A Markov Model of Bank Failure Estimated Using an Information-Theoretic Approach

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Abstract: In this paper, we develop an early-warning bank failure model (EWM) designed specifically to capture the dynamic process underlying the transition from financially sound to closure. We model the transition process as a stationary Markov model and estimate the transition probabilities using a Generalized Maximum Entropy (GME) estimation technique. The GME estimation method is a member of the class of information-theoretic methods, is semi-parametric, and is better suited for estimating models in which the data are limited (e.g., few events, and data availability problems), highly collinear, and measured with error – conditions that often exist with micro-level banking data. In addition, this method allows us to incorporate prior information and impose fewer distributional assumptions relative to conventional maximum likelihood (or full information maximum likelihood) methods. We report estimates of the transition probabilities for nine transition states for the population of nationally chartered banks incorporating the effect of bank-specific and macroeconomic variables from 1984 through 1999.

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I. Introduction

In this paper, we develop an early-warning bank failure model (EWM) designed specifically to capture the dynamic process underlying the transition from financially sound to failure/closure. We model the transition process as a first-order stationary Markov process in which failure is but one of several possible financial states (e.g., financially sound, distressed, insolvent, and closure). The multi-state design has several advantages over the more conventional binary-state approach generally found in the bank-failure literature (Altman, et al., 1981; Fissel, et al., 1996; Looney, et al., 1989; and Kolari, et al., 2001). First, the model is not dependent solely on an outcome state derived at the discretion of the regulators (i.e., failure/closure). Second, this approach captures the problem-bank/failure transition process over several states of financial distress; a model design that better serves the bank supervisors' early intervention objectives. Moreover, it differentiates between banks that remain healthy, those that exhibit distress but recover, and those that eventually fail.

We estimate the Markov transition model using a Generalized Maximum Entropy (GME) estimation procedure. The GME approach is an information-theoretic modeling technique that is better suited for estimating models, in which the data are limited (e.g., few events and data availability problems), highly collinear, and measured with error (Golan, et al., 1996a). The GME is a semi-parametric, robust estimator that is based on fewer distributional assumptions than conventional maximum likelihood (or full information maximum likelihood) methods. Furthermore, this approach allows us to incorporate prior information about the transition process directly into the estimation process.

More importantly for our purpose, the GME procedure allows us to incorporate bank-specific time-series, cross-section data as well as time-varying macroeconomics and financial market variables directly into estimation of the transition probabilities; a data design significantly broader than the cross-section data design generally found in the bank-failure literature. Our objective, then, is to model the transition process conditional on bank-specific characteristics, exogenous macroeconomic effects, and changing financial market conditions. The latter two categories are especially important because they capture the effects of changing market conditions on the failure probabilities generally ignored in the empirical bank-failure literature.

The remainder of the paper is organized as follows. In Section II we provide a brief overview of the limitations of the conventional EWM design. In Section III, we outline our proposed first-order Markov model and discuss in more detail the GME approach (see also Appendix 1). In Section IV we summarize the data and report our main results.

II. Limitations of Conventional Early-Warning Bank Failure Models

Although general agreement exists on the fundamental objective of an early warning model (i.e., a timely and accurate list of financially fragile banks), there is far less agreement on the development and design of these models (Stengel, et al., 2000). The early work in this area used conventional statistical techniques (e.g., discriminant analysis, logit models, and factor analysis) to develop models primarily for failure-classification purposes (see Altman, et al., 1981). Those models were designed specifically to partition the population of banks at a point in time into distinct groups that reflect the relative financial strength of the institutions. More recently, multinomial logit, survival analysis, neural networks, and other pattern recognition methods (Fissel, et al., 1996, Kolari, et al., 1996) have been used to improve on the classification power and overall reliability of these models.

The underlying data design supporting the estimation of those models, however, has not changed since the early work of Altman (1968), Beaver (1968), Sinkey (1975), and Eisenbeis (1977) on corporate bankruptcies. A cross-section sample of banks, at time t, is used to predict the performance (i.e., closure) over the interval $[t, t+n]$ (n=1,2 years forward) conditioned on initial balance sheet and income ratios. Under that design, the financial ratios are used to differentiate between those banks that correctly, from those that incorrectly, position themselves to withstand credit, liquidity, or market shocks.

Unfortunately, credit, liquidity, and market shocks are environment or state-specific events that are only observed over time. Models developed on cross-section data, by design, omit time-varying factors and, as a result, fail to capture the underlying dynamics of the failure/survival process.[1] Although these models tend to perform well at classifying banks within sample, they generally perform poorly out-of-sample, especially as banks reposition their portfolios and lending strategies to correspond to contemporaneous economic and industry conditions. For that reason, bank-failure classification models have not fared well as supervisory tools.

A time-series/cross-section sample design, however, introduces its own set of problems that makes it difficult to model performance using statistical procedures found in the bank-failure literature. Not all methods or techniques that are used with cross-section data work well with time-series/cross-section data. For example, survival analysis procedures could be extended to capture the impact of time-varying covariates on the probability of failure. However, those procedures require a relatively large number of events in each time period. Although the failure rates over the 1987-92 observation period were relatively large, the number of total failures since 1992 has declined dramatically making it difficult to satisfy this requirement.[2] In fact, the relatively small number of bank failures in general underlies the overwhelming choice of a pooled (often over several years), cross-section sample design in the bank-failure literature. That condition, by itself, suggests that a simple fail/nonfail model design is inadequate for our purpose.

For those reasons we propose an alternative model design in which we estimate the transition probabilities instead of failure-classification probabilities identified under a conventional EWM design. The multi-state, transition-based approach allows us to better capture the dynamic process leading to financial distress that cannot be achieved under a classification-based model design. Unfortunately, the multistate design does not address the data limitation problem. For that reason, we use a GME procedure

[1] For example, banks adjust their assets and liabilities in anticipation of specific movements in economic (i.e., business and interest-rate cycles) conditions. The success or failure of their asset-liability strategy (ex post) is conditional on the actual outcome of the macro economy over time (e.g., an asset-liability strategy in which a bank that maintains a high concentration of variable-rate C&I loans funded with seasoned long-term, fixed-rate liabilities will perform much differently in an expanding economy with interest rates rising than if the macro economy is slowing down and interest rates are falling). Models developed on cross-section data fail to capture this potentially important aspect of the failure process that is identified through behavior over time.

[2] For example, no national banks failed in more than 60 percent of the time periods from first quarter 1993 to fourth quarter 1999.

to recover the transition probabilities. The GME approach is better suited for estimating models in which data are limited (e.g., few events). Moreover, the GME procedure allows us to incorporate firm-specific strategic objectives (e.g., portfolio composition, funding, liquidity measures) and general economic conditions (e.g., economic growth, interest rate effects) at different points in the business cycle directly into the estimation process through the use of both time-series and cross-section data. Furthermore, the GME procedure uses minimal distributional assumptions, is easy to apply, and is computationally efficient.

III. An Alternative EWM Model: A Markov Transition Approach

We define the various states (e.g., financially sound, distressed, insolvent/failure, and closure) in terms of the equity-asset ratio, using a book-value equity measure. The transition probabilities measure the probability that a bank with equity capital $y_{t,j}$ (state *j*, in time *t*) will have equity capital $y_{t+1,k}$ (state *k*, in time *t+1*) in the next period. These transitions probabilities capture the likelihood that a bank will exhaust or increase its equity capital in period *t+n* conditional on its initial state, and that of the macro economy, in time *t*.

As a logical starting point for our analysis, we define the transition states using the book equity (i.e., leverage) capital zones defined under the prompt correction action provision of the Federal Deposit Insurance Corporation Improvement Act (FDICIA). In addition, we include an insolvency state (negative book-valued equity) and three terminal or absorption states – merged with an affiliated (i.e., within holding company) bank, merged with an unaffiliated bank, and failure for a total of nine financial states (see Table 1). The latter absorption state (i.e., failure) is of greatest interest to supervisors and the primary focus of the following analysis.

III.1 A Markov Transition Model: The Basic Model Design

We define the binary random variable y_{itj} = 1 if the i[th] bank (i=1, 2, …, n) is in state j (j=1, 2, …, K) at time t (t=1, 2, …, T), and $y_{itj} = 0$ for all other K-1 states. Under this assumption, our model is a generalization of a qualitative response variable model discussed in Amemiya (1985). However, to simplify the analysis, we formulate our model on the mean behavior of all banks within each state in time *t*, rather than the specific behavior of each bank. This allows us to capture the same information, but with a much lower dimensionality of data.

We define $\mathbf{y}_t$ as a K-dimensional vector of *proportions* in the *kth* Markov state in period *t*. The elements of $\mathbf{y}_t$ are the proportions of banks in each state, in period *t*, calculated directly from the sample data. Similarly, $\mathbf{y}_{t+1}$ is a K-dimensional vector of proportions representing the fraction of banks in the *kth* Markov state in period *t+1*. We start with periods two and three and condition the estimate of the transition probabilities on the first-period proportions. Our objective is to estimate the $K \times K$ Markov transition probabilities *P,* using information from the full sample 1984 through 1999.

Following convention, we represent the general linear relationship between $\mathbf{y}_{t+1}$, $\mathbf{y}_t$ and the matrix of transition probabilities *P* as

$$y_{t+1,j} = \sum_{k=1}^{K} p_{kj}\, y_{tk}\,, \tag{1a}$$

where the p_{kj} are the stationary Markov probabilities over the relevant periods, and

$$\sum_{j=1}^{K} p_{kj} = 1 \qquad \text{for } j, k = 1, 2, \ldots, K, \tag{1b}$$

imposes the condition that the p_{kj} are proper probabilities.

A model such as that proposed in equation (1), in which the only available information/data are given by the outcome states $\mathbf{y}_t$, is the most elementary design of a Markov probability model. This base-case model serves our purpose only insofar as it provides the foundation for the generalized model, which incorporates both bank-specific and macroeconomic factors.

Under this simple design, the number of unknowns may exceed the number of data points if the number of periods T is small. As a result, the base-case model is underdetermined in the sense that there are infinitely many stationary Markov solutions that satisfy the basic relationship in equation (1). An underdetermined model always exists regardless of the number of periods available when generalizing the approach to incorporate noise in the data. This type of problem can be addressed by (i) incorporating additional restrictions such as distributional assumptions, (ii) imposing a decision rule to select one of the infinitely many solutions, or (iii) both. The former approach is valid only when the assumptions or restrictions are consistent with the data generating process. Therefore, to avoid imposing arbitrary assumptions or restrictions, we employ the latter approach using an entropy decision criterion.

Using the entropy decision criterion to estimate the transition probability matrix P of equation (1), yields the classical maximum entropy (ME) method:

$$ME = \begin{cases} \hat{\mathbf{p}} = \arg\max - \sum_{k,j} p_{kj} \log p_{kj} \\ \text{s.t.} \\ y_{\mathrm{t+1,j}} = \sum_{\mathrm{k}} \mathrm{p}_{\mathrm{kj}} y_{tk} ; \ \sum_{j} p_{kj} = 1 \end{cases} . \tag{2}$$

The solution to the ME problem in equation (2) is

$$\hat{p}_{kj} = \frac{\exp\left(-\sum_{t} y_{tk} \hat{l}_{tj} \right)}{\sum_{j} \exp\left(-\sum_{t} y_{tk} \hat{l}_{tj} \right)} \tag{3}$$

where $\hat{l}$ is the vector of K Lagrange multipliers associated with equation (1a). See Appendix 1 for further background on the ME framework.

III.2 The General Model and Estimation Procedure

The simple ME model outlined in equation (2) forms the basis of a more general model, in which we incorporate bank-specific and macroeconomic variables. In addition to the information on the proportion of banks in the *kth* state in time t, we incorporate balance sheet and income statement information as well as macroeconomic variables by imposing additional constraints in the maximization problem. In this section, we outline the development of the full GME model used to estimate our bank-failure model. However, for a more detailed discussion, and additional variations of the model, see Glennon and Golan (2001).

Let $\mathbf{z}_{ti}$ be a G-dimensional vector of bank-specific covariates (e.g., asset, liability, and off-balance sheet composition, asset quality/non-performing indicators, etc.) with individual elements z_{gti}. In addition, for each period t, let $\mathbf{s}_t$ be an L-dimensional vector of macro-level variables.[3] It is important to note that the bank-specific covariates vary by both state and time; however, the macro-level variables vary only by time. We can represent the bank-specific and macro variables more compactly as $X = [Z, S]$ with elements x_{nti} (n = 1, 2, …, N; N = G+L) and the L macro variable held constant across states.[4] The exact functional relationship describing the effect of these two types of variables (i.e., Z and S) on the transition probabilities is unknown. For that reason, we follow the instrumental variable (IV) literature and capture the information related to these variables via the cross moments:

$$\sum_{t=2}^{T}\sum_{j} y_{tj} x_{ntj} = \sum_{t=1}^{T-1}\sum_{k=1}^{K} p_{kj} y_{tk} x_{ntk} \qquad (4)$$

The bank-specific data are measured at their historic or book, instead of market, values. In addition, the sample is composed of a large number of banks tracked over a relatively long period of time. As a result, the data are inherently noisy. Because we are interested in stationary estimates of the

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3 The macro-level variables may include macroeconomic, industry/market, and policy variables that either directly or indirectly affect the banking industry. For example, changes in national or regional income, oil shocks, or large swings in the federal funds rate (to name a few) are likely to affect the financial health of banks; the actual impact depends on the degree to which banks protect themselves from such shocks.

4 For a slightly more general approach for introducing the macro effects, see Glennon and Golan (2001). The approach we use here was tested against the more general one that was statistically rejected for these data.

transition probabilities, we must accommodate the noise as part of the modeling process. Unfortunately, the ME model in equation (2) does not capture the effect of random noise on the estimates. However, following Golan, Judge and Miller (1996a), we can incorporate the effect of noise in the data into the ME framework by reformulating the model as a generalized maximum entropy (GME) problem. More specifically, we can rewrite equation (1a) as

$$\begin{aligned} y_{t+1,j} &= \sum_{k=1}^{K} p_{kj} y_{tk} + e_{tj} \\ &= \sum_{k=1}^{K} p_{kj} y_{tk} + \sum_{m=1}^{M} w_{tjm} v_m \end{aligned} \tag{1a'}$$

where e_{tj} is a random error term for each state *j* and period *t*, $e_{tj} \equiv \sum_{m=1}^{M} w_{tjm} v_m$, $\boldsymbol{v}$ is a symmetric-around-zero support space for each random error e_{tj} , $\sum_{m=1}^{M} w_{tjm} = 1$, and $M \geq 2$. Furthermore, because $y_{tj} \in [0,1]$, it follows that $e_{tj} \in [-1,1]$ for all *t*, *j*, and that the errors' support has natural bounds $v_m \in [-1,1]$. Using this more general formulation of the basic Markov transition model, we reformulate the data in equation (4) to be consistent with the specification outlined in equation (1a'):

$$\begin{aligned} \sum_{t=2}^{T} \sum_{j} y_{tj} x_{ntj} &= \sum_{t=1}^{T-1} \sum_{k=1}^{K} p_{kj} y_{tk} x_{ntk} + \sum_{t=1}^{T-1} \sum_{i} e_{tj} x_{ntk} \\ &= \sum_{t=1}^{T-1} \sum_{k=1}^{K} p_{kj} y_{tk} x_{ntk} + \sum_{t=1}^{T-1} \sum_{k} \sum_{m} w_{tkm} x_{ntk} v_m \end{aligned} \tag{4a}$$

The data specified in equation (4a) incorporate both the bank-specific and macroeconomic data, and account for possible noise in the data. We refer to this version of the model as a generalized maximum entropy – instrumental variable (GME-IV) estimator.

Following the method proposed by Golan, Judge, and Miller (1996a), we select values for the support space v_m and use the modified or generalized ME method to estimate the values for the unknown p_{kj} and w_{tkm}. The resulting GME-IV estimation rule for our Markov problem is

$$GME-IV = \begin{cases} \underset{\{p,w\}}{Max}\left\{-\sum_{k,j} p_{kj}\log p_{kj} - \sum_{t,j,m} w_{tjm}\log w_{tjm}\right\} \\ s.t. \\ \sum_{t=2}^{T}\sum_{i} y_{tj}x_{ntj} = \sum_{t=1}^{T-1}\sum_{k=1}^{K} p_{kj}y_{tk}x_{ntk} + \sum_{t=1}^{T-1}\sum_{k}\sum_{m} w_{tjm}x_{ntk}v_{m} \\ \sum_{j} p_{kj} = 1; \sum_{m} w_{tjm} = 1 \end{cases} \tag{5}$$

The solution to the GME-IV problem is

$$\hat{p}_{kj} = \frac{\exp\left(-\sum_{t=1}^{T-1}\sum_{n} y_{tk}x_{ntk}\hat{l}_{jn}\right)}{\sum_{j}\exp\left(-\sum_{t=1}^{T-1}\sum_{n} y_{tk}x_{ntk}\hat{l}_{jn}\right)} \equiv \frac{\exp\left(-\sum_{t=1}^{T-1}\sum_{n} y_{tk}x_{ntk}\hat{l}_{jn}\right)}{\Omega_k} \tag{6}$$

and

$$\hat{w}_{tjm} = \frac{\exp\left(-\sum_{n} x_{ntk}v_m\hat{l}_{jn}\right)}{\sum_{m}\exp\left(-\sum_{n} x_{ntk}v_m\hat{l}_{jn}\right)} = \frac{\exp\left(-\sum_{n} x_{ntk}v_m\hat{l}_{jn}\right)}{\Psi_{itj}} \tag{7}$$

with $\hat{e}_{tj} = \sum_{m}\hat{w}_{tjm}v_m$.[5]

The concentrated-dual (unconstrained) GME-IV method can be derived from the Lagrangian of equation (5), and is

[5] Although we use discrete values for **w**, continuous values may be used as well (e.g., Golan and Gzyl, 2001).

$$\ell(l) = \sum_{t=2}^{T}\sum_{j=1}^{K}\sum_{n} y_{tj}x_{ntj}l_{nj} + \sum_{k}\log\left[\sum_{j}\exp\left(-\sum_{t=1}^{T-1}\sum_{n} y_{tk}x_{ntk}l_{nj}\right)\right]$$

$$+\sum_{t,j}\log\left[\sum_{m}\exp\left(-\sum_{n} x_{ntj}v_{m}l_{nj}\right)\right] \qquad (8)$$

$$=\sum_{t=2}^{T}\sum_{j=1}^{K}\sum_{n} y_{tj}x_{ntj}l_{nj} + \sum_{k}\log\Omega_{k}(l) + \sum_{t,j}\log\Psi_{tj}(l)$$

where Ω_k and Ψ_{tj} are defined in equation (6) and (7).

Minimizing equation (8) and solving for λ, yields the estimated $\hat{l}$, which in turn yield the optimal probabilities $\hat{p}_{kj}$ and $\hat{w}_{tjm}$ via equations (6) and (7). It is important to note that this model is computationally as efficient as the maximum likelihood (ML) approach. Moreover, because the estimates are unique functions of the Lagrange multipliers, λ, *this method (which is a generalization of the ML) has the same level of complexity as the ML.*

Interestingly, the solution to the maximum entropy (ME) problem in equation (2) subject to the moment constraints in equation (4) is equivalent to the maximum likelihood-logit (ML-logit) model (e.g., Amemiya, 1985, chapter 11). Moreover, the solution to the GME-IV problem in equation (5) converges to the ME/ML-logit solution as the noise approaches zero.[6] As a result, the GME-IV method can be viewed as a generalized ML-logit model, in which the conventional ML solution is a special case when all errors in equation (1a') are zero. That condition exists, however, if, and only if, the Markov process is stationary, and there is no noise in the data; an assumption that is generally inconsistent with the data.

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[6] The log-likelihood for a conventional logit model is:

$$\ln L = \sum_{i}^{n}[y_i \ln(\frac{e^{xB}}{1+e^{xB}}) + (1-y_i)\ln(\frac{1}{1+e^{xB}})] \quad = \quad \sum_{i}^{n}[y_i\{\ln(e^{xB}) - \ln(1+e^{xB})\} + (1-y_i)\{\ln(1) - \ln(1+e^{xB})\}]$$

$$= \sum_{i}^{n}[y_i xB - \ln(1+e^{xB})]$$

If we set j=2, e_{tj}=0, or $\mathbf{v}=\mathbf{0}$, for all t,j, redefine $B_j = -l_j$, and impose the standard identification condition that B_1=0, the dual GME-IV in equation (8) collapses to the log-likelihood of the conventional maximum likelihood-logit model (see Golan, et al., 1996a).

III.3 Introducing Prior Information – A General Cross-Entropy (GCE) Approach

Lastly, we consider the case in which prior knowledge or information on the underlying values of *P* are available. In general, such information may come from a previous sample, a previous study, or derived from the underlying theory. The prior information (P^0) is easily incorporated into the GME-IV approach via the cross-entropy or Kullback-Liebler information-divergence measure (see Appendix 1), and, as a result, represents a natural generalization of the entropy measure. Rewriting the objective function of our generalized entropy estimator in equation (5) as

$$I\left(P,W;P^0,W^0\right)=\sum_{k,j} p_{kj}\log\left(p_{kj}/p_{kj}^0\right)+\sum_{t,j,m} w_{tjm}\log\left(w_{tjm}/w_{tjm}^0\right) \tag{9}$$

and minimizing equation (9) with respect to (i) the moment constraints in equation (4a) and (ii) the requirements that p_{kj} and w_{tjm} are proper probabilities ($\sum p_{kj}=1$ and $\sum w_{tjm}=1$), yields the optimal solutions for the generalized cross entropy – instrumental variable (GCE-IV) problem

$$\tilde{p}_{kj}=\frac{p_{kj}^0\exp\left(\sum_{t=1}^{T-1}\sum_n y_{tk}x_{ntk}\tilde{l}_{jn}\right)}{\sum_j p_{kj}^0\exp\left(\sum_{t=1}^{T-1}\sum_n y_{tk}x_{ntk}\tilde{l}_{jn}\right)}\equiv\frac{p_{kj}^0\exp\left(\sum_{t=1}^{T-1}\sum_n y_{tk}x_{ntk}\tilde{l}_{jn}\right)}{\Omega_k} \tag{10}$$

and

$$\tilde{w}_{tjm}=\frac{w_{tjm}^0\exp\left(\sum_n x_{ntk}v_m\tilde{l}_{jn}\right)}{\sum_m w_{tjm}^0\exp\left(\sum_n x_{ntk}v_m\tilde{l}_{jn}\right)}\equiv\frac{w_{tjm}^0\exp\left(\sum_n x_{ntk}v_m\tilde{l}_{jn}\right)}{\Psi_{tj}} \tag{11}$$

where, as before, $\tilde{e}_{tj}\equiv\sum_m \tilde{w}_{tjm}v_m$, and the priors for the noise terms, w_{tjm}^0, are assumed to be uniformly distributed. A uniform distribution for the priors is used when prior information does not exist. In that case, the GCE-IV and GME-IV methods are equivalent.[7] Although additional information from

[7] Although prior information for the transition probabilities is available from a previous sample, priors for the error terms do not exist. For that reason, we use uniform priors for the error terms in equations (9) and (11). In that case,

economic theory, or institutional arrangements, can be incorporated easily by imposing additional restrictions (i.e., equalities, inequalities, etc.) when solving equation (9), we impose only information from the prior probabilities at this time.

Using the GME estimation rule in equation (5) with equation (9) substituted for the objective function, we derive the concentrated-dual GCE-IV

$$\ell(l) = \sum_{t=2}^{T}\sum_{j=1}^{K}\sum_{n} y_{tj} x_{ntj} l_{nj} - \sum_{k} \log \Omega_k - \sum_{t,j} \log \Psi_{tj}(l) \qquad (12)$$

where both normalization factors $\Omega_k(l)$ and $\Psi_{tj}(l)$ are defined in equations (10) and (11). Equation (12) is used to estimate the $\tilde{?}$. Substituting the estimated $\tilde{?}$ into equation (10) and using the mean values for the covariates, we can solve for the optimal transition probabilities for a set of representative or benchmark banks across all initial states.

IV. Data and Empirical Results

Bank-specific data were collected directly from the quarterly Call Reports for all national banks that existed from 1984.1 through 1999.4 (excluding a relatively small number of national banks that switched charters during the sample period). The full sample covers a total of 64 quarters: t = 1, 2, …, 64. Failed and merged banks were identified using the FDIC's Mergers and Failures data set. Closure due to merger was labeled as a within-holding company merger if the regulatory top holding company identifier of the closed bank matched that of the acquiring bank; all other closures because of merger were classified as mergers with nonaffiliated banks.[8]

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the results are the same with or without imposing the uniform prior. We include the priors for the error terms for completeness.

[8] At this stage in our development of a Markov transition-based EWM approach, we do not thoroughly investigate merger behavior. The current design recognizes that mergers within a holding company are more likely to be affected by changes in state banking laws and the potential lower costs associated with merging affiliates under a single corporate identity. Mergers among nonaffiliates however may represent a nonfailure exit for a bank that may have failed had it not merged. For those reasons, we differentiate between the two types of mergers, but postpone our analysis of this aspect of the EWM for future research.

IV.1 Bank-Specific Covariates and Macroeconomic Variables

Following the literature, we identified several bank-specific ratios based on publicly available data that reflect credit, interest rate, and liquidity risks. In addition, we evaluate the effect of several institutional factors on the dynamics of a bank's transition to alternative states, all else equal. We also selected a small set of macroeconomic variables that reflect general economic conditions, major industry effects, and interest rate expectations. Our initial selection of bank-specific and macroeconomic variables is consistent with those generally found in the bank-failure literature or commonly identified as indicators of a fragile/strong banking system.

We assume banks continuously reposition their portfolios and redevelop their lending/funding strategies in response to changes in both current and anticipated market conditions. These may involve modifying their credit risk exposure through adjustments in their concentration of assets in, say, C&I loans during the expansionary period of the business cycle, or controlling their interest-rate exposure by adjusting their reliance on core deposits and the average duration of their loan portfolio in anticipation of a flattening of the yield curve. In Table 2 we report the average value (on a pooled basis) for each bank-specific covariate included in the model for both the full sample and three sub-periods.[9] The trend in the average values across sub-periods suggests that, as a group, banks adjusted their behavior quite extensively to changes in market condition over the sample period; a result reflected in the rather pronounced change in the average values of the covariates between the 1987-92 and 1993-99 sub-periods.

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[9] The post 1992 sub-period begins soon after the passage of FDICIA, FIRREA, and implementation of the new risk-based capital rules. It covers a period in which relatively few national banks failed (i.e., on average, roughly four national banks failed per year during this period). The 1987-92 sub-period, however, was a period in which by historical standards a relatively large number of national banks failed (i.e., there were on average 70 national bank failures per year during this period with more than 100 in 1989 alone). The large number of failures during this period reflects the general state of the financial/banking market affected by the savings and loans crisis. A relatively moderate number of failures occurred during the pre 1987 sub-period (i.e., on average, roughly 35 per year). We use this sub-period to generate the prior transition probabilities in the following model.

In Table 3, we report the average values of the bank-specific covariates by failure status.[10] There are significant differences in the average (pooled) values by failure status over the full sample; a result that suggests that not all bankers correctly position themselves to survive credit, interest rate, and liquidity shocks. For example, failed banks over the full sample period were relatively more concentrated in C&I loans; held lower quality loans (i.e., higher non-performing to total loans); maintained lower levels of liquid assets to liabilities, net loans to deposits, market-sensitive assets (i.e., trading account assets and securities), and long-term assets; were less likely to be affiliated with a strong holding company; and were more likely to be a new bank relative to the average nonfailed bank.

We also report in Table 3 the average values of the covariates by failure status for each of the three sub-periods in which there were a moderate (i.e., 1984-86), large (i.e., 1987-92), and small (i.e., 1993-99) number of failures. These results show that the relationship between nonfailed and failed banks continues to hold over time, even as banks, as a group, adjust to changes in the market. The average values by failure status are significantly different for each sub-period, except for asset growth and long-term asset ratio in the later sub-period.

In Table 4 we list the macroeconomic variables used in the model. In addition, we report the average values over the full sample and sub-periods. These results show that the macroeconomic conditions have changed significantly over time – with lower unemployment, agricultural, oil, and interest rates, and tighter interest-rate spreads in the later period of our sample. The sub-periods were defined with respect to the volume of bank failures and, as such, do not correspond directly with the business cycle. However, the relatively large difference in the average values of the macroeconomic variables across the sub-periods suggests that macroeconomic conditions are contributing factors in the overall condition of the banking system; a result that implies traditional bank-failure or EWM models that omit macroeconomic variables are under-identified.

[10] We identify the timing of failure as the period in which the failed bank files its last Call Report. Banks that fail are considered "failed" for the full sample period. Under this classification rule, a bank that fails at any time during the sample period is classified as a failed bank. As a result, these banks contribute to the average for each quarter over the full sample period in which they file a Call Report, not merely the period in which they fail.

IV.2 Estimation Results

The GCE-IV procedure requires information on the prior transition probabilities (P^0), the row shares ($\mathbf{y}_t$), and bank-specific and macroeconomic data. The prior probabilities are derived from the quarterly transition frequencies over the first three years of our sample. More specifically, the prior transition probabilities (P^0) are calculated as the mean of the percentage of banks in state i in time t that fall in state j in time $t+1$, i,j = 1, 2, …, 9, and t = 1, 2, …, 12. The prior probability matrix, based on the transition frequencies over the sample period 1984.1 through 1986.4, is presented in Table 5.

The proportions of banks in the *kth* Markov state in each time t, t=13, 14, …, 64 (i.e., 1987.1 through 1999.4) are used to define the row shares $\mathbf{y}_t$. We estimated the transition probabilities using a GCE procedure with only the row shares and prior probabilities (i.e., excluding the instrumental variables). The estimated transition probabilities based on this information only are illustrative of the potential effect of a GCE approach. For example, we see in Table 6 that the likelihood of remaining in the same state increases (i.e., each p_{ii}, i=1, 2, …, 6 – the probabilities along the main diagonal – increases) and the failure probability declines over all six states (i.e., each $p_{i,9}$ for all i=1,2, …, 6, decreases) after incorporating the proportion of banks in each state over time – the row shares – via the constraints outlined in equation (1a'). These results reflect the relative stability of the banking industry since 1993. They do not, however, reflect the effect of bank-specific or macroeconomic conditions on the process that underlies the multistate Markov transition process.

The GCE-IV transition probabilities are estimated using equation (12). We introduce one-quarter *lagged*, bank-specific covariates and macroeconomic instruments through the moment constraints derived in equation (4a). We evaluated the effect of additional lags (up to two periods) for the macro level variables. However, we found that the additional information was not statistically significant and therefore was not included in the final model. Table 7 shows that the transition-probability mass becomes more concentrated around the main diagonal under a GCE-IV approach. For example, in Table 6, without taking into account bank-specific and macroeconomic conditions (i.e., the GCE approach), an undercapitalized bank (state 4) in time t has a 42 percent probability of remaining undercapitalized in time *t+1* (i.e, $p_{44|GCE}$ = 0.42996), a 11 percent probability of moving to a well capitalized state (i.e., $p_{46|GCE}$ = 0.1076), and a 10 percent probability of becoming critically undercapitalized (i.e., $p_{42|GCE}$ = 0.1037) within one quarter. After incorporating the bank-specific covariates and macroeconomic variables (Table 7), a representative bank in state 4 (i.e., undercapitalized) is significantly more likely to remain in state 4 (i.e., $p_{44|GCE\text{-}IV}$ = 0.674) and far less likely to become well capitalized (i.e., $p_{46|GCE\text{-}IV}$ = 0.0004) or critically undercapitalized (i.e., $p_{42|GCE\text{-}IV}$ = 0.0535) within one quarter. These results suggest that a bank's strategic structure (e.g., asset concentration and management efficiency) contributes significantly to the likelihood a bank will undergo an extreme jump in states over a short period of time (i.e., a single quarter).

The effect of a change in a bank-specific or macroeconomic variable on the estimates of the transition probabilities is difficult to sign *a priori*. In most cases, it will depend on the initial state. For example, the effect of an increase in asset growth on a well-capitalized (state 6), well-managed bank is likely to be different from, and, possibly, of different sign, than the same increase in the asset growth rate of a significantly undercapitalized (state 3), critically undercapitalized (state 2), or insolvent (state 1) bank.

The incremental or marginal effect of a change in bank-specific (z_{ntk}) or macroeconomic variable (s_{tl}) on the transition probabilities (p_{kj}) can be derived from equation (10). The marginal effect of each bank-specific covariate and macroeconomic variable on the estimated transition probabilities is derived by differentiating equation (10) with respect to x_{tin}:

$$\frac{\partial p_{kj}}{\partial x_{ntk}} = p_{kj}\, y_{tk}\left[l_{jn} - \sum_j p_{kj}\, l_{jn} \right] \tag{13}$$

and evaluating at the means, or at any other value of interest, to capture the "dynamic" effects of the market.

In Table 8 we report the marginal effects of including several of the bank-specific covariates and macroeconomic variables as factors in the estimates of the transition probabilities. The marginal effects show the direct effect of an increase in bank-specific covariates or macroeconomic variables on each of the transition probabilities. For example, the probability the representative bank in state 1 (i.e., book insolvent) in time *t* will remain in state 1 in *t+1* increases 0.000392 for a unit increase in a bank's concentration in C&I loan; and, the probability a critically undercapitalized bank (state 2) transition to a failure state (state 9) increases by 0.005149 for an unit increase in the quarter-over-quarter asset growth rate.

These results are especially interesting insofar as they show that the effect of a change in bank-specific and macroeconomic conditions is state specific. For example, in Table 8 we find that an increase in asset growth tends to increase the likelihood of failure for all banks except those that are initially well capitalized (i.e., state 6).[11] In addition, the results in Table 8 show that the probability that a well-capitalized bank in time *t* remains well-capitalized in time *t+1* (i.e., $p_{6,6}$) rises as asset growth increases. These results suggest that growth for a well-capitalized bank may improve the overall strength of the bank (at least in the short run). However, for banks that are initially significantly undercapitalized or worse (i.e., states 1-3), growth increases the probability of failure primarily by reducing the probability of remaining in the current state. Interestingly, asset growth increases the probability that a bank that is significantly undercapitalized or worse transitions to the higher state of undercapitalized (i.e., state 4) in a relatively short period of time. This suggests some problem banks may in fact improve their capital position through growth; however, relative to the initial transition probabilities, the impact is small especially when evaluated in terms of the effect that growth has on the likelihood of transitioning to a failure state.

Similar results hold for a decrease in the holding company-level equity-asset ratio. The resources of the holding company are expected to act as a source of strength for problem banks. As such, an increase in the equity-asset ratio of the holding company decreases the likelihood of transitioning to failure. Moreover, the results in Table 8 show that an increase in the holding company-level equity-asset ratio increase the transition probabilities associated with remaining in the same, or moving to, a higher capital state (i.e., $dp_{ij} > 0$ for $i \leq j$; $i,j = 1,2, \ldots, 6$) and decrease those associated with lower states (i.e., $dp_{ij} < 0$ for $i > j$; $i = 2, \ldots, 6$; $j = 1, 2, \ldots, 5$).

[11] See Table 8, Asset Growth series: column 9, rows 1 through 5 are positive, row 6 is zero

In general, an increase in the national unemployment rate lowers (raises) the likelihood of moving to a higher (lower) state. The results in Table 8 show that the likelihood of moving to a higher state from an initial state of significantly undercapitalized or better (i.e., states 3-6) declines, in general, and except for moving to the book-insolvent state, the likelihood of moving to a lower state increases. The decrease in the transition probability to a book-insolvent state is partially a reflection of the increased failure probabilities. Interestingly, the probabilities that book-insolvent or critically undercapitalized banks move to moderately higher states (i.e., states 2 and 3) rise as the national unemployment rate increases. These results appear to be counterintuitive; we expect the likelihood of moving to a higher state to fall as the unemployment rate increases. These results, however, are consistent with the hypothesis that a slow down in the macro economy is more likely to affect the ability of severely distressed banks (i.e., initial states 1 and 2) to move to the higher states 4 through 6. As a result, a proportion of the banks that would have transitioned to the higher-level states 4 through 6, in the absence of an increase in the national unemployment rate, are more likely to transition to less desirable higher states 2 and 3. As such, the decline in the likelihood of severely distressed banks reaching the higher states 4 through 6 has the effect of increasing the marginal effect on the intermediate-state transition probabilities (i.e., p_{12}, p_{13}, and p_{23}).

IV.3 Information, Diagnostics and Model Reliability

The amount of information captured by the GCE or GCE-IV model can be measured by using a normalized entropy (information) measure:

$$S(\tilde{P}) \equiv \frac{-\sum_k \sum_j \tilde{p}_{kj} \ln \tilde{p}_{kj}}{-\sum_k \sum_j \tilde{p}^0_{kj} \ln \tilde{p}^0_{kj}} \qquad (14)$$

with the $\tilde{p}_{kj}$ representing estimated transition probabilities under either a GCE or GCE-IV estimation rule, and the $\tilde{p}^0_{kj}$ are the prior probabilities. $S(\tilde{P})$ is bounded between 0 and 1, with 1 reflecting uniformity (complete ignorance) of the estimates and 0 reflecting perfect knowledge (Golan, Judge and

Perloff, 1996b). In that way, $S(\tilde{P})$ captures the amount of information in the data relative to the initial knowledge reflected in the priors (see also Soofi, 1996).

In addition, a likelihood ratio test of model fit can be constructed that is analogous to that developed under a maximum likelihood (ML) procedure. That is, let I_{Ω} be the value of the optimal GCE objective function where the data (or parameters of interest) are not constrained (e.g., equation 9). Let I_{w} be the constrained GCE model where, say, $\beta = \lambda = \mathbf{0}$, which is equivalent to minimizing (9) subject to no constraints. Then, the Entropy Ratio (ER) statistic is defined as $2|I_{\Omega} - I_{w}|$. Under the null hypothesis, ER converges in distribution to $\chi^2_{(K-1)}$. The ER ratio can also be used to derive a pseudo-R^2 measure (McFadden, 1974) based on the normalized entropy S:

$$\textit{pseudo-}R^2 \equiv 1 - S(\tilde{P});$$

a common measure of within-sample prediction (i.e., goodness-of-fit). Incorporating bank-specific and macroeconomic data into the model under a GCE-IV approach greatly increased the amount of information above that captured by the priors and row shares alone. We estimate $S(\tilde{P})|_{GCE-IV} = 0.657$; a normalized entropy measure that converts to a pseudo-$R^2 = 0.343$ and a high value for the ER statistic that exceeds the critical value of $\chi^2_{df=8,\alpha=0.05}=15.51$. In contrast, the normalized entropy for the GCE model using prior probabilities and row shares only (i.e., Table 6 results) is $S(\tilde{P})|_{GCE} = .9697$ with a corresponding pseudo-$R^2 = .0303$.

In addition, we evaluate the reliability of the estimated Markov transition matrix in terms of the predicted one-quarter, one-year, two-year, and three-year ahead failure rates. The cumulative probability of failure of banks initially in states 1-6 are plotted in Figure 1. The plots are consistent with our expectation that problem banks are more likely to fail, and that the likelihood of failure increases over time (all else equal). For example, although only one-in-five book-insolvent banks (initial state 1) fails within one quarter, nearly one-in-two fail within one year, and nearly three out of four fail within three years. These results suggest that, although in the short-run it appears that the resiliency of book-insolvent banks is relatively high, the long-run likelihood of recovery is low. In contrast, an adequately capitalized bank (initial state 5) has only a 30 bps likelihood of failing within one quarter, and a two percent likelihood of failure within one year. The likelihood that the average or representative, adequately-capitalized bank fails within three years, however, increases to nearly 13 percent of the initial number of banks within that state (all else equal).

In Figure 2, we show the cumulative probability of moving to a higher or nonfailure lower state, or remaining in an unchanged state conditioned on the bank's initial state (note that states 7, 8 and 9 are terminal states and, therefore, they cannot represent initial states). For example, the probability that the representative bank initially in state 3 will move to a higher state is 0.187 – the sum of the probabilities a bank initially in state 3 moves to either states 4, 5 or 6; the probability that a bank initially in state 3 moves to a lower state is 0.215 – the sum of the probabilities of moving to either states 1 or 2. These results show that the probability of moving to a lower state, short of failure, is greater than that of moving to a higher state for all banks irrespective of their initial condition, except for state 1 – the lowest non-failure state.

In Figure 3 we show the cumulative probability of remaining in the same or moving to a higher state within one quarter, one year, two years, and three years ahead, conditional on the bank's initial state. The one-quarter ahead results are equivalent to the sum of the unchanged and higher curves in Figure 2 and reflect the condition that in the short-run, a large percentage of the severely distressed banks tend to remain unchanged or move to a higher state. Similar results hold for undercapitalized banks (i.e., states 2 through 4). Over longer periods, however, the probability of remaining or moving to a higher state falls off for all, but the well capitalized banks (i.e., state 6).[12] As expected, the probability of remaining or moving to a higher state declines more quickly for distressed banks (i.e., banks initially in the lower financial states). By the end of year 1, one out of two of the undercapitalized or insolvent banks (i.e.,

[12] State 6 (i.e., well capitalized) is the highest financial state. For that reason, the probability of remaining in the same or moving to a higher state for each of the time horizons in Figure 3 is nearly equivalent to the probability of remaining in the same state.

states 1 through 4) move to a lower state or close. However, after three years, four out of five of the insolvent banks, but only two out of three of the undercapitalized and significantly undercapitalized banks, move to a lower state or close. Adequately capitalized banks (i.e., state 5) fare much better. Only one out of four after one year, and after three years (all else held constant) one out of two, move to a lower state or close.

The results to this point have been based on the mean values of the bank-specific variables within each financial state. However, within each of these states, there are banks better positioned, whether because of the composition of their portfolio, their asset-liability structure, or growth potential, to recover (or fail) over time. More specifically, we expect that banks in the upper tail of the distribution of values for some or all of the bank-specific variables to have lower probabilities of failure than that of the representative bank (or those with values for the bank-specific variables in the lower tail of the distribution). We test this hypothesis by constructing a set of hypothetical banks based on the observed 90^{th}- and 10^{th}-percentile values for a selected subset of bank-specific variables.

It is unlikely a specific bank will have values in the upper tail of the distributions for each of the bank-specific variables. For that reason, we substitute the upper (lower) values for only five of the eleven bank-specific variables used in our model: C&I loans to total loans, asset growth, income earned but not received to total loans, nonperforming loans to total loans, and the equity capital ratio of the holding company. We use the 10^{th}-percentile values to identify the upper tail for each of the covariates, except for the equity capital ratio of the holding company in which we use the 90^{th}-percentile value. We reverse the percentiles values to define the lower tail of the distributions.

To illustrate the impact, we graph the 1-quarter, 1-year, 2-year, and 3-year ahead failure probabilities using the upper, mean, and lower tail values for the bank-specific variables (all other values held constant at their mean values) for banks initial in states 2 and 3 (see Figures 4 and 5). The mean value in each graph are from Figure 1. The failure probabilities for the upper and lower tail banks are consistent with our expectations. The failure probabilities for the representative bank generally fall between those for the upper and lower tail banks. The effect is greatest for those in the upper tail; as reflected in the initial lower failure probability and a much flatter growth in the failure probabilities over time. The difference in the failure probabilities between the upper and lower tailed banks is nearly 20 percentage points for the hypothetical state 2 banks; and nearly more than 15 percentage points for the state 3 banks.

V. Conclusion

In this paper, we develop and estimate an alternative early-warning bank failure model. The model is designed as a stationary, first-order Markov transition process estimated, using a Generalized Maximum Entropy approach. We show that by incorporating bank-specific and macroeconomic variables through the constraints on the estimates of the transition probabilities, we can identify the marginal or incremental effects of a bank's financial structure and the economic environment on the transition probabilities.

We find that the effect of changes in the concentration of assets, management efficiency, and liquidity, and general economic conditions are conditional on the initial state of the banks – an intuitive result, but one that has not been empirically verified in the banking literature.

Table 1: Transition States[1]

States	Label/Zone	Criteria
1	Insolvent (Book-Value)	Equity/Assets < 0
2	Critically Undercapitalized	0 < Equity/Assets < 2%
3	Significantly Undercapitalized	2% < Equity/Assets < 3%
4	Undercapitalized	3% < Equity/Assets < 4%
5	Adequately Capitalized	4% < Equity/Assets < 5%
6	Well Capitalized	5% < Equity/Assets
7	Merged with Affiliate	Merged within the Holding Company
8	Merged with Non-Affiliate	Merged with Unaffiliated Bank
9	Failure	Closed by Primary Supervisor

1. States 7, 8, and 9 are absorbing states; banks never leave these states once they enter. For example, banks that fail in time t remain in that state in time t+1. Banks that begin the period in states 7 or 8 are censored and remain in that state in $t+\tau$, $\tau=1, 2, \ldots, \infty$.

Table 2: Bank-Specific Summary Statistics

Category[1]	*Description of the Measure*[2]	*Average Values*[3]			
		84.1-99.4	*84.1-86.4*	*87.1-92.4*	*93.1-99.4*
Credit Risk	Commercial and Industry Loans (C&I) / Loans	0.2129	0.2642	0.2149	0.1746
	Asset Growth (percentage change)	0.0089	0.0224	0.0080	0.0016
	Non-performing Loans / Loans	0.0218	0.0271	0.0262	0.0129
	Income Earned but Not Received / Loans	0.0127	0.0164	0.0126	0.0103
	Net Interest Income / Loans	0.0106	0.0005	0.0002	0.0302
Liquidity	On-Hand Liquid Assets / Liabilities	0.8480	0.6469	0.9014	0.9231
	Net Loans / Deposits	3.385	1.5722	3.0560	4.5337
	Trading Account and Securities / Assets	0.2815	0.2544	0.2808	0.3009
Interest Rate	Long-Term Assets / Assets	0.1262	0.1050	0.1229	0.1447
Institutional	Equity / Assets (within holding company)	0.0936	0.0878	0.0859	0.1067
	New Bank (less than five years)	0.1101	0.1822	0.1102	0.0604

1. Source: Quarterly Call Report data (Report of Condition and Income) filed by national banks 1983.1 – 1999.4.
2. Asset growth: quarter-to-quarter percentage change; on-hand liquidity: interest-bearing bank balances, federal funds sold and securities less federal funds purchased and pledged securities; net loans: total loans less allowance for loan and leases; long term assets: fixed- rate loans and securities with remaining maturity over five years and floating rate loans and securities with re-pricing frequency less than every five years; new banks: banks that were chartered within the past five years; equity and assets of the holding company: the sum over all national banks held under the same the holding company; and deposits: demand deposits , MMDA, NOW, and other savings deposits.
3. Based on the results of a pair-wise test of the hypothesis of no difference between sub-period means, we reject the null at the 5 percent level for all combinations except three cases: (i) the difference between means asset growth between 87.1-92.4 and 93.1-99.4; (ii) the difference between the mean values of net interest income ratio 84.1-86.4 and 87.1-92.4; and (iii) the difference in the mean liquid asset to liabilities ratio 87.1-92.4 and 93.1-99.4.

Table 3: Bank-Specific Summary Statistics by Failure States

Variables[1]	1984 Q1 – 1999 Q4		1984 Q1 – 1986 Q4		1987 Q1 – 1992 Q4		1993 Q1 – 1999 Q4	
	Non-Failed	*Failed*	*Non-Failed*	*Failed*	*Non-Failed*	*Failed*	*Non-Failed*	*Failed*
C&I / Loans[2]	0.207	0.331 *	0.254	0.358 *	0.210	0.304 *	0.174	0.254 *
Asset Growth	0.009	0.010	0.021	0.031**	0.009	-0.011 *	0.002	-0.004
Non-performing Loans / Loans	0.020	0.056 *	0.025	0.042 *	0.024	0.073 *	0.013	0.048 *
Income Earned but Not Received / Loans	0.013	0.016 *	0.016	0.019 *	0.013	0.014 *	0.010	0.012 *
Net Interest Income / Loans	0.012	-0.017 *	0.003	-0.019 *	0.001	-0.016 *	0.030	-0.007 *
On-Hand Liquid Assets / Liabilities	0.879	0.204 *	0.698	0.190 *	0.939	0.221 *	0.925	0.181 *
Net Loans / Deposits	3.519	0.683 *	1.669	0.710 *	3.560	0.651 *	4.644	0.728 *
Trading Account and Securities / Assets	0.289	0.132 *	0.268	0.132 *	0.289	0.129 *	0.301	0.201 *
Long-Term Assets / Assets	0.129	0.073 *	0.109	0.066 *	0.125	0.077 *	0.145	0.143
Equity / Assets (within holding company)	0.095	0.065 *	0.088	0.085 *	0.088	0.042 *	0.107	0.073 *
New Bank	0.098	0.359 *	0.155	0.426 *	0.100	0.298 *	0.060	0.045 *

1. See footnotes to Table 2.
2. H_0: $\mu_{nf} = \mu_f$ assuming unequal variance. * (**) denote significant at the 5 percent (10 percent) level.

Table 4: Macroeconomic Variables

Variables	Source	Description	Summary Statistics			
General Business Conditions			*84.1 - 99.4*	*84.1 - 86.4*	*87.1 - 92.4*	*93.1 - 99.4*
Unemployment Rate	BLS	Civilian Unemployment Rate (SA)	6.018	7.233	6.147	5.386
Agricultural Prices	USDA BEA	Agricultural prices received by farmers: all farms (1990-92 = 100) deflated by GDP price index	1.126	1.269	1.1671	1.030
Oil Prices	DOE BEA	Refiners' acquisition cost of crude oil: composite ($/bbl) deflated by GDP price index	21.575	31.947	21.615	17.052
Interest Rate/Yield Curve						
Bank Prime Loan Rate	FRB	Prime rate – average of daily figures	8.653	10.103	8.852	7.862
10-year Spread	FRB	Spread of 10-year Treasury rate over the 3-month Treasury rate	1.761	2.321	1.852	1.444
30-year Spread	FHLB FRB	Spread of 30-year mortgage rate over the 3-month Treasury rate	3.348	4.229	3.433	2.900

Table 5: Prior Transition Probabilities (P^0) - Average percentage of banks in each transition state 1984.1 through 1986.4

States		Book Insolvent	Critically Undercapital	Significantly Undercapital	Undercapital	Adequately Capitalized	Well Capitalized	Merged with Affiliate	Merged with Non-Affiliate	Failure
		time [t+1]								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	1	**0.4925059**	0.044193	0.0708333	0.0611111	0.0167625	0.0916362	0.0086207	0	0.2143373
	2	0.1581899	**0.4485589**	0.0572696	0.074592	0.0339627	0.0856976	0.0066667	0.012037	0.1230256
	3	0.0252646	0.2138889	**0.410961**	0.1496769	0.0523097	0.0944088	0	0.0027778	0.0507123
	4	0.0164488	0.1052244	0.0954008	**0.4109943**	0.212435	0.1288807	0	0.0036811	0.026935
time [t]	**5**	0.0044846	0.0193092	0.026702	0.0855492	**0.522016**	0.3195045	0.0058906	0.0040465	0.0124972
	6	0.0003299	0.0007536	0.0007154	0.0023621	0.0120795	**0.9757902**	0.002628	0.0049354	0.0004058
	7	0	0	0	0	0	0	**1**	0	0
	8	0	0	0	0	0	0	0	**1**	0
	9	0	0	0	0	0	0	0	0	**1**

1. The percentage of banks in state i during time [t] that fall in state j during time [t+1]; [t+1] represents one quarter ahead. States 7, 8, and 9 are terminal states; e.g., a bank in state 9 in time [t] will not transition to any other state in [t+1]. This implies that the transition probabilities along the main diagonal for these initial states are one and zero for the off diagonal elements.

Table 6: GCE Estimates – Based only on share data 1987.1-1999.4 and the prior states from 1984.1-1986.4

States		Book Insolvent	Critically Undercapital	Significantly Undercapital	Undercapital	Adequately Capitalized	Well Capitalized	Merged with Affiliate	Merged with Non-Affiliate	Failure
						time [t+1]				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	1	**0.504043**	0.045738	0.073228	0.062982	0.017972	0.085537	0.008504	0.0001	0.201895
	2	0.1596	**0.46297**	0.059293	0.078339	0.037775	0.075585	0.006385	0.011701	0.108351
	3	0.025035	0.216375	**0.416643**	0.153989	0.056369	0.083473	9.68E-05	0.002697	0.045322
	4	0.015764	0.103792	0.095017	**0.419958**	0.232337	0.107614	9.17E-05	0.003417	0.02201
time [t]	**5**	0.004088	0.018761	0.02639	0.089048	**0.617065**	0.228484	0.004837	0.003412	0.007915
	6	.000044	0.000199	0.000302	0.001054	0.003496	**0.993914**	0.000833	0.000158	0
	7	0	0	0	0	0	0	**1**	0	0
	8	0	0	0	0	0	0	0	**1**	0
	9	0	0	0	0	0	0	0	0	**1**

1. See footnote to Table 5.

Table 7: Generalized Cross-Entropy-Instrumental Variable Model – Incorporating P^0, bank-specific, and macroeconomic variables[1]

States		Book Insolvent	Critically Undercapital	Significantly Undercapital	Undercapital	Adequately Capitalized	Well Capitalized	Merged with Affiliate	Merged with Non-Affiliate	Failure
						time [t+1]				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	1	**0.593600**	0.112387	0.061766	0.024834	0.001318	0.000059	0.004713	0.000083	0.201241
	2	0.165668	**0.605370**	0.064082	0.051062	0.004946	0.000022	0.003897	0.010388	0.094566
	3	0.025340	0.189740	**0.547806**	0.165623	0.021031	0.000710	0.000082	0.002863	0.046805
	4	0.014130	0.053542	0.150941	**0.673824**	0.081888	0.000366	0.000072	0.003922	0.021316
time [t]	**5**	0.000320	0.000067	0.001285	0.059956	**0.926716**	0.000060	0.004980	0.004144	0.002473
	6	0	0	0.000008	0	0.000164	**0.999329**	0.000292	0.000208	0
	7	0	0	0	0	0	0	**1**	0	0
	8	0	0	0	0	0	0	0	**1**	0
	9	0	0	0	0	0	0	0	0	**1**

1. See Tables 3 and 4 for a list of bank-specific and macroeconomic variables.

Table 8: Marginal Effects – The incremental effect of a change in bank-specific and macroeconomic variables

Selected Series

	States	time [t+1]								
	time[t]	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
C&I Loans	**1**	**0.000392**	-0.001359	0.000168	0.000098	-0.000007	0.000050	0.000031	0.000000	0.000627
	2	0.002231	**-0.006228**	0.001109	0.001000	0.000011	0.000035	0.000096	0.000041	0.001706
	3	0.000006	-0.003606	**0.001844**	0.000864	-0.000186	0.000912	0.000001	-0.000022	0.000185
	4	-0.000066	-0.002125	0.000152	**0.002948**	-0.001735	0.000853	0.000001	-0.000074	0.000045
	5	0.000011	-0.000003	0.000059	0.003229	**-0.004092**	0.000319	0.000353	0.000005	0.000120
	6			-0.002445		-0.050131	**0.204352**	-0.088225	-0.063551	
Asset Growth										
	1	**-0.004731**	-0.000531	-0.000592	0.000292	-0.000014	0.000002	0.000028	0.000000	0.005547
	2	-0.001946	**-0.003448**	-0.000945	0.001275	-0.000083	0.000001	0.000055	-0.000059	0.005149
	3	-0.000178	-0.000399	**-0.005173**	0.003769	-0.000233	0.000033	0.000001	-0.000006	0.002184
	4	-0.000535	-0.001549	-0.006381	**0.010983**	-0.003702	0.000021	0.000000	-0.000113	0.001274
	5	0.000002	0.000002	-0.000004	0.007825	**-0.008988**	0.000014	0.000466	0.000115	0.000568
	6			-0.000106		-0.002213	**0.006909**	-0.002225	-0.002365	
Equity/Assets Hld Co.										
	1	**0.009308**	-0.023771	0.008310	0.007757	0.000756	0.000060	0.000643	0.000004	-0.003067
	2	0.035217	**-0.127552**	0.027814	0.039078	0.006190	0.000046	0.001705	0.002847	0.014655
	3	-0.003148	-0.088691	**0.030303**	0.053648	0.015106	0.000993	0.000005	-0.000213	-0.008002
	4	-0.009754	-0.070382	-0.054887	**0.084192**	0.068972	0.000761	-0.000026	-0.002351	-0.016525
	5	-0.001073	-0.000319	-0.003353	-0.089694	**0.129018**	0.000176	-0.012938	-0.013042	-0.008775
	6			-0.002562		-0.026320	**0.194003**	-0.092695	-0.072425	
Unemployment Rate										
	1	**-0.000299**	0.000106	0.000068	-0.000015	0.000000	0.000000	-0.000008	0.000000	0.000147
	2	-0.000337	**0.000402**	0.000060	-0.000112	-0.000005	0.000000	-0.000016	-0.000018	0.000025
	3	-0.000046	0.000070	**0.000323**	-0.000322	-0.000020	-0.000002	0.000000	-0.000005	0.000002
	4	-0.000013	0.000163	0.000521	**-0.000778**	0.000057	-0.000001	0.000000	-0.000002	0.000053
	5	-0.000001	0.000000	0.000008	-0.000235	**0.000278**	0.000000	-0.000051	-0.000010	0.000011
	6			0.000006		0.000052	**-0.000009**	-0.000082	0.000033	
10-year Treasury Spread										
	1	**-0.000350**	0.000043	-0.000006	0.000030	-0.000002	0.000000	-0.000021	0.000000	0.000306
	2	-0.000270	**0.000109**	-0.000046	0.000089	-0.000015	0.000000	-0.000035	-0.000050	0.000217
	3	-0.000032	0.000040	**-0.000282**	0.000243	-0.000052	0.000004	-0.000001	-0.000011	0.000090
	4	-0.000051	-0.000051	-0.000342	**0.000904**	-0.000476	0.000003	-0.000001	-0.000033	0.000046
	5	0.000001	0.000001	0.000009	0.000923	**-0.000850**	0.000002	-0.000102	-0.000027	0.000043
	6			-0.000011		-0.000298	**0.001613**	-0.000859	-0.000445	

Figure 1: Probability of Failure by Initial State

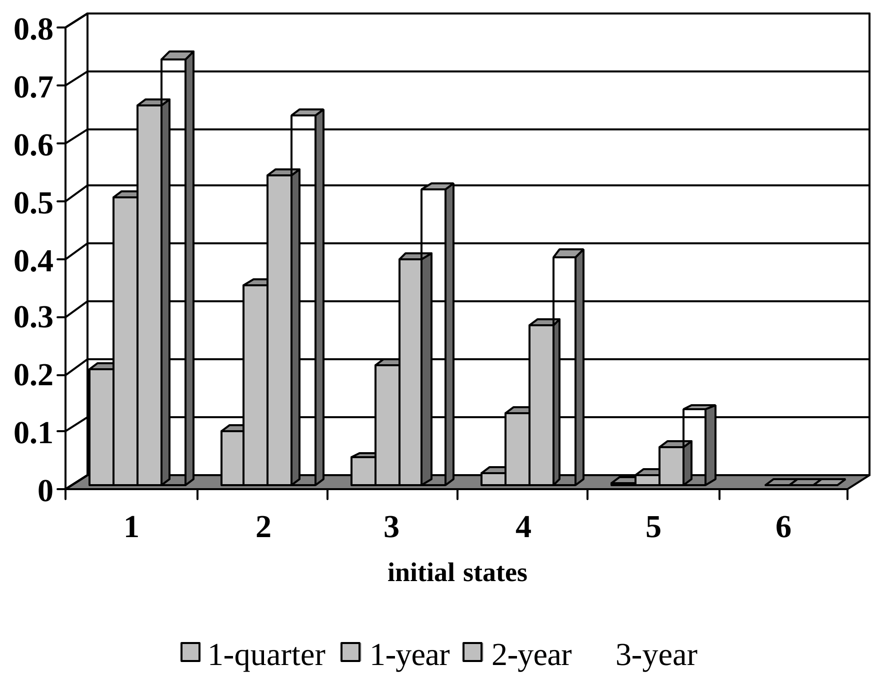


Figure 2: Probability of Moving to Higher, Lower, or Unchanged State (one-quarter ahead)

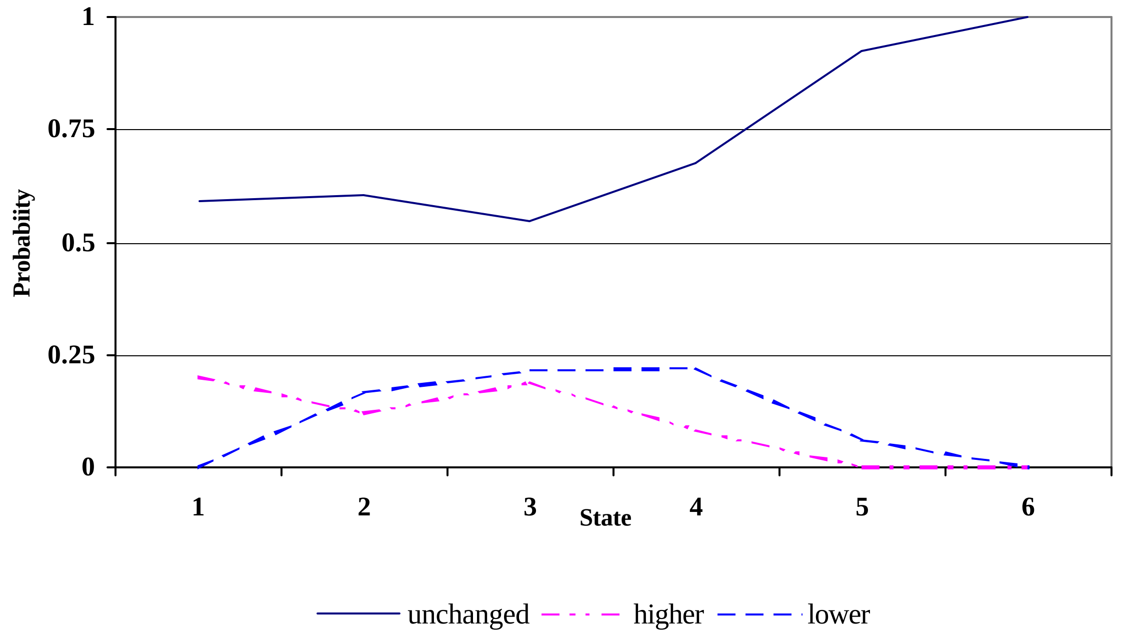


Figure 3: Probability of Remaining in or Moving to a Higher State

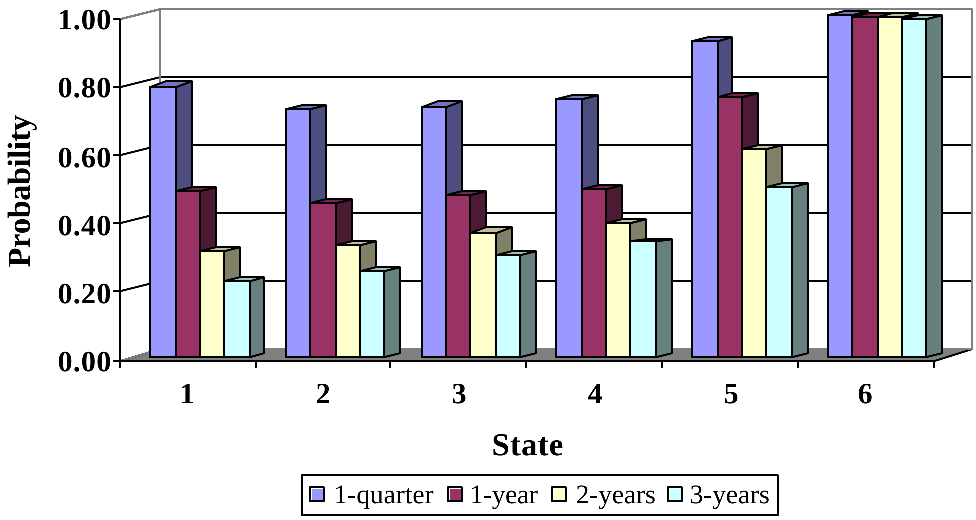


Figure 4: Upper and Lower Tails of the Distribution (State 2)

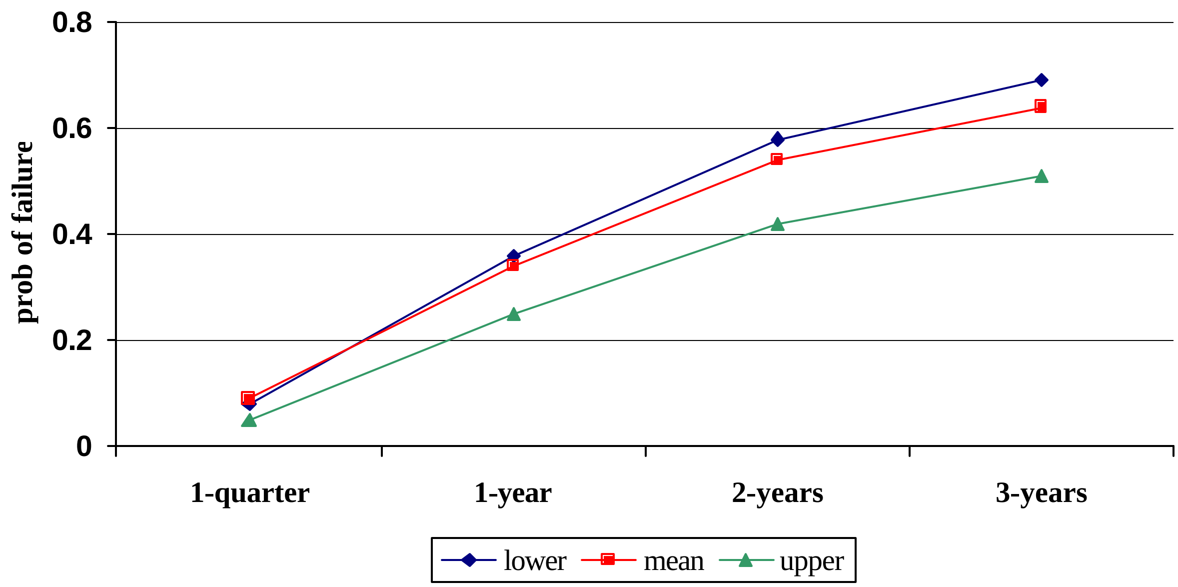
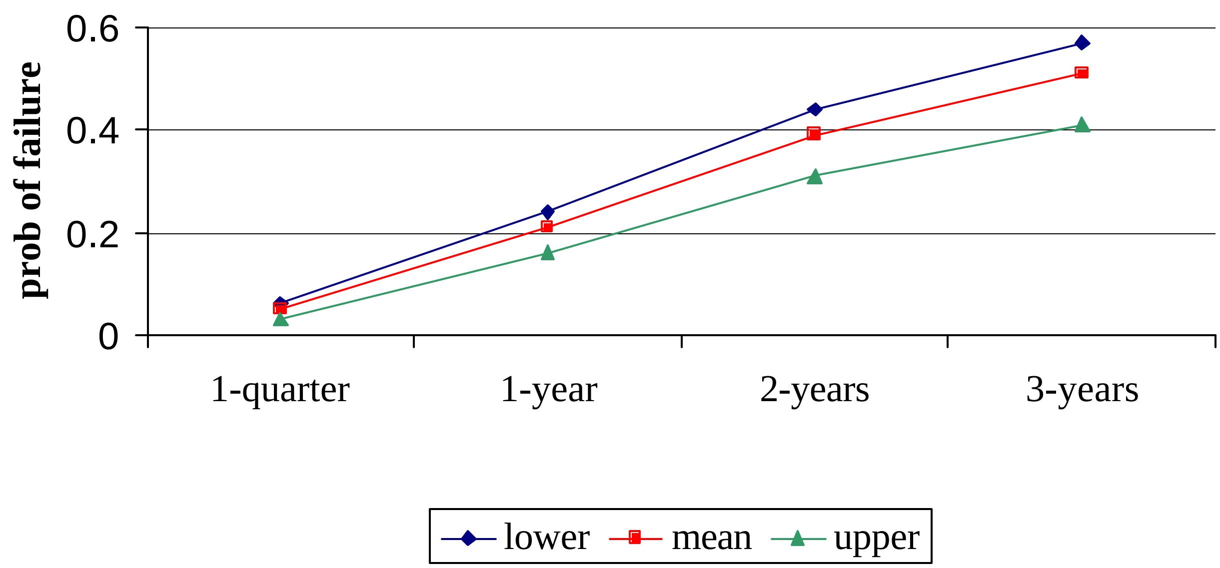


Figure 5: Upper and Lower Tails of the Distribution (State 3)



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Appendix 1. *The Classic Maximum Entropy Model: A Review*

To provide a basis for understanding the philosophy of the ME approach, we consider the following example. Let $\Theta = \{q_1, q_{2,} ..., q_M\}$ be a finite set and p be a probability mass function on Θ. The Shannon's (1948) information criterion, called entropy, is $H(\boldsymbol{p}) = -\sum_{i=1}^{M} p_i \log p_i$ with $0 \log 0 \equiv 0$. This information criterion measures the uncertainty, or informational content, in Θ which is implied by p. The entropy-uncertainty measure H(p) reaches a maximum when $p_1 = p_2 = ... = p_M = 1/M$ and a minimum with a point mass function. Given the entropy measure and structural constraints in the form of moments of the data (distribution), Jaynes (1957a, 1957b) proposed the maximum entropy (ME) method, which is to maximize H(p) subject to these structural constraints. If no constraints (data) are imposed, H(p) reaches its maximum value and the distribution of the p's is a uniform one. Thus, if we have partial information in the form of some moment conditions, Y_t (t=1, 2, …, T) , where T<M, the maximum entropy principle prescribes choosing the $p(q_i)$ that maximizes H(p) subject to the given constraints (moments) of the problem. The solution to this underdetermined problem is

$$\hat{p}(q_i) \propto \exp\left\{-\sum_t l_t Y_t(q_i)\right\} \tag{A1.1}$$

where l are the T Lagrange multipliers.

If prior information, q_i, concerning the unknown p_i exists, then one alternative to the ME approach is to minimize the Kullback-Leibler (K-L) entropy-distance between the post-data weights and the priors (Gokhale and Kullback, 1978). Under this criterion, known as cross entropy (CE), the problem of recovering p, may be formulated as minimizing the CE subject to the relevant structural constraints (moments). The resulting solution is

$$\tilde{p}(q_i) \propto q_i \exp\left\{\sum_t l_t Y_t(q_i)\right\}. \tag{A1.2}$$

When the prior information q_i has uniform mass, the optimal solutions of the ME and CE problems are identical.

To relate the ME formalism to the more familiar linear model, consider a special case of this model where there assumed to be no noise in the observed moments:

$$\mathbf{y}=X\mathbf{p} \tag{A1.3}$$

and **p** is a *K*-dimensional proper probability distribution. The ME formulation is

$$ME = \begin{cases} \hat{\mathbf{p}} = \arg\max\left\{-\sum_k p_k \log\, p_k\right\} \\ \text{s.t.} \quad \mathbf{y} = X\mathbf{p} \text{ and } \sum_k p_\text{k} = 1 \end{cases} . \tag{A1.4}$$

Similarly, the CE formulation is just

$$CE = \begin{cases} \tilde{\mathbf{p}} = \arg\min\left\{\sum_k p_k \log(p_k/q_k)\right\} \\ \text{s.t.} \quad \mathbf{y} = X\mathbf{p} \text{ and } \sum_k p_\text{k} = 1 \end{cases} \tag{A1.5}$$

where $I(\mathbf{p},\mathbf{q}) = \sum_k p_k \log(p_k/q_k)$ is the Kullback-Leibler information, or CE, measure.

The exact CE solution is

$$\tilde{p}_k = \frac{q_k \exp\left(\sum_{i=1}^{T} \tilde{l}_i x_{ik}\right)}{\sum_k q_k \exp\left(\sum_{i=1}^{T} \tilde{l}_i x_{ik}\right)} \equiv \frac{q_k \exp\left(\sum_{i=1}^{T} \tilde{l}_i x_{ik}\right)}{\Omega} \tag{A1.6}$$

The dual CE counterpart is

$$\underset{\mathbf{p}\in P}{Inf}\; I(\mathbf{p},\mathbf{q}) = \underset{l\in D}{Sup}\left\{1'\mathbf{y} - \log\Omega(X'1)\right\} \tag{A1.7}$$

where $P = \{\mathbf{p}\!: X\mathbf{p} = \mathbf{y}\}$ is a set of proper (normalized) distributions satisfying the linear constraints (A1.3), and D is the set $\{\, l \in \Re^T : \Omega(X'?) << \infty\}$. Having solved for $\tilde{1}$, one gets $\tilde{\boldsymbol{p}}$ via Eq. (A1.6).