17 September 2003
Ref. #: ANM-112N-03-11

CALCULATION OF BEND RADII FROM TENSILE ELONGATION DATA

1.0 INTRODUCTION

For decades, design and manufacturing organizations have been, and still are, using "cookbook" tables prescribing the recommended minimum bend radii for various metals and alloys. The tables typically list the bend radii corresponding to discrete thickness / diameter values that represent the commercially available standard mill products. There is no standardized set of tables that is embraced industry-wide and none of the available tables explain the basis on which they were obtained. As expected, then, the prescribed radii do vary from one table to the other. There is also a scarcity of data for thick and nonstandard-gage mill products, for mill products other than sheet and plate and for heat-treated alloys, with the possible exception of some aluminum alloys in certain tempers. Due to the limitations just cited, it is of interest to derive the necessary equations required to predict bend radii from the readily available % elongation data. [1] This would enable the prediction of bend radii for any situation that might be encountered in design or on the shop floor.

2.0 ANALYSIS

The calculation described here employs the bending configuration shown in Figure 1. In this configuration, the outermost fibers of the bent object are in tension, whereas the innermost ones are in compression. Since in most metallic materials, tensile strains control whether or not failure would take place, the calculation is for the outermost fibers. To preclude fracture, the strain that develops in the outermost fibers, as a result of bending, should not exceed the strain corresponding to the % elongation obtained in tensile testing of specimens parallel to the bend direction and representative of the product form and heat treatment of the object being bent. If required, the same type of calculation may be performed at the innermost fibers and, when a strain gradient is desired, also at various locations between the outermost and innermost fibers. Note that in Figure 1, "t" denotes the thickness or diameter of the object being bent. [2] Note also that the bending strains are geometric in nature and, as such, they will develop regardless of the bending temperature. As to what happens to these strains during or after hot bending will depend on the strain (bending) rate, temperature and on the metal / alloy in question and its heat-treatment.

<u>2.1 Strain Calculations</u>

In the equations that follow, "e" will denote the % elongation and ε the corresponding strain, where ε = e / 100; for example, a strain of 0.10 corresponds to a 10 % elongation (in the equations, e will be 10). This being so, and referring to Figure 1, we may proceed. The neutral axis is considered to undergo zero strain during bending; i.e., final length of neutral axis = its initial length = L_n. Before bending, the initial lengths of the innermost and outermost fibers, L_{ii} and L_{io}, respectively, are equal to that of the neutral axis; i.e., $L_{ii} = L_{io} = L_n$. After bending, the final lengths of the innermost and outermost fibers L_{if} and L_{of}, respectively, are represented by arc segments, where arc length = radius x θ, and θ is the bend angle. [3] Thus, $L_{of} = R\,\theta$ and $L_{if} = r\,\theta$, where R and r, respectively, are the radii of outermost and innermost fibers; r is also the bend radius. Similarly, the length of the neutral axis, L_n, may be represented by an arc segment; i.e., $L_n = r_n\,\theta$, where r_n is the radius at the neutral axis. For simplicity, the outermost and innermost fibers will be referred to as the outer and inner fibers, respectively.

If a tensile strain ε develops in the outer fiber as a result of the bend, then,

$$\varepsilon = (L_{of} - L_n) / L_n = (R - r_n)\,\theta / r_n\,\theta = \{R - [R - (t / 2)]\} / \{R - [R - (t / 2)]\}$$
$$= (t / 2) / [R - (t / 2)] = (t / 2) / [r + (t / 2)] \qquad \textbf{Equation 1}$$

We now define a bend factor F, where

$$F = r / t \qquad \textbf{Equation 2}$$

---

[1] The concept and the calculation method were introduced to the undersigned by Mr. W. D. Gaw, a fellow Space Shuttle team member at Rockwell International, during the 1980's and 1990's. The concept of relating bend radii to the % elongation is casually mentioned in the Rockwell International's design manual.
[2] The strain, which develops at the outermost fibers of a sheet or plate of thickness t, is the same as that which develops in a rod, bar or tube of diameter t, provided that the bend radius in all cases is the same.
[3] The bend angle, θ, is the angle which encloses the length of the bent object that is in full contact with the mandrel on which bending is performed. In Figure 1, the mandrel has a radius r, which is the bend radius, and the bend angle $\theta = \pi / 2$.

Equation 1 may be rewritten in terms of the bend factor F,

$e = 1 / (2F + 1)$ Equation 3

Rearranging the terms,

$F = [(1/e) - 1] / 2$ Equation 4

2.2 Usage

Equation 4 may be written in terms of e, the % elongation,

$F = [(100 / e) - 1] / 2$ Equation 5

Equation 5 may be used on a case by case basis to compute the bend factor corresponding to any given % elongation that represents the material of interest. This bend factor is then substituted in equation 2, to compute the bend radius required for the desired material thickness or diameter. To obtain conservative estimates of bend radii, minimum % elongation values should be used, preferably for products having the same thickness / diameter as the object being bend. These minimum values can be found in publications such as Mil-HDBK-5 (MMPDS Handbook), the applicable material specifications, and the Aluminum Association's Aluminum Standards and Data. Frequently, however, only typical % elongation values exist. In such cases, it is suggested that 85 % of the typical value be used. The corresponding bend factor equation then becomes,

$F = [(100 / 0.85\, e_{typical}) - 1] / 2$ Equation 6

For best ductility, bending is typically performed along the grain direction, i.e., in the longitudinal direction. In this case, longitudinal % elongation values may be used. Occasionally, however, bending is performed in other directions, e.g. the long transverse direction in sheet and plate. In such cases, the appropriate values of the % elongation should be used. A knock-down factor of say 10-15 % may be used in equations 5 and 6, to account for poor surface conditions, sheared edges (sheet) and other unforeseen effects.

2.3 Examples

Bend radius data for steel, especially for products of heavy gages and those that are heat treated, are not readily available. This being so, will consider two cases for AISI 4340 steel plate, 7/ 16 in. thick, for which no tables could be located.

<u>(a) Steel in normalized condition</u>: Only typical elongation values are available; e.g., 22 % elongation, for steel having an F_{tu} of 108 ksi in the longitudinal direction. Using equations 6 and 2, r = 0.95 in. Rounding up to the next higher number, a 1.0-in. bend radius should be used. Mil-HDBK-5 lists 16 % as the minimum transverse elongation for normalized low alloy steels. This value is smaller than the 18.7 % (0.85 X longitudinal e_{typ}) used in equation 6. One reason, of course, is that the latter is in the longitudinal direction, whereas the former is transverse. Another reason is that the handbook does not specify a particular steel or product form and, as such, the 16 % could be conservative for AISI 4340.

<u>(b) Steel heat-treated to F_{tu} 160-180 ksi</u>: Again, only typical elongation exist for the heat-treated steel; e.g., 15 % elongation, in the longitudinal direction. Using equations 6 and 2, r = 1.5 in. Mil-HDBK-5 lists 12 % as the minimum elongation for low alloy steels, heat-treated to F_{tu} 180 ksi. This value is slightly smaller than 0.85 e_{typ} (12.75), used in equation 6. Note, however, that the handbook does not specify a particular steel, product form or direction and, as such, the 12 % could be conservative for the case at hand.

2.4 Graphical Representation

As indicated in 2.2, the best use for equations 5 and 6 is the case by case hand computations; software can be designed to accomplish the task. If desired, tables may be constructed to list the bend factors corresponding to specific % elongation values. Graphical representations are only of academic value. For the sake of completeness, however, some of these representations are presented here. Figure 2 depicts the relationship between F and the minimum (equation 5) and typical (equation 6) % e values; note that F = 0 at 100 % elongation and infinity at 0 % elongation. No single relationship exists for the entire range of data shown in the figure. Power functions, however, provide an almost exact fit for either equation, in the 0-10 % elongation range, Figure 3. By contrast, only line segments can join the data points in the 30-100 % elongation range, Figure 4. Exponential functions provide a fairly good fit for 1 / F vs. % e in the 30-70 range, Figure 5. Figures 6 and 7, respectively, depict the r vs. t relationships for selected minimum and typical % elongation values. The resulting straight lines can be useful, especially in a shop setting.

3.0 OBSERVATIONS & COMMENTS

3.1 Effect of Bend Angle

In equation 1, it is seen that the bend angle (q) cancels out, indicating that it is not a determining factor in the strains that develop. In other words, bending a given object over a given radius will develop the same strains throughout the cross section of the bent length, regardless of the bend angle. More specifically, the resulting strain (% elongation) in the outer fibers of a given object, bent over some given radius, would be the same, regardless of whether the bend angle is 90^{O} or 180^{O}. This contradicts the commonly held view that larger bend radii would be required as the bend angle is increased. Most of the "cookbook" tables used in industry recommend 180^{O} bend radii that are much larger, up to twice, their counterparts for 90^{O} bends of the same alloy, heat treated to the same condition (i.e., the same % elongation). The undersigned, however, is of the opinion that increasing the bend angle only increases the length of the bent object that experiences the maximum strain (or % elongation). As such, it is felt that the data in said tables could be imposing unnecessary restrictions on design. It is unfortunate that the basis upon which these tables were constructed is not readily available.

3.2 Effect of Thickness / Diameter

Table 1: Bend radius data from two sources, Al alloy 2024-T3

Thickness, in.		1/64 (0.016)	1/32 (0.031)	1/16 (0.063)	1/8 - 3/16 (0.125-0.188)
Bend Radius, in.	Source 1	1.5t-3t	2t-4t	3t-5t	4t-6t
	Source 2	2.5t	3t	4t	5t

Cookbook bend radius tables often list bend radius, r, as a function of sheet / plate thickness, t . Specifically, the relationship takes the form r = Ft, where F is some number that may be larger or smaller than unity; note that this is equation 2. Table 1 lists data for aluminum alloy 2024-T3 from two such tables, referred to as sources 1 and 2. Source 1 lists bend radius ranges for the thicknesses indicated, without explaining what these ranges signify. It has been argued, however, that these ranges are intended to account for things like surface condition and bending in directions other than the longitudinal. To avoid this issue altogether, source 2 lists only the mid range radii. Clearly, the origin of the information in both sources is the same, most likely Alcoa. Figure 8 is an attempt to graphically represent the "midrange" data in a linear manner, similar to that shown in Figures 6 and 7; for the data to pass through the origin, at least two lines are needed. This aside, the data depicted in Table 1 shows a progressive increase in the bend factor (r / t) as t increases. This would be the case if the % elongation decreases with thickness, as is often the case. Let us now verify whether or not this is the case. Specification AMS-QQ-A-250/4 indicates that, for 2024-T3 sheet, the minimum elongation in the LT direction is 12 % for 0.010-0.020 in. thick sheet and 15 % for 0.021-0.249 in. thick sheet. [4] According to equation 5, the bend factor should be 3.6 for 1/64 in. thick sheet, and 2.83 for sheets in the 1/32-3/16 in. thickness range. Thus, the cookbook data appear to be liberal for the 1/64-in. sheet and, to varying degrees, conservative for thicker sheets. Liberal bend radii can lead to cracking, whereas conservative ones impose unnecessary restrictions on design. Similar issues exist for other aluminum alloy-temper combinations. It must be pointed out, however, that not all cookbook tables have the shortcomings just cited. Some tables, for aluminum and other metals and alloys, are more realistic and appear to be based on the approach described in this document.

Terry Khaled, Ph.D.
Chief Scientific / Technical Advisor, Metallurgy
Federal Aviation Administration

[4] *These are the S values listed in Mil-HDBK-5 (MMPDS Handbook).*

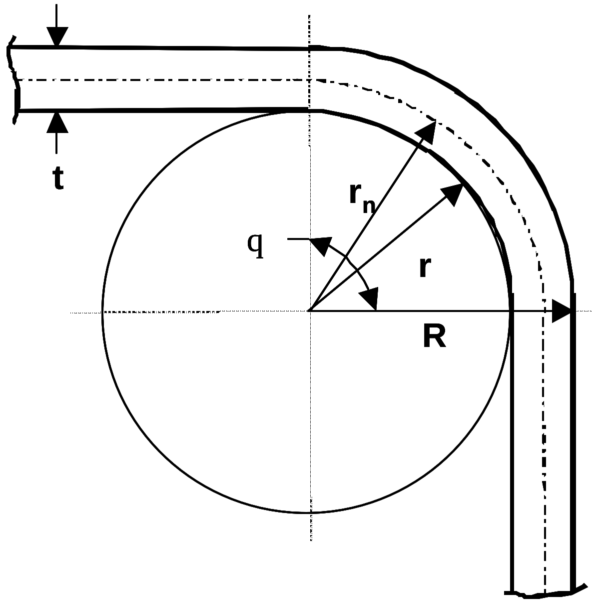

Figure 1: Bend Geometry

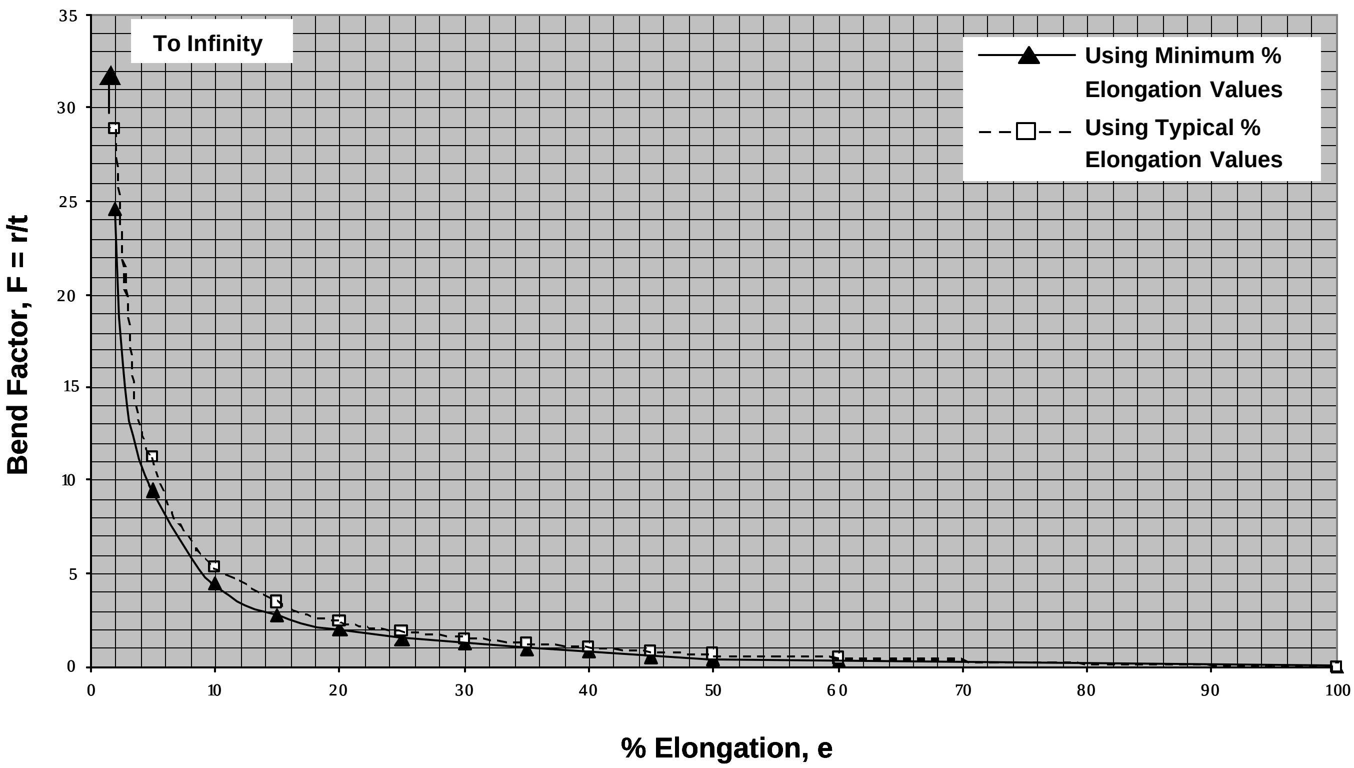

Figure 2: Bend Factor vs. % Elongation in the 2-100% Range.

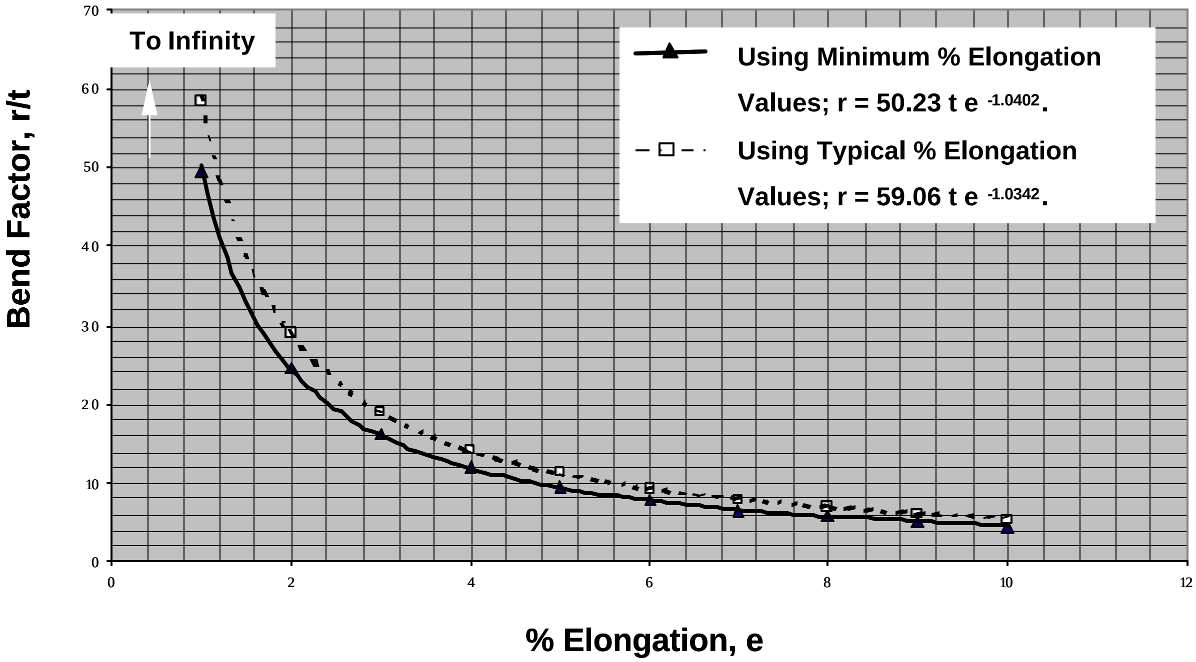

Figure 3: Bend Factor vs. % Elongation in the 1-10 % Range. Power Function Fit.

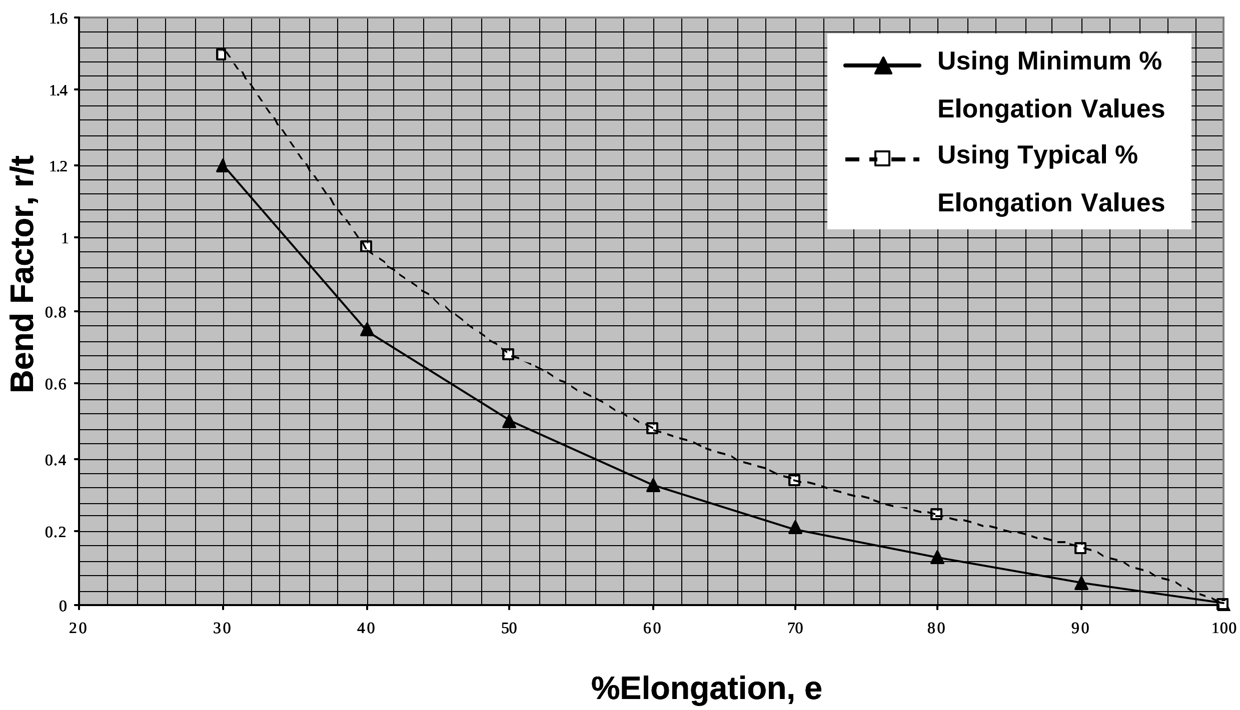

Figure 4: Bend Factor vs. % Elongation in the 30-100 % Range.

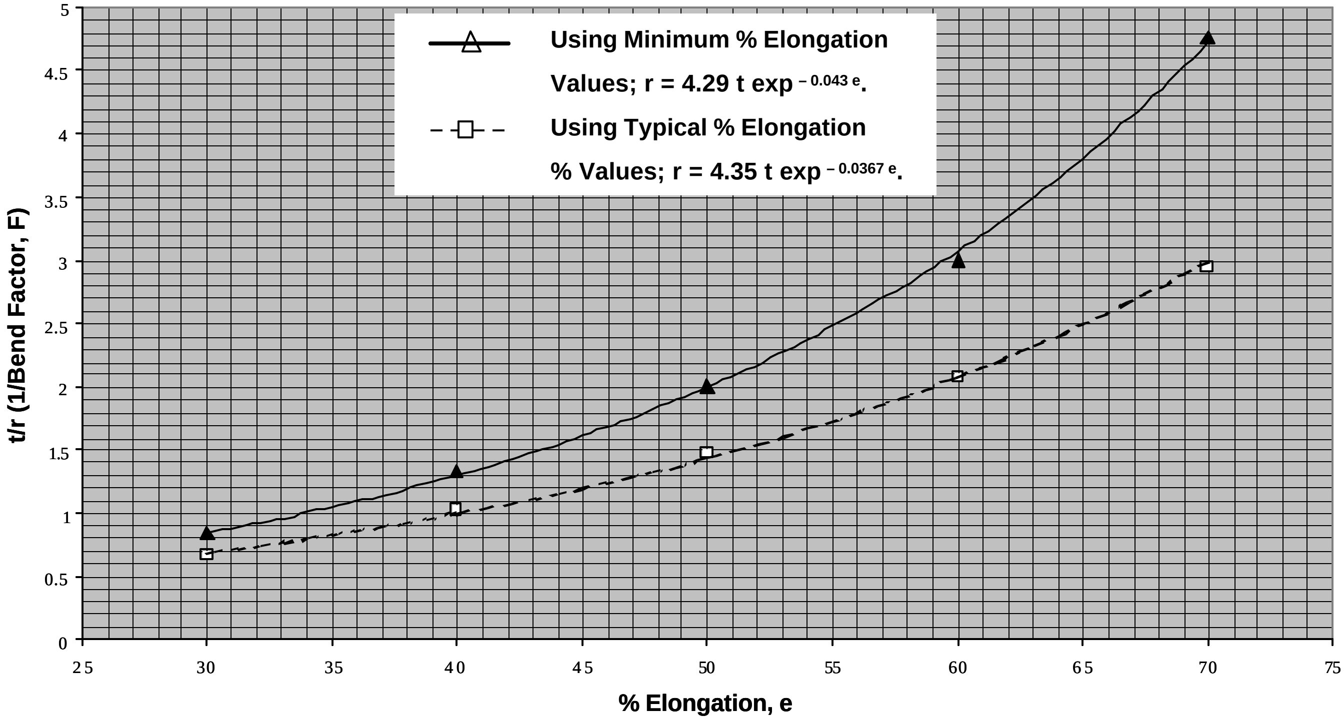

Figure 5: Inverse Bend Factor vs. % Elongation in the 30-70 % Range. Exponential Function Fit.

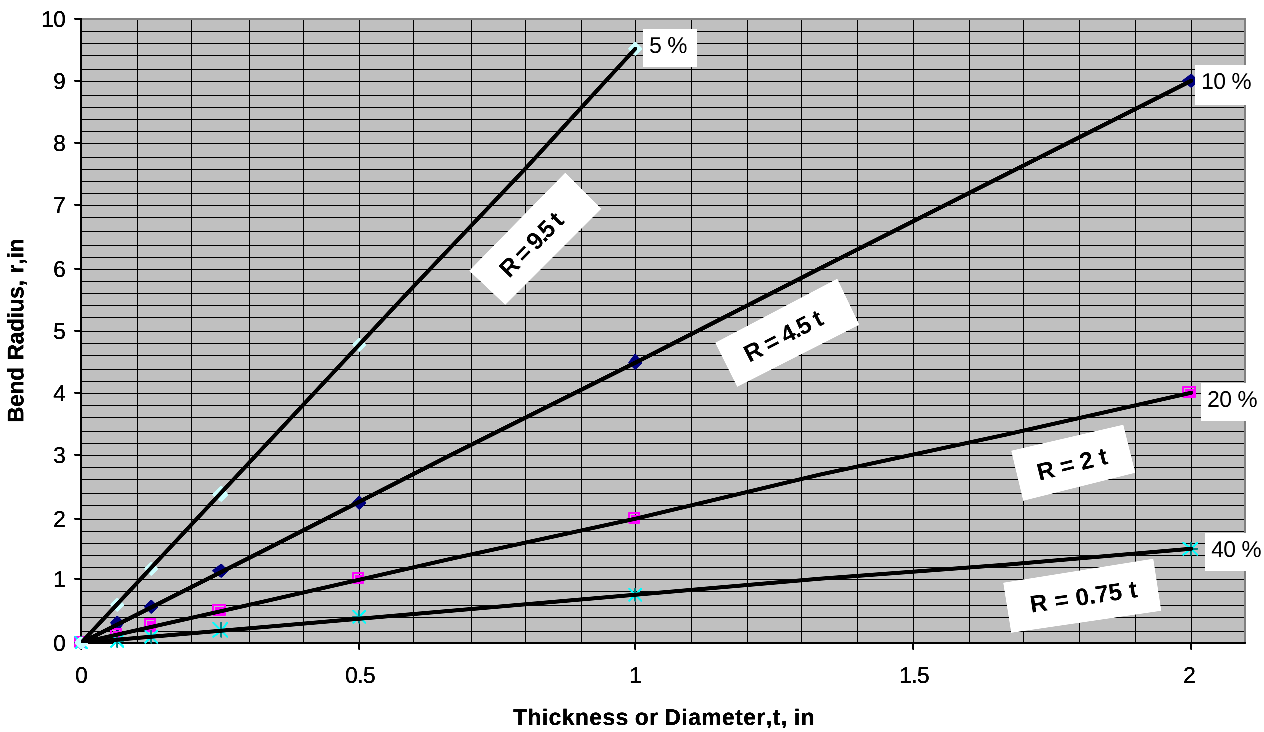

Figure 6: Linear Bend Radius vs. Thickness / Diameter Relationships For Selected Minimum % Elongation Values. Line Equations Shown.

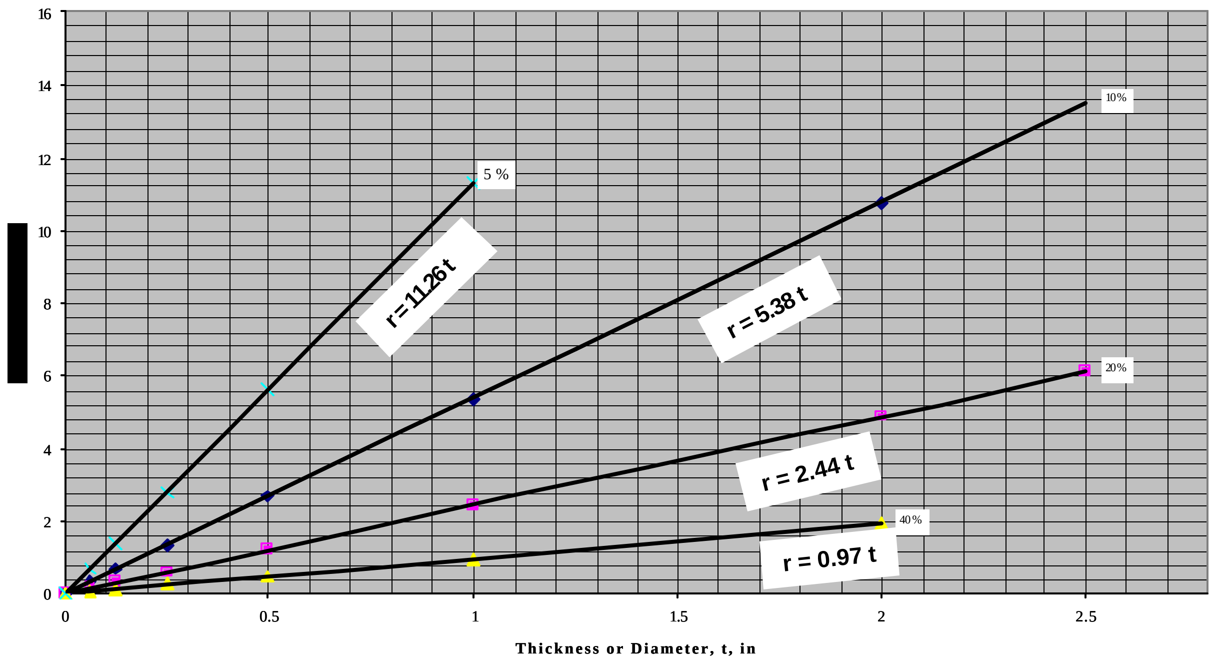

Figure 7: Linear Bend Radius vs. Thickness / Diameter Relationships For Selected Typical % Elongation Values. Line Equations Shown.

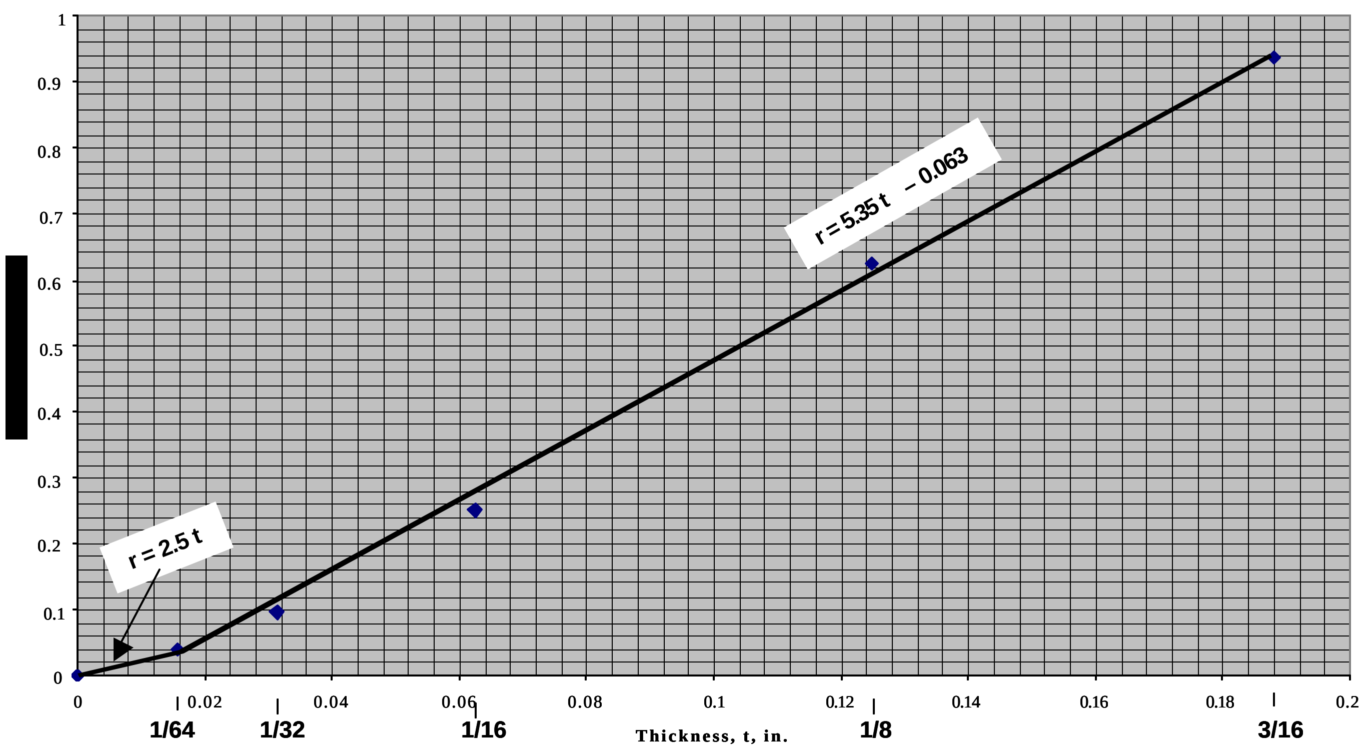

Figure 8: A Graphical Representation of Aluminum Alloy 2024-T3 Data.