This report is an excerpt from Argonne National Laboratory Progress Report for the period ending Mar. 28, 1949; Theoretical Nuclear Physics Division (ANL-4284 p/7-24).

ON CLOSED SHEIIS IN NUCLEI. II

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April., 1949

Feenberg[(1), (2)] and Nordlkeim[(3)] have used the spins and magnetic moments of the even-odd nuclei to determine the angular momentum of the eigenfunction of the odd particle. The tabulations given by them indicate that spin orbit coupling favors the state of higher total angular momentum, If - strong spin.orbit coupling' increasing with angular momentum is assumed, a level assignment encounters a very few contradictions. with experimental facts and requires no major crossing of the levels from those of a square well potential. The magic numbers O, 82, and 126 occur at the' place of the spin-orbit splitting of levels of high angular momentum,

Table 1 contains in column two in order of decreasing binding energy the levels of the square well potential. The quantum number gives the number of radial nodes. Two levels of the same quantum number gives the number of radial nodes. Two levels of the same quantum number cannot cross for any type of potential well, except due to spin-orbit splitting. No evidence of any crossing is founds Column three contains the usual spectroscopic designation of the levels, as used. by Nordheim and Feenberg. Column one groups together those levels which are degenerate for a three dimensional isotropic oscillator potential. A well with rounded corners

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(1)) Eugene Feenberg, PHYS, REV, 320, (1949)
(2) Eugene Feenberg, PHYS, REV, (1949)
(3): Iothar Nordheim, PHYS. REV, (1949)
The author is indebted to these authors for having obtained copies of both (2) and (3) before publication, xwW Is

will have a behAvior in between these two potentials. The shell grouping is given in column five, with the numbers of particles per shell and the total number of particles. up to and including each shell in column six and seven respectively.

Within each shell the levels may be expected to be close in energy, and not necessarily in the order of the table, although the order of levels of the same orbital angular momentum and different spin should be maintained. Two exceptions, $_{11}Na^{23}$ with spin 3/2 instead of the expected $d_{5/2}$, and $_{25}M^{55}$ with 5/2 instead of the expected f7/2, are the only violations.

Table 2 lists the known spins and orbital assignments from magnetic moments (*) when these are known and unambiguous, for the even-odd nucleii up to 83. Beyond 83 the data is limited and no exceptions to the assignment appear.

Up to Z or N = 20 the assignment is the same as that of Feenberg and No.rdheim. At the beginning of the next shell, $f_{7/2}$ levels occur at 21 and 23, as they should. At 28 the the $f_{7/2}$ levels should be filled, and no spins of 7/2 are encountered any more in this shell. This subshell may contribute to the stqbility of Ca 4^8. If the $g_{9/2}$ level did not cross the **$p_{1/2}$ or $f_{5/2}$ levels**, the first spin of 9/2 should occur at 41, which is indeed the case. Three nuclei with N or Z = 49 have $g_{9/2}$ orbits. No s or d levels should occur in this shell and there is no evidence for any.

The only exception to the proposed assignment in this shell is the spin 5/2 instead of 7/2 for Mq55 and the fact, that the magnetic moment of

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(4) H. H. Goldsmith and D. A. Inglis, the Properties of Atomic Nuclei. I., Information and Publications Division, Brookhaven National laboratory.

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$_{27}Co^{59}$ indicates a $g_{7/2}$ orbit instead of the expected f7/2.

In the next shell two exceptions to the assignment occur. *The spin* of 1/2 for Mo95 with 53 would be a violation, but. is experimentally doubtful. The magnetic moment of Eu^{153} indicates $f_{5/2}$ instead of the predicted $d_{5/2}$ No $h_{11/2}$ levels appear. It seems that these levels are filled in pairs only which does not seem a serious drawback of the theory as this tendency already shows up at the filling of the $g_{9/2}$ levels. Otherwise, the agreement is satisfactory. The shell begins with 51Sb, which has two isotopes with $d_{5/2}$ and $g_{7/2}$ **levels** respectively, as it should. The Thallium isotopes with 81 **neutrons and a** spin of 1/2 indicate a crossing of the $h_{11/2}$ and 3s **levels**. This is not surprising, since the energies of these levels are close together in the square well. The assignment demands that there be no spins of 9/2 in this shell, and none have been found. No f **or p levels** should occur and, except for Eu^{153}, there is no indication of are.

The spin and magnetic moment of 83k-, indicating an $h_{9/2}$ **state**.. is a beautiful confirmation of the correct beginning of the next shell. Here information begins to be scarce. The spin and magnetic moment of Pb^{207} with 125 neutrons interpret as $p_{1/2}$. This is the expected end of the shell since 7i and 4p have practically the same energy in the square well model. No spins of 11;2 and no s, d or g orbits should occur in this shell and the data indicates none.

The prevalence of isomerism in certain regions of the isotope chart, noticed by both Feenberg and Nordheim, is readily understood by **this assignment**. Long-lived isomeric states will occur in regions where levels of very different spin lie close together. These regions lie toward the end of the shells of 50, 82, and 126, where the levels of lowest angular momentum of one oscillator level almost coincide in energy with those of highest angular momentum from the next oscillator level. One is lead to the prediction that for nuclei of odd A isomerism should occur between $39 \leq Z$ or $N \leq 49$. This is the region where the $g_{9/2}$ and $p_{1/2}$ levels have closely the same energy and compete for the ground state. From 51 on there is a competition between $g_{7/2}$ and $d_{5/2}$, which would not lead to long-lived isomers. Later in the shell, from about 65 on., competition should occur between the $h_{11/2}$ and the $s_{1/2}$ and $d_{3/2}$ levels, and the occurrence of isomerism is predicted between $65 \leq Z$ or $N \leq 81$. The beginning of the new shell should again be free of isomerism. The experimental facts bear out the conclusions exceedingly well. Below are listed the number of long-lived isomeric states known and listed as A in the table by Seaborg and Perlman. Only isomers of odd A are used, and these are attributed to the odd one of the numbers N or Z.

N or Z =	29	31 - 37	39	41	43	45	47	49	51 - 61
# of isomers =	1	0	3	3	3	2	5	4	0

N or Z =	63	65	67	69	71	73	75	77	79	81	83
# of isomers =	1	1		4	1	2	2	2	2	2	0

In both regions, the level of high spin has opposite parity to the one of low spin. Consequently, one would expect electric 5th pole and magnetic 4th pole radiation to occur, but not electric 4th pole.

The assignment of orbits makes possible the prediction of spin and parity in cases in which the spin has not been observed. Since spin change and change of parity determine the selection rules for decay, it should be possible to test the theory against experiment. Work on this is in progress.

TABLE 1.

00c. No.	Square Well	Spect. termm	Spin term	Vo 0 of States	Shells	Total No.,
0	is	is	1 a1/2	2	2	2
			3- pl/2	44		
1	1P	2-p	"Pi/2	2J	v6	8
	id	3d	ld5/2	6		
2			Id3/2	4		
	A	2s	28 1/2	2		20
	if	4S	if7V/	8	8?	28?
3			If	6		
			2p5	4	22	
	C2p	A	77 1/2			
			47 2 V			50
		5g				
	(ig		19 7/2	8		
				6		
4	2d	4d	2d3/2	21	32	
	35	A	38 1/2			
			2	12)		82
		A				
			IN/2	10		
			29"a	8		
5	) 2f	A	2	6		
			3 f5/2			
			3P3/2	4		
	tL3p	4P	pj/2	2		
			11 i~3/2	14)		126
	{li	7i				
			U11/7-			
6	Ag	6g				
	3d	5d				
	A	A				

TABLE 2

SPINS OF EVEN-ODD NUCLEI

~ 5 5 e a	Odd Protons p~	3		i $		Odd Neutrons pp	y	s	p 1 1/2	0 • Neutron X • Proton SPIN d 2 3/2	f 3 5/2	` 7/2	h 9/2	e a	~
1	H	1	s	3	s	He	3	s	OX					1	is
3	Li	7	p							X				3	P3/2
5	B	11	p			Be	9			Ox				5	
7	N	15	p			C	13	p	Ox					7	P1/2
9	F.	19	•						X					9	
11	N	23								X				11	d 5/2
13	Al	27	d								X			13	
15	P	31							X					15	o 1/2
17	Cl	35	d	37	d	S	33			Ox				17	d 3/2
19	E	39	d	41	d					X				19	
21	Sc	45	f									X		21	f 7/2
23	V	51										X		23	
25	Mn	55									X			25	
27	Co	59	9									X		27	
29	Cu	63		65						X				29	P3/2
31	Ge	69								X				31	
33	A.	7S								X				33	
35	Br	79		81						X				35	f 5/2
37	Rb	8S	f(5/2)	87	p(3/2)	Zn	67	f		X	OX			37	
39	Y	89							X					39	P1/2
41	Cb	93	9										X	41	9 9/2
43	Tc					Se	77		O					43	
45	Rh													45	
47	Ag	107	p	109	p	Kr	83	9	X				O	47	
49	In	113	9	115	9	Sr	87	9					OX	49	
51	Sb	121	d(5/2)	123	g(7/2)						X	X		51	9 7/2
53	I	127				Mo	95		O		X			53	d 5/2
55	Cs	133	9	137	9							X		55	
57	La	139	9									X		57	
59	Pr	141									X			59	
61	PM													61	
63	Eu	151		153	f	Cd	111	0	O		X			63	
65	Tb	159				Cd	s, Sn	~	O	X				65	h 11/2
67	Ho	165				Sn	117	a	O			X		67	
69	Tm					Sn	119	s	Ox					69	
71	Lu	175	9									X		71	
73	T~	181	9									X		73	
75	Re	185		187		Xe	129	a	O		X			75	
77	Ir	191	(1/2)	193	(3/2)	Xe	131	d	X	OX				77	d 3/2
79	Au	197	d			Ba	135	d		OX				79	
81	T1	203		205		Be	137	d	X	O				81	s 1/2
83	BI	209	h										X	83	h 9/2