A Broad-Ranging Formulation of Lateral Mixing

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Abstract

A specification of lateral viscosity is developed, involving the combined application of biharmonic and Laplacian forms. Related to that of Chassignet and Garraffo [2001], our prescription is intended to be readily applicable across a wide range in model resolution.

Here, we apply this new approach in 0.1 degree simulations of Los Alamos' POP ocean model. A regional North Atlantic simulation is considered first. Based on experience gained with the regional model, we then apply our formulation to a fully global model.

Developed as a viscous parameterization, as presented here, the prescription may also offer a useful method for the scaling of lateral tracer mixing coefficients as a function of grid resolution.

Introduction

More of the mixing processes that occur in the ocean are explicitly included in strongly eddying models, yet the parameterization of mixing remains of foremost concern. In these models, biharmonic forms of lateral viscosity are most often used in order to avoid suppression of mesoscale variability. Along with higher levels of mesoscale variability, some of the longstanding deficiencies in mean circulation may be greatly improved, including the path and description of the Gulf Stream, North Atlantic and Kuroshio Currents, the Agulhas retroflection and the Antarctic Circumpolar Current.

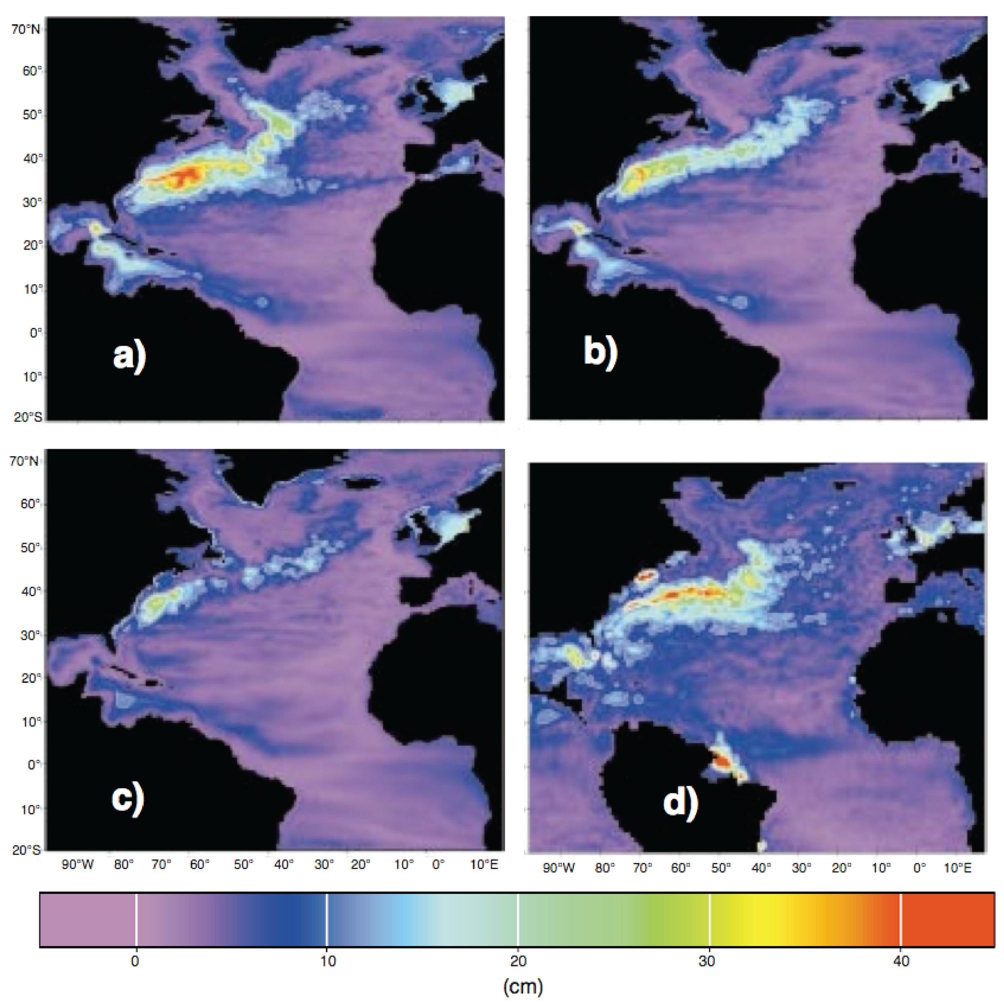

Figure 1: Sea surface height variability of North Atlantic models at resolutions of (a) 0.1, (b) 0.2, (c), 0.4 degrees, and (d) for a blended ERS-TOPEX/POSEIDON altimetric product (Le Traon et al. [1998]). Figure from Bryan and Smith [1998], reprinted in Hecht and Smith [2008].

The models, however, remain highly sensitive to the details of the closure schemes, and in particular to the form and strength of lateral dissipation (Chassignet and Garraffo [2001], hereafter CG01; Bryan et al. [2007], hereafter BHS07).

A number of simulations have used a value of biharmonic viscosity similar to that of Smith et al.[2000]. In that simulation, the coefficient of biharmonic friction was scaled with grid cell area as

$$\nu_4 = \nu_4(A_0)(A/A_0)^{3/2} \qquad (1)$$

in order to maintain a constant grid Reynolds number across the grid, and hence uniform capacity to suppress noise generated by the advective operator. Here, $\nu_4(A_0)$ refers to a reference value of biharmonic viscosity in a grid cell of area A_0.

Beyond noise control

The sensitivity of even strongly eddying models to the specification of lateral mixing suggests that the role of the mixing scheme is not limited to noise control.

In 1/12 degree simulations, CG01 found excessive variability with use of biharmonic viscosity, and suppression of variability with use of Laplacian viscosity. They achieved better results with combined use of both the Laplacian and biharmonic operators. The analysis which guided their choice of viscous coefficients was based on the analysis of damping times for monochromatic waves. For Laplacian and biharmonic dissipation, these damping times are

$$\tau_2 = \nu_2^{-1}\left(\frac{2}{\Delta x}\sin\left(\frac{k\Delta x}{2}\right)\right)^{-2} \qquad (2)$$

$$\tau_4 = \nu_4^{-1}\left(\frac{2}{\Delta x}\sin\left(\frac{k\Delta x}{2}\right)\right)^{-4} \qquad (3)$$

where ν_2 and ν_4 are Laplacian and biharmonic viscous coefficients, respectively, k is the wave number and Δx is the grid spacing. Using a small wave number approximation and setting the two damping times to be equal, we can solve for a crossover point, where the damping times of both operators are equal, as

$$\lambda_c = 2\pi\sqrt{\frac{\nu_4}{\nu_2}} \qquad (4)$$

For higher wave numbers, ν_4 provides the more rapid damping; for lower wave numbers ν_2 is dominant. A choice of crossover length scale of around 80 km at middle latitudes was made in CG01.

A specification which scales across resolutions

Whereas CG01 scaled both ν_4 and ν_2 for constant grid Reynolds number, resulting in the $A^{3/2}$ scaling of equation 1 and a slower $A^{1/2}$ scaling for ν_2, it is sufficient for one of the two terms to provide noise control. Accordingly, we reconsider the scaling of the Laplacian term. We note that the level of Laplacian viscosity in CG01 was sufficient to span the width of the viscous Munk layer of the western boundary

$$\delta_M = \left(\frac{\pi}{\sqrt{3}}\right)\left(\frac{\nu_2}{\beta}\right)^{1/3} \qquad (5)$$

with three grid lengths at Cape Hatteras. Indeed, one would expect this to be the case, or very nearly so, as this is a second requirement of the model viscosity, more physically-based, in addition to the purely numerical requirement of noise control.

We propose a more general prescription for the use of Laplacian and biharmonic viscosity together, consisting of:

-- Laplacian viscosity scaled as Δx^3, in order to span the width of the Munk layer with a fixed number of grid lengths (of order two or three) at midlatitudes, independent of overall model resolution, and

-- biharmonic viscosity scaled as $A^{3/2}$, for noise control, as in Smith et al. [2000] and CG01.

If the grid is one that maintains uniform aspect ratio, with $\Delta x = \Delta y$, then the scaling of the Laplacian coefficient is identical to that of the biharmonic coefficient.

If the approach of CG01 were applied to a grid of significantly coarser resolution, the prescription would result in a crossover length that grows as grid resolution is coarsened, and so is inconsistent with the widespread practice of using Laplacian viscosity at low resolution and biharmonic forms at high resolution. In contrast, the prescription we give above produces a crossover length which is independent of grid resolution, not just within some limited mid-latitude range on a particular grid, but from the strongly eddying regime shown here down to the coarse resolution (two, three or four degree) models used for long time-line paleoclimate study.

Evaluation within a regional model

Working within a 1/10 degree regional North Atlantic version of Los Alamos' POP model, we present results based on use of a crossover length scale (equation 4) of around 87 km with 2.5 grid lengths across the viscous Munk layer (equation 5}) at a latitude of 35 degrees N (see case 14x in Table 1).

If our crossover length scale (or that of CG01) is interpreted in terms of grid length, one sees that it falls toward the coarse end of what would be referred to as "eddy-permitting". Our prescription results in an active, primary role for the biharmonic viscosity operator in strongly eddying applications, and in a secondary role in non-eddying applications.

First, we compare domain-averaged kinetic energy with that of 3 cases from BHS07. The coefficient of biharmonic viscosity in our experimental case 14x is equal to that of their least viscous case (14c). Our domain-averaged kinetic energy is seen to drop to around the same level as that of case 14b of BHS07, where the fourth order dissipative coefficients were twice as large (see Figure 2).

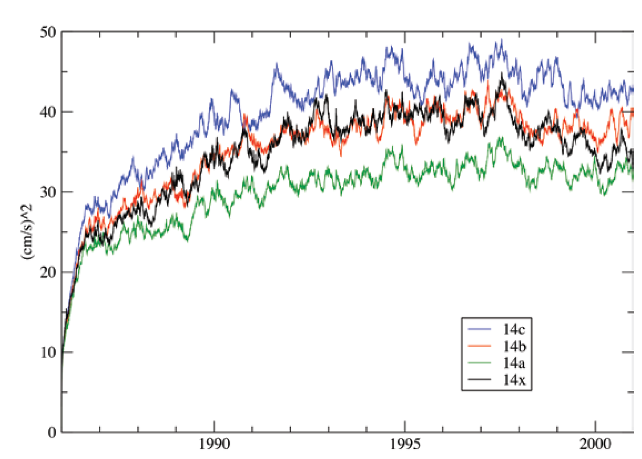

Figure 2: Mean kinetic energy, with the most viscous biharmonic case (14a) having the lowest kinetic energy, the least viscous case (14c) having the highest kinetic energy. Our test case (14x) has the the same biharmonic dissipation as case 14c, but with the addition of a Laplacian viscosity; its level of mean kinetic energy is similar to that of the intermediate case 14b. The four cases are described in Table 1.

Examining the surface expression of the circulation, the observed path of the North Atlantic Current is evident in satellite-based observations of altimetry, as in Figure 3 (panel at upper right). This branch of North Atlantic circulation is also produced in all the 0.1 degree model simulations shown except for the case with the highest values of dissipation (14a, upper left). Lower resolution simulations generally do not produce penetration of the NAC into the NWCorner (Figure 1). Our test case does well in this respect (lower right hand panel of Figure 3).

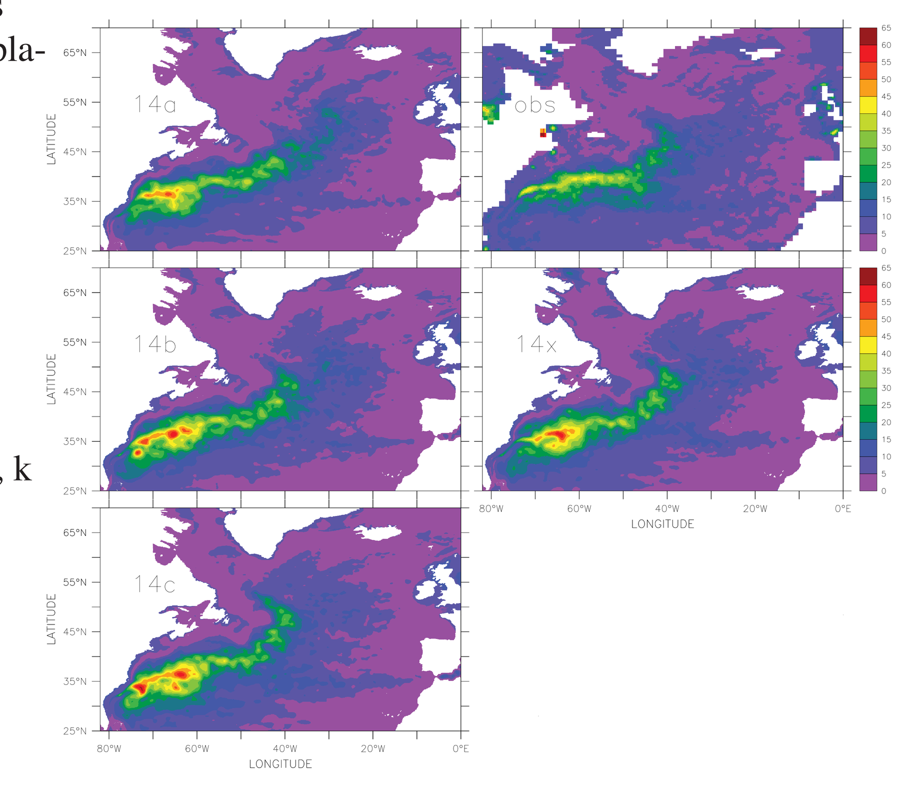

Figure 3: Time averaged North Atlantic sea surface height variability, from satellite altimetry (upper right), from the three biharmonic cases of BHS07 (left) and from our test case (lower right), all for the three year period 1998--2000. The anomalously high variability seen near the Gulf Stream's separation point in the less viscous 14b and 14c biharmonic cases is reduced in the more viscous case 14a, and in the 14x test case; the North Atlantic Current turns northward around the Grand Banks in the latter case.

Case	ν_4	ν_2
14a	$27.0 \times 10^9 m^4/s$	—
14b	$13.5 \times 10^9 m^4/s$	—
14c	$6.75 \times 10^9 m^4/s$	—
14x	$6.75 \times 10^9 m^4/s$	$35.5 m^2/s$
glc	$27.0 \times 10^9 m^4/s$	—
glx	$10.5 \times 10^9 m^4/s$	$105 m^2/s$

Table 1: Base coefficients of horizontal dissipation. Cases 14a, 14b and 14c are from Bryan et al. [2007], case 14x is our test case with the addition of Laplacian viscosity. The global control and experimental cases are referred to as glc and glx, respectively. Spatial scaling of coefficients is as per equation 1.

One of the points made in BHS07 is that deep penetration of eddy kinetic energy is correlated with reattachment of the Stream as it encounters the topography of the Southeast Newfoundland Rise (at around 48 degrees W). There is no obvious evidence of excessive suppression of deep eddy kinetic energy with inclusion of Laplacian viscosity in the 14x test case seen in Figure 4 at 50 degrees W; the only one of the four cases with significantly weaker penetration of deep eddy kinetic energy is the more dissipative case 14a, the one case in which the NAC failed to make the downstream turn northward around the topography of the Grand Banks (Figure 3).

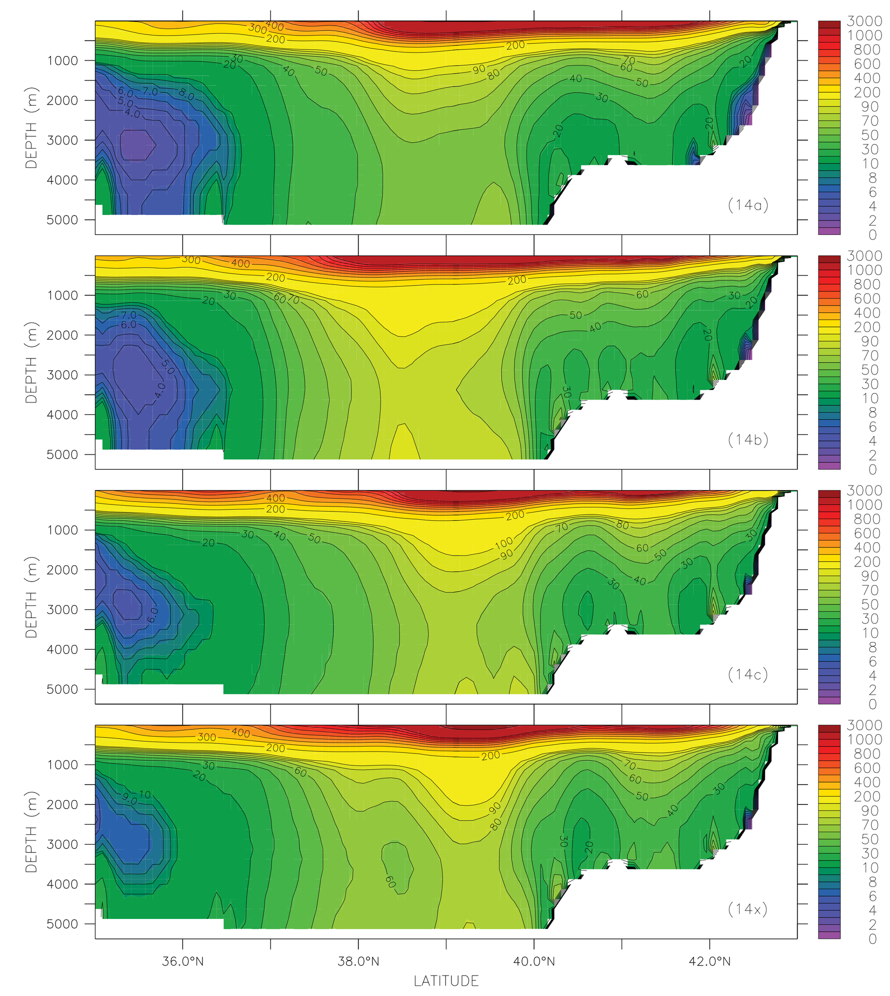

Figure 4: Eddy kinetic energies at 50 degrees W. Our test case 14x, despite having a less energetic (and more realistic) separation of the Gulf Stream from the North American coast at Cape Hatteras (Figure 3), appears to have sufficient vertical penetration of eddy kinetic energy at this downstream point in order to successfully reattach to the continental topography at the Grand Banks. In contrast, the most viscous biharmonic case (14a) has less vertical penetration of kinetic energy and fails to make the northward turn around the Grand Banks (as was seen, again, in the earlier figure).

Initial results in a global model

A control simulation with a new, tripolar grid version of the 0.1 degree POP model was produced with biharmonic dissipation (case glc of Table 1). The use of the tripolar grid with a smoother, partial cell representation of bottom topography results in a significantly better simulation of the Gulf Stream/North Atlantic Current system, compared with the earlier dipole grid version of Maltrud and McClean [2005], even with somewhat higher values of dissipation than we would use in the North Atlantic model, at least with full bottom cells (to which we ascribe much of the difference).

Our new prescription for combined use of Laplacian and biharmonic lateral viscosity produces some additional improvement in the North Atlantic, as in the regional study. In particular, the somewhat excessive variability downstream of Cape Hatteras is lessened (see middle panel of Figure 5, case glx).

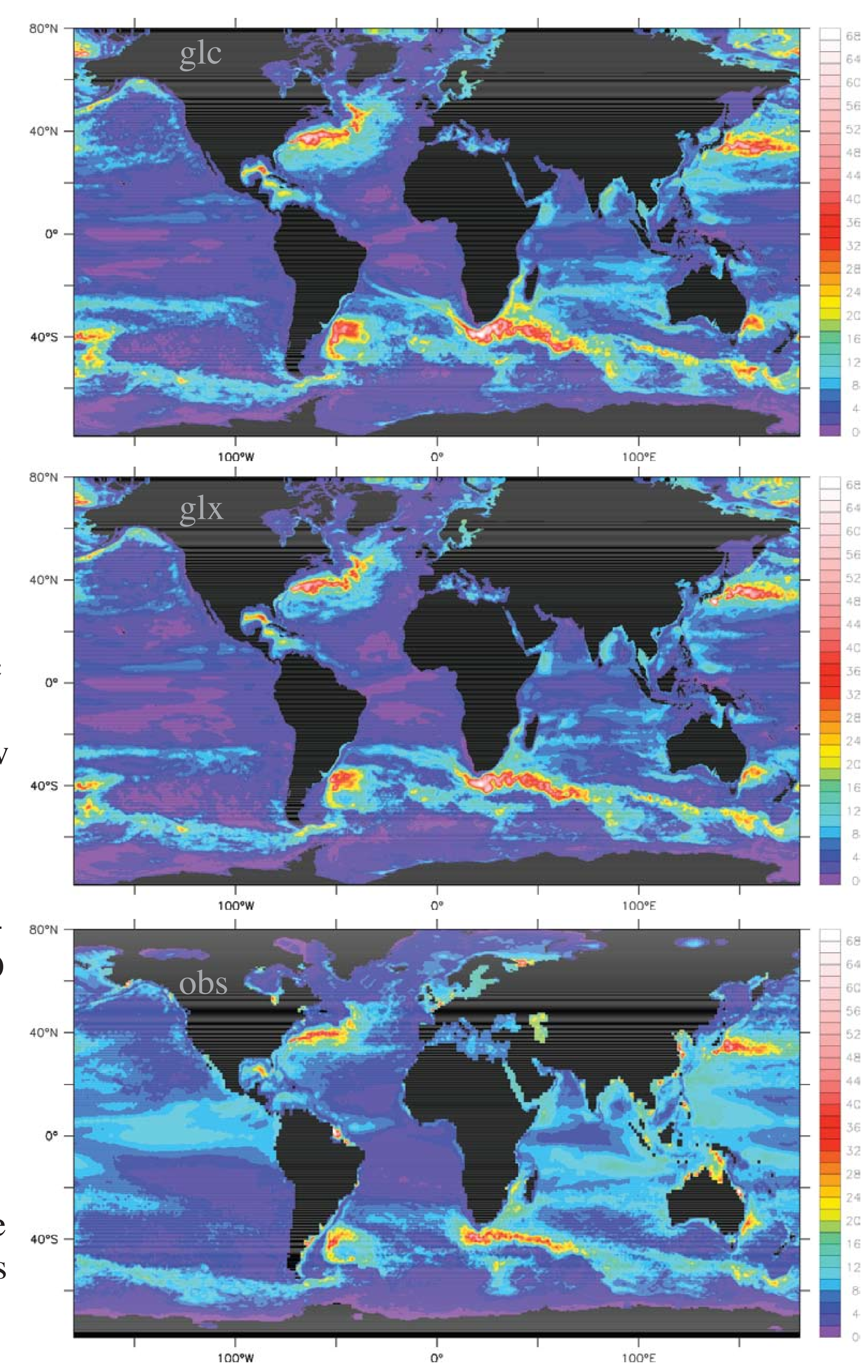

Figure 5: Sea surface height variability, from four year averages of 0.1 degree global simulations with (top) biharmonic lateral viscosity, and (middle) with our new prescription for combined Laplacian and biharmonic forms. If the variability is compared with the AVISO observations (bottom), not only is the apparently excessive variability reduced downstream of Cape Hatteras, but the path of Agulhas Rings is improved.

Another problem noted in Maltrud and McClean [2005] is addressed in our new case: The overly regular track of Agulhas Rings, clearly visible in our control case (upper panel of Figure 5), is much improved (compare the middle panel to the lower panel).

Concluding remarks

We have developed a simple method of combining Laplacian and biharmonic forms of viscosity, as an extension of the work of CG01, and we have found this prescription to result in improvements in both regional and global versions of a high-resolution, strongly-eddying model. Variability in the Gulf Stream region and the path of Agulhas Rings are two notable points of improvement.

Our prescription is designed for use over a broad range of resolutions, with biharmonic viscosity providing noise control and Laplacian viscosity providing viscous balance over the western boundary current regions. The basic scheme can be extended readily to more sophisticated anisotropic schemes, where our prescription for the scaling of the coefficient of Laplacian viscosity with grid resolution would provide the cross-stream or zonal component in the western boundary regions. Use in non-eddy-resolving models may require such an extension.