STATISTICAL CHARACTERIZATION OF NON-LINEAR ELASTIC PROPERTIES

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When a material is known lo exhibit non-linear elastic behavior, three methods have historically been used in wood engineering lo characterize its properties. Method 1 defines an initial slope, 1, and a maximum value, 2 (fig. 1). Method 2 determines a series of linear segments, as shown with 1 and 3. Method 3 uses a non-linear equation that models the entire curve.

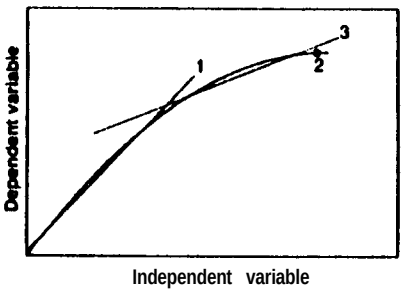

Figure 1 Representation of non-linear elastic behavior

When considering material variability, statistical methods are applied to these models by filling frequency distributions (Method 1). or determining an average piecewise-linear model (Method 2). Statistical models of Method 3 are not common practice. This paper discusses statistical procedures for models using Method 3.

The statistical non-linear modeling example used in this paper is the polynomial regression model represented by the following [1]:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \ldots + \beta_p x^p + e \qquad (1)$$

where y is the dependent variable. x is the independent variable, β_i are the model parameters, p is the order, and e is the error term. In matrix form this expression is:

$$Y = X\beta + E \qquad (2)$$

From this basic equation, the parameters (β) and their variance-covariance matrix ($Var(\beta_i)$) can be determined using (3) and (4), respectively.

$$\hat{\beta} = \left(X^T X\right)^{-1} X^T Y \qquad (3)$$

$$Var\hat{b} = \sigma^2\left(X^T X\right)^{-1} \tag{4}$$

where σ^2 is determined from the sum of squares of the residuals.

Given the properties in (3) and (4), multivariate Normal vectors of equation parameters (b_l) can be simulated using Monte Carlo simulation. If the b_l exhibit non-Normal frequency distributions, the multivariate-Normal vectors can be transformed.

Compression parallel-to-grain specimens were tested and the complete axial load-versus-deformation curves were recorded (fig. 2a). In this example, b_l and $Varb_l$ for a quadratic model were estimated, and Normality was assumed for distribution of each b_l. Monte Carlo simulation procedures were used lo generate vectors of b_l, resulting in the simulated data of fig. 2b.

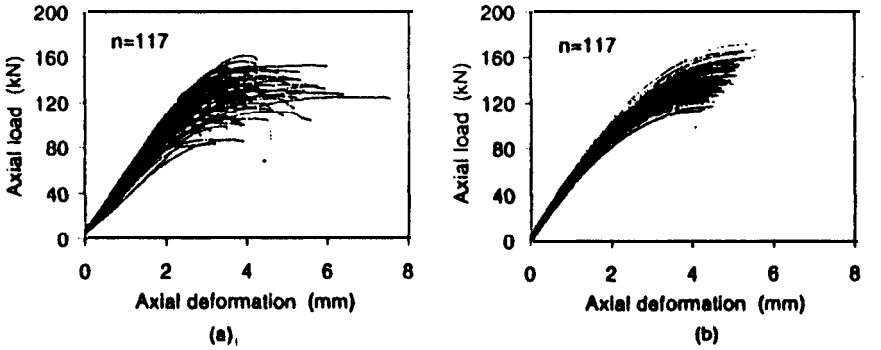

Figure 2 Plots of (a) Actual and (b) Simulated load-versus-deformation data

Thus, it was shown that statistical models could be applied lo characterize non-linear properties, as well as material variability. In this example, a 2nd-order polynomial equation was used. It is recommended that the researcher should determine the model most appropriate for their data before applying this statistical procedure.

[1] Milton, J.S., Arnold, J.C., *Probability and Statistics In the Engineering and Computer Sciences,* McGraw-Hill Book Company, New York, NY, 1986.

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ISBN 2-88074-387-7

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CH - 1015 Lausanne

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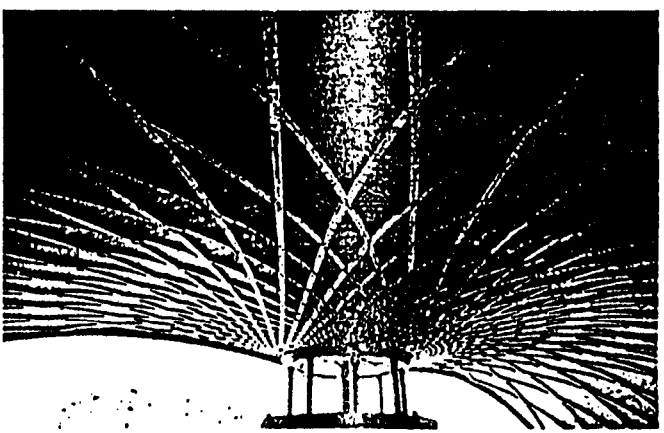

PROCEEDINGS

Volume 2

Edited by J. Natterer and J.-L. Sandoz

Presses polytechniques et universitaires romandes