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BENDING STRAIN CALCULATIONS FOR SHAPES

1.0 INTRODUCTION

Designs occasionally mandate forming of shapes, such as angles, U-channels, I-beams and the like. An earlier document, Reference 1, presented the concept of calculating bending strains from simple geometrical relationships, for standard mill products (sheet, plate, bar, rod and tube). Briefly, the neutral axis is considered to undergo zero strain during bending and the initial lengths of the inner and outer fibers (L_{ii} and L_{io}, respectively) are equal to that of the neutral axis; i.e., $L_{ii} = L_{io} = L_n$. All lengths after bending are represented as arc segments; i.e., arc length = (radius) (angle, θ), where θ is an arbitrary angle. Thus, final length of the outer fiber (L_{of}) = radius of outermost fiber (R) (θ). Similarly, final length of the inner fiber (L_{if}) = (radius of inner fiber, r) (θ). Finally, L_n = (r_n) (θ), where r_n is the radius of the neutral axis. The same type of calculation can be performed for shapes, using expressions of the neutral axes that can be found in texts such as References 2 and 3. In this document, the case of forming (bending) angles is treated.

<u>2.0 THE CASE OF THE ANGEL</u>

<u>2.1 General</u>:

In bending angles with identical legs, two orientations need to be considered; viz., those arising from placing the out of plane leg at the inside or outside of the bend (Figure 1). Furthermore, two arrangements need to be considered; viz., those arising from controlling the bend radius at the inside or outside surface. Thus, for angels with identical legs, four bending configurations are possible. For angels with non-identical legs, each leg generates four bend configurations, for a total of eight such configurations. In all configurations, the tensile and compressive strains, ε_O and ε_i, that develop as a result of bending, in the outer and inner fibers, respectively, may be represented by the following equations:

$\varepsilon_o = (L_{of} - L_n) / L_n = (R\,\theta - r_n\,\theta) / r_n\,\theta = (R - r_n) / r_n$ **Equation 1**

$\varepsilon_i = (L_{if} - L_n) / L_n = (r\,\theta - r_n\,\theta) / r_n\,\theta = (r - r_n) / r_n$ **Equation 2**

Note that the strain in equation 2 will have a negative value, indicating contraction as a result of the compressive stresses. The next task now is to find expressions for r_n. This, in turn, requires expressions for the neutral axes.

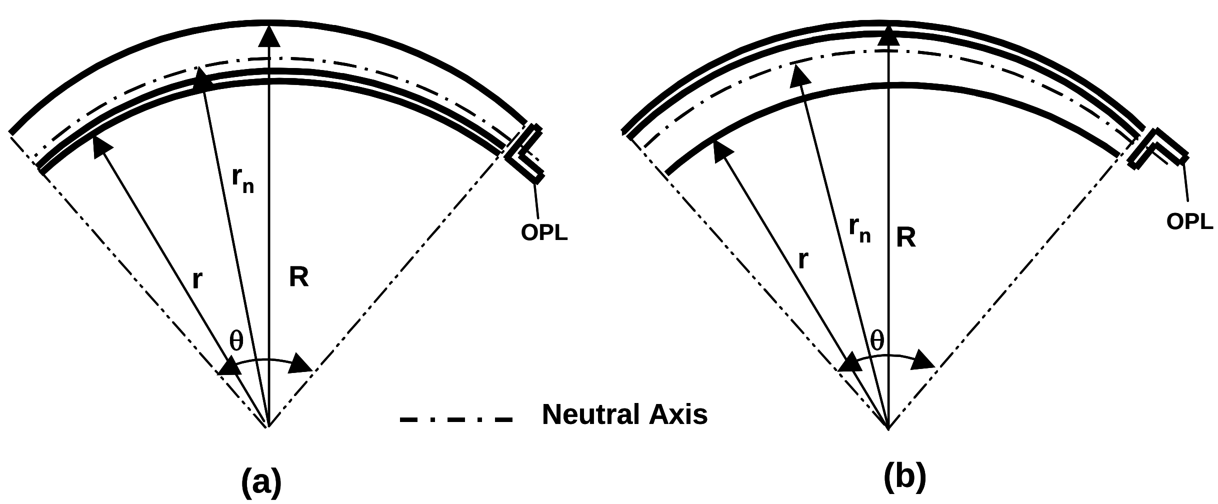


Figure 1: Orientation of Out of Plane Leg (OPL).
(a) At Inside Surface.
(b) At Outside surface.

2.2 Neutral Axes Expressions:

We refer to Figure 2, where lines x-x and y-y define the neutral axes. For the general case, where the legs have different lengths and thicknesses, the neutral axis expressions (Reference 2) are:

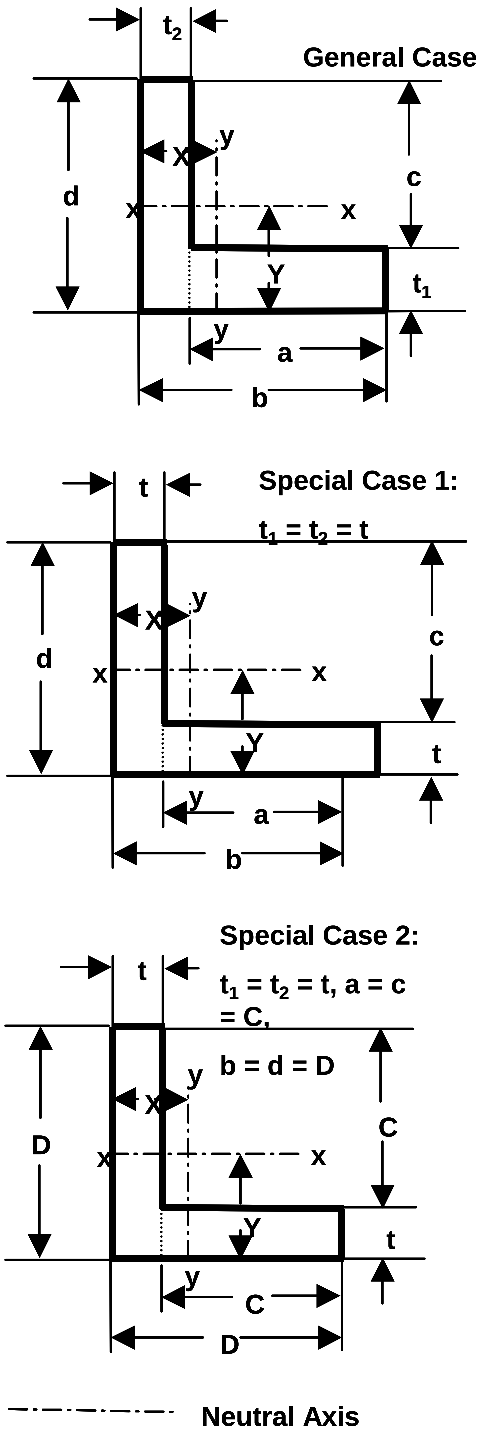


Figure 2: Geometry & Neutral Axes.

$X = (t_1 b^2 + c\, t_2^2) / 2\,(t_1 b + c\, t_2)$;

$Y = (t_2 d^2 + a\, t_1^2) / 2\,(t_2 d + a\, t_1)$ Equations 3

For the special case, where $t_1 = t_2 = t$, the neutral axis expressions (Reference 3) simplify to:

$X = (b^2 + c\,t) / 2(b + c)$;

$Y = (d^2 + at) / 2\,(b + c)$ Equations 4

The equations are further simplified for the special case where $t_1 = t_2 = t$, $a = c = C$ and $b = d = D$:

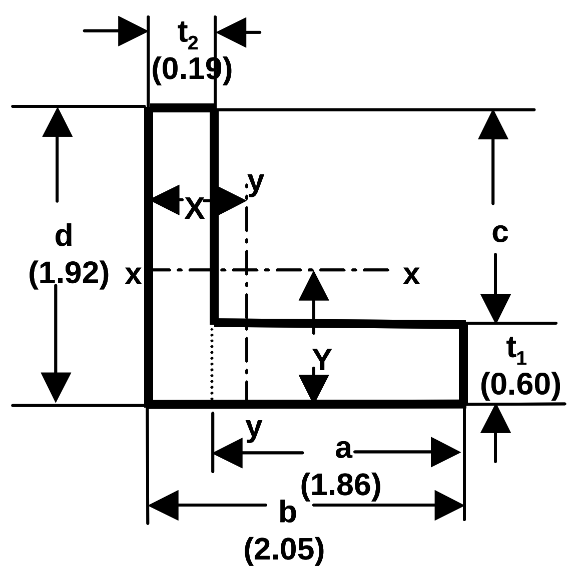
$X = Y = (D^2 + C\,t) / 2\,(D + C)$ Equations 5

2.3 Strain Calculations

We now consider a general angle (Figure 3) with $b = 2.05$, $d = 1.92$, $t_1 = 0.6$ and $t_2 = 0.19$; $a = 2.05 - 0.19 = 1.86$; $c = 1.92 - 0.60 = 1.32$. Using equations 3,

$X = (t_1 b^2 + c\, t_2^2) / 2\,(t_1 b + c\, t_2) = 0.87$

$Y = (t_2 d^2 + a\, t_1^2) / 2\,(t_2 d + a\, t_1) = 0.46$



$a = b - t_2 = 2.05 - 0.19 = 1.86$;

$c = d - t_1 = 1.92 - 0.60 = 1.32$

Figure 3: Selected Angle Section.

The eight bend configurations referred to in section 2.1 are shown in Figure 4. Using equations 1 and 2, the outer and inner fiber strains, respectively ε_o ε_i, for Figure 4 configurations may be represented by the following expressions:

Configurations A_o and A_i: $\varepsilon_o = (d - Y) / (R - d + Y)$; $\varepsilon_i = (- Y) / (r + Y)$
Configurations B_o and B_i: $\varepsilon_o = (Y) / (R - Y)$; $\varepsilon_i = (- d + Y) / (r + d - Y)$
Configurations C_o and C_i: $\varepsilon_o = (a - X) / (R - a + X)$; $\varepsilon_i = (- X) / (r + X)$
Configurations D_o and D_i: $\varepsilon_o = (X) / (R - X)$; $\varepsilon_i = (- a + X) / (r + a - X)$

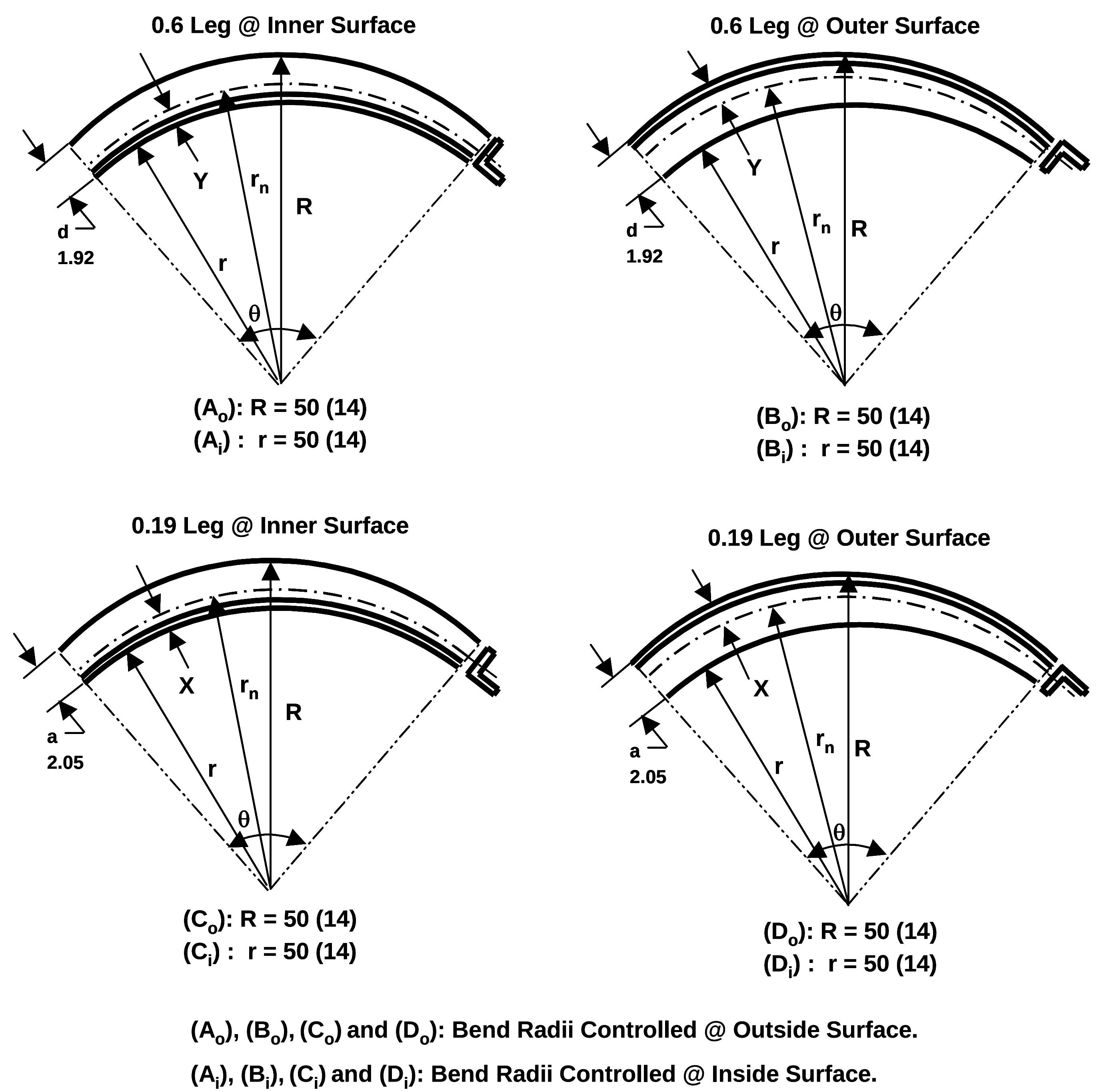


Figure 4: The Eight Configurations of Bending.

Two bend radii (50 and 14 in.) will be considered. When the bend radius is controlled at the outside surface, R = 50 (or 14). In this arrangement, r = R – d (configurations A_o and B_o) or R – a (configurations C_o and D_o). When the bend radius is controlled at the inside surface, r = 50 (or 14). Here, R = r + d (configurations A_I and B_i) or r + a

(configurations C_I and D_i). The calculated strains are multiplied by 100 to arrive at the corresponding % elongation values. A summary of the results is presented in Table 1.

Table 1: FIBER STRESSES

Bend Radius, in.	% Elongation [1]								
		Configuration (Figure 4)							
	T/C [2]	A_o	A_i	B_o	B_i	C_o	C_i	D_o	D_i
50	T	+ 3.0	+ 2.9	+ 0.9	+ 0.89	+ 2.4	+ 2.3	+ 1.8	+ 1.7
	C	- 0.94	- 0.93	- 2.95	- 2.8	- 1.8	- 1.7	- 2.4	- 2.3
14	T	+ 11.6	+ 10.0	+ 3.4	+ 3.0	+ 9.2	+ 7.9	+ 6.6	+ 5.7
	C	- 3.7	- 3.4	- 10.8	- 9.4	- 6.8	- 5.9	- 9.0	- 7.8

(1) Subscripts "o" nd "i," respectively, designate bend radii controlled at the outside and inside surfaces.
(2) "T" and "C," respectively, designate the tensile (outer) and compressive (inner) fibers.

2.4 Discussion

In examining the results presented in Table 1, some trends become evident. First, it is seen that controlling bend radii at the inner surface (configurations A_i, B_i, C_i and D_i) is accompanied by somewhat smaller % elongation (or contraction) values than their counterparts when the radii are controlled at the outer surface (configurations A_o, B_o, C_o and D_o). Second, placing the out of plane leg at the inner surface (configurations A_o, A_i, C_o and C_i) develops outer fiber % elongation values that are larger, and inner fiber % contraction values that are smaller, than their counterparts when that leg is at the outer surface (configurations B_o, B_i, D_o and D_i). Third, configurations A_o and A_i develop the highest values of outer fiber % elongation and the lowest inner fiber % contraction among all configurations. By contrast, configurations B_o and B_i develop the lowest outer fiber % elongation and the highest inner fiber % contraction values.

3.0 FORMING & ALLOWABLE STRAINS

If the maximum strain resulting from bending is less than the allowable strain, then the part will likely survive the forming operation without cracking. Noting that compressive strains do not initiate cracking, the focus must be on the outer fiber tensile strains (i.e., the outer fiber % elongation values). Assume that the angle in question is 2014-T6 aluminum alloy, and that forming will be performed at ambient temperature, per configuration A_o; this configuration gives rise to the highest tensile elongation values (see Table 1 and Figure 4). Further assume that the subject angle is an extrusion and, as such, forming would be performed with the grains, i.e., in the longitudinal direction (Figure 5); a similar situation would exist in those angles machined from bar stock. This being so, Mil-Handbook-5 (MMPDS-01) lists a minimum tensile elongation of 7% (A-value) in the longitudinal direction for extrusions in the thickness range of concern. [1]

<u>3.1 The 50 in. Bend Radius</u>

Using a 50 in. bend radius, the resulting outer fiber (tensile) elongation (Table 1) would be 3.0%. This is considerably lower than the allowable value (7%) and, as such, the part is likely to survive forming without cracking.

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[1] *In principal, individual lots can display properties that are far in excess of the allowable values listed here.*

3.2 The 14 in. Bend Radius

If a 14 in. bend radius were to be used, then the corresponding outer fiber elongation (Table 1) would be 11.6%. This is far in excess of the allowable value of 7%, indicating that the part is likely to crack during forming. To enhance the chances of survival, the following options need to be considered.

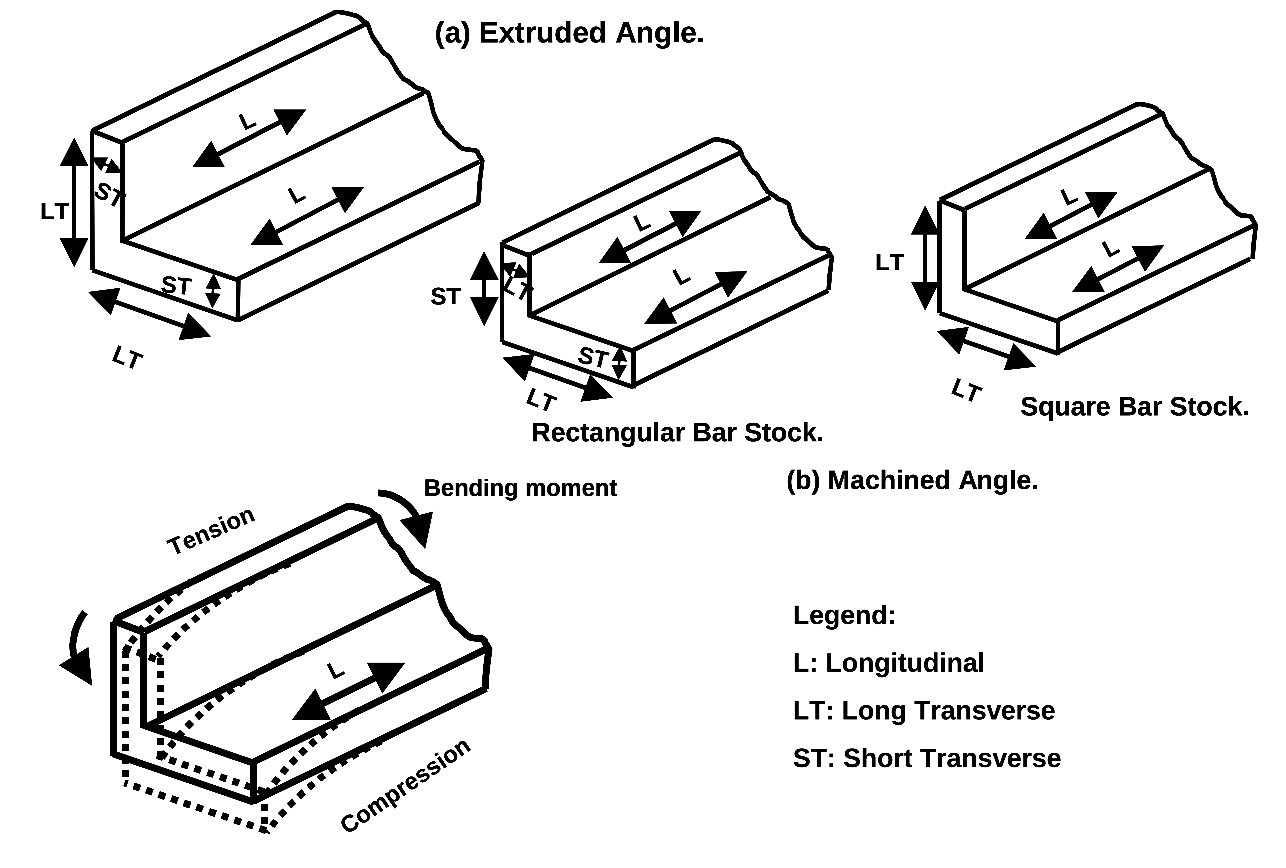

Figure 5 : Longitudinal Bending (Bending With Grains).

3.2.1 Forming in the - W Temper: The - W temper is obtained by solution treatment and quenching and it displays the best formability among all 2014 tempers. According to the Aerospace Structural Metals Handbook, the alloy remains in that temper for about 20 minutes at ambient temperature, before it gradually ages towards the -T4 temper. Data from ASM Handbook, vol. 4 (10 th Edition) indicate that the - W temper has a typical elongation of about 25%, which is sufficient to execute the required forming at ambient temperature without cracking. The part may be aged to the -T6 temper after forming. To adopt this option, the user would purchase the required angle stock in any available temper and cut to the desired lengths. This would be followed by solution treating, quenching and forming. Each angle must be subjected to this sequence individually, since forming must be accomplished within 20 minutes after quenching; the limiting factor here would be the availability of forming equipment. The angles may then be aged together to the -T6 temper. The problem with this scenario is the distortion that inevitably accompanies quenching and which may not be completely amenable to correction by forming. Ideally, forming would be performed at the mill, during stock production. Mills, however, do not usually engage in such custom processing.

3.2.2 Forming in the -T4 Temper: The -T4 temper is attained by allowing the alloy to age at ambient temperature for a minimum of 96 hrs, after quenching. AMS-QQ-200/2 specification indicates a minimum elongation of 12% for that temper. It may be possible, therefore, to form the part in the -T4 without cracking. Following forming, the part may be aged to the -T6 temper.

3.2.3 Hot Forming in the -T6 Temper: The information listed in the Aluminum Data and Standards (the Aluminum Association) indicate that typical elongation values for 2014-T6 increase from 13% at ambient temperature to 20% at 300 F to 38% at 400 F. Thus, forming in the 300-400 F range could accommodate the resulting strains [2] without cracking. According to the Aerospace Structural Metals Handbook, however, the part should not remain at temperature any longer than 10 minutes at 400, 1.5 hrs at 350 F and 10 hrs at 300 F, otherwise the –T6 properties would begin to degrade.

4.0 PRECAUTIONS

4.1 In the calculations presented herein, it is assumed that forming is with the grains, i.e., in the longitudinal direction. There are cases, however, where forming must necessarily be performed transverse to the grains, i.e., in the long transverse direction, e.g., Figure 6 b. In such cases, the appropriate values of the allowables need to

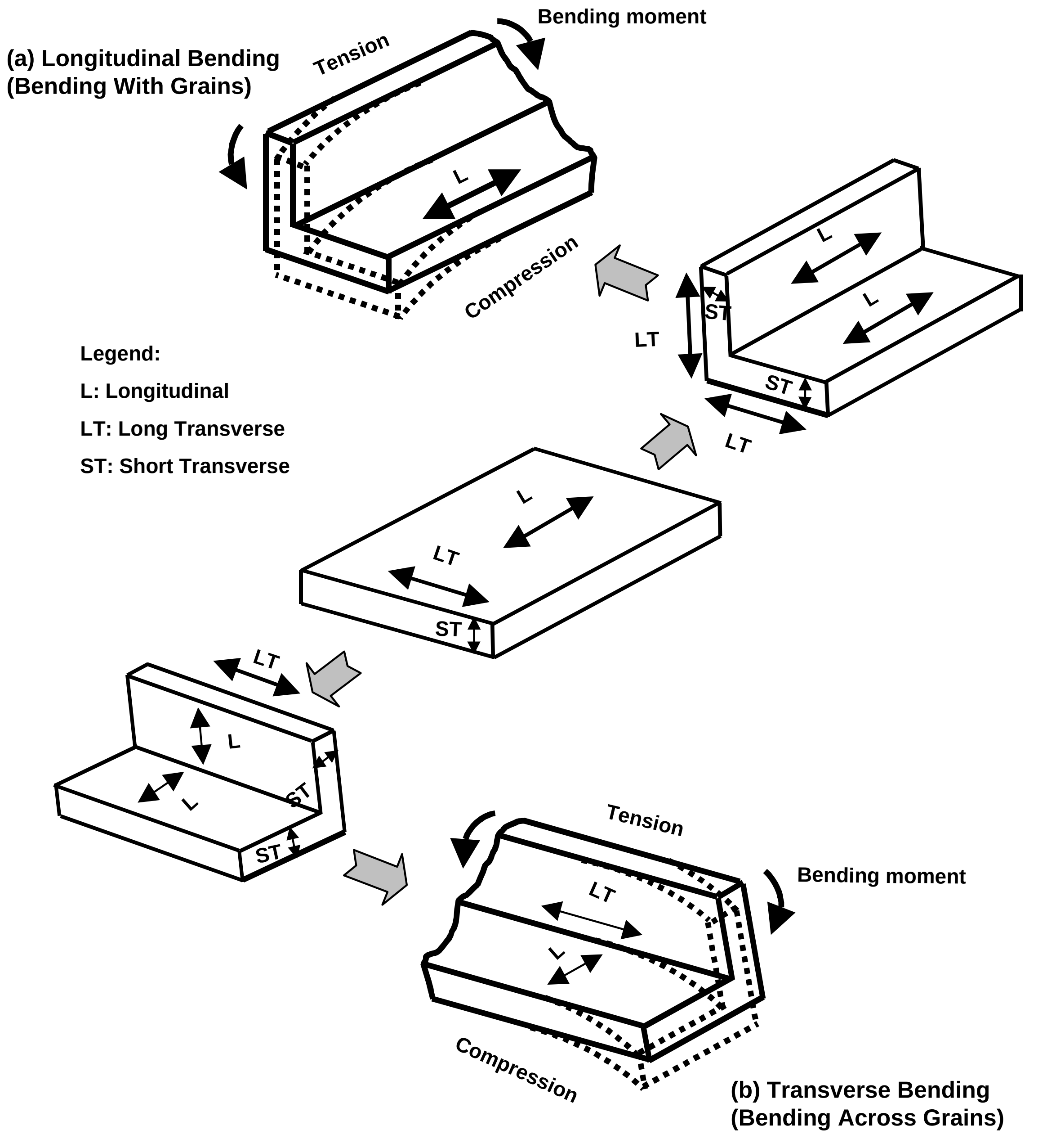

Figure 6: Longitudinal (a) and Transverse (b) Bending in Angle Made From Bent Plate.

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[2] *The assumption here is that forming is accomplished fairly fast, so as to develop the strains and % elongation values of section 2.3 and Table 1. Slow forming and the attendant low strain rates can lead to stress relief and creep. The net effect is to reduce the developing strains and enable forming around tighter bend radii, without cracking.*

be used. In all cases, Mil-HDBK-5 data and the minimum values listed in material specifications would be the most conservative. Typical data may also be used, if realistic knockdown factors are invoked.

4.2 It is important to note that all forming operations should be followed by nondestructive (NDI) inspection for cracking. Penetrant inspection is used for aluminum and other nonferrous alloys as well as for austenitic stainless steels. Magnetic particle inspection is used for carbon and alloy steels. It is wise to review the drawings and travelers to ensure that such inspection is required and that it is carried out in actual production.

4.3 Allowance should be made to compensate for springback. This is accomplished by overbending. Springback generally increases directly with the yield strength (3) and bend radius but inversely with stock thickness. Hot forming generally involves less springback than ambient temperature forming.

Terry Khaled, Ph.D.
Chief Scientific / Technical Advisor, Metallurgy
Federal Aviation Administration

References

1- T. Khaled, Report # ANM-112N-03-11, dated 17 September 2003.
2- Marks' Standard Handbook for Mechanical Engineers, 7th Edition, 1967, p 5-38.
3- E. F. Bruhn, " Analysis & Design of Flight Vehicle Structures," Tri-State Offset Company Publication, 1965, p A3.2.

[3] *That is to say that the - W, - O, -T4 and - T6 tempers, in that order, will involve progressively larger springback.*