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ON THE VARIATION OF η WITH ENERGY IN THE 100-1000 EV REGION

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1. The present report owes its origin to an informal discussion between H.A. Bethe, Harvey Brooks, and the writer at the General Electric Company. We were puzzled, at that time, by the apparent increase of η (the number of fission neutrons per neutron absorbed) in the aforementioned region, as appeared indicated by preliminary experiments at the G.E. laboratory. The qualitative concept which was put forward at that time was subject to a mathematical analysis during the writer's stay at the Brookhaven National Laboratory (summer 1949), and the present report records the calculations carried out there. It was thought, lately, when Rainwater and Havens' new measurement of the absorption cross section of 25 in the 1-100 ev region became available, that these measurements would furnish the constants necessary for the numerical evaluation of the following formulae. However, a review of these measurements undertaken recently in collaboration with Dr. Brooks, on the basis of his earlier work on the subject, led us to believe that the measurements are not yet accurate enough for this purpose. What would be necessary is, in particular, the measurement of the total and fission widths of a few low-lying levels, perhaps the first 5 to 10 levels. The present report is being written partly to stimulate further such measurements.

According to the usual theory, the integral of the fission cross section σ_f over a level, when expressed in barns, is

$$\int \sigma_f dE = (4.1 \times 10^6 \ f/E) \frac{\Gamma_n \Gamma_f}{\Gamma_n + \Gamma_r + \Gamma_f}. \tag{1}$$

In this, E is the energy of the level, Γ_n, Γ_r, and Γ_f are its neutron, radiation, and fission widths. All these quantities will be expressed, as usual, in electron volts; f is a factor to account for the spin, it is 1/2 for all practical purposes. For the radiative absorption cross section one has:

$$\int \sigma_r dE = (4.1 \times 10^6 \ f/E) \Gamma_n \Gamma_r/(\Gamma_n + \Gamma_r + \Gamma_f) . \tag{2}$$

Assuming that there are ν neutrons emitted per fission -- ν being assumed without too much justification to be the same for every level -- we obtain for the η of the processes which take place in the neighborhood of the level in question

$$\eta = \frac{\nu \int \sigma_f dE}{\int (\sigma_f + \sigma_r) dE} = \frac{\nu \Gamma_f}{\Gamma_f + \Gamma_r} \nu/(1 + \alpha); \quad \alpha = \frac{\Gamma_r}{\Gamma_f}. \tag{3}$$

Since Γ_f and Γ_r cannot be expected to change appreciably within the energy range of 1 to 10000 ev, and since ν is assumed to be a constant, it would seem that η will be independent of energy in the region in question.

There is, however, a flaw in the above argument, which makes a variation of η with energy indeed quite possible. If Γ_f varies from level to level, the η will have different values for different levels. Whether or not it is reasonable to assume such a variation of Γ_f from level to level will depend on the picture that one may form of the fission process. In particular, if the final state of fission is a small number of quantum mechanical states, i.e., if the different possible fission fragments are components of the same wave function (a possibility which was mentioned to me by F.J. Dyson), a considerable degree of variation of Γ_f from level to level can be expected. On the basis of the Bohr-Wheeler picture of fission, such variations are perhaps less likely. Variations of the above kind were demonstrated for Γ_n by T. Teichmann (Princeton dissertation, 1949), who showed that Γ_n shows irregular fluctuations from level to level by as large a factor as perhaps 8 both up and down from the average (Cf. also an earlier report by S. Dancoff, and my article Amer. Journ. of Physics 17, 99, 1949). An indication that there are, indeed, fluctuations in Γ_f from level to level may be seen in the fact that there is at least one level for which $\Gamma_f \ll \Gamma_r$, while for most level , apparently $\Gamma_f \gg \Gamma_r$. On the other hand, no large fluctuations in Γ_r can be expected because the transition takes place, in this case, to a wide variety of final states, so that the Γ_r is already an average.

The question whether Γ_f shows variations from level to level has been discussed above at some length, because the existence of such accidental variations is a condition for the systematic variation of the average value of η with energy by the mechanism to be considered below. Of course, if one had to average (3) giving each level the same weight, the existence of the accidental variations would remain irrelevant. However, (3) should be averaged giving each level, crudely speaking, a weight proportionate to the probability that a reaction will take place which is $(\sigma_f + \sigma_r)/(\sigma_f + \sigma_r + \sigma_s)$ where σ_s is the scattering cross section:

$$\frac{\int(\sigma_f + \sigma_r)dE}{\int(\sigma_f + \sigma_r + \sigma_s)dE} \approx \frac{\Gamma_f + \Gamma_r}{\Gamma_f + \Gamma_r + \Gamma_n} . \tag{4}$$

This probability is close to 1 at low energies, at which Γ_n is very small. However, around a few thousand volts, Γ_n becomes of the order of $\Gamma_f + \Gamma_r$, and will decrease the importance of those levels for which $\Gamma_f + \Gamma_r$ is small. Since Γ_r does not fluctuate from level to level, these are just those levels for which Γ_f is small. The decrease, with increasing energy, of the weighting factor (4) of those levels for which Γ_f is small will lead to an increase in the weighted mean of (3).

The rest of the present report is an attempt to formulate quantitatively the above qualitative consideration.

2. First, let us consider a simple absorption process, i.e., the neutron absorption of an element which does not undergo fission. Let us assume that the probability distribution of Γ_n is symmetric on the logarithmic scale, i.e., that the probability that Γ_n be between n and n + dn is

$$P(n)dn = f(n/n_o)\frac{dn}{n} , \tag{5}$$

where n_o is an average neutron width and $f(x) = f(1/x)$. This is a reasonable assumption for a strongly fluctuating quantity. For purposes of calculation, we shall use a special form of f, i.e., one which is finite only in a certain region around 1, zero outside. It must be admitted, of course, that this is only a crude approximation:

$$P(n)dn = \begin{cases} 0 & , \text{ for } n<n_o/s \\ (2n \ln s)^{-1} & , \text{ for } n_o/s<n<n_o s \\ 0 & , \text{ for } n>n_o s. \end{cases} \tag{5a}$$

The average absorption then becomes

$$\bar{\sigma}_r = \sigma_o \int_{-\infty}^{\infty} P(n) \frac{\Gamma_r n}{\Gamma_r + n} dn , \tag{6}$$

where $\sigma_o = 4.1 \times 10^6 f/DE$ barns, D being the average spacing of the levels which are assumed to be further from each other than their width is. Carrying out the integration in (6) yields

$$\bar{\sigma}_r = \sigma_o \frac{\Gamma_r}{2 \ln s} \ln \frac{\Gamma_r + n_o s}{\Gamma_r + n_o/s} . \tag{6a}$$

One will recognize in $n_o s$ the maximum, and in n_o/s the minimum neutron width of our model.

If the average neutron width n_o is much smaller than the radiation width Γ_r, (6a) gives an increased $\bar{\sigma}_r$, as compared to the usual formula which assumed $s = 1$. On the other hand, if $\Gamma_r << n$, the absorption is smaller than if all levels had the same neutron width n_o. In the limiting case of a very large s, the $\bar{\sigma}_r$ becomes independent of n_o and assumes the value $\sigma_o \Gamma_r/2$. This is, of course, larger than the usual value $\sigma_o \Gamma_r n_o/(\Gamma_r + n_o)$ if $n_o < \Gamma_r$, somewhat smaller if $n_o > \Gamma_r$. This qualitative behavior of $\bar{\sigma}_r$ is easily understandable if one considers that, in the case of $n_o < \Gamma_r$, the fluctuations of n occasionally give a wide level, and thus greatly increased absorption. The downward fluctuations of n_o do not affect the absorption enough to compensate for this. On the other hand, if $n_o > \Gamma_r$, the upward fluctuations have little effect, since the absorption is already almost as great as it would be for infinite neutron width. In this case the influence of the downward fluctuations of n is overwhelming.

3. For the actual problem at hand, we shall assume for the fission width a similar distribution as (5a) but, of course, with other constants. The probability that the fission width Γ_f be between f and f + df will be assumed to be

$$P_f(f)df = \begin{cases} 0 & , \text{ for } f<f_o/t \\ (2f \ln t)^{-1} & , \text{ for } f_o/t<f<f_o t \\ 0 & , \text{ for } f>f_o t. \end{cases} \tag{7}$$

However, as was mentioned before, while there is some experimental material to indicate that the spread s in the neutron width is of the order of 10, no such information is at present available for the spread t in the fission width. Thus, we have to leave t entirely free.

Next, we have to calculate the average neutron absorption and fission cross sections. For the former, we obtain:

$$\bar{\sigma}_a = \sigma_o \int_o^\infty \int_o^\infty P(n)P_f(f) \frac{n\Gamma_r}{n + f + \Gamma_r} dndf. \tag{8}$$

The integration over f can be carried out easily, and gives:

$$\bar{\sigma}_a = \sigma_o \frac{\Gamma_r}{2 \ln s} \ln \frac{\Gamma_r + n_o s}{\Gamma_r + n_o/s} - \frac{\Gamma_r \sigma_o}{4 \ln s \ln t} \int_{n_o/s}^{n_o s} \frac{dn}{\Gamma_r + n} \ln \frac{\Gamma_r + n + f_o t}{\Gamma_r + n + f_o/t} . \tag{8a}$$

Unfortunately, the last integral cannot be given a closed form, in spite of the simplicity of our choice for P and P_f. It was assumed, therefore, that $\ln t$ is a small quantity, and (8a) was evaluated under this assumption. It gave

$$\bar{\sigma}_a = \frac{\sigma_o \Gamma_r}{2 \ln s} \ln \frac{\Gamma_r + f_o + n_o s}{\Gamma_r + f_o + n_o/s} - \frac{\sigma_o \Gamma_r (\ln t)^2}{12 \ln s} \left\{ \frac{(\Gamma_r + n_o/s) f_o}{(\Gamma_r + f_o + n_o/s)^2} - \frac{(\Gamma_r + n_o s) f_o}{(\Gamma_r + f_o + n_o s)^2} \right\}. \tag{9}$$

A similar calculation gave for

$$\bar{\sigma}_f = \sigma_o \int_o^\infty \int_o^\infty P(n)P_f(f) \frac{nf}{n + f + \Gamma_r} dndf \tag{10}$$

the approximate result

$$\bar{\sigma}_f = \frac{f_o}{2 \ln s} \left(1 + \frac{1}{6}(\ln t)^2\right) \ln \frac{f_o + \Gamma_r + n_o s}{f_o + \Gamma_r + n_o/s}$$

$$+ \frac{f_o^2 (\ln t)^2}{12 \ln s} \left\{ \frac{3\Gamma_r + 2f_o + 3n_o s}{(\Gamma_r + f_o + n_o s)^2} - \frac{3\Gamma_r + 2f_o + 3n_o/s}{\Gamma_r + f_o + n_o/s} \right\}. \tag{11}$$

One obtains finally for $\alpha = \bar{\sigma}_a/\bar{\sigma}_f$ the rather complicated expression

$$\alpha = \frac{\Gamma_r}{f_o} \left[1 - \frac{1}{6}(\ln t)^2 \right.$$

$$\left. + \frac{1}{3}(\ln t)^2 \frac{\Gamma_r n_o (s - \frac{1}{s})}{(\Gamma_r + f_o)^2 + n_o^2 + n_o(\Gamma_r + f_o)(s + \frac{1}{s})} \left(\ln \frac{\Gamma_r + f_o + s n_o}{\Gamma_r + f_o + n_o/s} \right)^{-1} \right], \tag{12}$$

which is correct up to $(\ln t)^2$. For $t = 0$ one obtains, naturally, the usual expression Γ_r/f_o.

The energy dependence of (12) is caused by the energy dependence of n_o, which is proportional to the square root of the energy E. For very large E, above a few 10000 ev, the second term of (12) can be neglected and α, which can be accurately calculated in this case, becomes

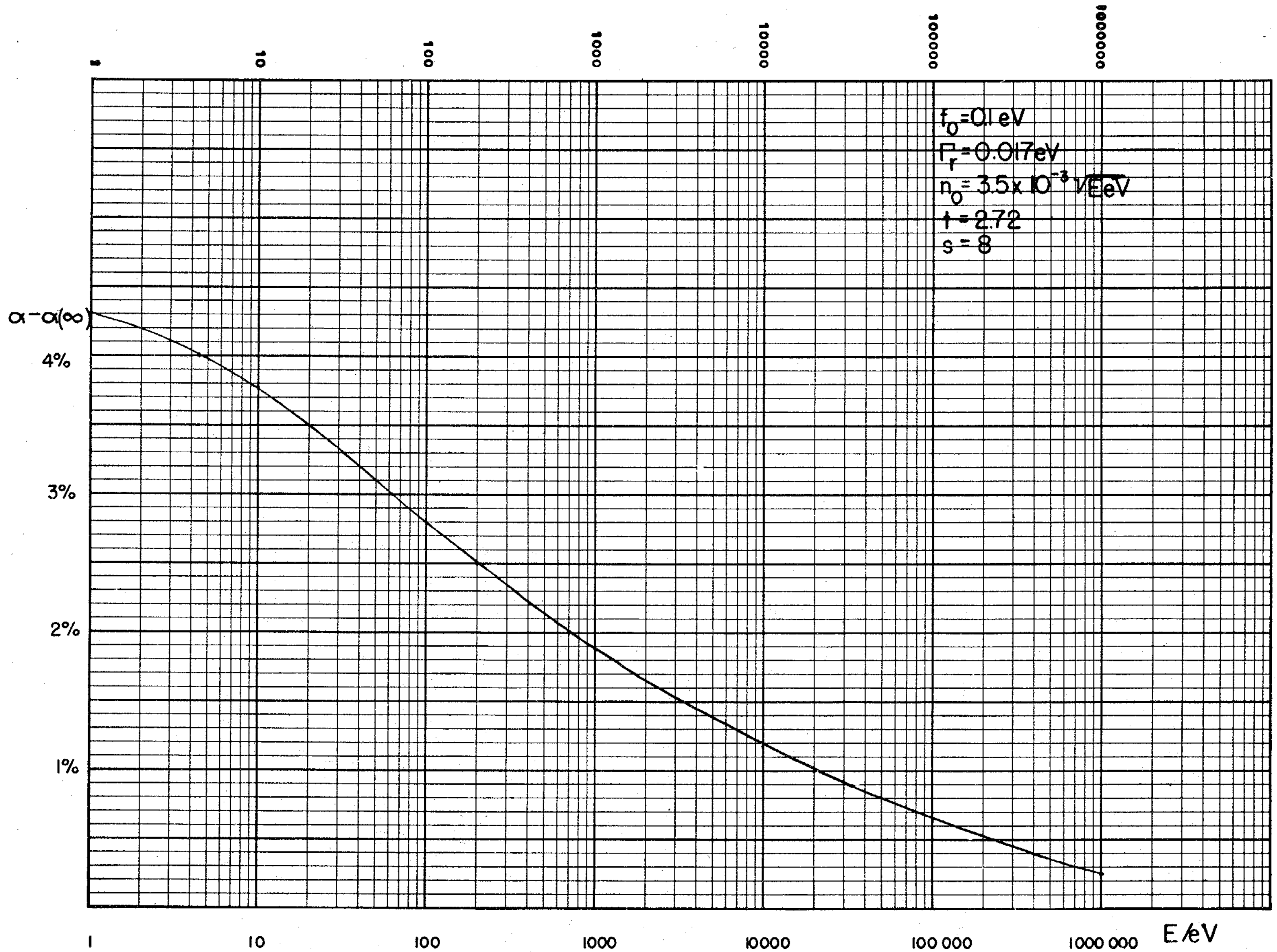
$$\alpha = \frac{\Gamma_r}{f_o} \frac{2 \ln t}{t - 1/t} \qquad (\text{for } n_o >> \Gamma_r,\ f_o). \tag{12a}$$

It is the ratio of Γ_r to the average value $\int P_f(f) f df$ of Γ_f. For low energies, the second term of (12) will be positive, so that α can be expected to increase with decreasing energy. At a hundred volts or so, n_o will be negligible as compared to Γ_r and f_o. In this case, it can be calculated again accurately for our model, and becomes

$$\alpha = \frac{2 \ln t}{\ln \left(\frac{\Gamma_r + f_o t}{\Gamma_r + f_o/t} \right)} - 1 \qquad (\text{for } n_o << \Gamma_r,\ f_o). \tag{12b}$$

The total decrease of α with increasing energy, which is caused by the increase of the neutron width, is the difference between (12b) and (12a). It is, of course, independent of the average neutron width itself. The corresponding increase of η can be calculated from (3).

Fig. 1 illustrates the decrease of α with energy for a typical case. Fig. 2 illustrates, as a function of t, the total decrease of α from very low (below 100 ev) to very high (above 10000 ev) energies. Two curves are given, one for $f_o/\Gamma_r = 3.5$, the other for $f_o/\Gamma_r = 7$. It should be remembered, of course, that the α for thermal energies is characteristic only for one level and has, therefore, a somewhat accidental value. Its most probable value is, in fact, f_o/Γ_r. The values for α given in (12), (12a) and (12b) are, on the other hand, averages taken over many levels. It would be necessary, to obtain these averages, to know Γ_r, f_o, and t. All these quantities could be obtained, in principle, from measurements on a few near thermal resonances, so that such measurements can be expected to give an indication of α and its energy variation up to a few dozens of kilovolts. Beyond that energy range, between 100 kev and 1 mev, all quantities of the theory, such as f_o, n_o, etc., can be expected to undergo unpredicatble changes, and higher angular momentum phenomena also will make their appearance.



f0=0.1 eV
Γr=0.017eV
n0=3.5x10^-3 √E eV
t=2.72
s=8
α-α(∞)
4%
3%
2%
1%
1
10
100
1000
10000
100 000
1000 000
E/eV

FIG. I

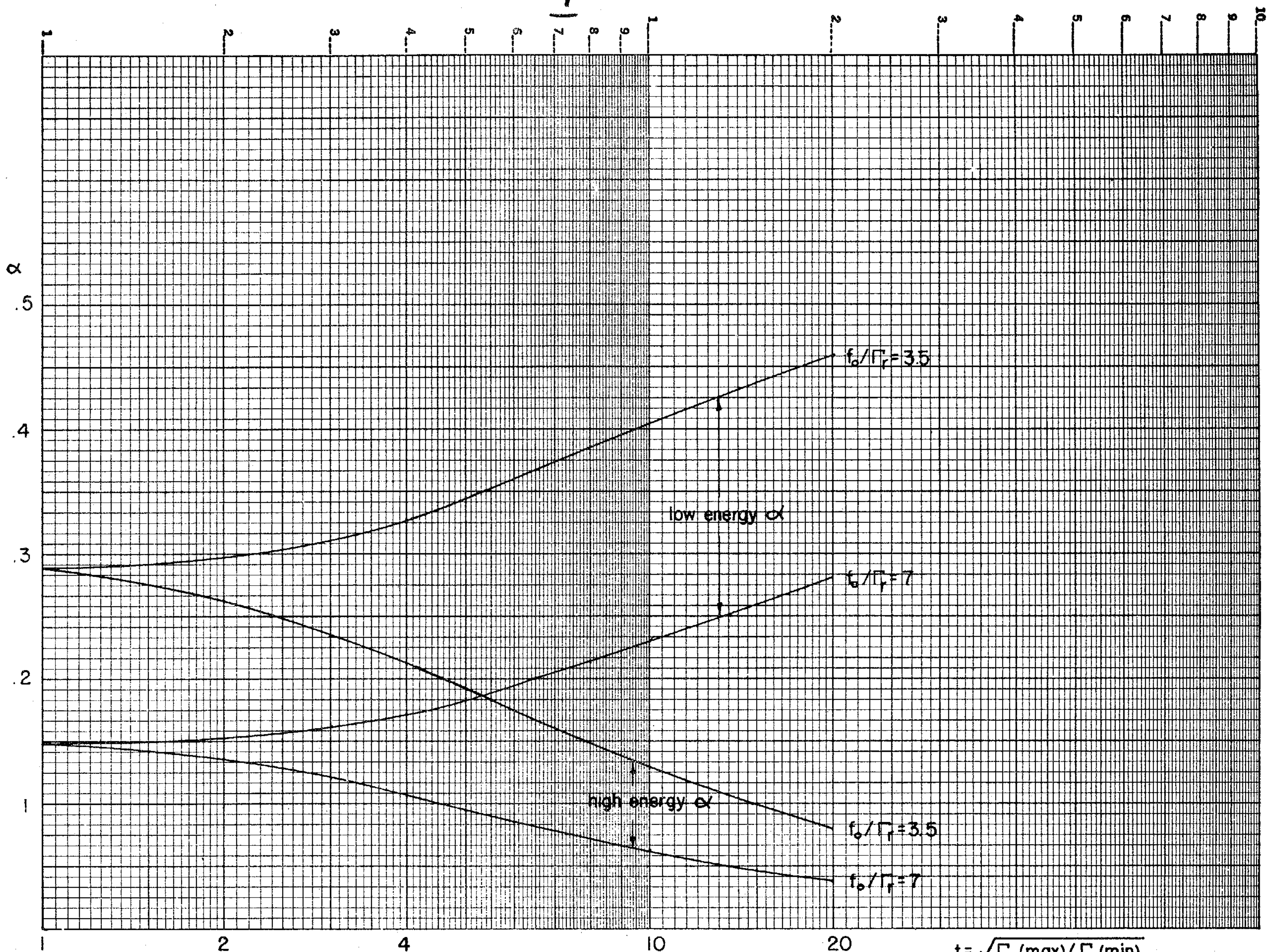

α
.5
.4
.3
.2
1
$f_o/\Gamma_r = 3.5$
low energy α
$f_o/\Gamma_r = 7$
high energy α
$f_o/\Gamma_r = 3.5$
$f_o/\Gamma_r = 7$
1
2
4
10
20
$t = \sqrt{\Gamma_f\,(max)/\Gamma_f\,(min)}$

FIG. 2

I-III And
1-7

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