

# **LECTURE 5**

## **EARTHQUAKE**

### **SCALING LAWS**

# EARTHQUAKE SOURCE PARAMETERS

- We seek to understand the properties of very large earthquakes. However, they are very rare.
- Thus, we look at patterns in the *growth* of earthquakes
- We examine the various parameters describing the earthquake source.
- Recall

$$M_0 = \mu \cdot S \cdot \Delta u = \mu \cdot L \cdot W \cdot \Delta u$$

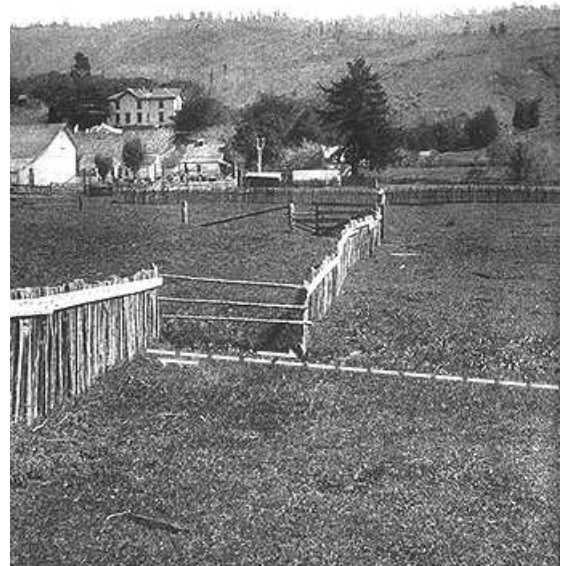
→ Can we measure these terms independently?

\* *FAULT SLIP*  $\Delta u$



**Imperial Valley, 1979**

$M \approx 6$ ;  $\Delta u = 25 \text{ cm}$



**San Andreas, 1906**

$M \approx 8$ ;  $\Delta u = 2.6 \text{ m}$

# EARTHQUAKE SOURCE PARAMETERS

## \* *FAULT LENGTH $L$*

- It is some times possible to follow an earthquake rupture on the field, and to gain an estimate of its length  $L$ .



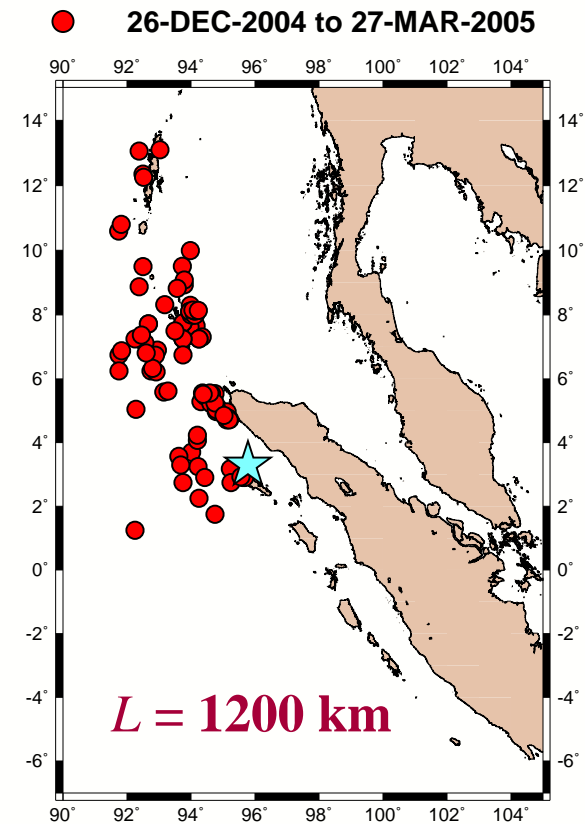
**Borah Peak,  
Idaho;  
1983**



**Landers,  
Calif.;  
1992**

- *Aftershocks* are universally used as expressing the extent of the rupture zone of a major earthquake.

This approach also yields an estimate of the transverse dimension (width  $W$ ).

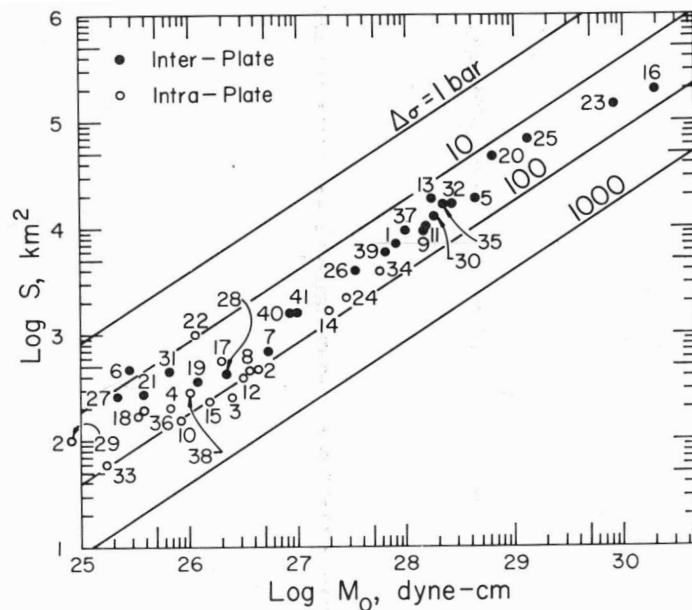


**Principal Aftershocks of the 2004 Sumatra earthquake**

# GROWTH of PARAMETERS with EARTHQUAKE SIZE

- Empirical evidence verifies that parameters such  $\Delta u$ ,  $L$ ,  $S$ , perhaps  $W$ , grow with the size of the earthquake, expressed by its seismic moment.

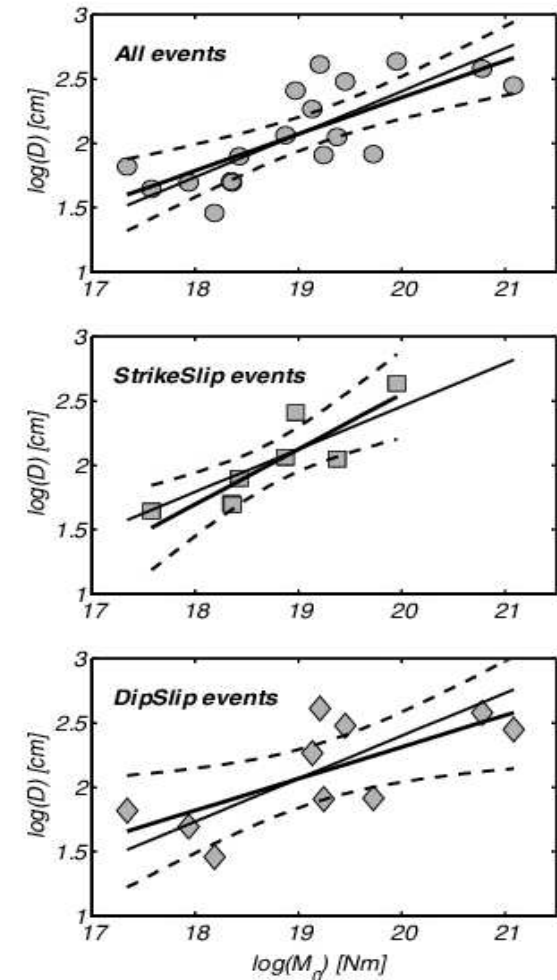
$S$  vs.  $M_0$



[Kanamori and Anderson, 1975]

$\Delta u [D]$  vs.  $M_0$

Mean Slip vs. Moment



[G. Beroza, [www.stanford.edu](http://www.stanford.edu)]

# SIMPLE IDEAS TOWARDS SCALING LAWS

1. As the source grows,  $\mu$ , a material property, should remain *invariant*.

2. The *shape* of the fault zone may remain constant (as long as one does not reach the physical limits of the seismogenic zone — stay tuned). [The rupture can grow in all directions on the fault plane]. Hence  $W \sim L$ .

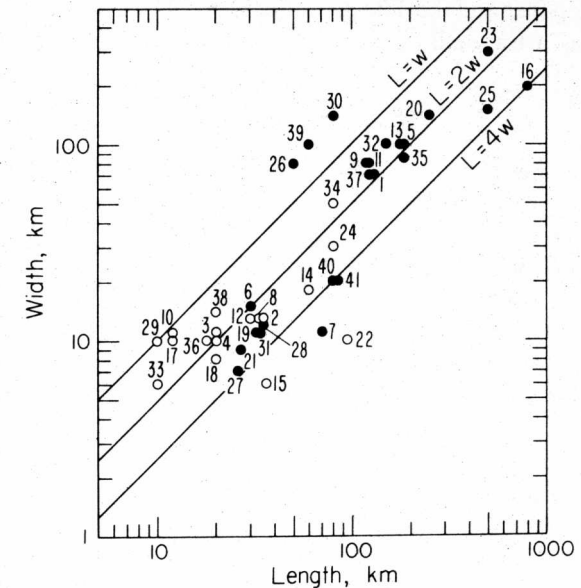
3. The rock cracks because it has accumulated too much *strain*  $\epsilon$ . The latter is measured by the ratio  $\Delta u / L$ , or perhaps  $\Delta u / W$ . Such ratios should also be invariants, related to the *strength* of the rock, which ruptures at a certain, probably universal,  $\epsilon_{\max}$ .

4. Thus, one predicts that the seismic moment  $M_0$  should grow as the cube of the linear size of the earthquake:

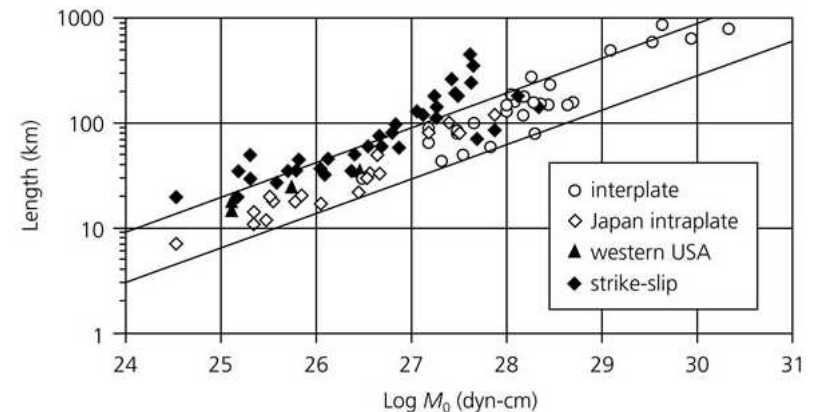
$$M_0 \sim L^3$$

VERDICT: about right (Slope close to 1/3).

(At least for reasonably sized events).



[Geller, 1976]



[Stein and Wyssession, 2002]

# SCALING LAWS and $b - [\beta]$ VALUES

## *Frequency–Size Distributions*

- It is known that there are more small earthquakes than large ones.

Why ? and can it be quantified ?

→ *Gutenberg and Richter* [1954] proposed  $\log_{10} N = a - b \cdot M$ , with  $b \approx 1$ .

- *JUSTIFICATION*: [Rundle, 1989] Rupture is a scale-invariant process, or "All elements of a fault have the same probability of being released by an earthquake of any size".

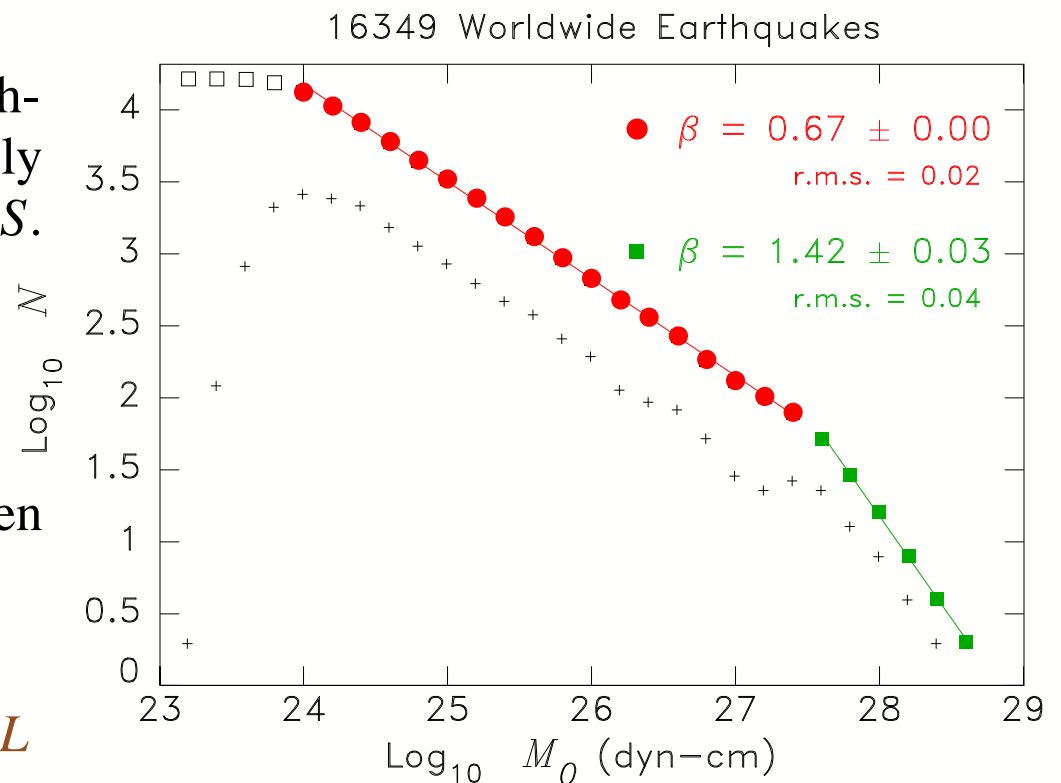
This suggests that the number of earthquakes of any given size  $N$ , is inversely proportional to the area of rupture,  $S$ . Hence  $N \sim 1/S$ , or as  $M_0 \sim S^{3/2}$ ,

$$\log_{10} N = a - \beta \cdot \log_{10} M_0 \quad \beta = \frac{2}{3}$$

[ and if one uses a slope of 3/2 between  $M_0$  and a magnitude  $M$ , then  $b = 1$ . ]

*UPHELD SPECTACULARLY WELL*

[at least for "not too large" earthquakes]

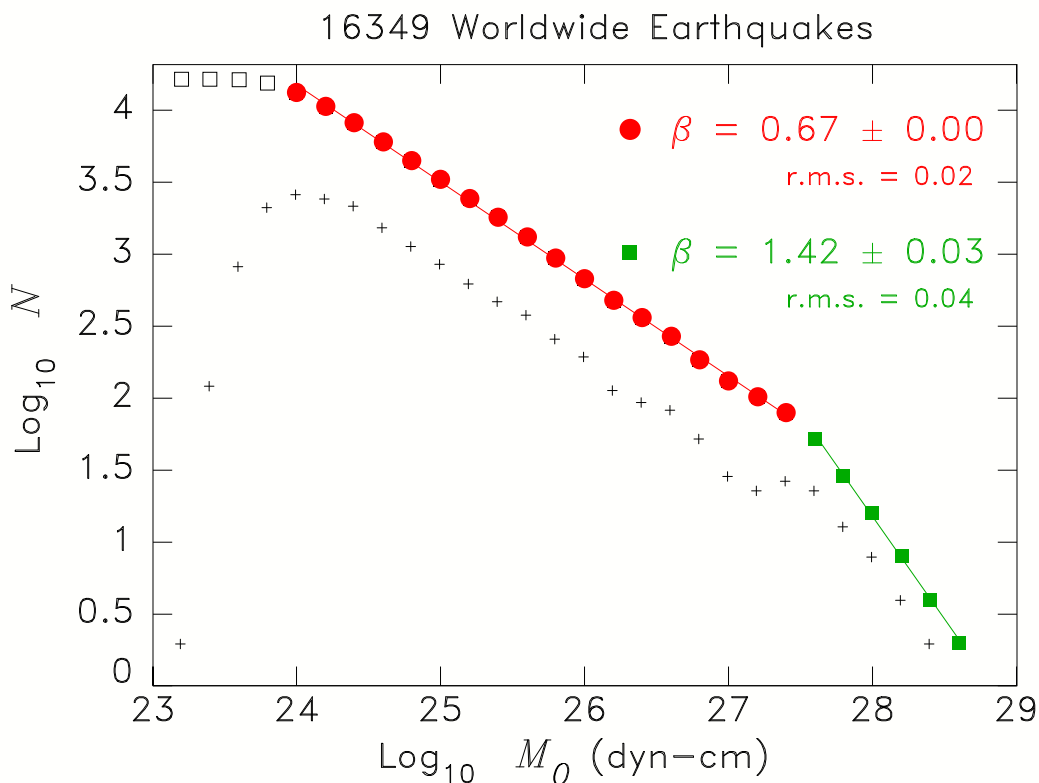


[Okal and Sweet, 2007]

# BREAKDOWN of SCALING LAWS

## *at Large Moments*

- The seismogenic zone is limited in space, principally the parameter  $W$ , due to the *increasing temperature at depth* in the Earth; the material ceases being *brittle*.
- $\Delta u$  may also stop growing with earthquake size, to keep the strain  $\varepsilon = \Delta U / W$  invariant.
- Then one predicts  $M_0 \sim L$ , and  $\beta = 1$ .



→ Rather **WELL VERIFIED**, but **CONTROVERSIAL** (the population of large events is small and may be heterogeneous).

# SOURCE FINITENESS and GROWTH

→ To understand the properties of waves (seismic or tsunami) from great earthquakes, we must remember that

*A GREAT EARTHQUAKE IS  
EXTENDED IN TIME and SPACE*

*(it needs Room and it needs Time)*

- *RISE TIME*  $\tau$  is the time necessary for walls of the fault to move with respect to each other.
- *RUPTURE TIME (or DURATION)*  $T_R$  is the time it takes for the cracking to propagate from one end of the fault to the other.

→ **SIMPLE IDEAS:**

- If the motion of the particles along the fault is at a constant velocity, then

$$\tau \sim \Delta u \sim M_0^{1/3} = \text{a few seconds}$$

- If the propagation of the rupture along the fault is at a constant velocity, then

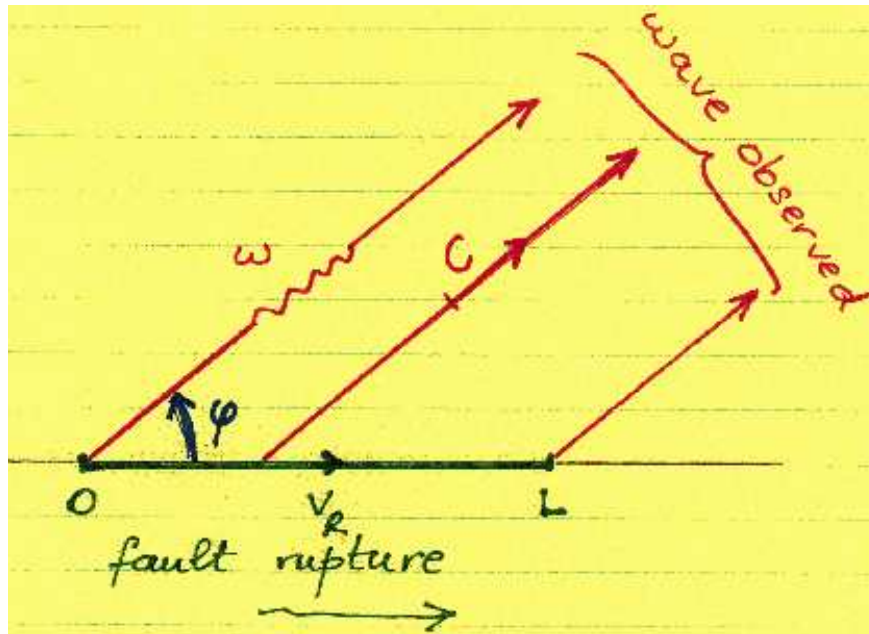
$$T_R \sim L \sim M_0^{1/3} = \text{tens of seconds}$$

( $\geq 500$  seconds for Sumatra, 2004).



# FAR FIELD: THE BASICS of DIRECTIVITY

[Ben Menahem, 1962]



If a source propagating a length  $L$  at velocity  $V_R$  in the direction  $x$  generates a wave traveling at phase velocity  $C$  observed at an angle  $\phi$  from  $x$ , then the amplitude of the wave is affected by a *DIRECTIVITY* function  $D$

$$D = \frac{\sin Y}{Y} \quad \text{with} \quad Y = \frac{\omega L}{2C} \cdot \left[ \frac{C}{V_R} - \cos \phi \right]$$

This formula simply expresses that the various elements of the source always interact destructively at high enough frequencies, *except when the wave propagation compensates exactly the offset of source time*

( $\sin Y / Y$  maximum requires  $Y = 0$ .)

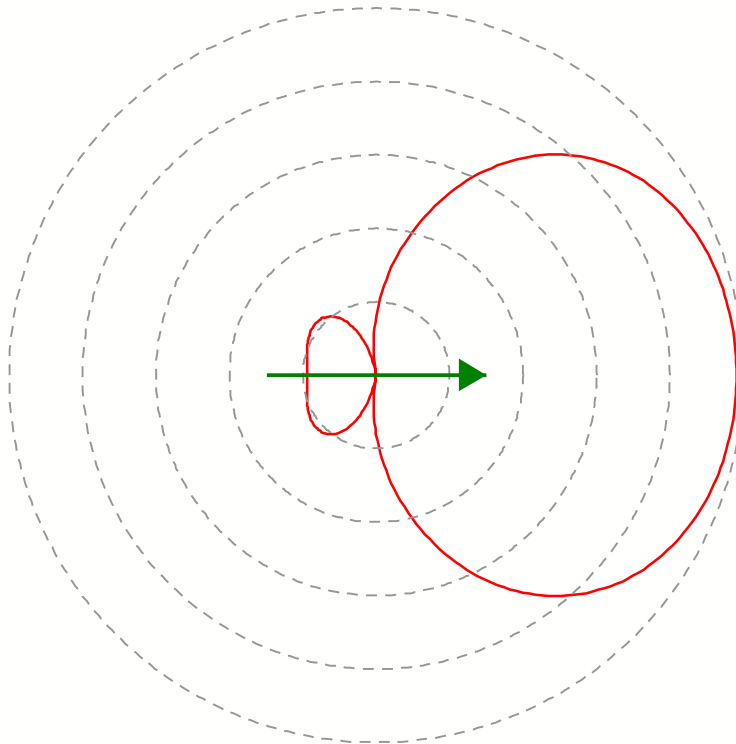
$$D = \frac{\sin Y}{Y} \quad \text{with} \quad Y = \frac{\omega L}{2C} \cdot \left[ \frac{C}{V_R} - \cos \phi \right]$$

Then several scenarios can take place

- *Seismic surface wave generated by a seismic dislocation*

Then,  $V_R$  is close to  $C$  (3.5 to 4 km/s), and the maximum of directivity is *in the direction of propagation*.

120 s; 300 km;  $V_R = 3.5$  km/s;  $C = 4$  km/s



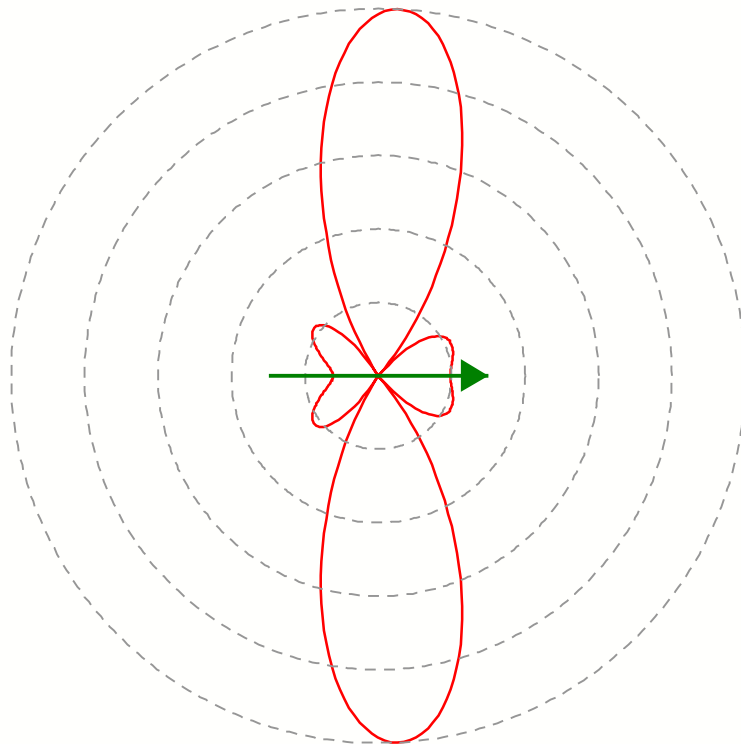
*(A classic result in Seismological Source theory)*

$$D = \frac{\sin Y}{Y} \quad \text{with} \quad Y = \frac{\omega L}{2C} \cdot \left[ \frac{C}{V_R} - \cos \phi \right]$$

- *Tsunami generated by a seismic dislocation*

Then,  $V_R$  is always much greater than  $C$ , and the maximum of directivity is *at right angles to the fault strike*.

900 s; 300 km;  $V_R = 3.5 \text{ km/s}$ ;  $C = 0.2 \text{ km/s}$



[*Ben-Menahem and Rosenman, 1972*]

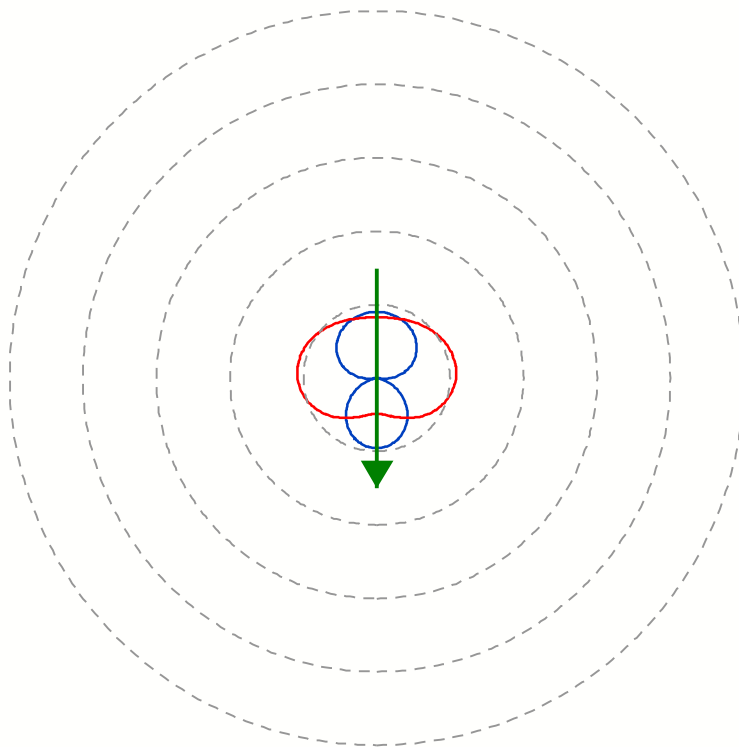
The tsunami is so slow that the source appears instantaneous, and the interference is constructive only in a direction where distance is stationary along the fault line.

$$D = \frac{\sin Y}{Y} \quad \text{with} \quad Y = \frac{\omega L}{2C} \cdot \left[ \frac{C}{V_R} - \cos \phi \right]$$

- *Tsunami generated by a landslide*

Then,  $V_R$  is always much *SMALLER* than  $C$ , and the interference is always destructive (for long enough sources).

600 s; 25 km;  $V_R = 0.04$  km/s;  $C = 0.2$  km/s  
 900 s; 50 km;  $V_R = 0.04$  km/s;  $C = 0.2$  km/s

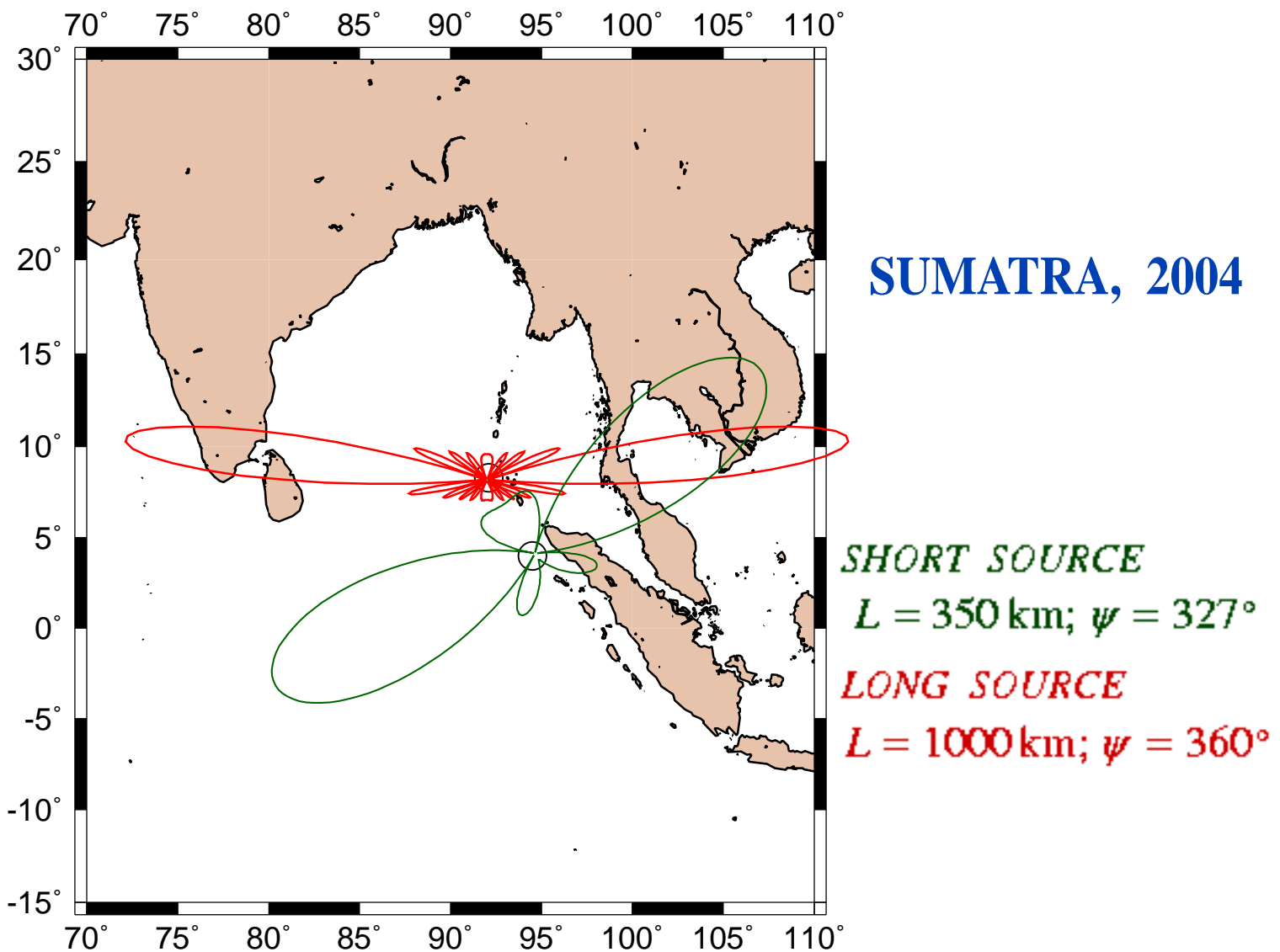


The rupture is so slow (with respect to the wave) that there are no directions in which it can be compensated by the variations of phase due to propagation.

**LANDSLIDES CANNOT GENERATE  
 FAR-FIELD DIRECTIVITY**

## Note in particular

- Even *slow* earthquake rupture velocities (**1 km/s**) are hypersonic with respect tsunami propagation.
  - Even the *fastest recognized* submarine landslide velocities (**50 m/s**) are considerably slower than tsunami velocities.
- Directivity lobes for tsunami become **narrower** as Earthquake size increases [*Okal and Talandier, 1991*].



## FROM FINITENESS to SATURATION

- In general, any seismic wave (body or surface) of (angular) frequency  $\omega$  will have a spectral amplitude directly proportional to the seismic moment, or

$$X(\omega) \sim M_0 \sim L^3$$

→ Effect of directivity:

$$D = \frac{\sin Y}{Y} \quad \text{with} \quad Y = \frac{\omega L}{2c} \cdot \left[ \frac{C}{V_R} - \cos \phi \right]$$

For small events (small  $L$ ),  $Y \rightarrow 0$  and  $D \rightarrow 1$ .

For big events (large  $L$ ),  $Y \rightarrow 0$  and  $D \sim \frac{1}{L}$ .

We anticipate  $X(\omega) \sim L^2$ .

→ But, there should also be a similar effect along the width  $W$  of the fault. Hence an additional factor  $D_W \sim 1/W$  for large events.

→ And the source has a rise time  $\tau$ , which also grows with earthquake size, leading to yet another function

$$D_\tau = \frac{\sin Y_\tau}{Y_\tau} \sim \frac{1}{\tau}$$

**In the end, the spectral amplitude of a wave is expected to grow like**

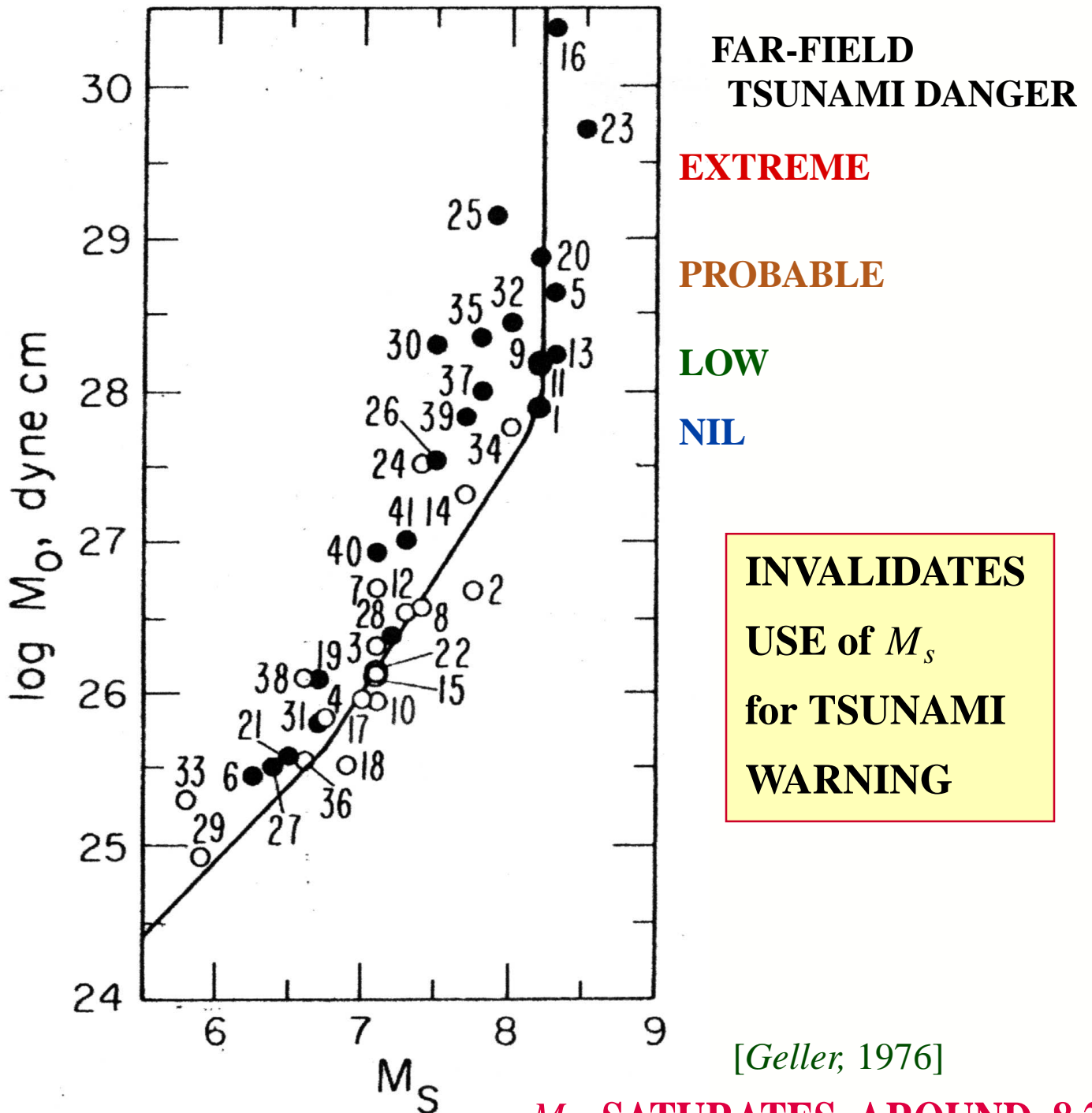
$$X(\omega) \sim \frac{M_0}{L \cdot W \cdot \tau} \sim \frac{L^3}{L^3} = \text{constant}$$

**WE PREDICT TOTAL SATURATION !!**

# SATURATION OF $M_s$

Any magnitude scale measured using a constant period  $T$  (20 s for  $M_s$ ) will saturate for large enough earthquakes, namely when the *duration* of the source becomes longer than  $T$ .

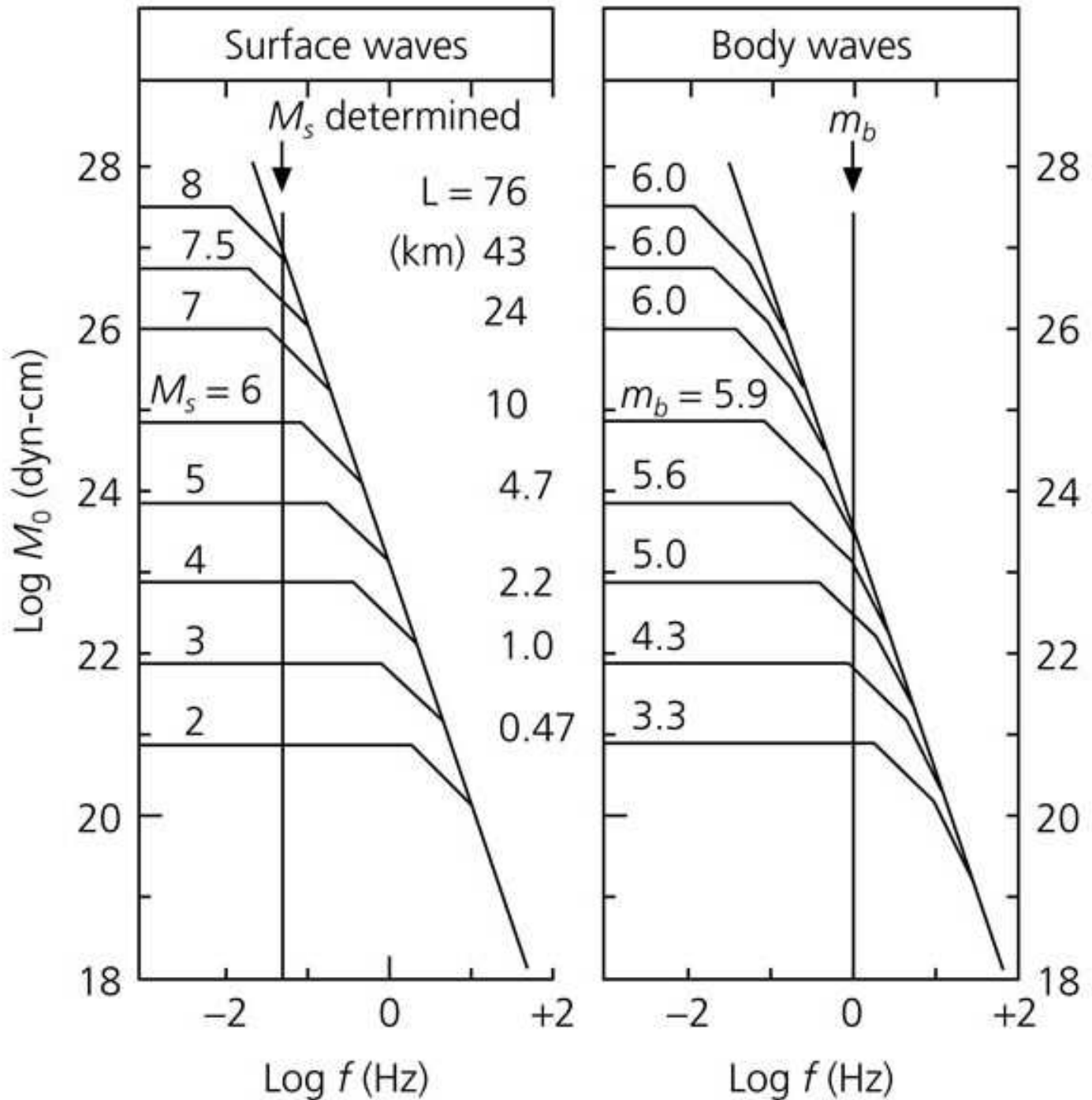
- In REMARKABLE agreement with OBSERVATIONS.



**$M_s$  SATURATES AROUND 8.2**

# ALL CONVENTIONAL MAGNITUDES SATURATE

It is only a question of the period  $T$  which they use.



- $m_b$ , measured at 1 s, would saturate event earlier (at  $m_b = 6$  if properly measured at exactly  $T = 1$  s).



# SCALING TSUNAMIS in the NEAR FIELD

*Okal and Synolakis [2004]*

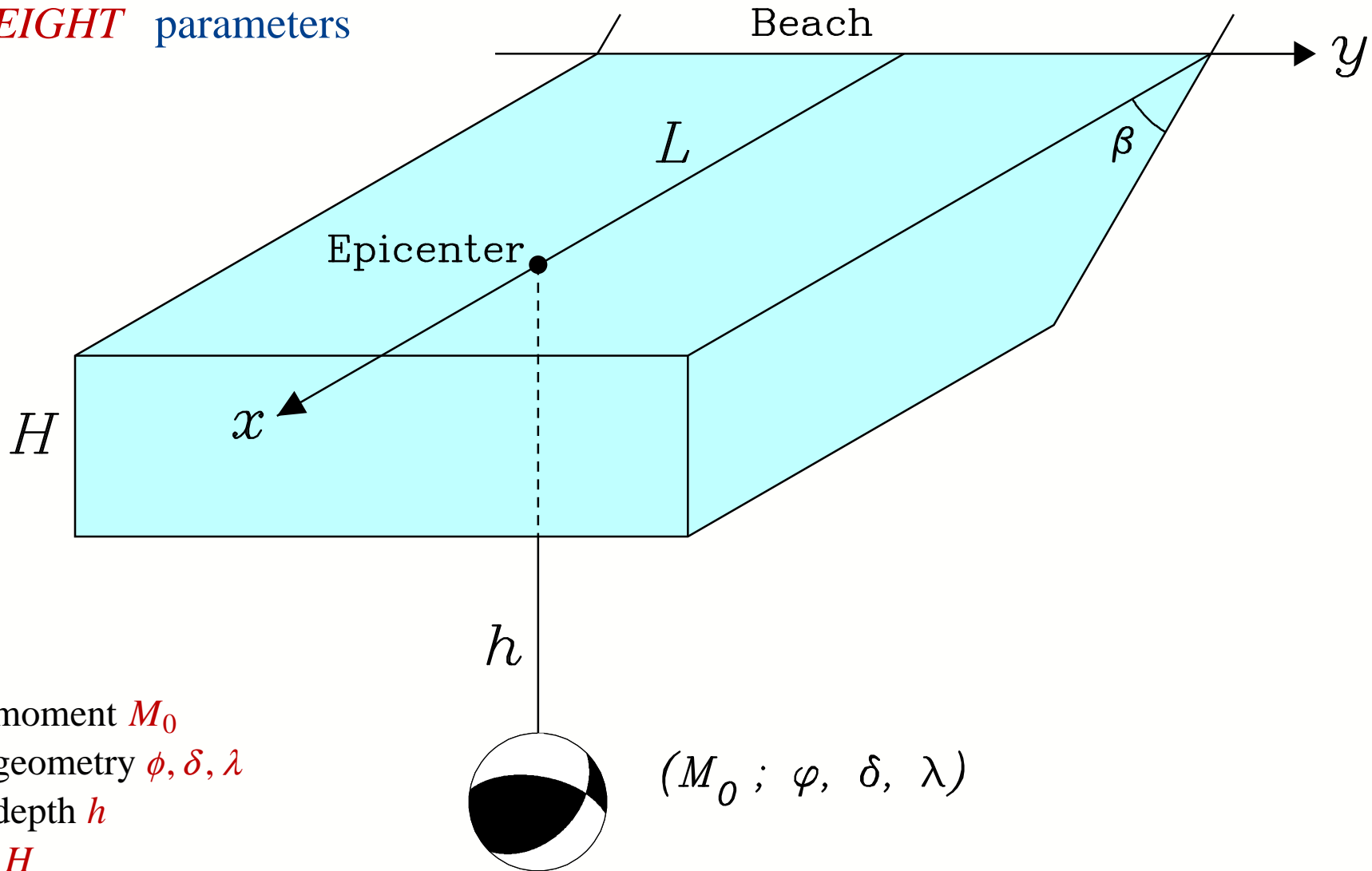
- **SIMPLE IDEAS:** Consider a seismic source
  - Everything else being equal, the maximum value of run-up on a beach should grow like the slip,  $\Delta u$ .
  - Everything else being equal, the lateral extent of run-up on the beach should grow like the size of the fault,  $L$ .
  - The ratio of the two, which is the *aspect ratio* of the distribution of run-up along the beach, should behave like  $\Delta u / L$ , which being the strain released,  $\varepsilon$ , should be invariant under seismic scaling laws.
- Thus we predict that all earthquakes should feature the *same distribution of run-up along a beach in the near field*.
  - TEST this theoretically.
  - COMPARE with data from tsunami surveys.
- If this invariant is violated, it means the source does not scale like an earthquake.

It probably is not one !

[ *LANDSLIDE ?* ]

# GENERIC DISLOCATION in the NEAR FIELD

Involves *EIGHT* parameters



Earthquake moment  $M_0$

Earthquake geometry  $\phi, \delta, \lambda$

Earthquake depth  $h$

Water depth  $H$

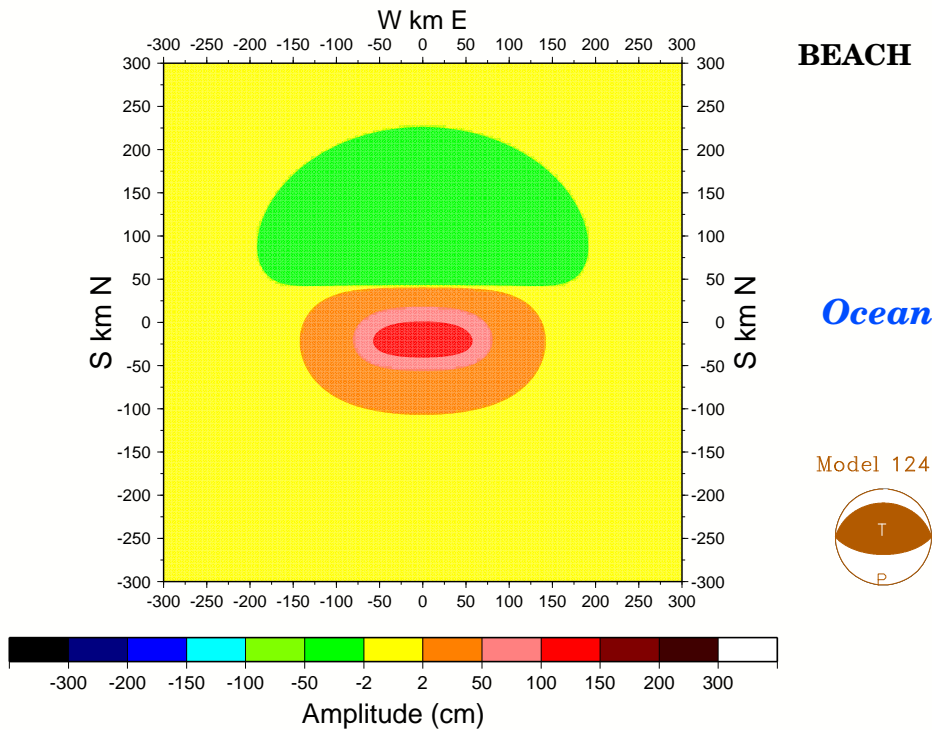
Epicentral distance to shore  $L$

Beach slope  $\beta$

$(M_0 ; \phi, \delta, \lambda)$

# NEAR-FIELD: *The Earthquake Dislocation*

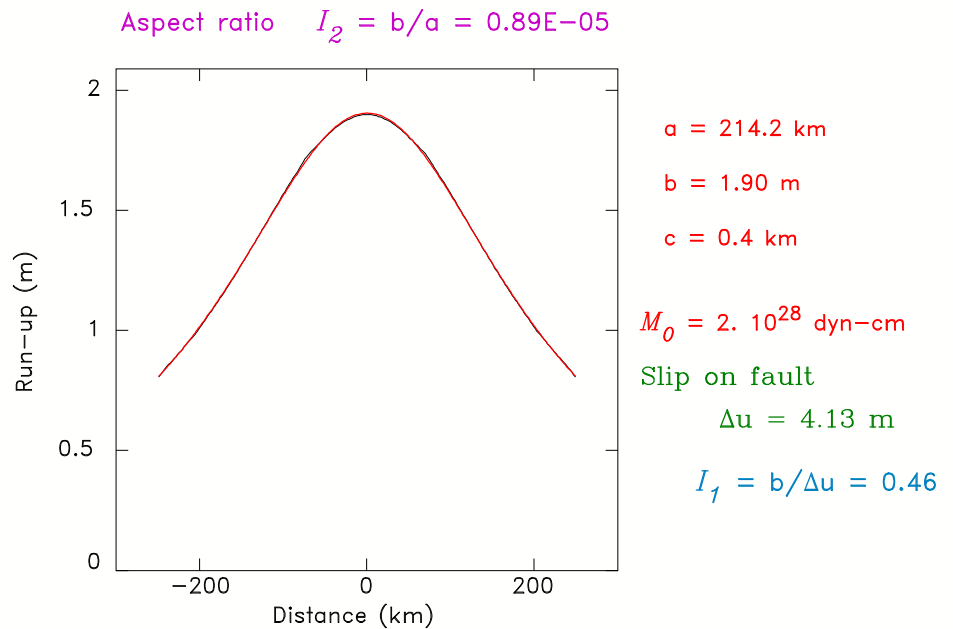
- Compute Ocean-Bottom Deformation due to Dislocation



- Simulate Tsunami Propagation to Beach and Run-up

- Fit Bell Curve

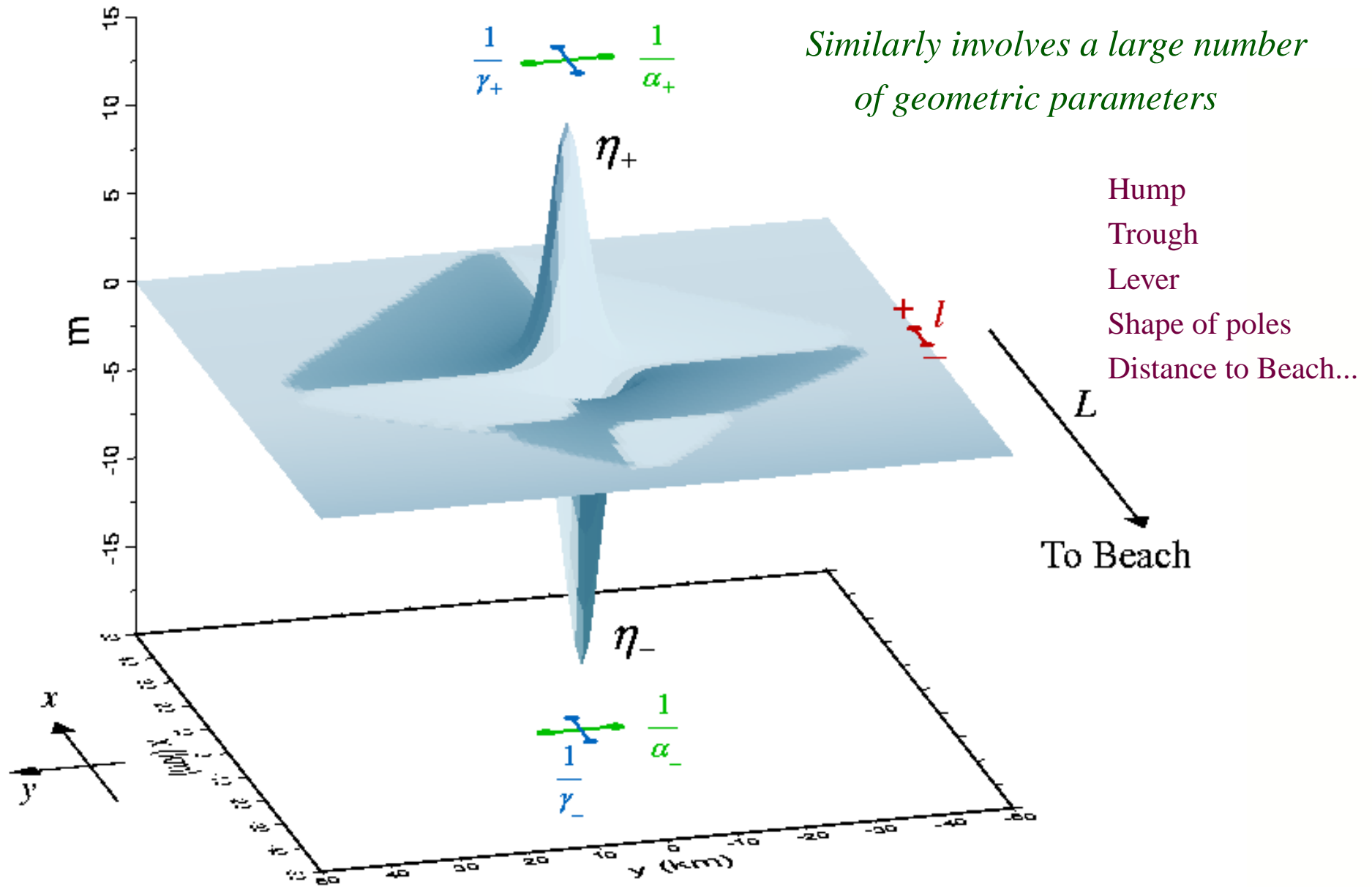
$$\zeta = \frac{b}{\left(\frac{x-c}{a}\right)^2 + 1}$$



- Retain aspect ratio  $I = b/a$

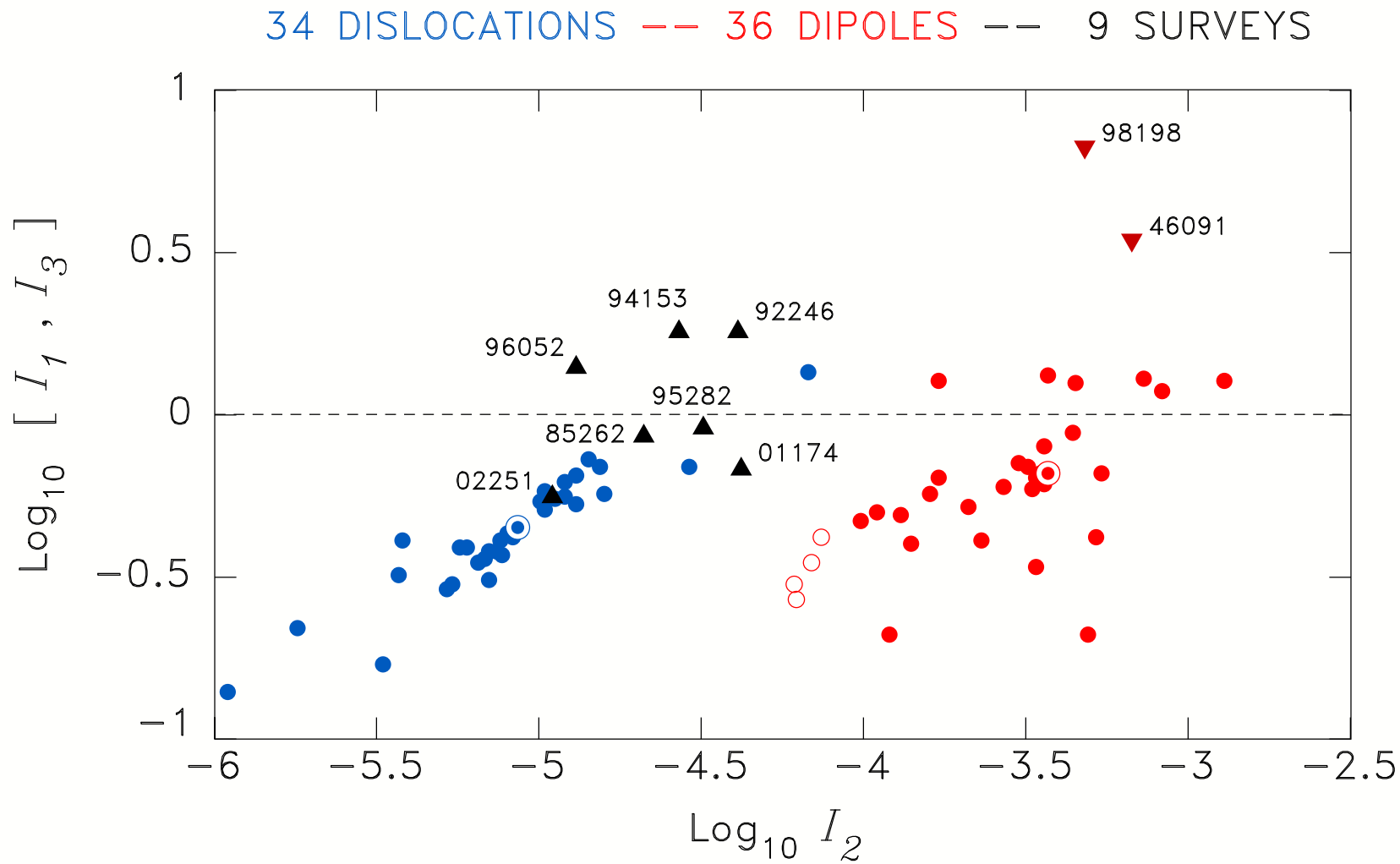
- Vary source parameters:  $I$  no greater than  $2.3 \times 10^{-5}$ .

# THE DIPOLAR SOURCE (Landslide)



[Okal and Synolakis, 2004]

**MAX. RUN-UP SCALED TO FAULT SLIP**  
**MAX. RUN-UP SCALED TO INITIAL TROUGH**

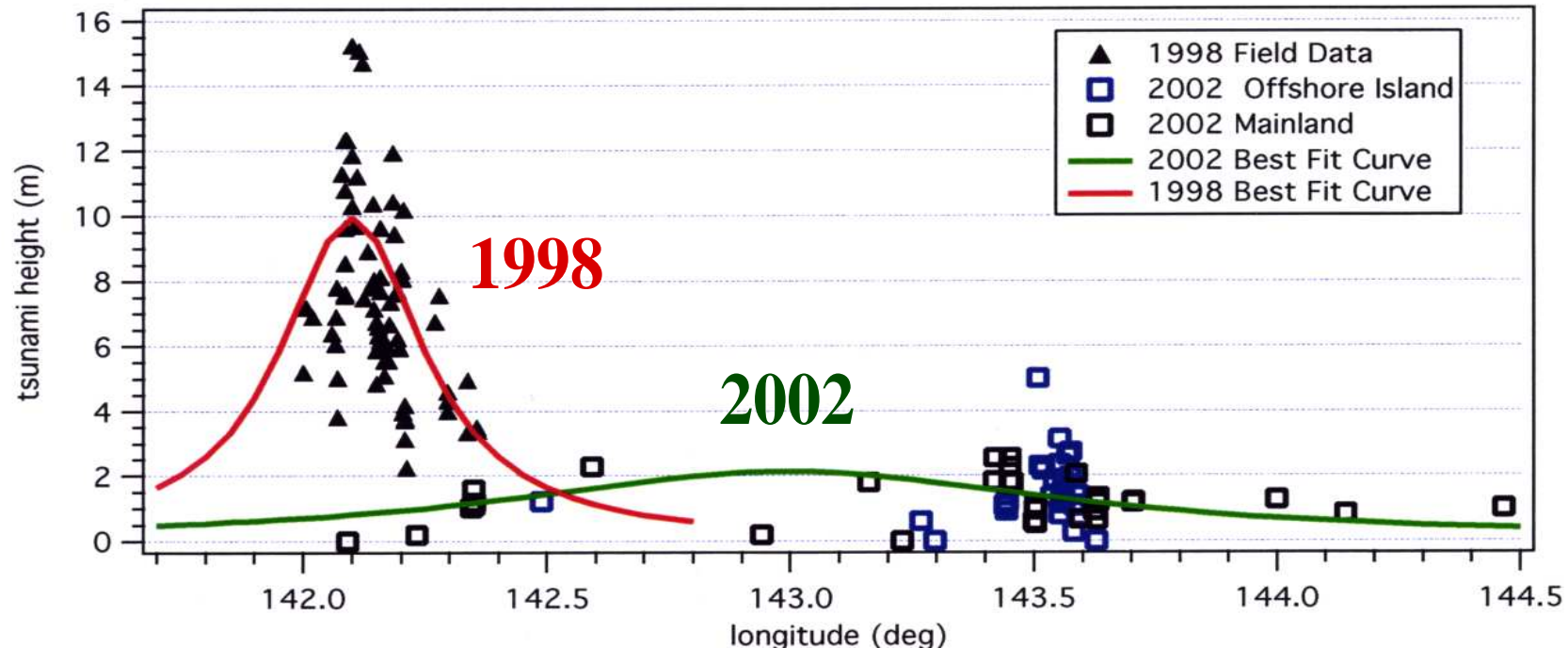
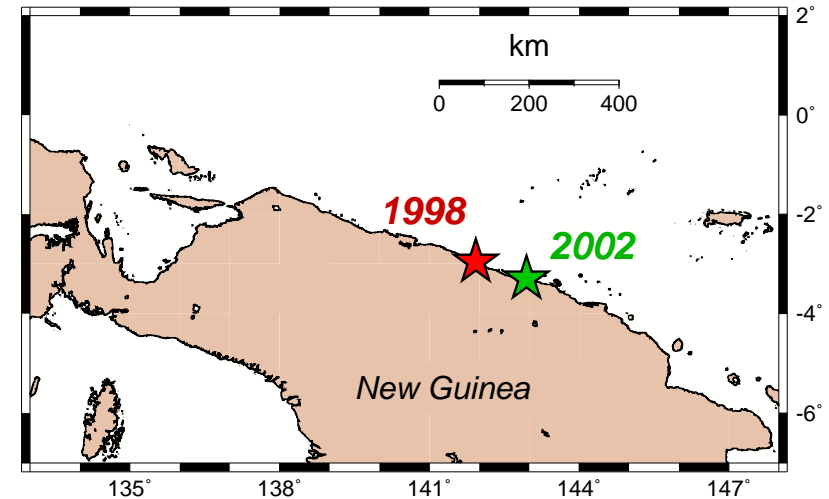


**ASPECT RATIO OF RUN-UP DISTRIBUTION ALONG BEACH**

*[Okal and Synolakis, 2004]*

# PAPUA NEW GUINEA: A TALE of TWO EARTHQUAKES

- 08 SEP 2002: Regular Earthquake,  $A.R. = 2.6 \times 10^{-5}$   
*No tsunami deaths.*
- 17 JUL 1998: Landslide Tsunami,  
 $A.R. = 4.8 \times 10^{-4}$   
*2200 Tsunami Deaths*



# VIOLATORS of SEISMIC LAWS

Apart from non-earthquakes [landslides, volcanic eruptions, etc.], seismic events will violate scaling laws if *invariants are not followed*.

- Anomalous material properties ( $\mu$ ; *weak* sediments)
- Anomalous shapes of fault zones ( $W/L$ ; *ribbon like; shallow strike-slip events*)
- Anomalous rupture velocities ( $V_R$ ; *slow or irregular, jagged ruptures*).

→ It is important to detect such events because

- (i) we may not catch the true size of the source by using conventional methods;
  - (ii) their tsunami potential may be enhanced.
- In general, all "*Tsunami Earthquakes*" are violators of scaling laws.

# THE INFAMOUS "TSUNAMI EARTHQUAKES"

- A particular class of earthquakes defying seismic source scaling laws. Their tsunamis are much larger than expected from their seismic magnitudes (even  $M_m$ ).
- Example: Nicaragua, 02 September 1992.

*THE EARTHQUAKE WAS NOT FELT AT SOME BEACH COMMUNITIES,  
WHICH WERE DESTROYED BY THE WAVE 40 MINUTES LATER*

170 killed, all by the tsunami, none by the earthquake



El Popoyo, Nicaragua



El Transito, Nicaragua



# "TSUNAMI EARTHQUAKES"

- *The Events:*

1896 Sanriku, Japan

1946 Aleutian

1923 (13 April) [*probably*] Aftershock of large Kamchatka earthquake

1932 (22 June) [*Probably*] Aftershock of Jalisco, Mexico earthquake

1963 (20 Oct.) Aftershock of great Kuriles earthquake

1975 Kuriles (following regular 1973 Nemuro-Oki event)

1982 Tonga

1992 Nicaragua

1994 Java                    **2006 Java** (cc. of 1994)

1996 Chimbote, Peru

2004 Sumatra (?; *features some slowness*)

# "TSUNAMI EARTHQUAKES"

- *The Cause:* Earthquake has exceedingly slow rupture process releasing very little energy into high frequencies felt by humans and contributing to damage [Tanioka, 1997; Polet and Kanamori, 2000].

→ Rupture in weak sedimentary material on splay fault through accretionary prism.

Candidates: Kuriles, 1963, 1975; Sanriku, 1996

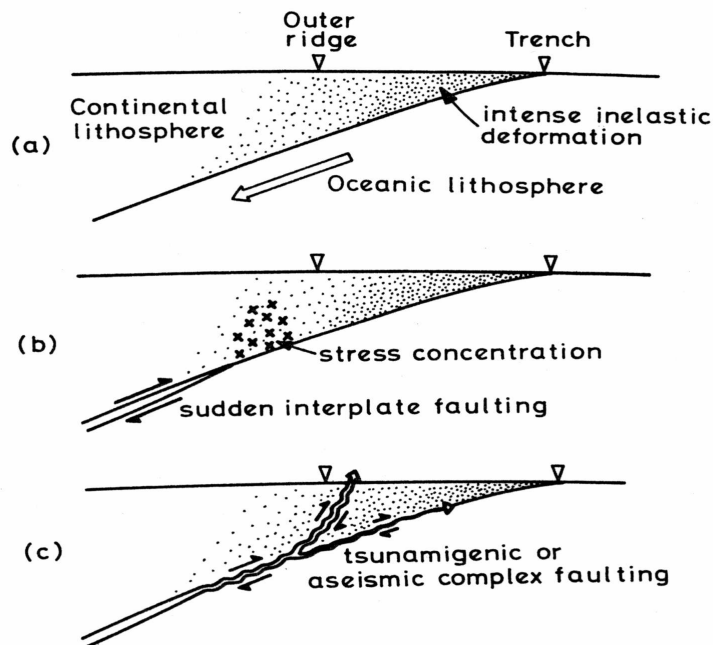


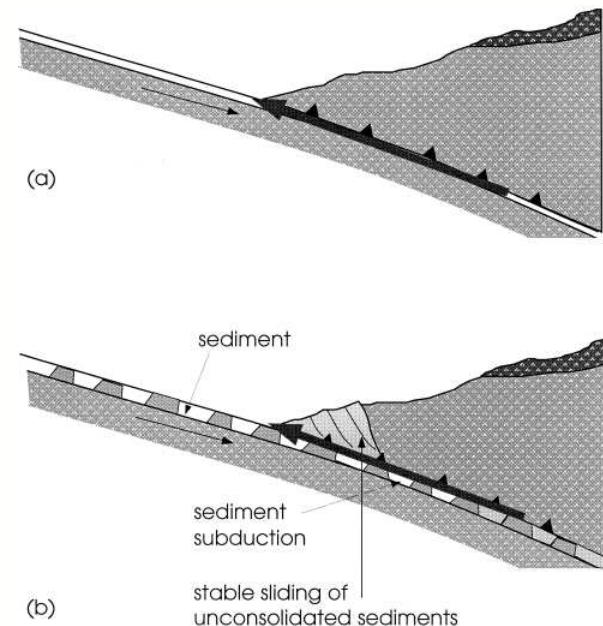
Fig. 19. A model for a great earthquake sequence showing (a) interseismic stage, (b) coseismic stage, and (c) postseismic stage. See the text for details.

[Fukao, 1979]

- *The Origin:* Generally interpreted as involving rupture in anomalous situations, which could involve

→ Rupture in jagged mode along corrugated interface poorly coupled due to sediment starvation [Tanioka et al., 1997].

Candidates: Nicaragua, 1992; Chimbote, Peru, 1996



[Polet and Kanamori, 2000]

# "TSUNAMI EARTHQUAKES"

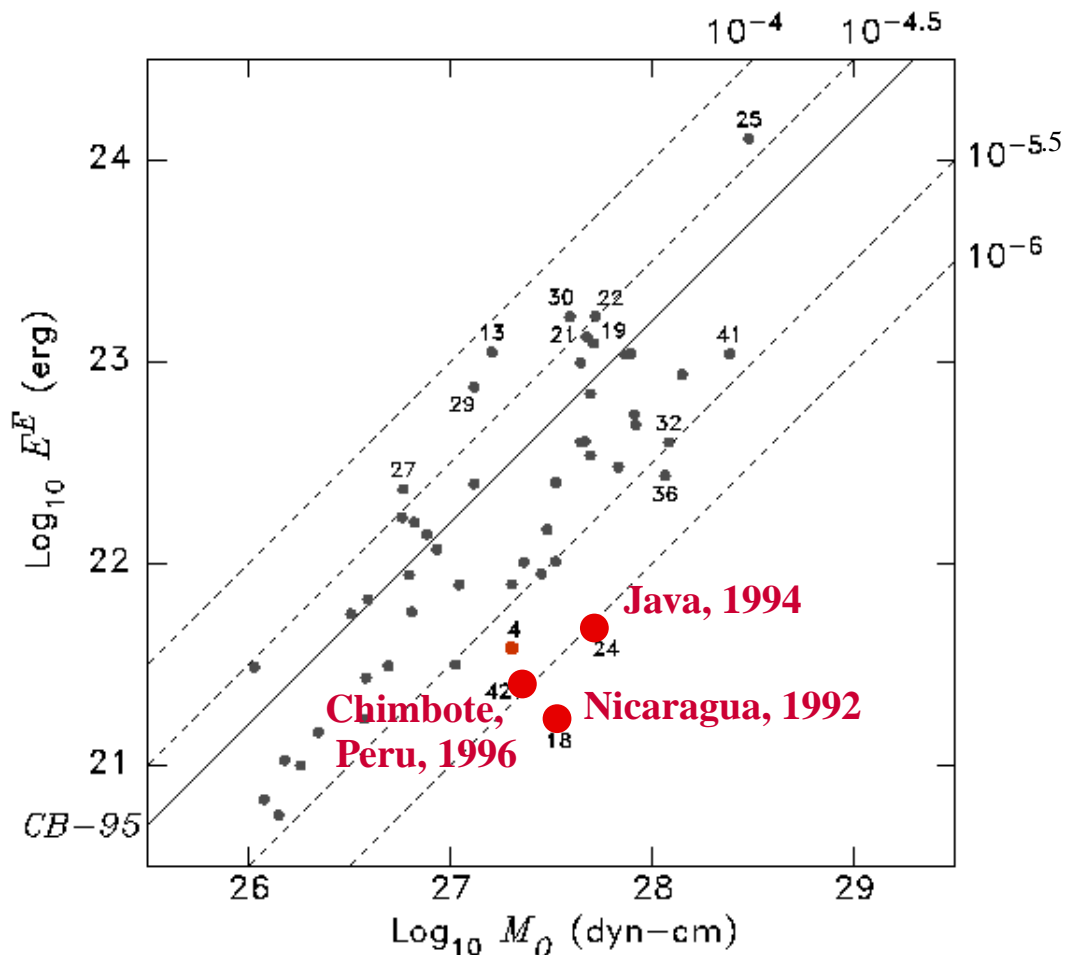
→ Define *Estimated Energy*,  $E^E$

$$E^E = (1 + q) \frac{16}{5} \frac{[a/g(15; \Delta)]^2}{(F^{est})^2} \rho \alpha \int_{\omega_{\min}}^{\omega_{\max}} \omega^2 |u(\omega)|^2 e^{\omega t^*(\omega)} \cdot d\omega$$

→ Scale to Moment through  $\Theta = \log_{10} \frac{E^E}{M_0}$

→ Scaling laws predict  $\Theta = -4.92$ .

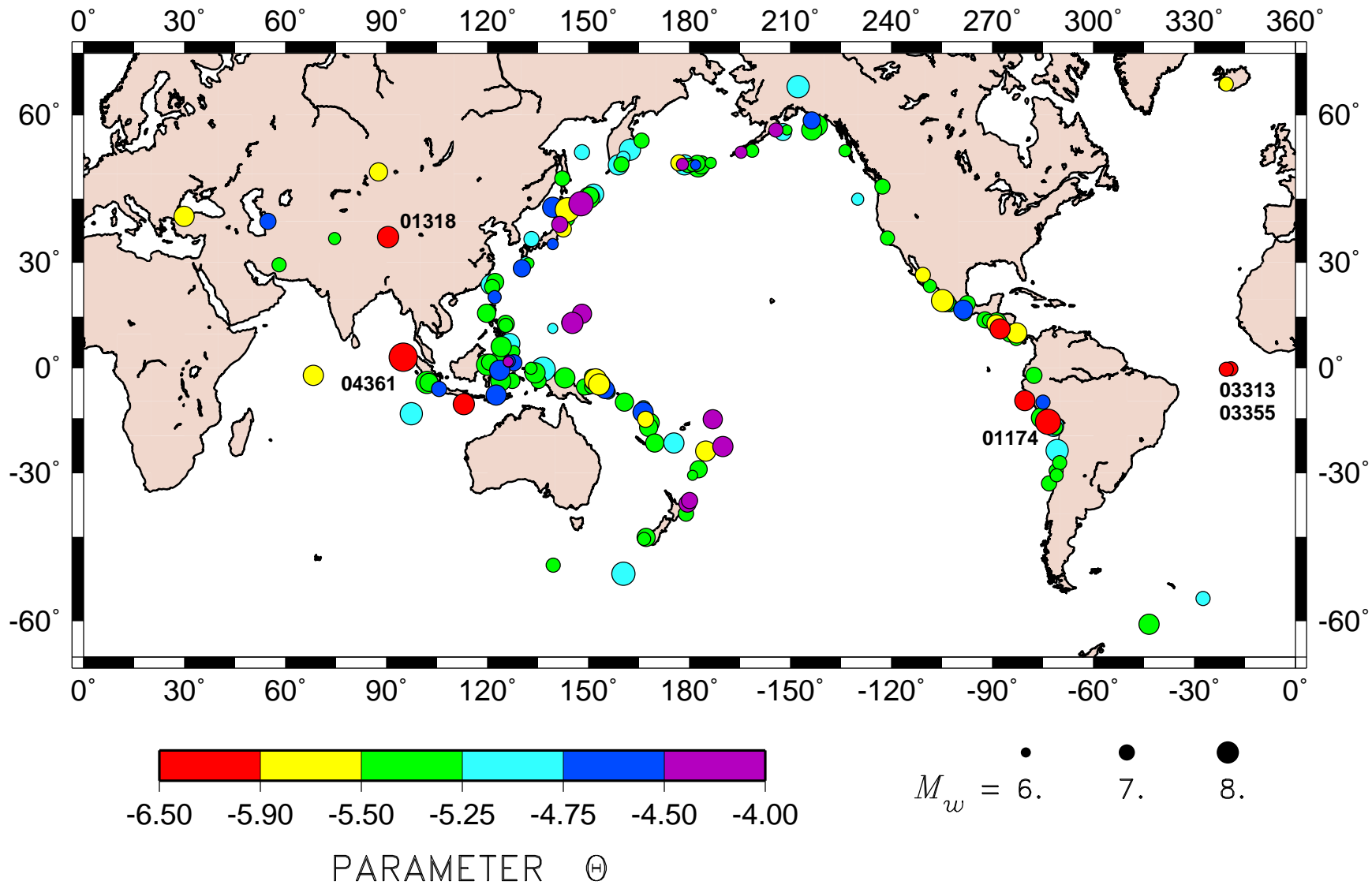
- **Tsunami earthquakes characterized by Deficient  $\Theta$  (as much as 1.5 units).**



*Now being implemented at Papeete and PTWC*

# COMPUTATION of $\Theta$ OPERATIONAL at PTWC since 2001

## COMBINED PTWC AND NEWMAN & OKAL [1998] DATASET



# OTHER PROXIES for "TSUNAMI EARTHQUAKES"

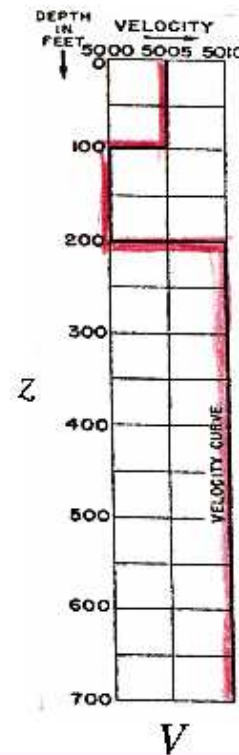
- Use hydroacoustic  $T$  phases propagating in water column at high frequencies ( $f \geq 3$  Hz) to explore relative properties of earthquake source in different frequency windows and detect any anomalous behavior.
- Define  $T$ -PHASE ENERGY FLUX (TPEF) using algorithm similar to  $E^E$  and scale to moment  $M_0$  to obtain new slowness parameter  $\gamma$ .
- Define Amplitude-Duration discriminant  $D$  to characterize slowness of events.
- Examine correlation between  $\Theta$ ,  $\gamma$  and  $D$ .

[Talandier and Okal, 2003] [Okal et al., 2003]

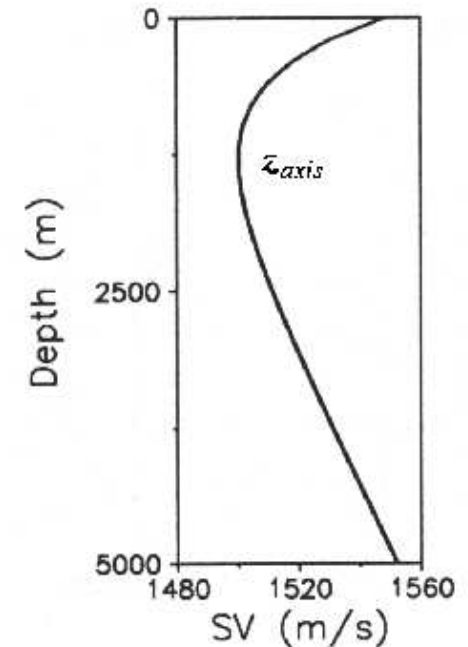
## THE SOFAR CHANNEL

- Variations in pressure, temperature and salinity of seawater with depth create a *channel of minimum velocity* around  $z = 1000$  m.
- This acts as a **WAVEGUIDE** allowing exceptionally efficient propagation of acoustic energy in the ocean basins ( $f \geq 3$  Hz).

[Pekeris, 1948]



[Munk, 1972]



# T PHASE ENERGY FLUX (TPEF) and PARAMETERS $\Gamma$ ( $\gamma$ )

[Okal et al., 2003]

- We seek to combine the amplitude and duration information to retrieve a measure of source size.
- Recall the definition of *Seismic Energy radiated into Body Waves* [Boatwright and Choy, 1986; Newman and Okal, 1998]: integrate energy flux at receiver; correct for distance.
- Define  $TPEF = \rho \alpha \int_W [\dot{u}(t)]^2 \cdot dt$ ,

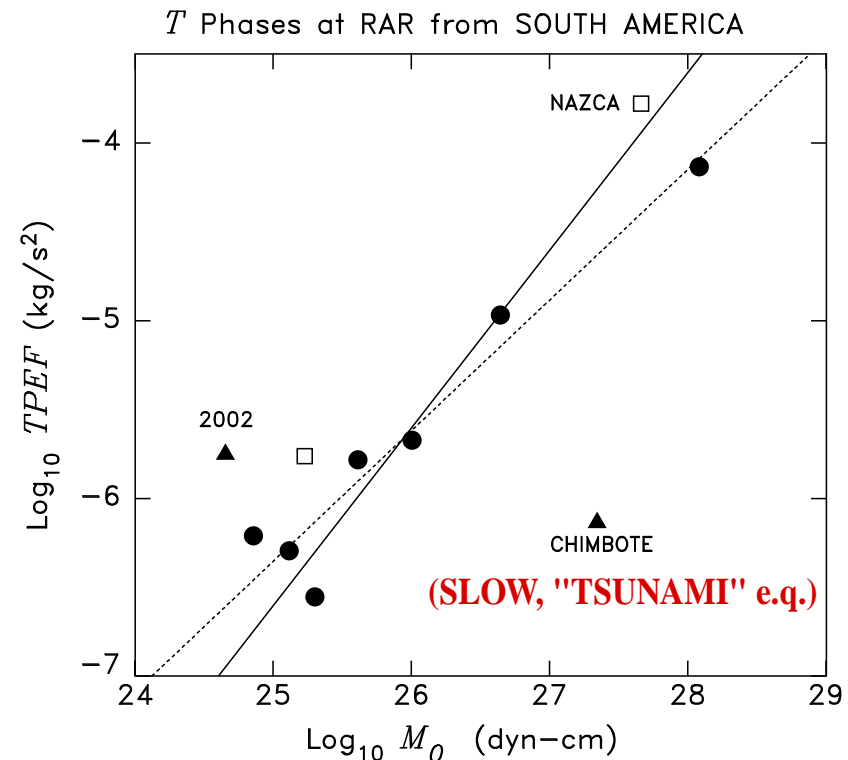
which is more readily computed in the Fourier domain as

$$TPEF \approx \frac{\rho \alpha}{\pi} \int_{\omega_{\min}}^{\omega_{\max}} \omega^2 |U(\omega)|^2 \cdot d\omega$$

- To eliminate receiver effects, *use ONLY TO COMPARE RECORDS AT SAME RECEIVING STATION*
- Then TPEF scales with *MOMENT*. Define

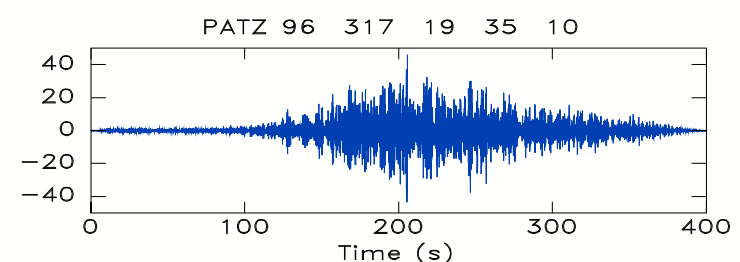
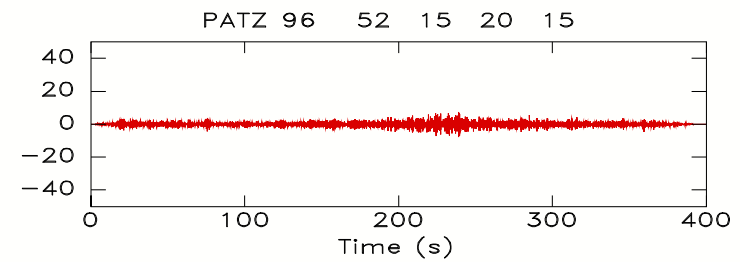
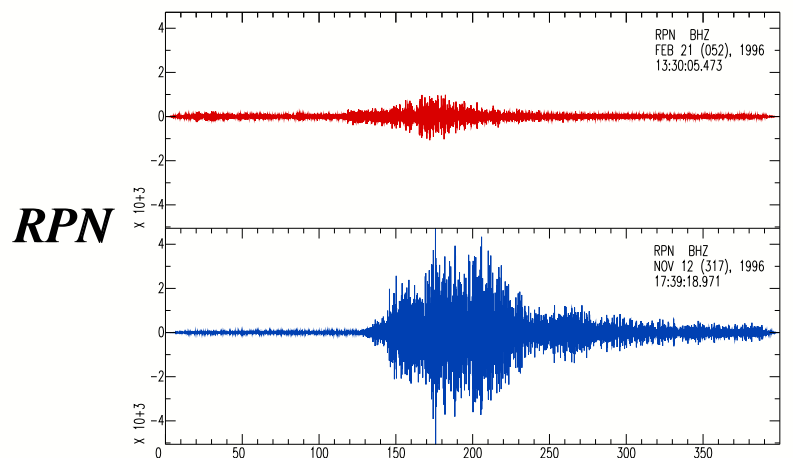
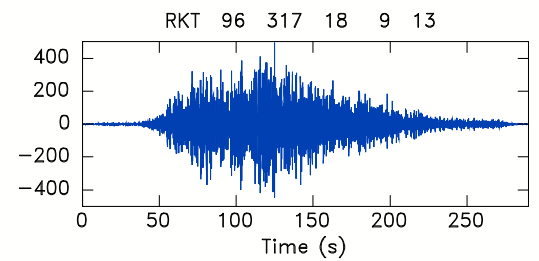
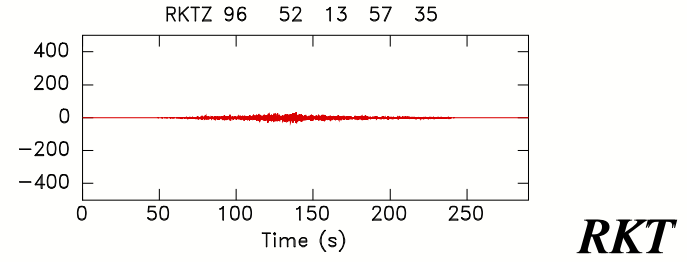
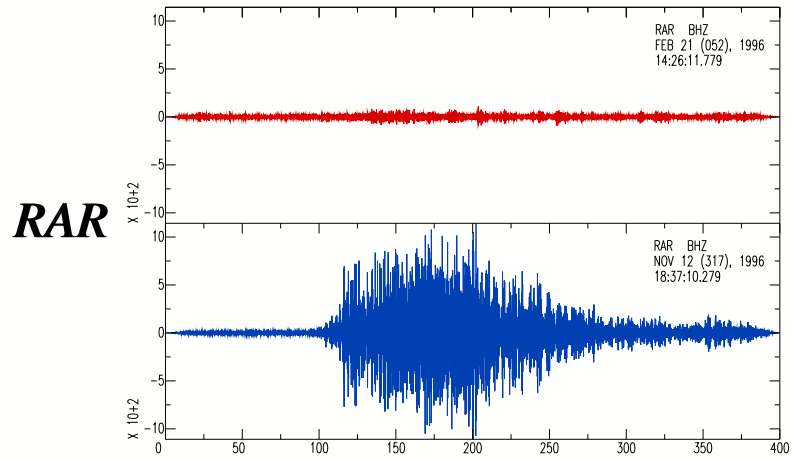
$$\Gamma = \frac{TPEF}{M_0} \quad \text{and} \quad \gamma = \log_{10} \Gamma + 30$$

$\Gamma$  is invariant for constant source–receiver geometries.



# TPEF ( $\Gamma$ ; $\gamma$ ) IDENTIFYING SLOW ("TSUNAMI") EARTHQUAKES

Contrast **CHIMBOTE, Peru (21 FEB 1996)** and **NAZCA, Peru (12 NOV 1996)**



[Okal et al., 2003]

$$\gamma_C - \gamma_N = -1.14 \text{ to } -2.29 \text{ log. units}$$

# T WAVES as a PROXY to SOURCE SLOWNESS

- Use  $T$  phases at RAR from a series of regular, fast, and slow earthquakes in Peru and Chile.
- Compare the three parameters
  - \*  $\Theta = \log_{10} E^E / M_0$   
Energy-to-moment ratio, characterizing slowness of the source.  
[Newman and Okal, 1998]
  - \*  $\gamma = \log_{10} \frac{TPEF}{M_0} + 30$ :  
 $T$  -phase efficiency of the source
  - \*  $D$ : amplitude-duration discriminant.

A remarkable correlation exists between all 3.

