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Weak Interaction Models with New Quarks and Right-Handed Currents*

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ABSTRACT

We discuss various weak interaction issues for a general class of models within the $SU(2) \times U(1)$ gauge theory framework, with special emphasis on the effects of right-handed, charged currents and of quarks bearing new quantum numbers. In particular we consider the restrictions on model building which are imposed by the small K_L-K_S mass difference and by the $\Delta I = \frac{1}{2}$ rule; and we classify various possibilities for neutral current interactions and, in the case of heavy mesons with new quantum numbers, various possibilities for mixing effects analogous to K_L-K_S mixing.

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I. INTRODUCTION

The recently discovered ψ and ψ' resonances[1] have been widely interpreted as evidence for new ingredients of hadronic matter, going beyond the "usual" SU(3) triplet of p, n and λ quarks. The possibility of such a proliferation of quarks had already suggested itself on the basis of other, earlier considerations. A fourth, charmed quark[2] had been introduced to resolve certain problems in the weak interactions (suppression of $\Delta S = 2$ nonleptonic transitions and of $\Delta S = 1$ neutral currents); and a color tripling of quarks had been proposed to deal with various issues in the strong interactions (restoration of the connection of spin and statistics for the quark model of hadrons, implementation of the ideas of asymptotic freedom, etc.) The four quark scheme (with color tripling) emerged as the simplest picture which, qualitatively at least, incorporates the standard phenomenology of weak interactions involving ordinary hadrons. In this framework the ψ resonances have been described, alternatively, as color singlets formed of charmed quark-antiquark pairs, or as color octets. It is by no means clear yet that either interpretation will prove to be tenable; and indeed the new discoveries have already spawned various schemes involving still more quarks.[3] Further progress awaits the discovery of hadronic states which more directly suggest the existence of new quantum numbers and correspondingly, on the quark picture, of new quark types.

We wish to consider here some of the constraints on the underlying quark structure of matter that might be extracted from weak phenomena, for processes involving possible hadrons with new quantum numbers but also processes involving only ordinary hadrons. The latter, of course, provide only indirect information, but this can be useful. The point is that new, heavy quarks can have important effects on the weak interactions of the light, p, n, λ quarks:

(1) Nonleptonic weak decays and certain higher order weak effects probe momenta of order M_W, the intermediate vector boson mass. At these momenta all quarks of mass $< M_W$ can have important dynamical effects, and the ideas of asymptotic freedom provide a rough quantitative basis for assessing these effects.

(2) Neutral current phenomena, even for ordinary hadrons, take on a structure which is, of course, set by the nature of the weak couplings of the ordinary quarks, among themselves but also in combination with new quarks.

Throughout this paper we adopt for the weak interactions the general $SU(2) \times U(1)$ gauge-theory framework of Weinberg and Salam,[4] assigning all quarks to singlets or doublets of the weak SU(2). Quark charges are either 2/3 or −1/3. When questions of strong interaction dynamics arise we adopt a color gauge theory of the strong interactions, with all quarks assigned to triplets of color SU(3). The options, therefore, have to do with the number of quark types introduced as a basis for hadronic matter and with their assignments in the weak interactions to weak SU(2) multiplets, left vs. right-handed, singlet vs. doublet. In this enlarged framework we take up in Sec. II the familiar question of K^0-$\bar{K}^0$ mixing - more generally, the issue of higher order effects leading to $\Delta S = 2$ transitions. The smallness of the K_L-K_S mass difference imposes a severe constraints on the structure of weak interaction theories; and indeed, it was in order to meet this constraint that one was led, via the GIM mechanism,[2] to the postulate of a fourth, charmed quark. In the standard model the quarks enter into weak SU(2) doublets only of the left-handed variety, and the mixing effects can be sufficiently suppressed if the mass of the charmed quark is not too large. In models involving both left- and right-handed doublets, we find that the constraints become much more severe. In Sec. II we also discuss the analog of K^0-$\bar{K}^0$ mixing for mesons carrying new quantum numbers. Here we illustrate various possibilities, ranging from negligible mixing effects (decay time much shorter than switching time) to nearly complete mixing. In Sec. III we discuss the question of octet enhancement for weak nonleptonic decays. This effect is often attributed to the dominance of octet operators in the short distance expansion of products of currents. In the standard model, however, based on left-handed currents, the enhancement suggested by such considerations seems to be too modest, as is well known. Right-handed currents could be of great help here, but these tend to run into trouble with the K_L-K_S mass difference. We also discuss the nonleptonic decays of hadron bearing new quantum numbers. Again there are enhancement effects, but the group structure is simple only if there is no interaction between right and left-handed currents. Finally we remark on the possibility of intensifying all of the above enhancement effects by introducing a multitude of quarks into the strong interactions (with masses $< M_W$), independently of whether they couple to the light quarks in the weak interactions. In Sec. IV we consider several experimental signatures bearing on the various alternatives discussed in the earlier sections, in particular, the production of muon pairs (of opposite or same signs) in neutrino reaction, and neutral current effects in neutrino reactions. Appendix I contains the details of a renormalization group analysis of K^0-$\bar{K}^0$ mixing for situations where one has both left and right-handed currents. Appendix II contains some details of the renormalization group analysis of nonleptonic weak decays.

II. K_L-K_S MASS DIFFERENCE AND RELATED MIXING EFFECTS

The smallness of the K_L-K_S mass difference has long had a constraining role in weak interaction model building.[5] It was this constraint, in part, that led to the introduction of a fourth, charmed quark.[2] In the standard model, where the charged currents are exclusively left-handed, the mass difference is perhaps sufficiently suppressed provided the charmed quark mass is $\lesssim$ a few GeV.[6] We want to consider here what happens when one expands the model to include right-handed currents.[7] The situation is well illustrated by a scheme[8,9] in which the right-helicity states p'_R and n_R, rather than being taken to be two weak SU(2) singlets as in the standard model, are instead grouped into a weak SU(2) doublet. In this example the charged current is

$$j_\mu = \bar{p}_L \gamma_\mu n_L^c + \bar{p}'_L \gamma_\mu \lambda_L^c + \bar{p}'_R \gamma_\mu n_R \quad ,$$

where $n^c = n \cos\theta_c + \lambda \sin\theta_c$, $\lambda^c = \lambda \cos\theta_c - n \sin\theta_c$, with θ_c the Cabibbo angle; and where the subscripts L and R refer to left and right-handed helicity projections. The last term is absent in the standard model.

The K_L-K_S mass difference is determined by diagrams involving a pair of W boson exchanges. The potentially dangerous contributions arise from the domain of large boson momenta, of order M_W; we deal with this by looking for an effective Lagrangian that describes the process $n + n \to \lambda + \lambda$ in the approximation that the strong interactions are switched off at large momenta. To within logarithmic corrections, this is in the spirit of asymptotic freedom provided M_W and $m_{p'}$ are large compared to typical hadronic masses (we assume $M_W >> m_{p'}$ and $m_{p'} >> m_p$). With the strong interactions switched off there are two diagrams to be considered, as shown in Figs. 1a and 1b. It is only Fig. 1a, where vertices involve exclusively left-handed currents, that enters into the standard model; Fig. 1b involves both left- and right-handed currents. The effective Lagrangian is computed to lowest order in $m_{p'}/M_W$.

For the left-left case the computation has been carried out by Gaillard and Lee, who find

$$\mathscr{L}_{\text{eff}}^{LL} = -\frac{G_F}{\sqrt{2}}\,\frac{\alpha}{16\pi}\left(\frac{m_{p'}}{M_W}\right)^2 \frac{\sin^2\theta_c \cos^2\theta_c}{\sin^2\theta_W}\,\bar{\lambda}_i\gamma_\mu(1+\gamma_5)n_i\,\bar{\lambda}_j\gamma_\mu(1+\gamma_5)n_j + \text{h.c.}$$

where θ_c is the Cabibbo angle, θ_W the Weinberg angle, G_F the Fermi coupling constant, and where the indices i and j run over the three colors.

For the left-right case of Fig. 1b we find by similar methods

$$\mathscr{L}_{\text{eff}}^{L-R} = +\frac{G_F}{\sqrt{2}}\,\frac{\alpha}{16\pi}\left(\frac{m_{p'}}{M_W}\right)^2 2\left(\ln\frac{M_W}{m_{p'}} - 1\right)\frac{\cos^2\theta_c}{\sin^2\theta_W}\left\{4\,\bar{\lambda}_i(1-\gamma_5)n_i\;\bar{\lambda}_j(1-\gamma_5)n_j\right.$$

$$\left. + \,\bar{\lambda}_i\,\sigma_{\mu\nu}(1-\gamma_5)n_i\;\bar{\lambda}_j\,\sigma^{\mu\nu}(1-\gamma_5)\,n_j\right\} + \text{h.c.}$$

In order to compute the short-distance contribution to the K_L-K_S mass difference one requires the matrix element of the four-quark operators that appear in $\mathscr{L}_{\text{eff}}$. No exact methods are available. However, for the left-left case considered by them, Gaillard and Lee[5] have argued that a reasonable estimate can be obtained by inserting the vacuum state between two currents, in all possible ways, so that one encounters (suppressing color indices)

$$\langle\bar{K}|\,\bar{\lambda}\gamma_\mu\gamma_5 n\,|0\rangle\;\langle 0|\,\bar{\lambda}\,\gamma_\mu\gamma_5 n\,|K\rangle = f_K^2 m_K^2 \;.$$

On this basis they then find an upper limit of about 1.5 GeV on the mass $m_{p'}$.

Proceeding in the same vacuum insertion approximation for the operator in $\mathscr{L}_{\text{eff}}^{L-R}$, we encounter

$$4 \langle \bar{K} | \bar{\lambda} \gamma_5 n | 0 \rangle \langle 0 | \bar{\lambda} \gamma_5 n | K \rangle = f_K^2 m_K^2 \left(\frac{2m_K}{m_\lambda + m_p} \right)^2 ,$$

where use has been made of the relation

$$\bar{\lambda}\gamma_5 n = (m_\lambda + m_p)^{-1} \partial_\mu \bar{\lambda} \gamma_\mu \gamma_5 n \quad .$$

The factor $m_K/(m_\lambda + m_p)$ is presumably of order unity, or perhaps even somewhat larger, so we conclude the matrix elements of the four-quark operators in Eqs. (2) and (3) are roughly comparable. However, the co-factor of the operator in $\mathscr{L}_{eff}^{L-R}$ is larger than that in $\mathscr{L}_{eff}^{L-L}$ in the ratio $2 (\ln \frac{M_W}{m_{p'}} -1) \sin^{-2} \theta_c \simeq 50 (\ln \frac{M_W}{m_{p'}} - 1)$. Relative to the situation for the standard model, therefore, this implies for the nonstandard model under consideration a reduction of the upper bound on $m_{p'}^2$, by about two orders of magnitude! Despite the crudity[10] of the matrix elements estimates, this would appear to be unacceptable and to rule this model out of serious consideration.

The model that we have discussed here, and rejected, represents a very simple variation on the standard scheme: no additional quarks have been introduced but one has grouped p'_R and n_R into a weak right-handed doublet, leaving p_R and λ_R as singlets (in the standard model the right-handed quarks all enter as singlets.). An equally simple alternative would group p'_R and λ_R into a doublet, leaving p_R and n_R as singlets. For this case $\mathscr{L}_{eff}^{L-R}$ is again given by the right-hand side of Eq. (3), multiplied however, by a factor $\sin^{-2} \theta_c \approx 0.04$. The contributions to the K_L-K_S

mass difference coming from $\mathscr{L}_{\text{eff}}^{L-R}$ and $\mathscr{L}_{\text{eff}}^{L-L}$ are now more nearly comparable, the former being enhanced mildly by the factor $2(\ln \frac{M_W}{m_{p'}} - 1)$. The mild reduction in the upper bound on $m_{p'}$ which this enhancement implies may be tolerable.

The discussion so far has been conducted in an approximation where one neglects the strong interactions, insofar as these determine the effective Lagrangian. In the context of asymptotic freedom and to within logarithmic corrections, this requires that the dominant contributions come from large momenta in the diagrams that we have been considering. For $\mathscr{L}_{\text{eff}}^{L-L}$ there are in fact important contributions from low momenta,[11] owing to the highly convergent character of the integral. The effective Lagrangian $\mathscr{L}_{\text{eff}}^{L-R}$ is on a somewhat more secure footing -- the logarithmic factor $\ln \frac{M_W}{m_{p'}}$ in Eq. (3) signifies the dominance of large momenta.

The notions of asymptotic freedom suggest that strong interaction effects at large momenta introduce corrections which are only logarithmic, i.e., powers of $\log \frac{M_W^2}{\mu^2}$, where μ is a scale factor. The second order weak effects that we are concerned with involve products of four current operators. If we focus on the contributions to $\mathscr{L}_{\text{eff}}^{L-R}$ coming from the domain where all space-time separations are small, then what is needed is a generalization of the Wilson expression for products of four operators. The machinery for a careful analysis is not yet available. Nevertheless, an intuitive, though perhaps crude basis for estimating the corrections to Eq. (3) would be as follows. The four quark operator which we meet in

Eq. (3) involves a combination of scalar and tensor coupling terms (we subsume the $1 - \gamma_5$ factors in this characterization). The mass factor $m_{p'}^2$, also appears in Eq. (3). It is suggestive on the ideas of asymptotic freedom to treat this mass factor as a function of loop momentum and similarly to multiply the four quark operators by momentum dependent factors which reflect their anomalous dimensions (actually there are two different linear combinations of the scalar and tensor operators which constitute the objects having definite anomalous dimension). This approach ignores other operators that might be important at small distances, e.g., operators involving the strong interaction gluon fields, but it does incorporate gluon effects for the operators that are retained. The question, then, is whether the momentum dependent factors introduced by these considerations are substantial enough to modify Eq. (3) seriously. A discussion along these lines is carried out in Appendix I. It turns out that the anomalous dimensions of the mass operator and of the four quark operators combine to produce what is only a rather modest momentum dependence. There is therefore no obvious reason to expect a substantial change in the estimate of Eq. (3) -- what is at stake, recall, is two orders of magnitude.

We turn next to the analog of K^0-$\bar{K}^0$ mixing for mesons composed of a heavy and light quark pair, i.e., mixing of $D^0 \equiv (p'\bar{p})$ and $\bar{D}^0 \equiv (\bar{p}'p)$, where p' is some new heavy quark. In the present absence of experimental information on such states we are no longer dealing with constraints, but rather with possibilities. For present purposes we ignore effects which might arise from CP violation. Suppose that the state D^0 is produced at the initial time, so that initially the decay $\mu^+ + \bar{\nu}_\mu$ + ordinary hadrons is allowed, the decay $\mu^- + \nu_\mu$ + ordinary hadrons forbidden. As time goes on that state acquires an admixture of $\bar{D}^0$, for which the selection rules are reversed. With an eye to later phenomenological applications we therefore take as a conventional measure of mixing the time integrated ratio of μ^- and μ^+ events,

$$r = \frac{N(\mu^- + \bar{\nu}_\mu + X)}{N(\mu^+ + \nu_\mu + X)} .$$

It is the CP even and odd combinations of D^0 and $\bar{D}^0$ that have definite decay rates and masses. Let λ devote the average of the two decay rates, $\Delta\lambda$ the differences; and let Δm denote the mass difference. The central mass is presumably large compared to the masses of ordinary mesons, so one expects that there are many open(and closed) channels available that couple importantly to D^0 and/or $\bar{D}^0$. The differences $\Delta\lambda$ and Δm arise only from these transitions, real and virtual, which couple D^0 and $\bar{D}^0$ to common states.

In the absence of mixing, $r = 0$. For small mixing ($\Delta\lambda/\lambda \ll 1$, $\Delta m/\lambda << 1$) one has[12]

$$r \approx \frac{1}{8}\left(\frac{\Delta\lambda}{\lambda}\right)^2 + \frac{1}{2}\left(\frac{\Delta m}{\lambda}\right)^2 .$$

For the K^0-$\bar{K}^0$ system there is the special circumstance that, overwhelmingly, the most important open channel is the 2π state, and this is even under CP. This gives $\Delta\lambda/\lambda \approx 1$. For a massive $D^0, \bar{D}^0$ system, on the other hand, it is likely that many open channels, with both signs for CP, are important and that $\Delta\lambda/\lambda$ is small. As for $\Delta m/\lambda$ the second order contributions which come from channels which are nearly open, i.e., from intermediate states near the mass shell, are similarly expected to be small. The issue of substantial mixing therefore hinges on the following two effects: (i) Substantial second order contributions from closed channels ranging far off the mass shell -- in the extreme, contributions from the ultra-violet region, analogous to the effects discussed earlier for the $K^0, \bar{K}^0$ system, where the object was to suppress such effects. (ii) Direct first order interactions which couple D^0 and $\bar{D}^0$, such as would arise if there were "charm" changing neutral currents;[13] this would produce essentially complete mixing, $\Delta m/\lambda >> 1$, hence $r \approx 1$. $r \approx 1$ was suggested ~~by~~ DeRujula, Georgi, and Glashow[9] as a possible explanation of "wrong-sign" dimuon events.[14]

Let us now consider several models which may serve to illustrate the various possibilities. We characterize the models sufficiently by displaying the weak SU(2) doublets, left- and right-handed.

$$A: \quad \begin{pmatrix} p \\ n^c \end{pmatrix}_L \, , \quad \begin{pmatrix} p' \\ \lambda^c \end{pmatrix}_L$$

$$B: \quad \begin{pmatrix} p \\ n^c \end{pmatrix}_L \, , \begin{pmatrix} p' \\ \lambda^c \end{pmatrix}_L \, ; \begin{pmatrix} p \\ n \end{pmatrix}_R \, .$$

$$C: \quad \begin{pmatrix} p \\ n^c \end{pmatrix}_L \, , \begin{pmatrix} p' \\ \lambda^c \end{pmatrix}_L \, ; \begin{pmatrix} p' \\ x \end{pmatrix}_L \, ; \begin{pmatrix} p \\ x \end{pmatrix}_R \, , \begin{pmatrix} p' \\ n \end{pmatrix}_R \, , \begin{pmatrix} p'' \\ \lambda \end{pmatrix}_R$$

$$\mathrm{D}:\quad \begin{pmatrix} p \\ n^c \end{pmatrix}_L , \quad \begin{pmatrix} p' \\ \lambda^c \end{pmatrix}_L ; \begin{pmatrix} p \sin\alpha + p' \cos\alpha \\ x \end{pmatrix}_R .$$

Model A: This is the standard GIM model and the mixing effects, which have already been discussed in the literature, need only be briefly summarized. This model has no charm-changing neutral currents and the second-order contributions to $\Delta m/\lambda$ from the ultraviolet region are small. Therefore $\Delta m/\lambda$ should be comparable to $\Delta\lambda/\lambda$. The off-diagonal elements in the decay ray matrix are suppressed relative to the diagonal elements by $\tan^2\theta_c$ and also by a factor which measures breaking of strong SU(3) symmetry. We therefore expect that $\Delta\lambda/\lambda$ is at most of order $\tan^2\theta_c$. Altogether, therefore, we expect $r \lesssim \tan^4\theta_c \sim 10^{-3}$.

Model B: This is the model[9] aready discussed above in connection with the K_L-K_S mass difference, where it runs into trouble. For the question of D^0-$\bar{D}^0$ mixing, on the other hand, the ultraviolet contribution to Δm is small: $\mathscr{L}_{eff}$ is again given by an obvious variation on Eq. (3), with the field operators λ and n replaced by p' and p and $m_{p'}$ replaced by $m_n \ll m_{p'}$. Also, there are no charm-changing neutral currents in this model. Substantial D^0, $\bar{D}^0$ mixing is therefore not to be expected. The contribution from the open and nearly open low mass channels could conceivably give contributions to $\Delta m/\lambda$ which, though not large, might be of order unity; but as we have argued qualitatively, when there are many open channels involving states with both signs of CP, we in fact, expect that $\Delta m/\lambda$ and

$\Delta\lambda/\lambda$ will be small, so that $r \lesssim 0.1$ seems a reasonable upper limit.

Model C: This model illustrates what can happen if we begin a serious proliferation of quarks; in addition to the standard four, two new quarks p'' and X are introduced here. In order to suppress the K_L^0-K_S^0 mass differences we require that $m_{p''}$ not exceed a few GeV -- for this equation p'' plays the role assigned to p' in model A. Here, however, we are concerned with D^0-$\bar{D}^0$ mixing, where there are bound states of $(p', \bar{p})$ and $(\bar{p}', p)$ pairs. The point of the present model is that if m_X is made large enough one can achieve a large mass difference Δm, as we see from the obvious generalization of Eq. (3). That is, with large enough m_X one can achieve large $\Delta m/\lambda$ and therefore substantial mixing, $r \approx 1$.

Model D: This model is designed to produce a non-diagonal neutral current with the quantum numbers of $\bar{p}'p + \bar{p}p'$. Thus Δm is first order weak, whereas $\Delta\lambda$ is second order weak, so $\Delta m/\lambda \gg 1$ and the mixing is essentially complete, $r \approx 1$. The mass of the X quark plays no role here. For economy we might be tempted to identify X with the "usual" λ quark. This would introduce right-handed currents for ordinary semileptonic $\Delta S = 0$ or $\Delta S = 1$ processes. Experimentally there is perhaps room for such currents at the 10% level. This degree of suppression could of course be achieved by choosing a small enough value for the mixing angle α, $\alpha \lesssim 0.1$.

Let us abstract some lessons from these models and from others that one can contemplate in the general $SU(2) \times U(1)$ framework that we have been considering. If mixing effects beyond $r \approx 10^{-3}$ were to be observed for the D^0-$\bar{D}^0$ system (the signatures will be discussed later on), this would almost surely make it necessary to abandon the standard GIM scheme (model A). The simplest generalization, model B, could produce an appreciable mixing effect, $r \lesssim 0.1$, but this model is in trouble with the K_L-K_S mass difference. Any mixing much above $r \sim 10^{-3}$ would, therefore, signal the need for more than four quarks; or, as in model D with $X = \lambda$, for small right-handed current effects in ordinary semileptonic processes. The alternative represented by model D, whether or not the X particle is identified with an ordinary quark, introduces another feature that could be observationally significant, namely an off-diagonal neutral current which allows neutral current semileptonic production and decay of charmed hadrons.[13]

III. WEAK NONLEPTONIC DECAYS

We turn now to first order nonleptonic decays; in particular, the matter of octet dominance for ordinary $\Delta S = 1$, charm conserving transitions. Our discussion is predicated on the assumption that octet dominance is governed by the short distance properties[15] of products of current, hence by the properties of strong interaction dynamics at large momenta. Concerning this dynamics, we accept the nonabelian gauge theory structure which is suggested by the observation of Bjorken scaling and by the renormalization group analysis of renormalizable field theories. Nonleptonic decays have been analyzed in this framework by Gaillard and Lee and by

Altarelli and Maiani.[16] Let us summarize the results of this analysis.

Neglecting the possibility[17] of important contributions from scalar Higgs meson exchange and neglecting contributions from neutral currents (which have to be banished for $\Delta S \neq 0$ transition), they consider the short distance character of W boson exchange and write the effective Lagrangian in the form

$$\mathscr{L}_{eff} = \frac{G_F}{\sqrt{2}} \sum_K C_K \left(\ln \frac{M_W^2}{\mu^2}\right)^{\phi_K} \theta_K + \text{"small" corrections,}$$

where μ is a scale parameter and θ_K runs over locally gauge invariant operators of dimension five or six. The coefficients C_K and ϕ_K can be evaluated in a perturbation expansion in the strong interaction effective coupling constant, which is small at large momenta. The local operators that we encounter are formed out of quark and gluon fields. In particular, suppressing color and strong SU(3) and charm indices, we encounter the operators

$$\theta^{(0)}_{LR} = \bar{\psi}\, \nabla^2\, \psi$$

$$\theta^{+-}_{LL} = (\bar{\psi}_L \gamma_\mu \psi_L \cdot \bar{\psi}_L \gamma^\mu \psi_L)^{\pm} \text{ and } L \to R$$

$$\theta^{+-}_{LR} = (\bar{\psi}_L \gamma_\mu \psi_L \cdot \bar{\psi}_R \gamma_\mu \psi_R)^{\pm} .$$

where the $\pm$ symbol indicates that there are two possible color structures for the operators and ∇ is the covariant derivative; as usual, the subscripts L and R refer to left- and right-handed projections of the quark fields.

Of course, operators involving the right-handed fields become relevant only in models which contain right-handed charged currents. For the anomalous dimension coefficients ϕ_K one finds

$$\phi^+_{LL} = \frac{12}{33 - 2n}, \qquad \phi^-_{LL} = -\frac{6}{33 - 2n}$$

$$\phi^+_{LR} = \frac{24}{33 - 2n}, \quad \phi^-_{LR} = -\frac{3}{33 - 2n}, \quad \phi^{(0)}_{LR} = +\frac{4}{33 - 2n}$$

where n is the number of quark color triplets that enter in the strong interactions. The coefficient $\phi^{(0)}_{LR}$, which is relevant for the operator[18] $\theta^{(0)}_{LR}$, was not computed in the above papers. We have found that its Wilson coefficient C_K vanishes up to second order in the momentum dependent strong coupling constant. This becomes small at large momenta and it therefore seems reasonable to drop the operator $\theta^{(0)}_{LR}$ from consideration. We note that the operator θ^+_{LL} is symmetric in color indices; θ^-_{LL} antisymmetric. The former belongs to the $\underline{8}$ representation of strong SU(3), the latter also to the $\underline{27}$ representation. In $\theta^{(+)}_{LR}$ the color indices of the right-handed quarks are contracted with those of the left-handed quarks; θ^-_{LR} has a more complicated color structure. Both θ^+_{LR} and θ^-_{LR} belong to strong SU(3) octets in the models considered.

The question of octet enhancement[15] depends on the logarithmic factors in Eq. (10), on the Wilson coefficients C_K, and on the matrix elements of the operators θ_K. It is in the spirit of the present discussion to suppose that the matrix elements of the various θ_K are comparable[19] for all K. Similarly, the Wilson coefficients are all comparable, apart from obvious factors like sin θ_c, cos θ_c, etc. that arise from the structure of the weak interaction model and which distinguish one model from another. It is these factors, together with the logarithmic factors in Eq. (10), that we want to invoke for octet enhancement. To get some idea of the importance of these logarithmic terms, let us take M_W = 60 GeV, μ = 1.0 GeV and compute the quantities $\left(\ln \frac{M_W^2}{\mu^2}\right)^{\phi_K}$. The results are set out in Table I for n = 4,6,10 . The relevant ratios for dominance of $\underline{8}$ over $\underline{27}$ interactions are $\theta^+_{LR}/\theta^-_{LL}$ and $\theta^+_{LL}/\theta^-_{LL}$. The observed enhancement is perhaps of order 20 or more in many processes, although for $K \to 3\pi$ decays the $\Delta I = \frac{1}{2}$ rule seems to fare rather badly.[20]

Since a general discussion becomes cumbersome, let us once more consider several illustrative models and then try to extract some general lessons.

Model A:

This is the standard GIM scheme, set out in Eq. (9). There are no right-handed (charged) currents and the number of quark species is n = 4. The enhancement of octet over $\underline{27}$ interactions is only of order 4 or 5, probably not enough to account for the observed enhancement unless we invoke, additionally, extra enhancements[21] in the matrix elements of the octet operator θ^+_{LL}. Such extra effects, which go outside of the spirit of asymptotic freedom, are of course, entirely possible; however, the game here is to see what can be done without invoking such possibilities. There is an alternative possibility, which is in the spirit of the present discussion and which could preserve the otherwise attractive features of the model; namely, one can imagine a substantial increase in the number n of color triplets of quarks that enter into the strong interactions. This would intensify octet enhancement; yet other effects on present weak interaction phenomenology would be negligible if the new quarks were not coupled with the light ones in the weak interactions.

Model B: In this model[9] octet enhancement has a chance to be quite substantial. The point is that a right-handed current appears in the model, hence an operator θ^{+}_{LR} associated with a large logarithmic enhancement, as we see from Table 1. Moreover, the Wilson coefficient which multiplies this operator does not contain the Cabibbo factor $\sin \theta_c$ that appears in the coefficient of θ^{-}_{LL}. The net enhancement (for n = 4) is therefore of order $12.5/\sin \theta_c \approx 60$. Unfortunately, as we've already noticed, this model runs into trouble with the K_L-K_S mass difference.

In Sec. II we also commented on a variant of model B, obtained by replacing the right-handed doublet (p'_R, n_R) with (p'_R, λ_R). This produces the enhancement factor 12.5 and does not involve any obvious difficulties for the K_L - K_S mass difference.

Model C: This model is identical with the variant of model B just discussed as far as octet enhancement is concerned, except now n = 6.

The enhancement factor is 20.

Model D: If 'X' in this model is n, then we need $\alpha \lesssim 0.1$ to agree with weak interaction phenomenology, and therefore we are essentially back to model B, with good octet enhancement, but troubles in the K^0-$\bar{K}^0$ system. If 'X' is λ we get back essentially to the $(p'\lambda)_R$ variant of model B. If X is some new quark, then the right-handed currents are irrelevant for octet enhancement, so we have essentially model A. Finally X could be some linear combination of n or λ (not both!) and a new quark. We leave it to the reader to contemplate this possibility.

Finally, we wish to mention one more model:

Model E:
$$\begin{pmatrix} p \\ n_c \end{pmatrix}_L \quad \begin{pmatrix} p' \\ \cos\alpha\,\lambda_c + \sin\alpha\, X \end{pmatrix}_L \quad \begin{pmatrix} p'' \\ -\sin\alpha\,\lambda_c + \cos\alpha\, X \end{pmatrix}_L$$
$$\begin{pmatrix} p' \\ n\cos\beta - X\sin\beta \end{pmatrix}_R \quad \begin{pmatrix} p'' \\ n\sin\beta + X\cos\beta \end{pmatrix}_R$$

This model is cooked up to show one way of avoiding trouble in K^0-$\bar{K}^0$ mixing while insuring a large octet enhancement. The point is that in the mixing problem quark masses appear multiplying the couplings of the currents, while in the Wilson coefficient relevant for octet enhancement questions no such factors occurs. In the present case if

$$m_{p'}\cos\beta\cos\alpha = m_{p''}\sin\beta\sin\alpha$$

there is no dangerous contribution to K^0-$\bar{K}^0$ mixing. If α and β are small, there is large octet enhancement.

Let us summarize this discussion of octet enhancement. If one sticks to left-handed currents only, then manifestly the octet and 27 pieces have the same Cabibbo factors and we can only get enhancement dynamically. In models with four quarks, the dynamical calculation of Refs. 15, 16 does not seem to yield a large enough factor. The situation improves if we add more quark species, even if they do not couple to the light quarks via the weak interactions, and with 10 or so quark species one gets enough. With right-handed currents, one has bigger dynamical enhancement factors and the possibility of an extra $1/\sin\theta_c$ since the left-right current product need not see a Cabibbo angle accompanying $\bar{\lambda}n$. As we have seen in Sec. II, these same circumstances tend to cause trouble in the K^0-$\bar{K}^0$ system (c.f. Model B above). It is possible that if one relies only on the dynamical enhancement, and not on the Cabibbo factor (as in Model C) one might avoid this difficulty. The dynamical factors in the two cases are different while the 'kinematical' Cabibbo factors are the same. As mentioned in footnote 19 we have some general doubts whether octet enhancement can be obtained by adding right-handed currents coupling light to heavy quarks. The point is that operators like $\bar{n}\,\gamma_\mu(1+\gamma_5)p'\bar{p}'\,\gamma_\mu(1-\gamma_5)\lambda$, while formally 'enhanced' in the sense of appearing with large coefficients in the effective Hamiltonian for weak $\Delta S = 1$ decays, may be expected to have small matrix elements between low-mass hadron states. We also demonstrated one tricky way of suppressing the contribution to K^0-$\bar{K}^0$, by having mixing angles related to quark masses (Model E).

The success of current algebra[22] in describing some features of nonleptonic decays also puts some restrictions on attempts to obtain octet dominance by adding right-handed currents coupled to n quarks. If the effective Hamiltonian for $\Delta S = 1$ nonleptonic weak decays involves right-handed n quarks, then the soft pion predictions for nonleptonic weak decay amplitudes have an opposite sign from what one gets with left-handed n quarks. The reasonable predictions are the same as long as one has purely left-handed or purely right-handed quarks in the Hamiltonian, but are changed if one has both. We must therefore make sure not only that the LR operators dominate over the LL 27-plet piece, but also that they dominate the LL octet by a substantial factor.

Finally we should remark on the analogue of octet enhancement for weak nonleptonic decays of heavy hadrons. Simple selection rules[23] only occur when the dominant operator for these processes is of LL or RR type since only in these cases does the effective Hamiltonian have two identical currents and a simple symmetry structure in the color indices. In all cases the nonleptonic decays should be very much enhanced over the semileptonic decays. If this turns out not to be true experimentally, it probably means that the short-distance explanation of the $\Delta I = \frac{1}{2}$ rule is on the wrong track.

IV. PHENOMENOLOGY

What is common to almost all schemes of the weak interactions, in the $SU(2) \times U(1)$ framework discussed here but more generally, is the appearance of at least one new quark going beyond the usual triplet p, n, λ. This implies the existence of hadronic states with new quantum numbers. On this view the unravelling of the weak (and strong) interactions will rest on discovery of these new particles and detailed measurements of their production and decay properties. The GIM scheme (model A) represents the simplest and perhaps most attractive possibility that accords with standard weak interaction phenomenology. This scheme introduces a single new quantum number, charm (C). The weak interaction structure is marked by charged currents that are exclusively left-handed and a neutral current that is diagonal ($\Delta S = 0$, $\Delta C = 0$). The weak charm-changing nonleptonic interactions are dominated by $\Delta C = \Delta S$ couplings; the mixing effects discussed in Sec. II are expected to be small ($r \lesssim 10^{-3}$); and with respect to SU(3) structure, these interactions are expected to be dominated by terms belonging to the $6 + \underline{6}$ representation.

Let us now turn to several rather gross phenomenological signatures that might serve as tests for this model and the other, alternative possibilities that we have been considering. We focus especially on inclusive neutrino reactions.

a) Right-handed current coupling to n quarks and dimuon production: In the usual GIM charm model, production of p' quarks requires either a $\sin^2 \theta_c$ (for production off n quarks) or production off 'sea' quarks (λ), which is expected to be small. The observed number of $\mu^+\mu^-$ events[26] would therefore require a very large semileptonic branching ratio for charged particle decays. As we have seen in Sec. III, this conflicts with the short-distance analysis understanding of octet enhancement, according to which nonleptonic decays should be dominant. One is therefore lead to consider new couplings of n-quarks, which involve right-handed currents. As we have seen, if one identifies a quark having a substantial coupling to right-handed n quarks with the charmed quark of GIM, one runs into difficulties explaining the smallness of K^0-$\bar{K}^0$ mixing. One therefore probably requires a new quark. This could be accomplished most simply by adding a new right-handed doublet $\binom{p''}{n}_R$ to the usual left-handed doublets $\binom{p}{n_c}_L$, $\binom{p'}{\lambda_c}_L$. A p'' quark coupled in this way, if it were degenerate in mass with the p' quark, could also help to explain the large value of the ratio R(s) in electron-positron annihilation, without requiring the existence of new narrow resonances in e^+e^- annihilation[25] (which have not been observed).[26] Production of particles containing the new quark would be copious in high-energy ν scattering, so a small semileptonic branching ratio (as expected theoretically)[27] would suffice to give a substantial number of dimuon events with a fast μ^-. The x-distribution of these events would be normal, since we are scattering off valence quarks. A clean test for right-handed currents is to plot the y-distribution for the dimuon events, which should come out $(1-y)^2$. The ratio $\sigma_\nu / \sigma_{\bar{\nu}}$ at very high energies would tend to 4/1, from 3/1, (provided no other quarks start getting produced!) according to the super-naive parton model.

b) Right-handed coupling to p quarks:

This has similar effects in $\bar{\nu}$ scattering at high energy. The dimuons would be produced with a fast μ^+. The apparent breakdown of scaling would be dramatic, since now 1 is competing with $(1-y)^2$ instead of the other way round. $\sigma_\nu / \sigma_{\bar{\nu}}$ would approach 3/4, from 3/1! The y-distribution for dimuon events alone would be flat. There is presently little experimental support for this possibility.

Of course, one can combine a) and b).

c) Production of baryon resonances:

In the ordinary charm picture, production of baryon resonances by neutrino scattering off nucleons is a $\sin^2\theta_c$ process. If some new quark coupled to a right-handed n quark (again, this probably could not be the usual GIM charmed quark) then one could get substantial resonance production. If some new quark couples to a right-handed p quark, similar remarks apply to $\bar{\nu}$ scattering off nucleons.

d) Diagonal neutral currents:

We consider the effects of adding extra right-handed pieces to the neutral current in the context of the parton model with valence quarks only. In the SU(2) $\times$ U(1) theories under consideration the neutral current involving valence quarks will take the form

$$\bar{p}_L\gamma_\mu p_L - \bar{n}_L\gamma_\mu n_L + \alpha\, \bar{p}_R\gamma_\mu p_R - \beta\, \bar{n}_R\gamma_\mu n_R - 2\sin^2\theta_W j_\mu^{e.m.}$$

with $0 \leq \alpha, \beta \leq 1$. The standard GIM model has $\alpha = \beta = 0$.

In the super-naive parton model under consideration one obtains for the ratio of neutral to charged current cross sections in deep inelastic scattering off an isoscalar target

$$R^{\nu} = \frac{X}{3}\left[\left(\frac{1+\alpha}{2} - \frac{4}{3}\sin^2\theta_W\right)^2 + \left(\frac{1-\alpha}{2}\right)^2 + \left(\frac{1-\alpha}{2}\right)\left(\frac{1+\alpha}{2} - \frac{4}{3}\sin^2\theta_W\right)\right.$$
$$\left. + \left(\frac{1+\beta}{2} - \frac{2}{3}\sin^2\theta_W\right)^2 + \left(\frac{1-\beta}{2}\right)^2 + \left(\frac{1-\beta}{2}\right)\left(\frac{1+\beta}{2} - \frac{2}{3}\sin^2\theta_W\right)\right]$$

$$R^{\bar{\nu}} = X\left[\left(\frac{1+\alpha}{2} - \frac{4}{3}\sin^2\theta_W\right)^2 + \left(\frac{1-\alpha}{2}\right)^2 - \left(\frac{1-\alpha}{2}\right)\left(\frac{1+\alpha}{2} - \frac{4}{3}\sin^2\theta_W\right)\right.$$
$$\left. + \left(\frac{1+\beta}{2} - \frac{2}{3}\sin^2\theta_W\right)^2 + \left(\frac{1-\beta}{2}\right)^2 - \left(\frac{1-\beta}{2}\right)\left(\frac{1+\beta}{2} - \frac{2}{3}\sin^2\theta_W\right)\right]$$

where X is a parameter depending on the neutral vector boson mass; in the simplest Weinberg-Salam theory $X = 1$. These numbers have been plotted in Fis. 2 as a function of $\sin^2\theta_W$ for the extreme cases $\alpha, \beta = 0, 1$. As experimental results on the neutral current become more precise, we can expect that α and β will be severely constrained.

Further tests of the structure of the neutral current, for reactions other than deep inelastic and without using the parton model so naively could be carried out following Ref. 28.

e) Wrong-sign dimuons and trimuons:

As was alluded to above, D^0-$\bar{D}^0$ mixing provides a possible mechanism[29] which would account for the observation of $\mu^-\mu^-$ events recently reported in[14] a ν scattering experiment. The D^0 produced in the "primary" reaction $\nu + N \to \mu^- + D^0 + X$ may switch into a $\bar{D}^0$ which then decays into $\mu^- + \bar{\nu} + X'$. It is important to substantiate these events since, as we have seen, in order to obtain an adequate D^0-$\bar{D}^0$ mixing one needs either more than four quarks or a charm-changing neutral current. It should be noted however, that (e.g., model D) with a charm-changing neutral current D^0 and $\bar{D}^0$ may decay into a $\mu^+\mu^-$ pair (perhaps accompanied by hadrons). One would thus expect to observe "trilepton" events

$$\nu + N \to \mu^-\mu^+\mu^- + X.$$

Such events have not been seen thus far.[14] Of course, the neutral charm-changing current can be made small (i.e., $\alpha \approx 0$ or $\frac{\pi}{2}$ in model D) without substantially changing ν from 1.[30]

In summary, the observation of substantial number of $\mu^-\mu^-$ events (compared to $\mu^+\mu^-$ events say) and the nonobservation of $\mu^-\mu^+\mu^-$ events would suggest the presence of degrees of freedom "beyond charm."

f) Off-diagonal neutral currents:

Other effects of a charm-changing neutral current include direct $\Delta C = 2$ decay of charm baryons (not so easy to observe!) and apparent lepton nonconservation such as in the process

$$\nu + N \to \mu^+ + X.$$

This process could come about by neutral current production of D^0

$$\nu + N \to \nu + D^0 + X'(C = 0)$$

followed by $D^0 \to \bar{D}^0 \to \mu^+ + \bar{\nu} + X''$.

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APPENDIX I

We will outline here the calculation of the correction factors due to a asymptotically free strong interaction. The theory to be considered is the standard gauge theory with SU(3) color triplets. In this theory[31]

$$\beta(g) = b g^3 + \ldots$$

where
$$\begin{aligned} b &= -\frac{1}{16\pi^2}\,\frac{1}{3}\,[11\, C_2(G) - 4\,T(R)] \\ &= -\frac{1}{16\pi^2}\,\frac{1}{3}\,[33 - 2n] \end{aligned} \qquad (1)$$

n is the number of quark types.

The effective amplitude contributing to the K_L-K_S mass difference is given by the time-ordered product of four currents joined by intermediate vector boson propagators. In order to proceed, we have to make the (dynamical) assumption[32] that the region in which the space-time locations of the four currents are close together yields the dominant contribution and that the operator product expansion may be used. As was discussed in the text there is no rigorous argument to support the validity of this assumption. There are suggestive arguments, however. In any case, we will proceed with this assumption.

Our task is to find firstly, the operators θ_A which have definite anomalous dimensions γ_A. The operators θ_A will be a linear combination of the operators S and T defined by

$$\begin{aligned} S &\equiv \bar{\lambda}_i(1+\gamma_5)n_i \quad \bar{\lambda}_j(1+\gamma_5)n_j \\ T &\equiv \bar{\lambda}_i\,\sigma_{\alpha\beta}(1+\gamma_5)n_i\,\bar{\lambda}_j\,\sigma^{\alpha\beta}(1+\gamma_5)n_j \end{aligned} \qquad (2)$$

(The indices i and j refer to color).

According to the standard renormalization group analysis[33] the coefficient function C_A appearing in the operator product expansion corresponding to the operators σ_A satisfies the equation

$$C_A(X/\lambda,\ g,\ m,\ M)$$
$$\left[\exp -\int_1^{\lambda} \frac{d\lambda'}{\lambda'}\ \gamma_A\left(g(\lambda')\right)\right] C_A\,(X, g(\lambda), m(\lambda), M)\ , \tag{3}$$

Here M is the renormalization mass. $g(\lambda)$ is the solution of the equation

$$\lambda\,\frac{dg(\lambda)}{d\lambda} = \beta\left(g(\lambda)\right). \tag{4}$$

For $\beta(g) = bg^3 + \ldots$ one finds $g^2(\lambda) \to \frac{-1}{2b \log \lambda}$ $(b < 0)$. The effective mass $m(\lambda)$, as is well known, behaves like

$$m(\lambda) \sim (\log \lambda^2)^{-12/25} \text{ for } \lambda \to \infty \tag{5}$$

for the standard color-triplet theory. The two operators S and T, which we will refer to collectively as P_J, $J = 1, 2$ are multiplicatively renormalized by a four-by-four matrix $\underset{\sim}{Z}$. To determine $\underset{\sim}{Z}$ we compute the matrix element $<T\ P_J\ \bar{\psi}\,\psi\bar{\psi}\,\psi>$ and require $Z_2^2\ Z_{JK}\ <T\ P_K\ \bar{\psi}\,\psi\bar{\psi}\,\psi>$ to be cut-off independent. The matrix element $<T\ P_J\ \bar{\psi}\,\psi\bar{\psi}\,\psi>$ are given to one-loop order by the graphs in Fig. 3. The computation of these graphs consists of simple exercises in manipulating identities involving color SU(3) and Lorentz indices. The result is

$$\underset{\sim}{Z} = \underset{\sim}{1} - \frac{g^2}{16\pi^2} \log \frac{\Lambda^2}{\mu^2} \left(\underset{\sim}{B} - \frac{8}{3}\underset{\sim}{1}\right)$$

where

$$\underset{\sim}{B} \equiv \begin{pmatrix} 23/3 & 1/12 \\ -20 & -3 \end{pmatrix} \tag{6}$$

(The basis vector is $\{S, T\}$). The anomalous dimension matrix $\underset{\sim}{\gamma}$ is given by

$$\begin{aligned} \underset{\sim}{\gamma} &= -\underset{\sim}{Z}^{-1} \mu \frac{\partial}{\partial \mu} \underset{\sim}{Z} \\ &= -2 \frac{g^2}{16\pi^2} \left(\underset{\sim}{B} - 8/3\ \underset{\sim}{1}\right) \quad . \end{aligned} \tag{7}$$

The operators θ_A are determined thus by the eigenvectors of $\underset{\sim}{B}^T$. γ_A are the eigenvalues of $\underset{\sim}{\gamma}$. They may be computed to be

$$\begin{aligned} \gamma_1 &= (7.51 - 8/3) \quad (-2g^2/16\pi^2) \\ \gamma_2 &= (-2.84 - 8/3) \quad (-2g^2/16\pi^2). \end{aligned} \tag{8}$$

The corresponding eigenoperators are (the coefficient of S defines the normalization)

$$\begin{aligned} \theta_1 &= S + 0.008\ T \\ \theta_2 &= S + 0.53\ T \quad . \end{aligned}$$

These equations may be inverted to give

$$\begin{aligned} 4S + T &= 2.14\ \theta_1 + 1.87\ \theta_2 \\ &\equiv \sum_{A=1}^{2} f_A \theta_A \quad . \end{aligned}$$

Associated with each operator θ_A is an enhancement factor

$$\exp - \int_1^\lambda (d\lambda'/\lambda')\gamma_A \, g(\lambda') \to \eta_A (\log \lambda)^{C_A/2b} \quad , \ \lambda \to \infty , \quad (9)$$

according to Eq. (3). Here C_A is defined by $\gamma_A = C_A g^2 + 0(g^4)$ and η_A is an unknown constant reflecting the contribution of the low momentum region.

In summary, the effect of an asymptotically free strong interaction is to modify the free quark effective interaction as follows: replace m_c by

$$m_c(M_W^2) \sim \eta \, m_c \left(\log \frac{M_W^2}{\mu^2}\right)^{-12/(33-2n)} ,$$

where η is an unknown constant and the operators 4S + T by

$$\sum_{A=1}^{2} f_A \, \eta_A \left(\log \frac{M_W^2}{\mu^2}\right)^{C_A/2b} \theta_A \ .$$

Thus, the operator that dominate as $M_W \to \infty$ is θ_1 with the dominance factor

$$\left(\log \frac{M_W^2}{\mu^2}\right)^{3(7.51-8/3)/(33-2n)} .$$

For a four-quark model n = 4 and the dominance factor is $(\log M_W^2/\mu^2)^{0.6}$. It is amusing to note that the free quark effective operator 4S + T has been transformed almost completely into the scalar operator:

$$4S + T \to 6.6 \, \eta_1 \theta_1 = 6.6 \, \eta_1 \, [S + 0.008\, T] \ .$$

Hopefully, the unknown constant η_1 is of order one and may be absorbed into the unknown matrix element of θ_1. Thus, in conclusion, an asymptotically free strong interaction modifies the free quark result only slightly.

One should perhaps emphasize here a rather obvious point: namely that as in all asymptotic application of the renormalization group the various results will be strictly correct if all large parameters, such as m_W^2 in the present context, are mathematically infinite. In actuality, m_W^2/μ^2 is certainly large, of order 1000, but finite. With $\log M_W^2/\mu^2 \sim 7$, one may be concerned that the next-to-leading operator θ_2 in Eq. (11) may not be completely negligible since $(\log M_W^2/\mu^2)^{C_2/2b^2} \sim (\log \frac{M_W^2}{\mu^2})^{-0.66}$. However, one must be warned that various terms down by inverse power of $(\log \frac{M_W^2}{\mu^2})$ had already been dropped in the renormalization group analysis of course. Thus strictly speaking the contribution of θ_2 cannot be taken seriously.

APPENDIX II

It may appear at first sight that various two-body operators can contribute to nonleptonic decay. Let us list all two-body operators with canonical dimension ≤ 6 here:

$$\theta_1 = \bar{n}\lambda, \qquad \theta_2 = \bar{n}\not{D}\lambda, \qquad \theta_3 = \bar{n}D_\mu D^\mu\lambda$$

$$\theta_4 = \bar{n}D_\mu\not{D}D^\mu\lambda, \qquad \theta_5 = g\bar{n}\gamma_\mu D_\nu\lambda F^{\mu\nu},$$

$$\theta_6 = \epsilon_{\mu\nu\rho\sigma}\bar{n}\gamma^\mu D^\nu D^\rho D^\sigma\lambda.$$

Here D_μ represents the covariant derivative. It is well-known that θ_1 and θ_2 may be transformed away. The Wilson coefficient of θ_3 in the relevant operator-product expansion is of order g^2 and hence suppressed in an asymptotically free theory. By using the equation of motion θ_3 is also related to the operator $g\bar{n}\sigma_{\mu\nu}\lambda F^{\mu\nu}$. θ_4 is related to $g\bar{n}\gamma_\mu D_\nu\lambda F^{\mu\nu}$ and has a Wilson coefficient of order g^0 but is suppressed by the GIM mechanism. The Wilson coefficient of θ_5 is of order g^2. The operator θ_6 is related to θ_5. There are other operators which we have not listed, e.g., $\epsilon^{\mu\nu\rho\sigma}\bar{n}\gamma_\mu\gamma_\nu D_\rho D_\sigma\lambda$. However, by using the equation of motion, one can easily reduce these operators to the ones listed. Finally the operator $\bar{\psi}D^2\psi$ does not mix with the four-fermion operators to lowest order.

We also remark that in the nonleptonic decay of charmed particles the GIM mechanism may not be available to suppress the charmed analogue of θ_4. For example, in model D of the text the nonleptonic decay of charmed hadrons will receive a contribution from the SU(3) triplet[34] operator $\bar{c}D_\mu\not{D}D^\mu p$.

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In this decay we meet the matrix element of a four-quark operator similar to the one appearing above in K^0-$\bar{K}^0$ mixing, composed of four light quark fields, although the Lorentz structure is different. If we use PCAC on one of the outgoing pions, we meet the matrix element of the same operator between a kaon and a pion. We propose that this matrix element should be of the same order as the matrix element we want between K^0 and $\bar{K}^0$. The experimental numbers then lead to an estimate similar to the one in the text. However, $K^+ \to \pi^+\pi^0$ is absolutely forbidden by current algebra in the (unphysical) limit when both pions are soft, forbidden by SU(3) and CP invariance, and finally according to the short-distance analysis of nonleptonic decays (see Sec. III) $\Delta I = 3/2$ transitions have an extra suppression factor. All these effects would tend to increase our estimate of the matrix element, so the estimate above might if anything be too small.

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FIGURE CAPTIONS

Fig. 1 Effective four-quark interaction contributing to K_L-K_S mass difference. (a) appears in the standard theory; (b) reports an additional contribution appearing in certain models involving right-handed currents.

Fig. 2 Behavior of R^{ν}, $R^{\bar{\nu}}$ and $\sigma^{\nu}/\sigma^{\bar{\nu}}$ as functions of Weinberg's angle plotted here for typical values of α and β, two parameters defined in the text. Dashed line: $R_{\bar{\nu}}$; Dot-dash line: R_{ν} Solid line: $\sigma_{\nu}/\sigma_{\bar{\nu}}$

Fig. 3 The graphs relevant to calculating the anomalous dimensions of four-quark operators.

Table 1.

Logarithmic factors $(\ln \frac{M_W^2}{\mu^2})^{\phi_K}$ multiplying various operators θ_K in the effective Lagrangian, and ratios of factors relevant for the question of octet enhancement; with M_W = 60 GeV, μ = 1.0 GeV. Here n is the number of color triplets of quarks that enter into the strong interaction.

	n = 4	6	10
θ^+_{LR}	7.5	11.1	48.5
θ^-_{LR}	0.78	0.74	0.62
θ^+_{LL}	2.7	3.3	7.0
θ^-_{LL}	0.60	0.55	0.38
$\theta^+_{LR}:\theta^-_{LL}$	12.5	20.2	128.0
$\theta^+_{LL}:\theta^-_{LL}$	4.5	6.1	18.4

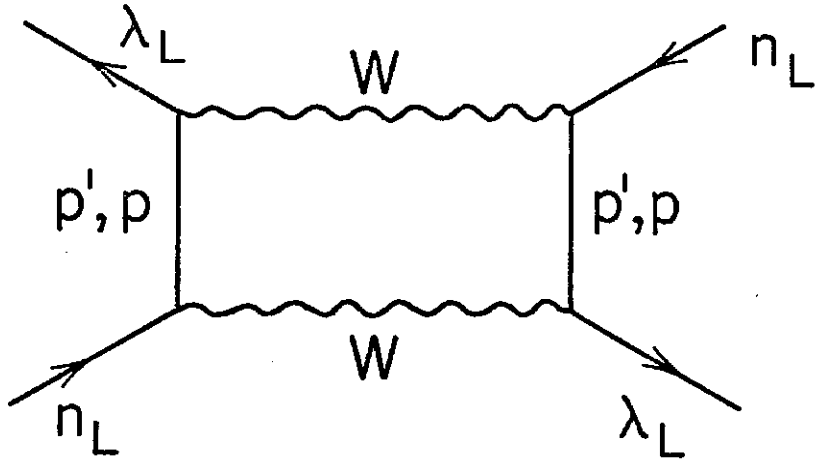


Fig. 1a

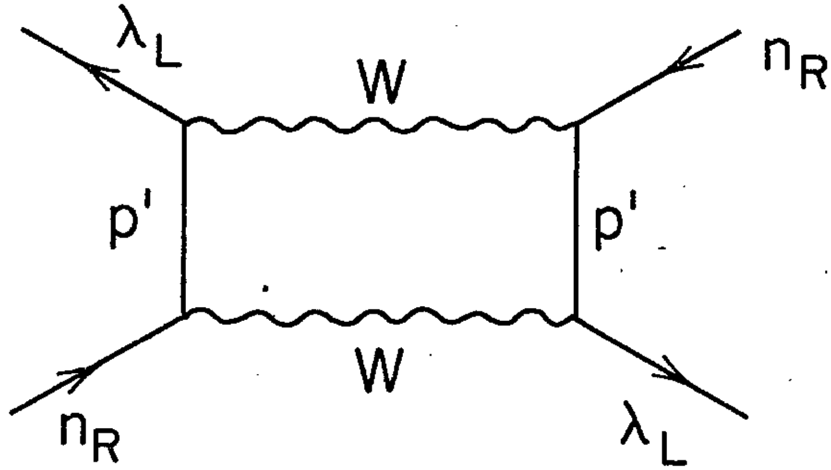


Fig. 1b

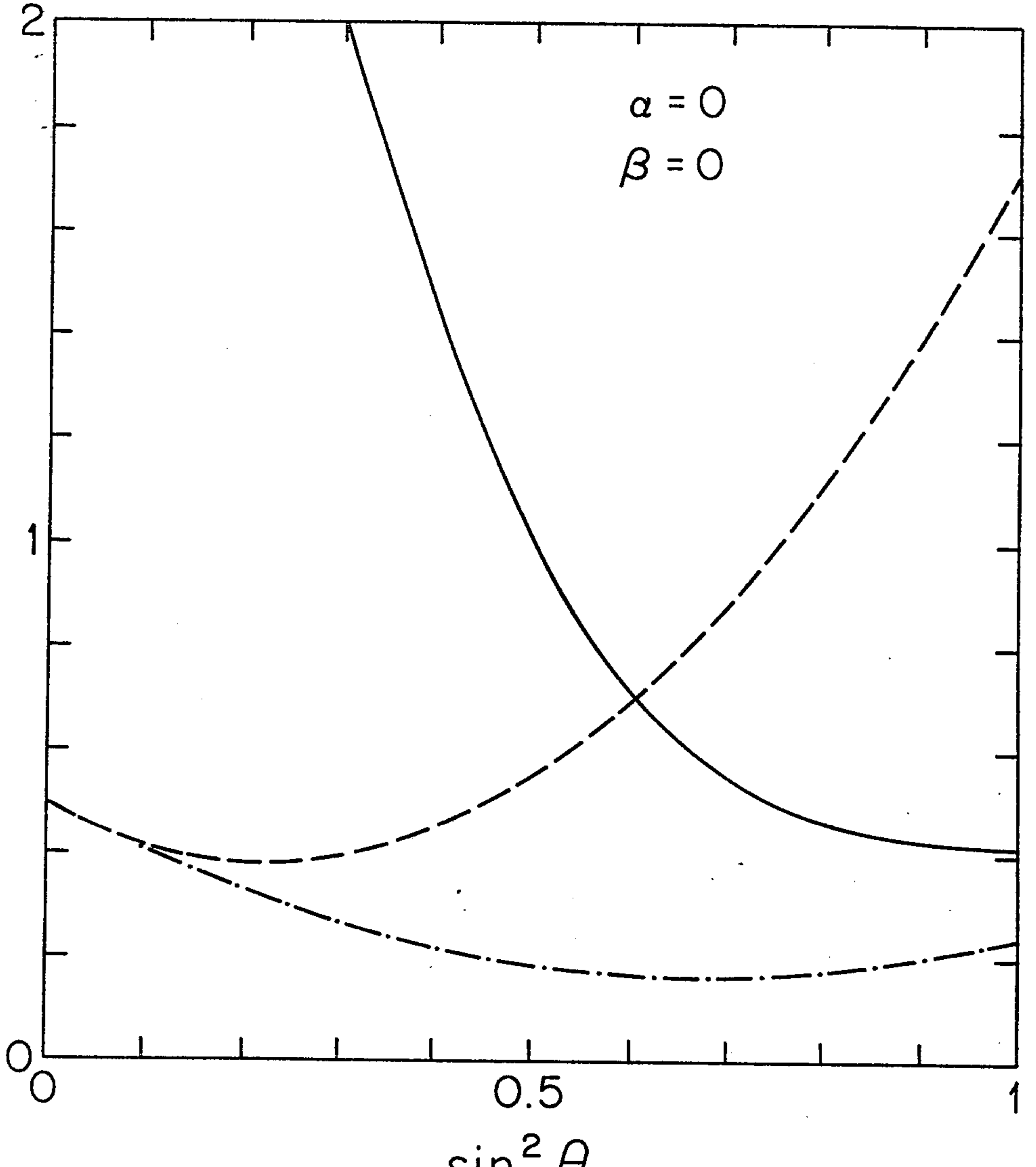


Fig.2(a)

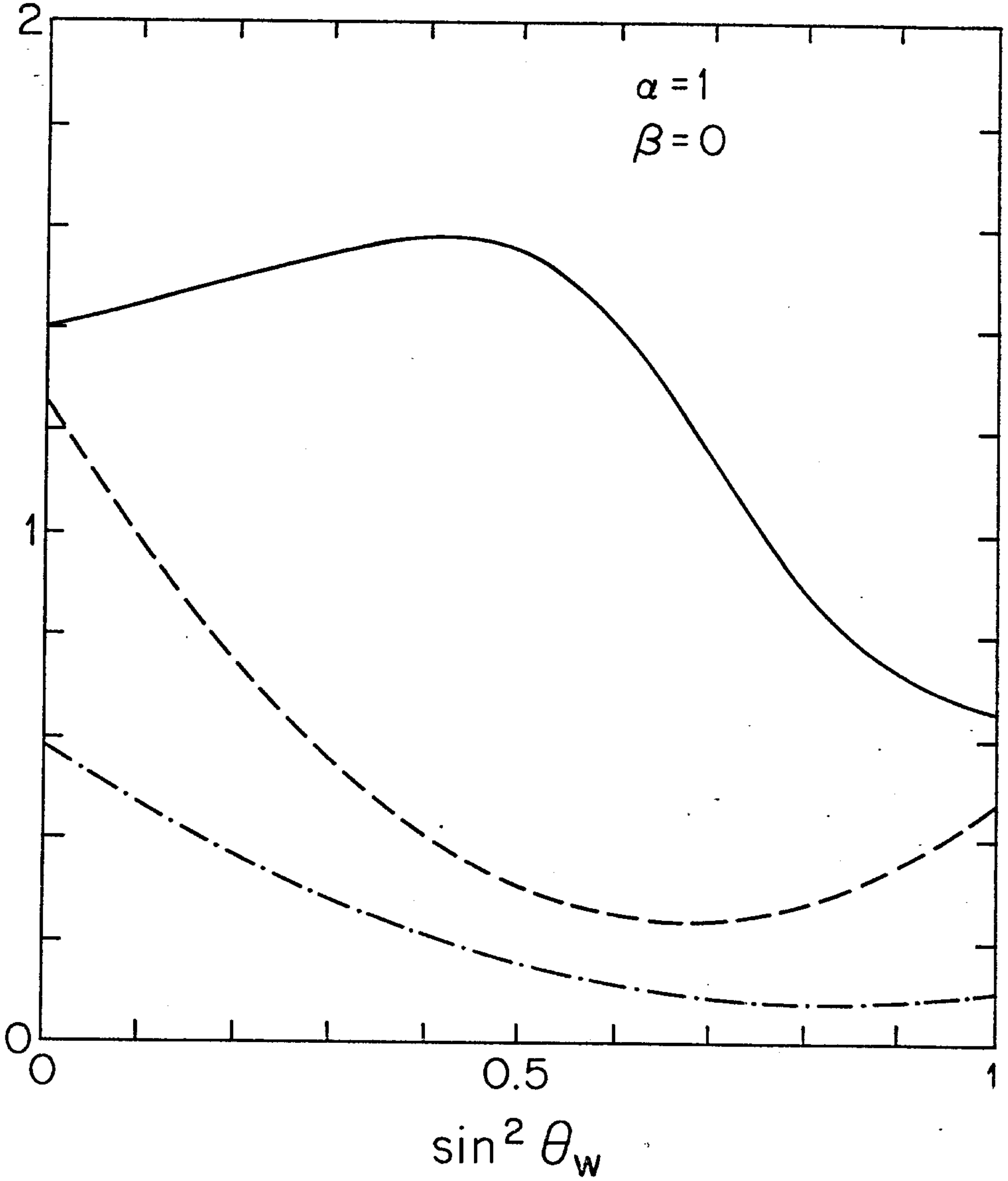


Fig. 2(b)

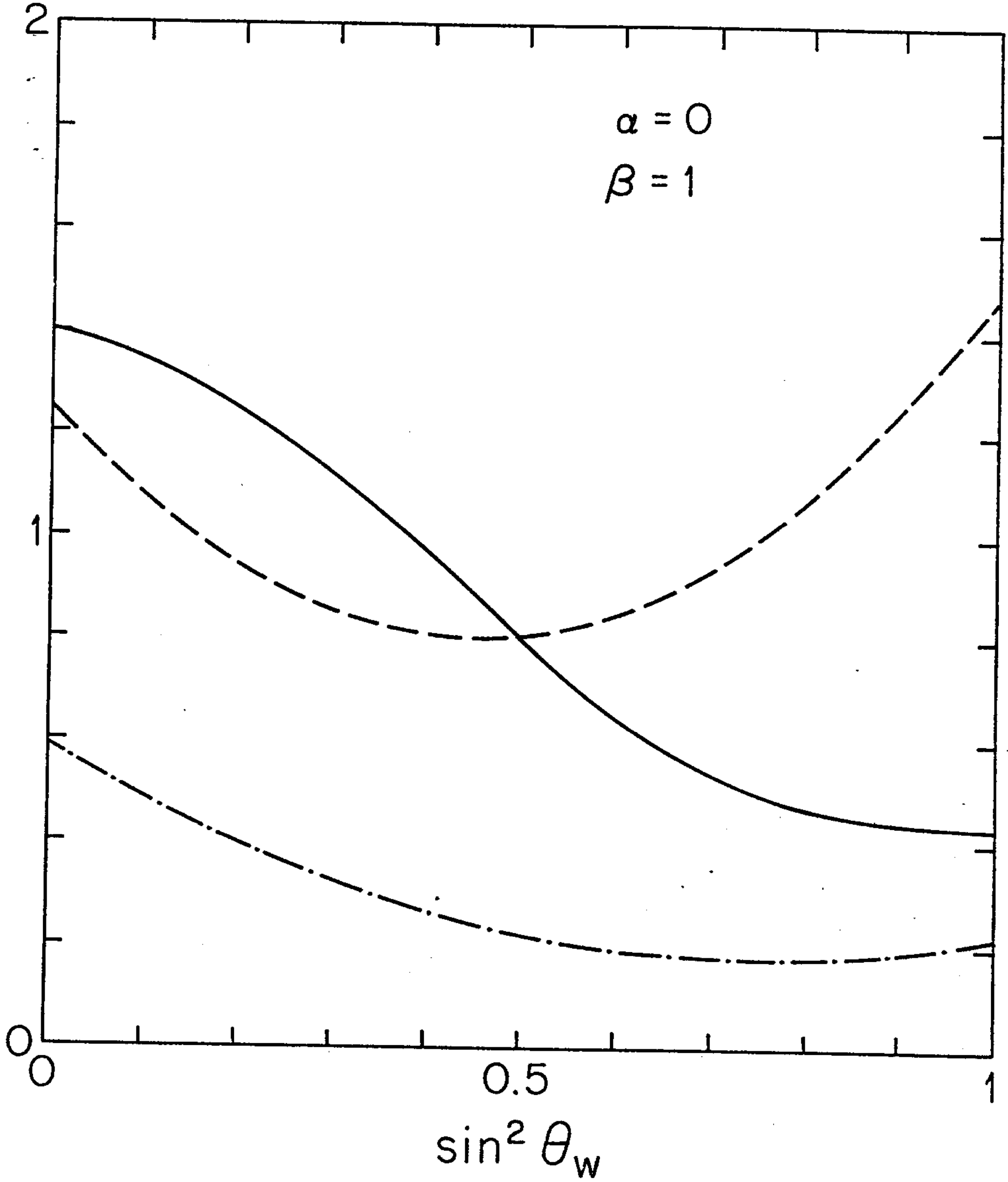


Fig. 2c

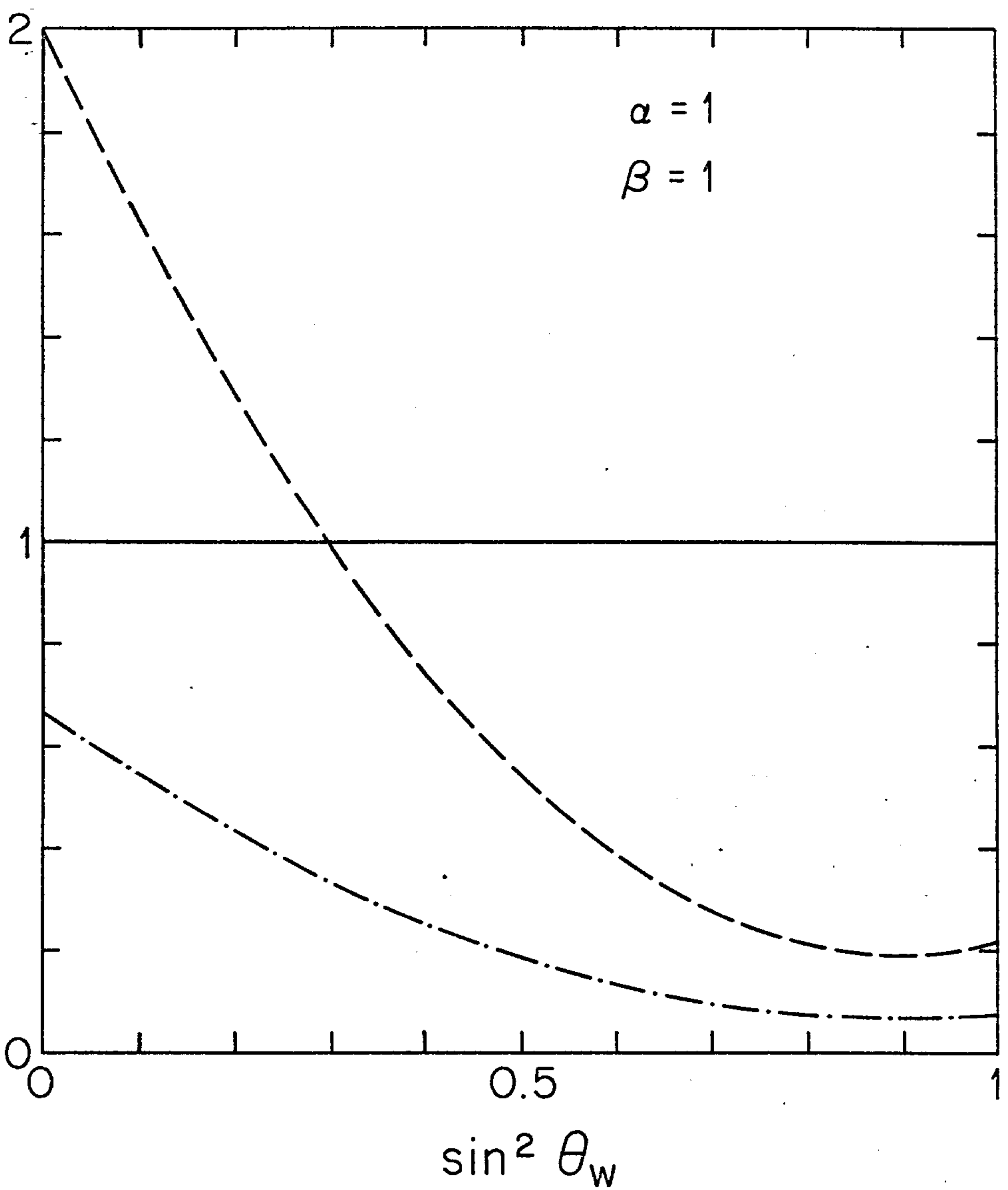
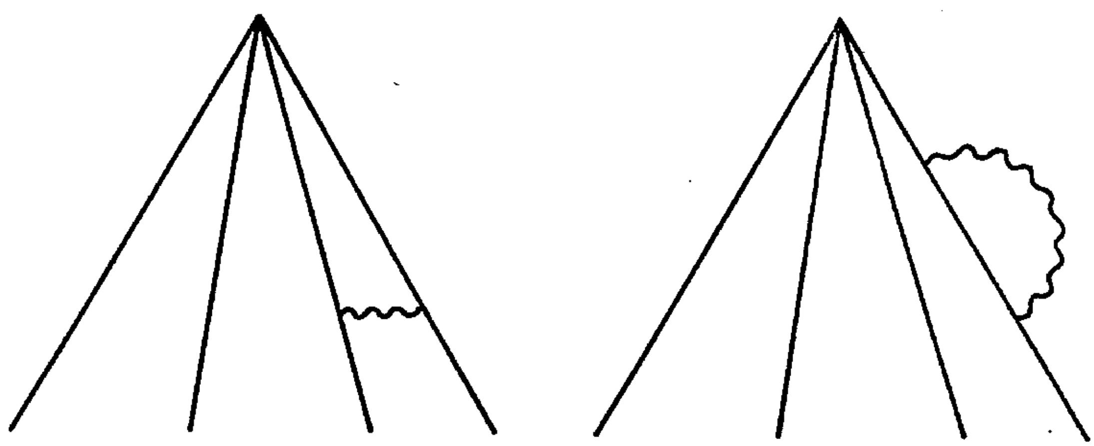


Fig. 2(d)



+ permutations

Fig. 3