ANALYSIS OF OFF-AXIS TENSION TEST OF WOOD SPECIMENS

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ABSTRACT

This paper presents a stress analysis of the off-axis tension test of clear wood specimens based on orthotropic elasticity theory. The effects of Poisson's ratio and shear coupling coefficient on stress distribution are analyzed in detail. The analysis also provides a theoretical foundation for the selection of a 10° grain angle in wood specimens for the characterization of shear properties. The Tsai–Hill failure theory is then applied to derive a formula for predicting shear strength. Existing strength data for Sitka spruce (*Picea sitchensis* Carr.) were used in a numerical analysis. Because of the large discrepancies in published test data from different sources, the accuracy of the formula is limited to the data used to derive it. However, the procedures are believed to be accurate. The off-axis tension test is attractive mainly because of its economy and ease of application. This research promises to pave the way for the adoption of the off-axis tension test for characterizing the shear properties of clear wood by the practicing engineer once representative input data become available.

Keywords: Off-axis tension test, orthotropic elasticity, shear strength, tensile strength, Tsai–Hill failure theory.

INTRODUCTION

The off-axis tension test has become attractive for determining the shear properties of composite materials because of its relatively low cost and operational simplicity. Pagano and Halpin (1968) studied the problem of specimen end constraints and the effect of the length-to-width ratio. Wu and Thomas (1968) discussed a rotating clamp fixture design. The results of Pagano and Halpin (1968) were verified and extended by Richards et al. (1969) and Rizzo (1969). Chamis and Sinclair (1977) proposed a 10° off-axis tensile specimen for characterizing fiber–composite intralaminar shear.

More recently, Pindera and Herakovich (1986) studied the relation between end constraints and accurate determination of shear modulus and the necessity of including shear coupling in stress analyses. Pierron and Vautrin (1996) discussed the use of oblique tabs proposed by Sun and Chung (1993). Pierron et al. (1998) then performed a whole-field assessment of the effects of end constraints on the strain field and reported that oblique tabbing can lead to a homogeneous strain field. Yoshihara and Ohta (2000) conducted an off-axis tension test for shear strength of Sitka spruce (*Picea sitchensis* Carr.) and other species. Their sample size was relatively small, nor did they apply any failure criteria to analyze their data. Consequently, their results seem to be somewhat inconclusive.

In the present study, we performed a detailed stress analysis for the off-axis tension test for clear Sitka spruce specimens to evaluate the effects of Poisson's ratio and shear coupling coefficient on stress distribution. We then applied the Tsai–Hillfailure criterion using test data from Yoshihara and Ohta (2000), the Forest Products Laboratory (FPL 1999),

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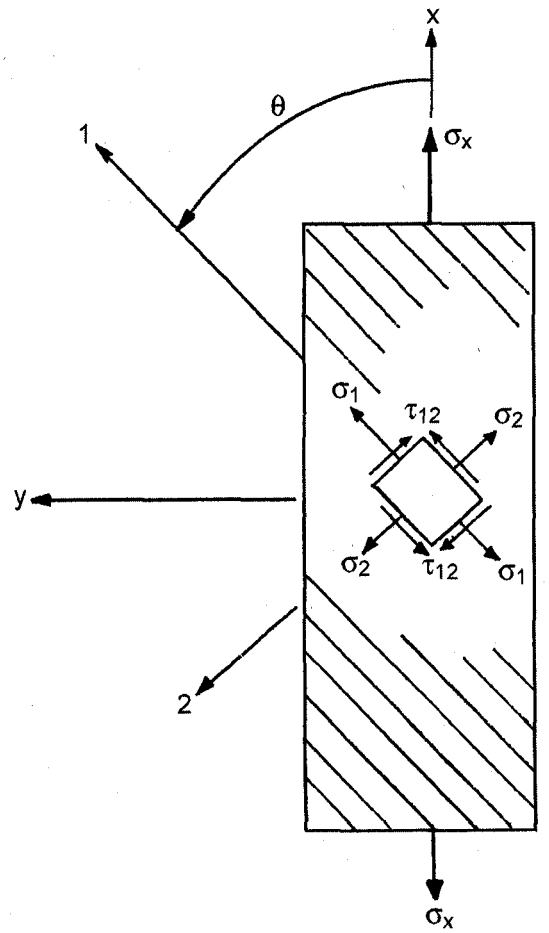

FIG. 1. Schematic of off-axis tension specimen. (*x*,*y* are geometrical axes; 1,2 are material axes.)

and Liu and Floeter (1984). A procedure to use the 10° off-axis tension test for the determination of clear wood shear strength is suggested.

STRESS-STRAIN RELATIONS

Let the 1–2 coordinate system represent the principal material axes and the *x*–*y* coordinate system the geometrical axes with angle q from the *x* axis to the 1 axis, as shown in Fig. 1. The normal stresses σ_1 and σ_2 in the 1 and 2 axes, respectively, and the shear stress τ_6 or τ_{12} in the 1–2 plane can be written as

$$\sigma_1 = \sigma_x m^2 \tag{1a}$$

$$\sigma_2 = \sigma_x n^2 \tag{1b}$$

$$\tau_6 = -\sigma_x mn \tag{1c}$$

where

$$m = \cos q \quad \text{and} \quad n = \sin q \tag{2}$$

The relations in Eq. (1) are independent of material properties; i.e., they are the same for isotropic or anisotropic materials.

For an anisotropic or orthotropic material such as wood, the applied stress σ_x in Fig. 1 produces the following strains:

$$\epsilon_x = \frac{\sigma_x}{E_x} \qquad \epsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x \qquad \gamma_s = \frac{\eta_{xs}}{E_x}\sigma_x \tag{3}$$

In Eq. (3), the Poisson's ratio ν_{xy}, corresponding to stress in the *x*-direction and strain in the *y*-direction, is the negative ratio of the transverse strain ϵ_y to the axial stain ϵ_x. The shear coupling coefficient η_{xs}, corresponding to normal stress in the *x*-direction and shear strain in the x-y plane, is the ratio of the shear strain γ_s or γ_{xy} to the axial strain ϵ_x.

The strain components referred to the 1 and 2 axes can be expressed in terms of those referred to the *x*, *y* axes by the following transformation relations:

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \frac{1}{2}\gamma_6 \end{bmatrix} = [T] \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \frac{1}{2}\gamma_s \end{bmatrix} \tag{4}$$

where the transformation matrix [*T*] is given by

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \tag{5}$$

The stress-strain relations in the 1-2 coordinate system are

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \frac{1}{2}\gamma_6 \end{bmatrix} \tag{6}$$

The reduced stiffnesses Q_{11}, Q_{22}, Q_{12}, and Q_{66} can be related to the basic engineering constants E_1, E_2, G_{12}, and ν_{12} as follows:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12} \tag{7}$$

From Eqs. (3), (4), and (6) by means of Eqs. (5) and (7), we obtain expressions for σ_1, σ_2, and τ_6 in terms of the engineering constants, which should be equal to the corresponding expressions in Eq. (1). These expressions can be put in dimensionless form as follows:

$$\frac{\sigma_1}{\sigma_x} = \frac{E_1}{E_x(1 - \nu_{12}\nu_{21})} \times [m^2 + \nu_{21}n^2 - \nu_{xy}(n^2 + \nu_{21}m^2) + \eta_{xs}(1 - \nu_{21})mn] = m^2 \tag{8a}$$

$$\frac{\sigma_2}{\sigma_x} = \frac{E_2}{E_x(1 - \nu_{12}\nu_{21})} \times [\nu_{12}m^2 + n^2 - \nu_{xy}(\nu_{12}n^2 + m^2) + \eta_{xs}(\nu_{12} - 1)mn] = n^2 \tag{8b}$$

$$\frac{\tau_6}{\sigma_x} = \frac{G_{12}}{E_x}[-2mn(1 + \nu_{xy}) + \eta_{xs}(m^2 - n^2)] = -mn \tag{8c}$$

In Eq. (8), the transformed elasticity modulus E_x, the Poisson's ratio ν_{xy}, and the shear coupling coefficient η_{xs} are as follows (Jones 1975):

$$\frac{1}{E_x} = \frac{1}{E_1}m^4 + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1}\right)m^2n^2 + \frac{1}{E_2}n^2 \tag{9}$$

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{12}}{E_1}(m^4 + n^4) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}}\right)m^2n^2 \tag{10}$$

$$\frac{\eta_{xs}}{E_x} = 2\left[\left(\frac{1}{E_1} + \frac{\nu_{12}}{E_1} - \frac{1}{2G_{12}}\right)m^2 - \left(\frac{1}{E_2} + \frac{\nu_{12}}{E_1} - \frac{1}{2G_{12}}\right)n^2\right]mn \tag{11}$$

TSAI–HILL FAILURE CRITERION

The Tsai–Hill criterion for orthotropic materials (Daniel and Ishai 1994) is

$$\frac{\sigma_1^2}{F_1^2} + \frac{\sigma_2^2}{F_2^2} + \frac{\tau_6^2}{F_6^2} - \frac{\sigma_1\sigma_2}{F_1^2} = 1 \tag{12}$$

in which F_1, F_2, and F_6 are the values of σ_1, σ_2, and τ_6 at failure, respectively, and the normal stress and strength in any one direction must be either tensile or compressive. Substituting Eq. (1) into Eq. (12) with the applied stress σ_x replaced by F_x, we obtain

$$\frac{1}{F_x^2} = \frac{m^4}{F_1^2} + \frac{n^4}{F_2^2} + \left(\frac{1}{F_6^2} - \frac{1}{F_1^2}\right)m^2n^2 \tag{13}$$

Note that F_x corresponding to any specified angle θ can be predicted once F_1, F_2, and F_6 are known. The Tsai–Hill theory allows for considerable interaction among the stress components σ_1, σ_2, and τ_6. The requirement that each normal stress and the corresponding strength should be either tensile or compressive poses no problem because an off-axis tension test involves only tensile stresses and strengths.

RESULTS AND DISCUSSION

We obtained numerical results to demonstrate the effects of Poisson's ratio and shear coupling coefficient on stress distribution in defect-free Sitka spruce based on Eq. (8). Identifying the longitudinal axis with the 1 axis and the radial axis with the 2 axis, the basic engineering constants for Sitka spruce (Liu and Ross 1998) are E_1 = 11,800 MPa, E_2 = 2,216 MPa, G_{12} = 910 MPa, and ν_{12} = 0.37. Using these values as input, E_x, ν_{xy}, and η_{xs}, which are needed in Eq. (8), are first calculated.

Figure 2 presents the variation of the ratio

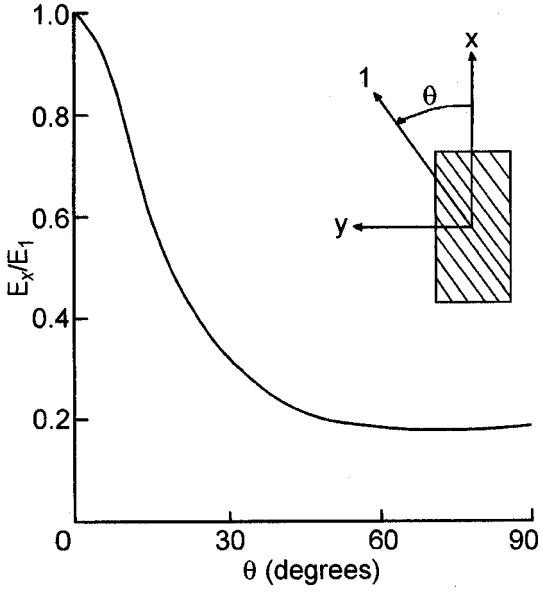

FIG. 2. Elasticity modulus of Sitka spruce as a function of grain angle. E_1 = 11,800 MPa, E_2 = 2,216 MPa, G_{12} = 910 MPa, and ν_{12} = 0.37.

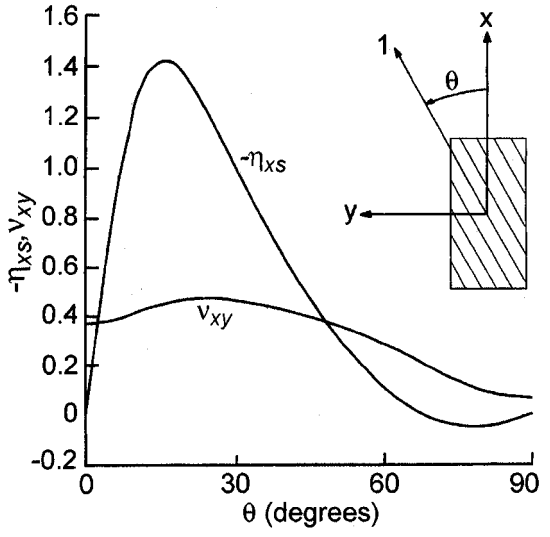

FIG. 3. Shear coupling coefficient and Poisson's ratio of Sitka spruce as a function of grain angle.

E_x/E_1 with q. The maximum value occurs at q = 0°. It decreases drastically to q = 30° and reaches a constant value of E_2/E_1 at about q = 55°. At q = 10°, the decrease in E_x is about 23% of E_1.

Figure 3 shows that h_{xs} is negative at 0° ≤ q ≤ 69° with a minimum value of -1.42 at q = 15°. At 69° < q ≤ 90°, h_{xs} is positive with a maximum value of 0.04 at 0 = 79°. The Poisson's ratio starts from 0.37 at q = 0°, reaches a maximum value of 0.47 at q = 25°, and drops to 0.07 at q = 90°.

Using these data, we found that all of equations (8) are satisfied for any value of q. We then plotted from Eq. (8a) the sum of terms containing h_{xs}, the sum of terms containing ν_{xy}, and the total of all terms, i.e., m^2, versus q (Fig. 4). As Fig. 4 shows, h_{xs} has a stronger effect than ν_{xy} on s_1/s_x at 3° ≤ q ≤ 45°. Beyond that range, the reverse is true.

Figure 5 shows the results from Eq. (8b). Here the sum of terms containing h_{xs} has the same sign as the total, n^2, while that of ν_{xy} has the opposite sign in the range of q < 67°. Beyond that range, both have insignificant effects on the total.

The ratio of t_6/s_x in Eq. (8c) plotted in Fig. 6 reveals that up to q » 10°, the total *mn* is only slightly above the sum of terms containing h_{xs} and the two vary linearly with q, indicating that h_{xs} has a predominant effect on t_6 when q = 10°. Assuming that when s_x increases to cause material to fail and the same

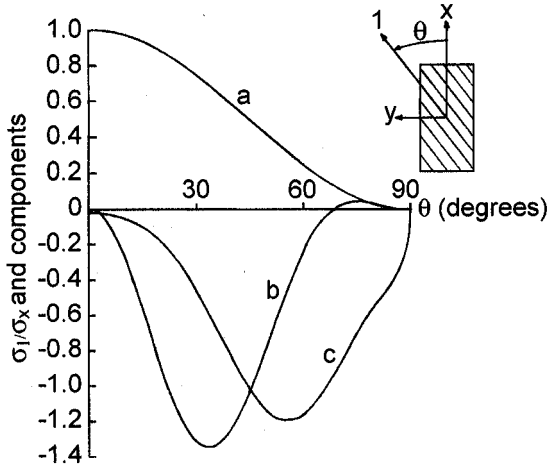

FIG. 4. Variations of s_1/s_x with q showing effects of h_{xs} and ν_{xy} (Eq. 8a). a = m^2, b = terms containing h_{xs}, c = terms containing ν_{xy}.

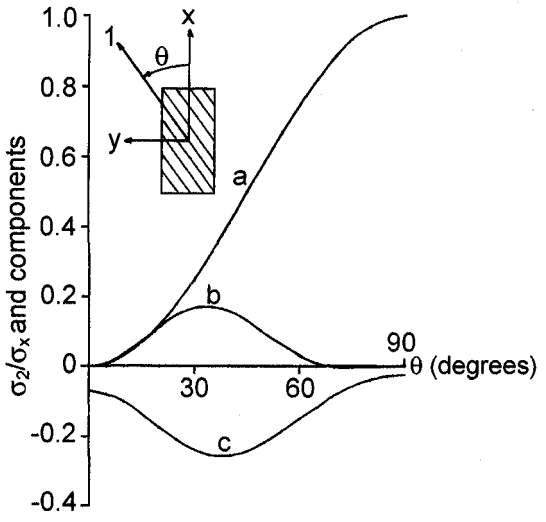

FIG. 5. Variations of (s_2/s_x with q showing effects of h_{xs} and v_{xy} (Eq. 8b). a = n^2, b = terms containing h_{xs}, c = terms containing v_{xy}

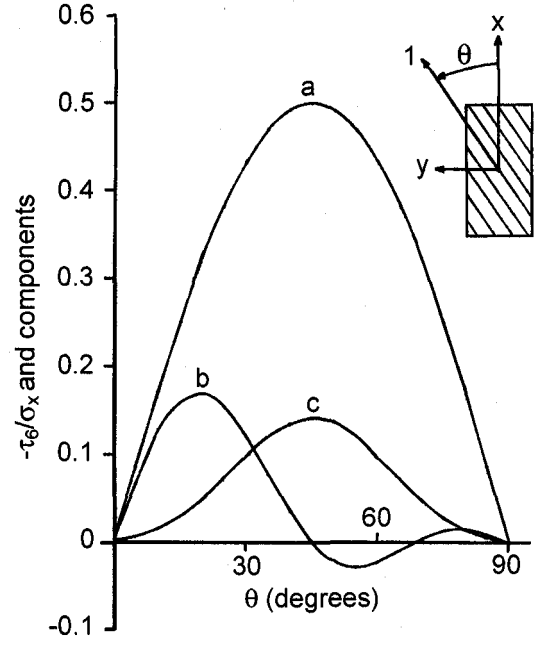

FIG. 6. Variations of — t_6/(swith q showing effects of h_{xs} and v_{xy} (Eq. 8c). a = *mn*, b = terms containing h_{xs}, c = terms containing v_{xy}.

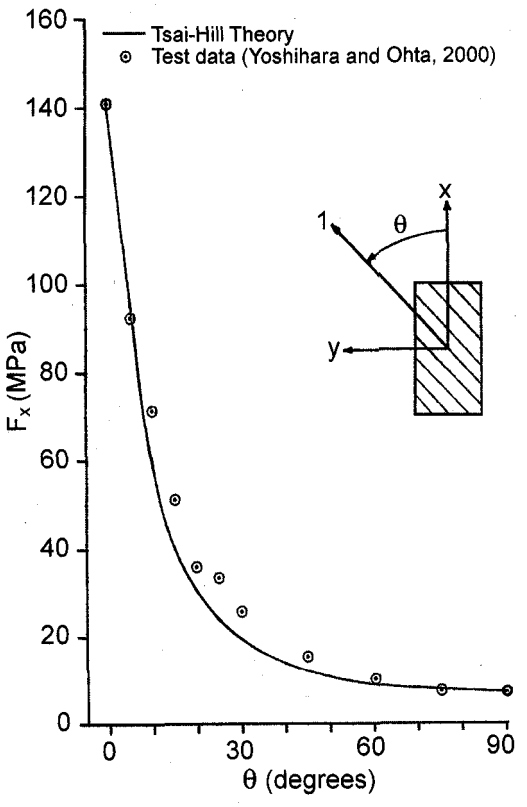

FIG. 7. Failure curve based on Tsai–Hill theory for F_1 = 141 MPa, F_2 = 7.4 MPa, and F_6 = 11.3 MPa and test data for Yoshihara and Ohta (2000).

relation between s_x and t_6 still holds, the value of t_6 at that instant must have a definite relation with its value at the yield limit.

The Tsai–Hill failure criterion in Eq. (13) is based on assumptions of homogeneity (Perkins, Jr. 1972) and linear stress–strain behavior to failure as are almost all other macromechanical failure theories. It shows that for the prediction of failure load F_x corresponding to any specified value of q, only the values of F_1, F_2, and F_6 are needed. Yoshihara and Ohta (2000) recently published these values for Sitka spruce as F_1 = 141 MPa, F_2 = 7.4 MPa, and F_6 = 11.3 MPa. Using these values in Eq. (13), the failure curve obtained is shown in Fig. 7 together with the test data of F_x at several q values. Within the range 110° < q < 75°, the test data are significantly above the predicted values. With the F_x values from the

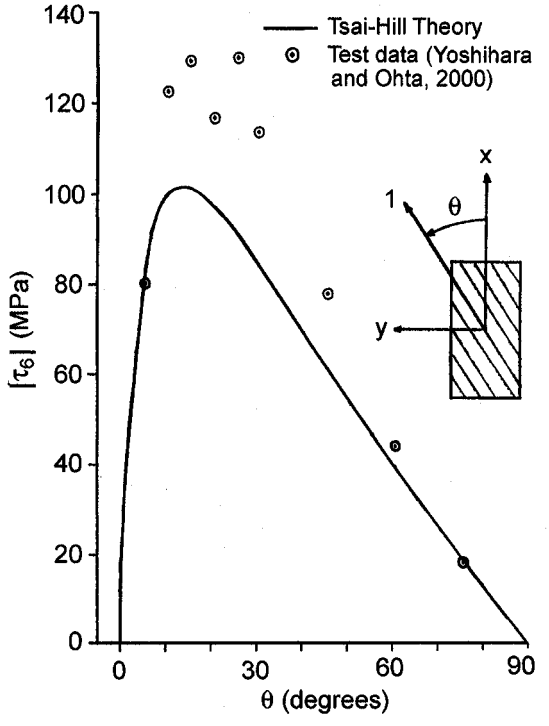

FIG. 8. Variation of $|t_6|$ with q at occurrence of F_x.

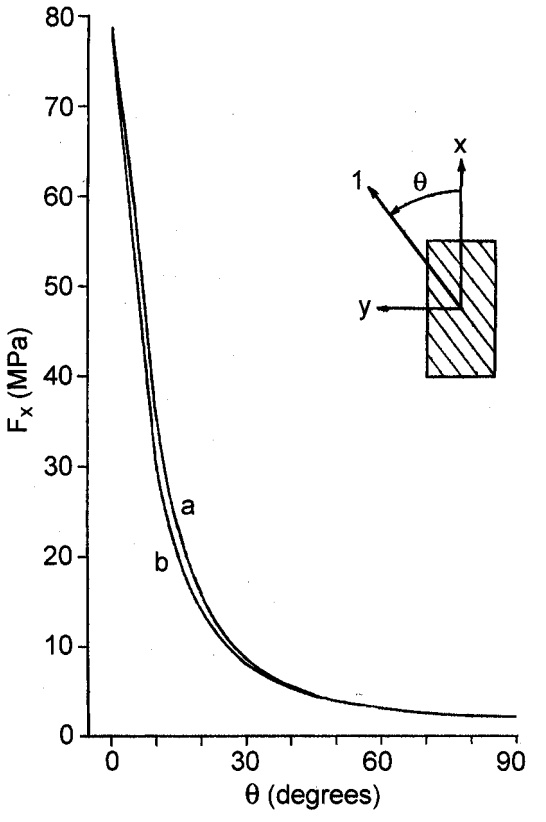

FIG. 9. Failure curve based on Tsai–Hill theory for two sets of F_x values: (a) F_1 = 78.3 MPa, F_2 = 2.55 MPa, and F_6 = 7.93 MPa (FPL 1999). (b) F_1 = 78.3 MPa, F_2 = 2.55 MPa, and F_6 = 6.25 MPa (Liu and Floeter 1984).

Tsai–Hill theory and the test data in Fig. 7 representing the corresponding value of s_x in Eq. (1c), the shear stresses $|t_6|$ at failure are plotted in Fig. 8. (Note: In the Tsai–Hill theory, the sign for t_6 makes no difference. We therefore use its absolute value.) At q = 10°, $|t_6|$ is only about 1.2% less than its maximum value, which occurs at q » 13°, based on the Tsai–Hill theory. However, the test data are more prominently above the theoretical predictions at 10° < q < 45°. It is against any theory for $|t_6|$ to be larger than F_6 in an off-axis tension test. When that happens, the test facilities, specimen conditions, sample size, and other variables may all need to be reexamined. However, if we use the predicted value of F_x = 58.57 MPa at q = 10° and substitute it into Eq. (1c) for s_x, we obtain $|t_6|$ = 10 MPa, which is about 13% less than F_6 = 11.3 MPa, and seems to be reasonable. So, the important question is how reliable are the F_1, F_2, and F_6 values that were used in Eq. (13).

In the *Wood Handbook* (FPL 1999), F_1, F_2, and F_6 values for Sitka spruce are 78.3, 2.55, and 7.93 MPa, respectively, which are considerably lower than the values found by Yoshihara and Ohta (2000).[2] Using these values in Eq. (13), we obtain F_x = 36.43 MPa at q = 10° (Fig. 9). The corresponding value of $|t_6|$ from Eq. (1c) is 6.23 MPa, which is about 27% less than F_6 = 7.93 MPa.

Liu and Floeter (1984) reported that F_6 = 6.25 MPa for Sitka spruce. Using F_1 = 78.3 MPa, F_2 = 2.55 MPa, and F_6 = 6.25 MPa, and following the same steps as described in the preceding text, we obtain F_x = 31.05 MPa at q = 10° and $|t_6|$ = 5.31 MPa, which is about

[2]The *Wood Handbook* F_1 value is adjusted based on data from dry specimens.

18% less than F_6 = 6.25 MPa. Note the close proximity of the two failure curves in Fig. 9.

We are now ready to discuss the procedures to determine F_6 from the 10° off-axis tension test. Assuming the set of data F_1 = 141 MPa, F_2 = 7.4 MPa, and F_6 = 11.3 MPa is valid, we have found that $F_x(q = 10°)$ = 31.05 MPa and $|t_6|$ = 5.31 MPa, which is about 18% less than F_6 = 6.25 MPa. Since 13% and 18% are close to one another, we select their mean, which is about 15%. We therefore obtain

$$F_6 \approx F_x(\theta = 10°) \times \sin 10° \cos 10° \times 1.15$$
$$= F_x(\theta = 10°) \times 0.2 \quad (14)$$

Also, from these two sets of data, we find $F_x(q = 10°) \approx 0.4 \times F_1$. We can replace Eq. (14) by

$$F_6 = 0.08 \times F_1 \quad (15)$$

Our discussion is based on the Tsai–Hill theory and limited test data for Sitka spruce from different sources. Although there are considerable discrepancies between the two sets of test data, the formulas seem to be valid for both. The discrepancies may need to be reconciled, but the procedures presented are believed to be useful in applying the off-axis tension test for the characterization of wood shear properties.

CONCLUSIONS

1. In an off-axis tension test of clear wood specimens, at a 10° grain angle, the shear stress in the principal material plane is due mainly to the shear coupling coefficient.
2. In applying the Tsai–Hill theory for tensile strength prediction of wood at any grain angle, the strength data in the principal material axes and plane must be reliable to produce accurate results.
3. The 10° off-axis tension test can be used to predict clear wood shear strength in a principal material plane of wood.
4. A formula has been derived for shear strength prediction of Sitka spruce based on input data in the literature.

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