	-0.1973	0.0410	-0.1658	0.1313	0.2656	-0.3284	0.4029	0.0970	0.0320	-0.2285	0.0287	-0.1690	1.0000	0.9480	0.7821
	CHF	-0.1110	0.0157	-0.0788	0.2146	0.1769	-0.2842	0.4599	-0.0012	-0.0406	-0.1278	0.0159	-0.1737	0.9480	1.0000	0.7399
	GPB	-0.2338	-0.1090	-0.2077	0.0544	0.2291	-0.1911	0.2107	0.2654	-0.0347	-0.1588	-0.1547	-0.1733	0.7821	0.7399	1.0000

Orange or dark shading denotes correlation coefficients $\geq |0.20|$.

Table 4
Correlation Matrix for 1996

		USD			DEM			CHF			GBP					
		PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	DEM	CHF	GPB
	PC1	1.0000	0.0000	0.0000												
USD	PC2	0.0000	1.0000	0.0000												
	PC3	0.0000	0.0000	1.0000												
	PC1	0.1835	0.1280	-0.0053	1.0000	0.0000	0.0000									
DEM	PC2	-0.3702	-0.0632	-0.0320	0.0000	1.0000	0.0000									
	PC3	0.1874	-0.0636	-0.0646	0.0000	0.0000	1.0000									
	PC1	0.1885	0.0743	0.0426	0.2008	-0.5270	0.0857	1.0000	0.0000	0.0000						
CHF	PC2	0.0028	0.0696	-0.0599	0.2304	0.1536	-0.3383	0.0000	1.0000	0.0000						
	PC3	-0.1078	-0.0376	-0.2738	-0.2031	0.2122	0.1032	0.0000	0.0000	1.0000						
	PC1	0.2976	-0.0748	-0.1314	0.0387	-0.3411	0.2986	0.1992	-0.1477	-0.2207	1.0000	0.0000	0.0000			
GBP	PC2	-0.3897	0.1677	-0.1583	0.0812	0.2347	-0.3899	-0.2045	0.2512	0.0176	0.0000	1.0000	0.0000			
	PC3	-0.0415	-0.1877	0.0432	-0.1639	0.1666	-0.0255	0.1491	-0.2525	0.1632	0.0000	0.0000	1.0000			
	DEM	-0.1648	0.0493	-0.0433	-0.0331	0.3982	-0.0179	-0.0414	0.1605	0.2381	-0.3227	0.1500	0.0664	1.0000	0.9082	0.5467
	CHF	-0.1446	0.1074	-0.0079	-0.0147	0.2580	0.0678	0.1123	0.0395	0.2135	-0.2431	0.1968	0.0786	0.9082	1.0000	0.3939
	GPB	0.0158	-0.1582	0.0315	-0.1182	0.4321	0.1001	-0.2499	0.2005	0.1301	-0.3019	-0.1001	-0.1275	0.5467	0.3939	1.0000

Orange or dark shading denotes correlation coefficients $\geq |0.20|$.

Table 5
Correlation Matrix for 1997

		USD			DEM			CHF			GBP					
		PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	DEM	CHF	GPB
	PC1	1.0000	0.0000	0.0000												
USD	PC2	0.0000	1.0000	0.0000												
	PC3	0.0000	0.0000	1.0000												
	PC1	0.2038	0.2331	-0.2225	1.0000	0.0000	0.0000									
DEM	PC2	-0.4812	-0.0061	-0.0237	0.0000	1.0000	0.0000									
	PC3	0.3724	-0.2849	0.1437	0.0000	0.0000	1.0000									
	PC1	-0.0521	0.2049	-0.0963	0.1181	-0.3524	-0.3831	1.0000	0.0000	0.0000						
CHF	PC2	-0.2835	-0.1072	0.2664	-0.3935	0.2929	-0.2211	0.0000	1.0000	0.0000						
	PC3	0.1981	-0.0974	-0.1996	0.0757	-0.0847	-0.0414	0.0000	0.0000	1.0000						
	PC1	0.3952	0.1504	0.0119	0.1930	-0.4372	0.3689	0.0344	-0.2904	0.1653	1.0000	0.0000	0.0000			
GBP	PC2	-0.4174	0.3071	-0.0839	-0.1467	0.2206	-0.3406	-0.0340	0.1567	0.1784	0.0000	1.0000	0.0000			
	PC3	-0.1641	0.1786	0.0832	0.1539	-0.1095	-0.2622	0.1541	0.0241	-0.3206	0.0000	0.0000	1.0000			
	DEM	-0.0067	0.1331	-0.2346	0.0235	0.0594	-0.1519	0.0677	-0.0728	0.0815	-0.1494	-0.0170	0.0382	1.0000	0.8619	0.5048
	CHF	0.0445	0.0899	-0.2982	0.0595	0.0594	-0.1570	0.1325	-0.1244	0.0245	-0.2129	-0.0648	0.0383	0.8619	1.0000	0.5042
	GPB	0.0636	0.0025	0.0284	0.1569	-0.1351	-0.1548	0.2061	-0.1543	0.0300	-0.1859	-0.2212	0.2953	0.5048	0.5042	1.0000

Orange or dark shading denotes correlation coefficients $\geq |0.20|$.

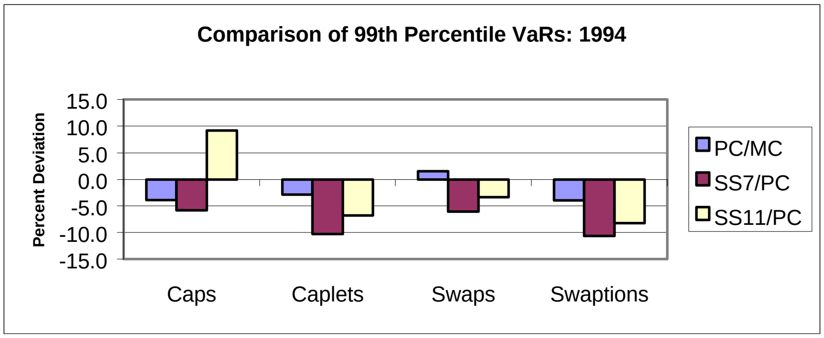
Table 6

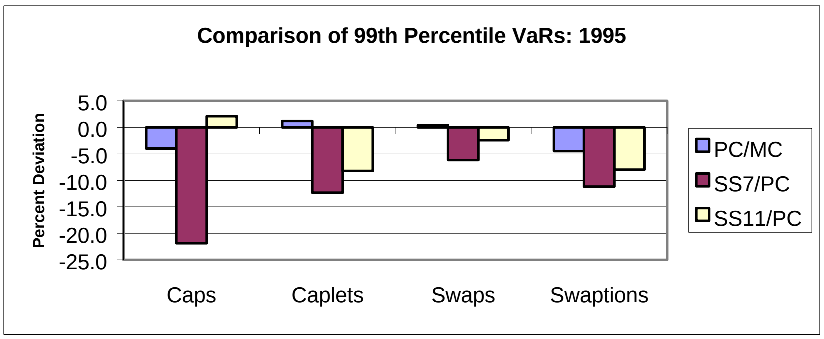
Correlation Matrix for 1998

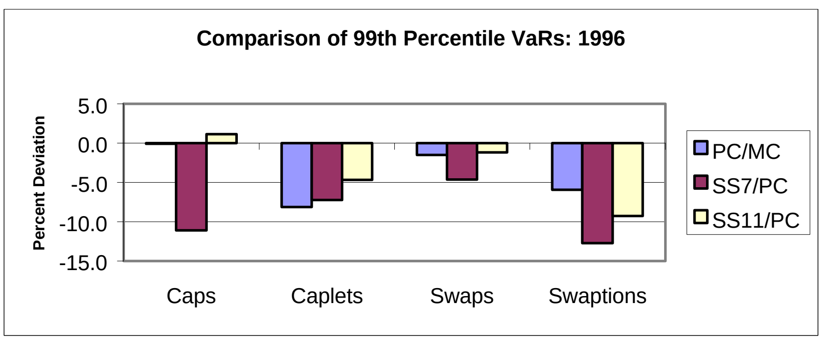
		USD			DEM			CHF			GBP					
		PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	PC1	PC2	PC3	DEM	CHF	GPB
	PC1	1.0000	0.0000	0.0000												
USD	PC2	0.0000	1.0000	0.0000												
	PC3	0.0000	0.0000	1.0000												
	PC1	0.5895	0.1920	0.1399	1.0000	0.0000	0.0000									
DEM	PC2	-0.1843	0.3053	-0.1411	0.0000	1.0000	0.0000									
	PC3	-0.0164	0.1013	0.4459	0.0000	0.0000	1.0000									
	PC1	0.2457	0.2168	-0.3287	0.2157	0.1724	-0.3276	1.0000	0.0000	0.0000						
CHF	PC2	-0.4962	0.1380	-0.3000	-0.5829	0.2149	0.1660	0.0000	1.0000	0.0000						
	PC3	-0.0526	-0.3017	0.2418	-0.0585	-0.2248	-0.0174	0.0000	0.0000	1.0000						
	PC1	0.3325	0.0345	0.2870	0.4608	-0.3084	0.1082	0.0504	-0.4726	0.1368	1.0000	0.0000	0.0000			
GBP	PC2	0.0275	0.2709	-0.1490	-0.0509	0.2179	-0.0809	0.0849	0.1430	0.0367	0.0000	1.0000	0.0000			
	PC3	-0.0440	-0.1809	0.3284	-0.0185	-0.1246	0.1316	-0.1899	-0.1496	0.0957	0.0000	0.0000	1.0000			
	DEM	0.2576	0.0872	-0.3963	0.1789	-0.0970	0.0143	0.2737	0.0900	-0.3002	0.2587	0.0303	-0.1092	1.0000	0.9109	0.5094
	CHF	0.2706	0.0921	-0.4253	0.2371	-0.0515	-0.1046	0.4369	-0.0251	-0.2704	0.3278	-0.0036	-0.1872	0.9109	1.0000	0.3912
	GPB	0.1794	-0.0708	-0.2995	0.0843	-0.0901	-0.1089	0.1925	0.1464	-0.1175	-0.2163	0.0882	-0.0387	0.5094	0.3912	1.0000

Orange or dark shading denotes correlation coefficients $\geq |0.20|$.

Chart 1

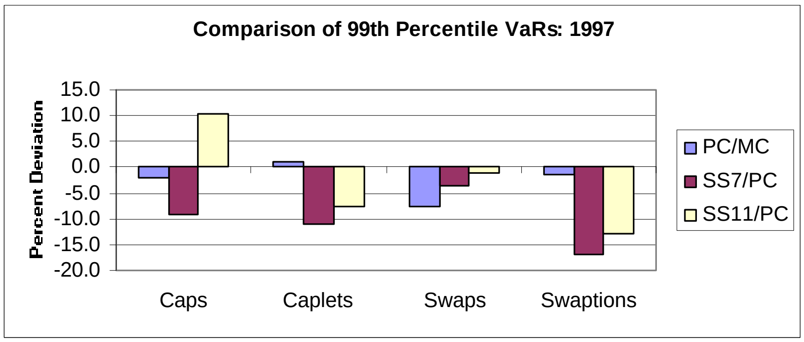


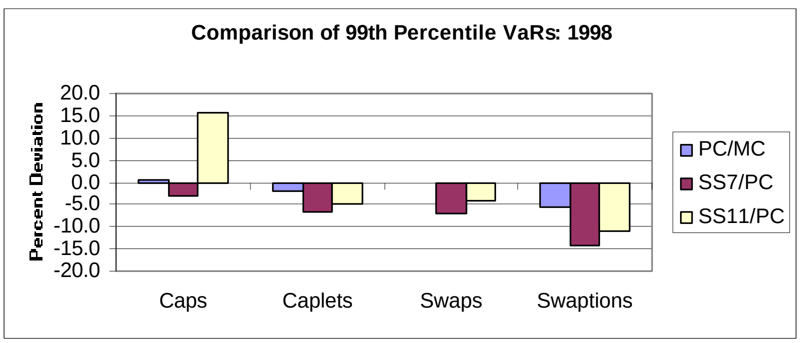




PC/MC denotes the percentage deviation of the principal components VaR from the Monte Carlo VaR; SS7/PC denotes the $7\times5\times3\times3$ scenario simulation VaR relative to principal components VaR; SS11/PC denotes the $11\times7\times5\times5$ scenario simulation VaR relative to principal components VaR.

Chart 1, continued





PC/MC denotes the percentage deviation of the principal components VaR from the Monte Carlo VaR; SS7/PC denotes the $7\times5\times3\times3$ scenario simulation VaR relative to principal components VaR; SS11/PC denotes the $11\times7\times5\times5$ scenario simulation VaR relative to principal components VaR.

Table 7
Four-Currency Interest Rate Derivate Portfolios

1994	Caps/Floors		Caplets/Floorlets		Swaps		Swaptions	
	99th	95th	99th	95th	99th	95th	99th	95th
PC/MC	-3.90	-1.28	-2.89	-1.35	1.52	-1.06	-3.95	-3.12
SS7/PC	-5.82	-3.93	-10.29	-5.86	-6.08	-2.24	-10.65	-5.12
SS11/PC	9.14	15.21	-6.78	-4.19	-3.33	-0.19	-8.25	-8.35

1995	Caps/Floors		Caplets/Floorlets		Swaps		Swaptions	
	99th	95th	99th	95th	99th	95th	99th	95th
PC/MC	-3.99	-5.09	1.22	-2.18	0.44	1.02	-4.45	-3.82
SS7/PC	-21.88	-17.74	-12.35	-4.77	-6.18	-3.65	-11.17	-6.48
SS11/PC	2.09	13.47	-8.23	-3.76	-2.40	-0.54	-7.98	-9.87

1996	Caps/Floors		Caplets/Floorlets		Swaps		Swaptions	
	99th	95th	99th	95th	99th	95th	99th	95th
PC/MC	-0.09	-0.99	-8.14	-9.52	-1.52	-4.30	-5.92	-4.28
SS7/PC	-11.11	-7.49	-7.26	-5.29	-4.62	-1.37	-12.71	-7.90
SS11/PC	1.13	7.24	-4.69	-4.84	-1.17	0.91	-9.27	-8.67

1997	Caps/Floors		Caplets/Floorlets		Swaps		Swaptions	
	99th	95th	99th	95th	99th	95th	99th	95th
PC/MC	-2.00	-5.75	0.94	-4.24	-7.56	-1.95	-1.32	-6.21
SS7/PC	-9.30	-3.56	-11.06	-6.19	-3.44	-3.71	-16.83	-8.56
SS11/PC	10.48	22.80	-7.56	-4.61	-0.98	-1.46	-12.76	-10.49

1998	Caps/Floors		Caplets/Floorlets		Swaps		Swaptions	
	99th	95th	99th	95th	99th	95th	99th	95th
PC/MC	0.67	-1.62	-2.02	-2.57	-0.17	-2.83	-5.76	-4.11
SS7/PC	-2.93	0.66	-6.66	-3.75	-7.06	-2.61	-14.40	-8.32
SS11/PC	15.74	23.78	-4.87	-2.97	-4.01	-0.20	-10.82	-10.19

PC/MC is the percentage deviation of the principal component VaR from the Monte Carlo VaR. SS7/PC is the percentage deviation of the $7\times5\times3\times3$ scenario simulation VaR from the principal component VaR. SS11/PC is the percentage deviation of the $11\times7\times5\times5$ scenario simulation VaR from the principal component VaR.

Table 8
Detailed Results

1994: Caps/Floors

	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	159.153	111.127	152.946	109.700	144.040	105.383	166.926	126.387
Std Dev.	8.127	3.466	9.734	3.325	1.970	1.089	2.087	0.787
Std Error	1.817	0.775	2.177	0.743	0.440	0.243	0.467	0.176

1994: Caplets/Floorlets

	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	2.674	1.437	2.597	1.418	2.330	1.334	2.421	1.358
Std Dev.	0.227	0.081	0.225	0.079	0.057	0.022	0.058	0.028
Std Error	0.051	0.018	0.050	0.018	0.013	0.005	0.013	0.006

1994: Swaps

	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	940.829	648.902	955.140	642.013	897.066	627.647	923.356	640.762
Std Dev.	59.107	23.946	50.360	24.845	13.049	8.117	13.351	7.966
Std Error	13.217	5.355	11.261	5.556	2.918	1.815	2.985	1.781

1994: Swaptions

	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	130.812	65.544	125.649	63.502	112.269	60.251	115.281	58.198
Std Dev.	10.996	4.782	12.274	4.701	3.219	1.445	3.010	1.009
Std Error	2.459	1.069	2.744	1.051	0.720	0.323	0.673	0.226

Results reported in dollars. The standard error is for the VaR at a given percentile, computed based on 20 runs of 1,000 iterations each for Monte Carlo and principal component VaR and 10,000 iterations for scenario simulation.

Table 8, continued
Detailed Results

1995: Caps/Floors	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	73.824	52.779	70.880	50.091	55.374	41.205	72.359	56.837
Std Dev.	4.667	1.910	3.777	2.125	0.814	0.379	0.845	0.361
Std Error	1.044	0.427	0.844	0.475	0.182	0.085	0.189	0.081

1995: Caplets/Floorlets	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	1.896	1.013	1.919	0.991	1.682	0.943	1.761	0.954
Std Dev.	0.275	0.048	0.205	0.105	0.035	0.019	0.036	0.019
Std Error	0.061	0.011	0.046	0.023	0.008	0.004	0.008	0.004

1995: Swaps	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	491.033	333.107	493.177	336.520	462.707	324.244	481.345	334.692
Std Dev.	19.749	15.063	29.659	16.037	8.865	4.157	7.262	5.181
Std Error	4.416	3.368	6.632	3.586	1.982	0.930	1.624	1.159

1995: Swaptions	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	59.544	30.463	56.895	29.298	50.539	27.401	52.357	26.408
Std Dev.	8.161	2.243	4.390	1.780	1.418	0.429	1.415	0.549
Std Error	1.825	0.502	0.982	0.398	0.317	0.096	0.316	0.123

Results reported in dollars. The standard error is for the VaR at a given percentile, computed based on 20 runs of 1,000 iterations each for Monte Carlo and principal component VaR and 10,000 iterations for scenario simulation.

Table 8, continued
Detailed Results

1996: Caps/Floors								
	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	81.816	58.560	81.744	57.980	72.660	53.638	82.666	62.176
Std Dev.	3.240	1.912	3.975	1.953	0.965	0.389	1.468	0.531
Std Error	0.724	0.428	0.889	0.437	0.216	0.087	0.328	0.119

1996: Caplets/Floorlets								
	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	2.070	1.096	1.901	0.992	1.763	0.939	1.812	0.944
Std Dev.	0.157	0.088	0.244	0.050	0.045	0.019	0.048	0.018
Std Error	0.035	0.020	0.054	0.011	0.010	0.004	0.011	0.004

1996: Swaps								
	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	479.380	333.362	472.100	319.029	450.281	314.660	466.572	321.944
Std Dev.	24.958	12.000	29.377	12.295	5.680	3.803	5.553	3.963
Std Error	5.581	2.683	6.569	2.749	1.270	0.850	1.242	0.886

1996: Swaptions								
	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	59.738	30.539	56.200	29.233	49.059	26.924	50.993	26.698
Std Dev.	7.046	2.134	5.280	2.087	1.149	0.413	1.309	0.609
Std Error	1.575	0.477	1.181	0.467	0.257	0.092	0.293	0.136

Results reported in dollars. The standard error is for the VaR at a given percentile, computed based on 20 runs of 1,000 iterations each for Monte Carlo and principal component VaR and 10,000 iterations for scenario simulation.

Table 8, continued
Detailed Results

1997: Caps/Floors	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	74.880	53.855	73.385	50.759	66.557	48.951	81.074	62.333
Std Dev.	4.696	2.266	4.007	2.293	0.998	0.405	1.111	0.458
Std Error	1.050	0.507	0.896	0.513	0.223	0.091	0.249	0.102

1997: Caplets/Floorlets	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	2.259	1.189	2.281	1.139	2.028	1.068	2.108	1.086
Std Dev.	0.218	0.076	0.232	0.063	0.048	0.027	0.084	0.025
Std Error	0.049	0.017	0.052	0.014	0.011	0.006	0.019	0.006

1997: Swaps	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	558.799	368.531	516.547	361.331	498.765	347.917	511.459	356.071
Std Dev.	19.522	14.999	30.331	19.930	6.500	5.030	9.029	3.694
Std Error	4.365	3.354	6.782	4.456	1.453	1.125	2.019	0.826

1997: Swaptions	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	68.428	35.532	67.523	33.326	56.162	30.472	58.906	29.829
Std Dev.	4.922	2.109	6.603	2.784	1.571	0.574	1.485	0.708
Std Error	1.101	0.472	1.476	0.622	0.351	0.128	0.332	0.158

Results reported in dollars. The standard error is for the VaR at a given percentile, computed based on 20 runs of 1,000 iterations each for Monte Carlo and principal component VaR and 10,000 iterations for scenario simulation.

Table 8, continued
Detailed Results

1998: Caps/Floors	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	-59.022	-42.418	-59.416	-41.731	-57.677	-42.005	-68.770	-51.656
Std Dev.	3.485	1.903	3.451	1.855	0.838	0.319	0.931	0.476
Std Error	0.779	0.426	0.772	0.415	0.187	0.071	0.208	0.106

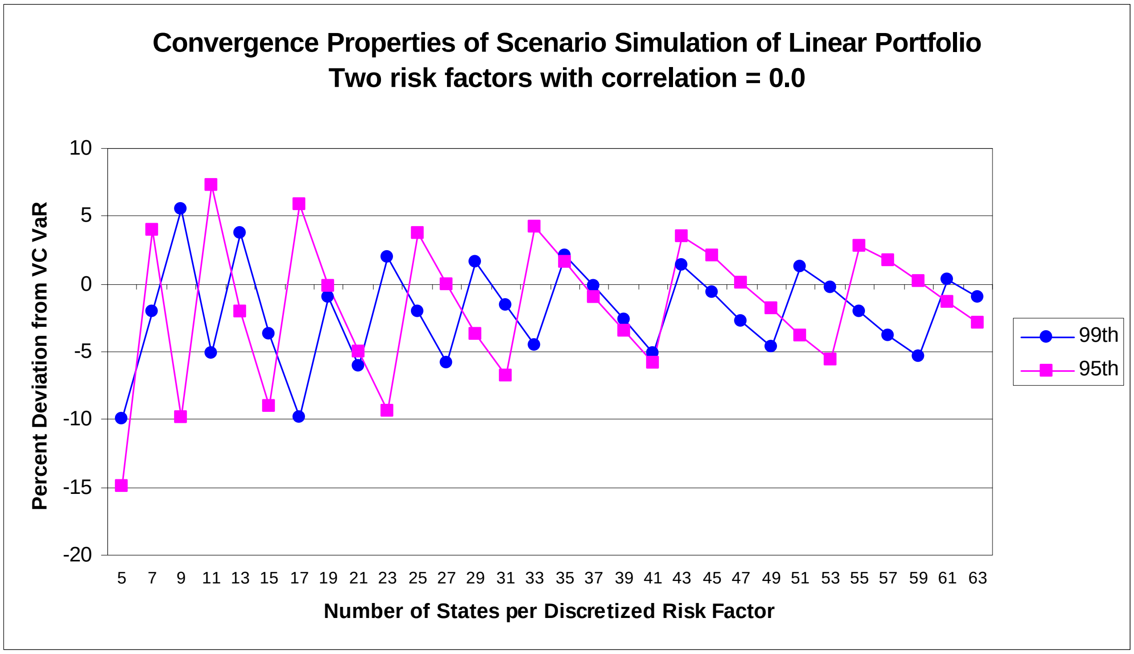
1998: Caplets/Floorlets	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	-2.122	-1.091	-2.080	-1.063	-1.941	-1.023	-1.978	-1.032
Std Dev.	0.252	0.098	0.235	0.070	0.065	0.021	0.045	0.021
Std Error	0.056	0.022	0.053	0.016	0.014	0.005	0.010	0.005

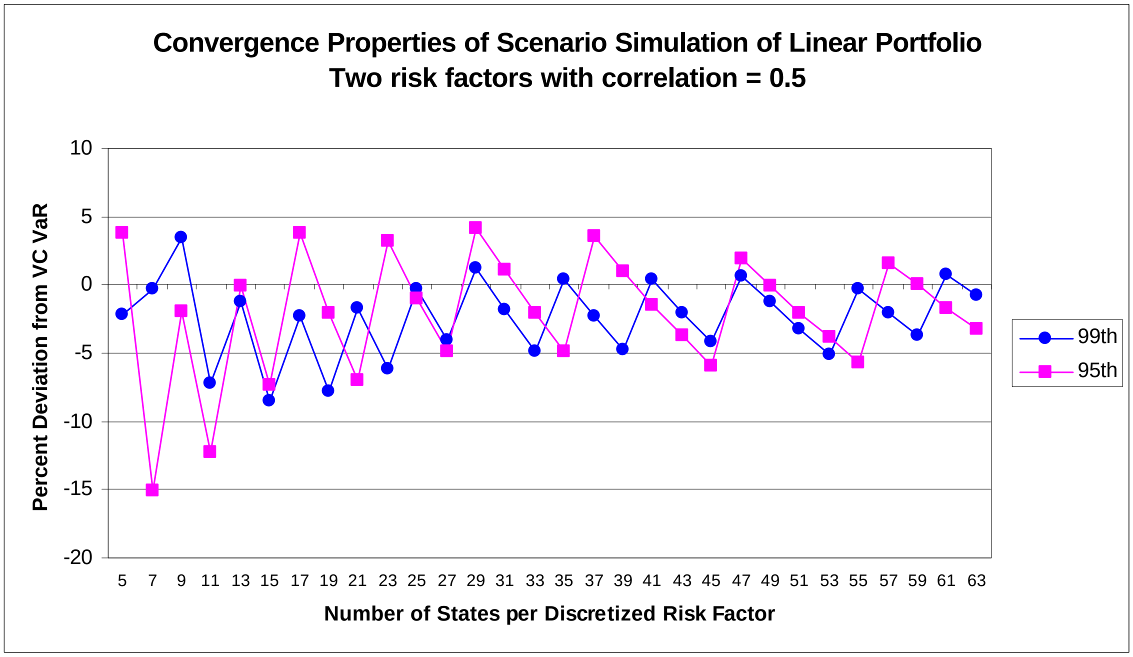
1998: Swaps	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	-481.901	-331.331	-481.069	-321.947	-447.116	-313.537	-461.794	-321.300
Std Dev.	17.660	13.655	25.037	13.649	7.782	4.240	7.434	3.308
Std Error	3.949	3.053	5.598	3.052	1.740	0.948	1.662	0.740

1998: Swaptions	Monte Carlo		PC4		7x5x3x3		11x7x5x5	
	99th	95th	99th	95th	99th	95th	99th	95th
VaR	-60.657	-30.439	-57.164	-29.187	-48.933	-26.759	-50.979	-26.211
Std Dev.	6.669	2.145	4.641	2.284	1.092	0.588	1.163	0.524
Std Error	1.491	0.480	1.038	0.511	0.244	0.132	0.260	0.117

Results reported in dollars. The standard error is for the VaR at a given percentile, computed based on 20 runs of 1,000 iterations each for Monte Carlo and principal component VaR and 10,000 iterations for scenario simulation.

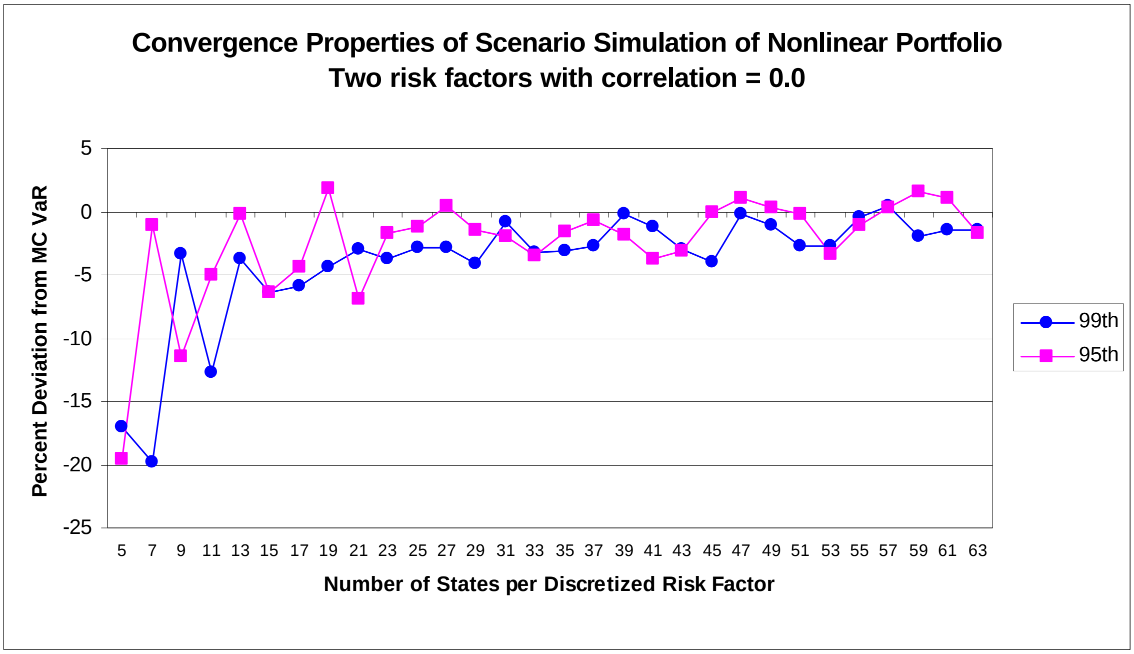
Chart 2

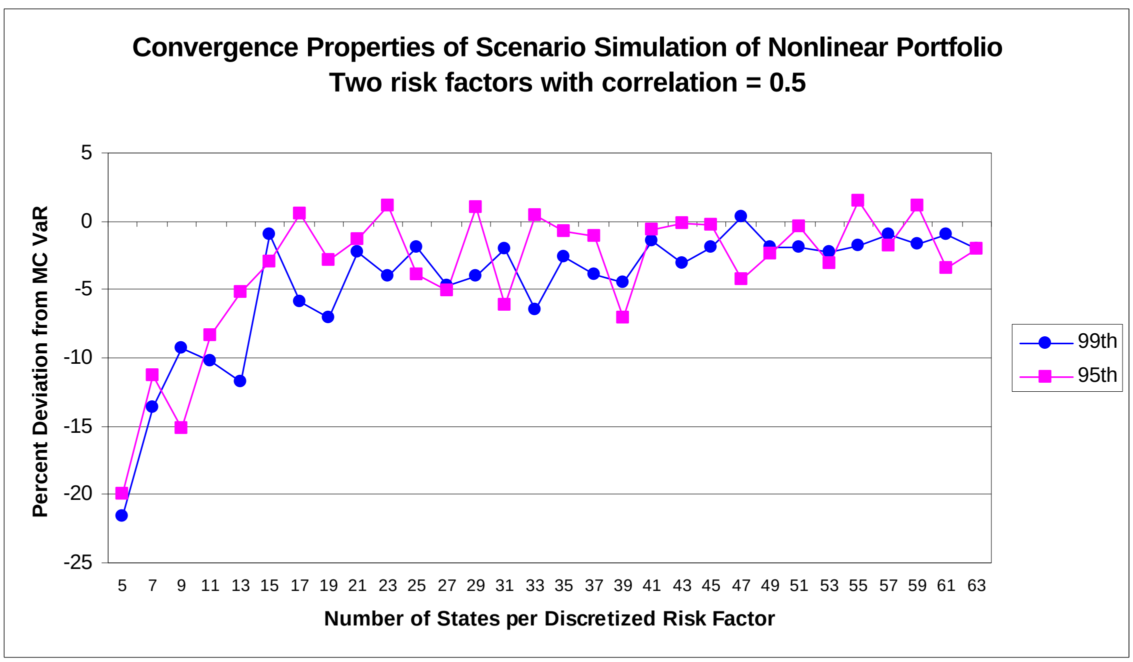




Each pair of VaRs at a given discretization level is derived from a simulation of 1,000,000 iterations.

Chart 3





Each pair of VaRs at a given discretization level is derived from a simulation of 1,000,000 iterations.