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Triplets and Triality*

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This essay forms the introduction to Section VII of the forthcoming Benjamin book "The Eightfold Way", by M. Gell-Mann and Y. Ne'eman. The same material was presented to the International High Energy Physics Conference, Dubna, U.S.S.R., August 1964.

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DISCLAIMER

VII. TRIPLETS AND TRIALITY

We now return to the discussion, begun in the first section, of whether or not triplets (or, in general, spinor representations of SU(3)) will show up among hadrons.

Let us first be more precise about the notion of spinor representations. All representations of SU(3) can be constructed by reducing direct products of the fundamental triplet representation $\mathbf{3}$ multiplied by itself any number of times. Thus we have

$$\mathbf{3} \times \mathbf{3} = \mathbf{3}^* + \mathbf{6} \quad ,$$

where $\mathbf{3}^*$ is the representation to which an anti-triplet belongs. (Our definition of which is triplet and which is anti-triplet is given below.) We have also

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{8} + \mathbf{8} + \mathbf{10} \quad ,$$

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{3} + \mathbf{3} + \mathbf{3} + \mathbf{6}^* + \mathbf{6}^* + \mathbf{15} + \mathbf{15} + \mathbf{15} + \mathbf{15}' \quad ,$$

(where $\mathbf{15}$ and $\mathbf{15}'$ are two different 15-dimensional representations), and so forth.

If we continue this process indefinitely, we will discover that the representations $\mathbf{3}$, $\mathbf{6}^*$, $\mathbf{15}$, $\mathbf{15}'$, etc., can occur only in the reduction of a product of $3n + 1$ triplets, with n integral, while $\mathbf{3}^*$, $\mathbf{6}$, $\mathbf{15}^*$, $\mathbf{15}'^*$, etc., only come from $3n - 1$ triplets and $\mathbf{1}$, $\mathbf{8}$, $\mathbf{10}$, etc., only come from 3n triplets. Following Baird and Biedenharn (Proceedings of the 1964 Coral Gables Conference on Symmetry Principles at High Energy, W. H. Freeman Publishers, p. 58), we can define the triality τ to be 1, -1, and 0 respectively in these three cases. Every representation then has a value of τ and if representation $\underset{\sim}{c}$ occurs in the reduction of $\underset{\sim}{a} \times \underset{\sim}{b}$, we have

$$\tau_c = \tau_a + \tau_b \pmod 3 \quad .$$

All the representations that have been identified among the hadrons have triality 0. (These are the ones we have referred to as tensor rather than spinor representations.) The first possibility we may consider is that the other representations, with $\tau = \pm 1$, do not occur. This situation may be perfectly compatible with the bootstrap hypothesis and it may very well be the case in reality.

In a Lagrangian field theory, a fairly complicated picture is necessary, as remarked in the first section, to arrive at SU(3) with $\tau = 0$ only; for example, there is the original hypothesis of eight baryons and eight Yang-Mills type vector mesons. A Lagrangian field theory with a fundamental triplet of fields could give a world without real particles of $\tau = \pm 1$ only if something very peculiar happens, which amounts to giving the triplets infinite mass but also infinite binding energy when they combine to form $\tau = 0$ particles!

The remainder of our discussion will be devoted to the hypothetical case that real hadrons exist with $\tau = \pm 1$. Such a situation is clearly compatible with a theory, say a Lagrangian field theory, based on a fundamental triplet (with perhaps some other basic entities) and it may also be compatible with the bootstrap hypothesis.

We shall make two simplifying assumptions that have characterized all work on triplets so far. One is that, in the limit of SU(3) symmetry, not only is τ conserved modulo 3 but also a quantum number with eigenvalues . . . -4, -3, -2, -1, 0, +1, +2, +3, +4 . . . is conserved that is equal to τ modulo 3. We call this quantum number $n_t - n_{\bar{t}}$ (which stands for number of triplets minus number of anti-triplets). The other assumption is that there is no other quantum number involved, besides familiar ones.

Let us now consider the case $n_t - n_{\bar{t}} = 1$. All representations with $\tau = +1$ are possible for $n_t - n_{\bar{t}} = 1$ if we include enough particles. The representation $\underset{\sim}{3}$ will occur, containing an isotopic doublet and a singlet, with the electric charge of the singlet in units of e equal to some number, which we call z. The electric charges of the doublet members, in units of e, are z and z+1. The fact that we take z+1 instead of z-1 defines what we mean by a triplet t (τ = +1) rather than an anti-triplet $\bar{t}$ (τ = -1); the definition is, of course, arbitrary.

For $n_t - n_{\bar{t}} = -1$, the anti-triplet representation $\underset{\sim}{\bar{3}}$ will occur, containing an isotopic singlet with charge -z and an isotopic doublet with charges -z and -z-1. In general, for any supermultiplet, the electric charge Q in units of e qill be given by the formula

$$Q = F_3 + \frac{F_8}{\sqrt{3}} + (z + \tfrac{1}{3})\,(n_t - n_{\bar{t}}) \quad .$$

We may now enumerate four interesting physically distinct situations:

a) The quantum number $n_t - n_{\bar{t}}$ is not a new one, but is proportional to baryon number $n_B - n_{\bar{B}}$ and thus absolutely conserved. Since the known baryons have triality zero, all baryons must have triality zero in this case, and thus $(n_B - n_{\bar{B}})/(n_t - n_{\bar{t}})$ is an integral multiple of 3. The simplest choice is the one for which the ratio is +3; the triplets are then "quarks" as discussed by Gell-Mann (Physics Letters 8, 214 (1964)). In the formula for the electric charge, we know there is not a term proportional to $n_B - n_{\bar{B}}$, so in this model the coefficient of $n_t - n_{\bar{t}}$ must be zero, giving $z = -1/3$. The quarks thus consist of an isotopic singlet with $Q = -1/3$ and a doublet with $Q = -1/3$ and $Q = 2/3$, all with baryon number $1/3$. The known baryons transform under SU(3) like tri-quarks.

b) The quantum number $n_t - n_{\bar{t}}$ is absolutely conserved but is distinct from baryon number. In this case, z can have any value whatever, including simple integral values like -1 and 0. Here, the known mesons transform like combinations $\bar{t}t$, while the known baryons transform like $b\bar{t}t$, $b\bar{t}t\bar{t}t$, etc., where b is a conventional neutral singlet baryon. States of any baryon number and any triality occur in this picture. Some $\tau \neq 0$ particles are stable, as in (a).

c) The quantum number $n_t - n_{\bar{t}}$ is not absolutely conserved, but the violation occurs only through a weak interaction, with triality changing by one unit. Since particles with $\tau = \pm 1$ can now turn into conventional particles with $\tau = 0$, we must have integral z, for example $z = -1$ as in a theory discussed below by Gell-Mann. Again, the known baryons transform like $b\bar{t}t$, $b\bar{t}t\bar{t}t$, etc., but the baryons with $\tau \neq 0$ can decay into them by weak interactions.

d) The conservation of the quantum number $n_t - n_{\bar{t}}$ is violated by the part of the strong interaction that breaks SU(3) symmetry. The symmetry-violating term thus has triality ± 1; only in second order does it give the usual symmetry violation with $\tau = 0$. We want the latter to transform like $\underset{\sim}{8}$, so the $\tau = \pm 1$ symmetry violation must go like $\underset{\sim}{3}$ and $\underset{\sim}{\bar{3}}$, as if it transformed b into t and vice versa. Since the strong violation conserves isotopic spin, an isotopic singlet must go into an isotopic singlet; thus the isotopic singlet in t must have zero charge and hence $z = 0$.

In the foregoing treatment, triplets as such have been discussed mainly for pedagogical reasons; the essential points are the nature of the quantum number $n_t - n_{\bar{t}}$, its relation to triality τ, and the degree of its conservation. These symmetry questions determine the readily observable properties of the hadrons with $\tau \neq 0$, if such exist. The questions of which particles are "fundamental", if any, and how a detailed Lagrangian field theory model could be identified in practice are subtle ones and perhaps even meaningless. Certainly any attempt to describe the known hadrons dynamically as simple pairs or triads of very heavy "fundamental objects" is doomed to failure, since higher-order corrections in the field theory sense will be of the greatest importance and all these dynamical effects would be largely subsumed, in the sense of dispersion theory, in the lower threshold energy channels such as are included in approximate bootstrap calculations.

Nevertheless, a number of Lagrangian field theory models have been discussed by various authors. Gell-Mann in the accompanying paper treats two, a quark model of type (a) and a model of type (c) with $z = -1$. His main purpose in introducing the models is to abstract from them mathematical relations (involving the weak currents) and symmetry principles like the ones treated above; he is then content to discard the Lagrangians.

Lee and Gürsey (Phys. Rev., in press) consider a field theory model of type (b) with integral but otherwise unspecified charge. For some reason, they employ as basic units not t and b, but a fermion t and a boson, which transforms like a combination of t and b. These fermion and boson triplets are just like the triplets ℓ and L of the original "Eightfold Way" paper.

Schwinger (Phys. Rev. Letters 12, 237 (1964)) has put forward a rather complicated field theory model which seems to be of type (d).

A model of type (d) is favored by Z. Maki (Prog. Theor. Phys. 31, 331 (1964)). The (d) situation is also discussed in the framework of the bootstrap idea by Tarjanne and Teplitz (Phys. Rev. Letters 11, 447 (1963)).

Tarjanne and Teplitz emphasize a point that is applicable to any scheme of types (b), (c), and (d), whether in the bootstrap theory or in a theory with a fundamental triplet and singlet. The point is that there may be approximate symmetry under SU(4) when particles with $\tau \neq 0$ are included. SU(4) allows another commuting quantum number along with I_z and Y, which can be just $n_t - n_{\bar{t}}$.

The exciting questions are, of course, experimental. Will hadrons of $\tau \neq 0$ be found and, if so, of what type? If they are of type (d), they may be quite inconspicuous and some of them may already have been discovered. (See, for example, one of the interpretations of $\kappa(725)$ in an earlier paper.) If they belong to type (c), then there exist metastable particles with $\tau \neq 0$, decaying by weak interactions into conventional hadrons; we would be dealing with strange particles of a new kind. The most fascinating possibility is that hadrons of type (a) or (b) will be found; for then there must be particles with $\tau \neq 0$ that are absolutely stable.

Such new stable particles must be present on the earth if they exist. Presumably, the cosmic process that is responsible for the presence of ordinary matter has also produced some admixture of $\tau \neq 0$ matter, but even if it has not, the cosmic radiation, through pair production processes in the atmosphere, must be depositing some $\tau \neq 0$ matter near the surface of the earth. Assuming that these particles exist but are too heavy to be made in present accelerators, they can be sought in analyses of terrestrial material, in cosmic ray experiments, or in accelerator experiments with machines of higher energy.