# Nomenclature

$\alpha$ = Angle of Forward Strap of After Pontoon w.r.t. the Horizontal
$\beta$ = Angle of Backward Strap of After Pontoon w.r.t. the Vertical
$\gamma$ = Angle of Inclination of the Ramp Boom w.r.t. the Horizontal
$\theta$ = Angle Associated with Depth of Submersion of After Pontoon
$\rho$ = Density
$b$ = Depth of Submersion of After Pontoon
$c$ = Distance Associated with Depth of Submersion of After Pontoon
$r$ = Radius of Pontoons
$t$ = Projected Distance of Center of Gravity of Foam Blocks
$u$ = Hypotenuse of the Triangular Foam Blocks
$v$ = Projected Distance of Center of Gravity of Ramp Boom Sheet
$x$ = Projected Distance of Center of Gravity of Ballast Weight
$y$ = Projected Distance of Center of Gravity of Aluminum Stiffeners
$z$ = Overall Length of Ramp Boom Sheet
$C_L$ = Lift Coefficient
$F_{Block}$ = Buoyancy of Syntactic Foam Blocks
$F_1$ = Buoyant Force of Forward Pontoon
$F_2$ = Averaged Buoyancy of Syntactic Foam Blocks
$F_b$ = Buoyant Force of After Pontoon
$F_c$ = Lift Force due to Water Current
$F_{ch}$ = Horizontal Lift Component due to Water Current
$F_{cv}$ = Vertical Lift Component due to Water Current
$F_{pb}$ = In-water Weight of Ballast Material
$F_{pl}$ = Weight of Ramp Boom Sheet
$F_s$ = In-water Weight of Aluminum Stiffener
$L_1$ = Tensile Force Required to Restrain the Boom
$T_1$ = Tensile Force Induced on the Forward Strap of After Pontoon
$T_2$ = Tensile Force Induced on the Backward Strap of After Pontoon
$V$ = Velocity of Water Current
$\vec{X}$ = Vector of Design Variables

# Force Analysis on the Ramp Boom Surface

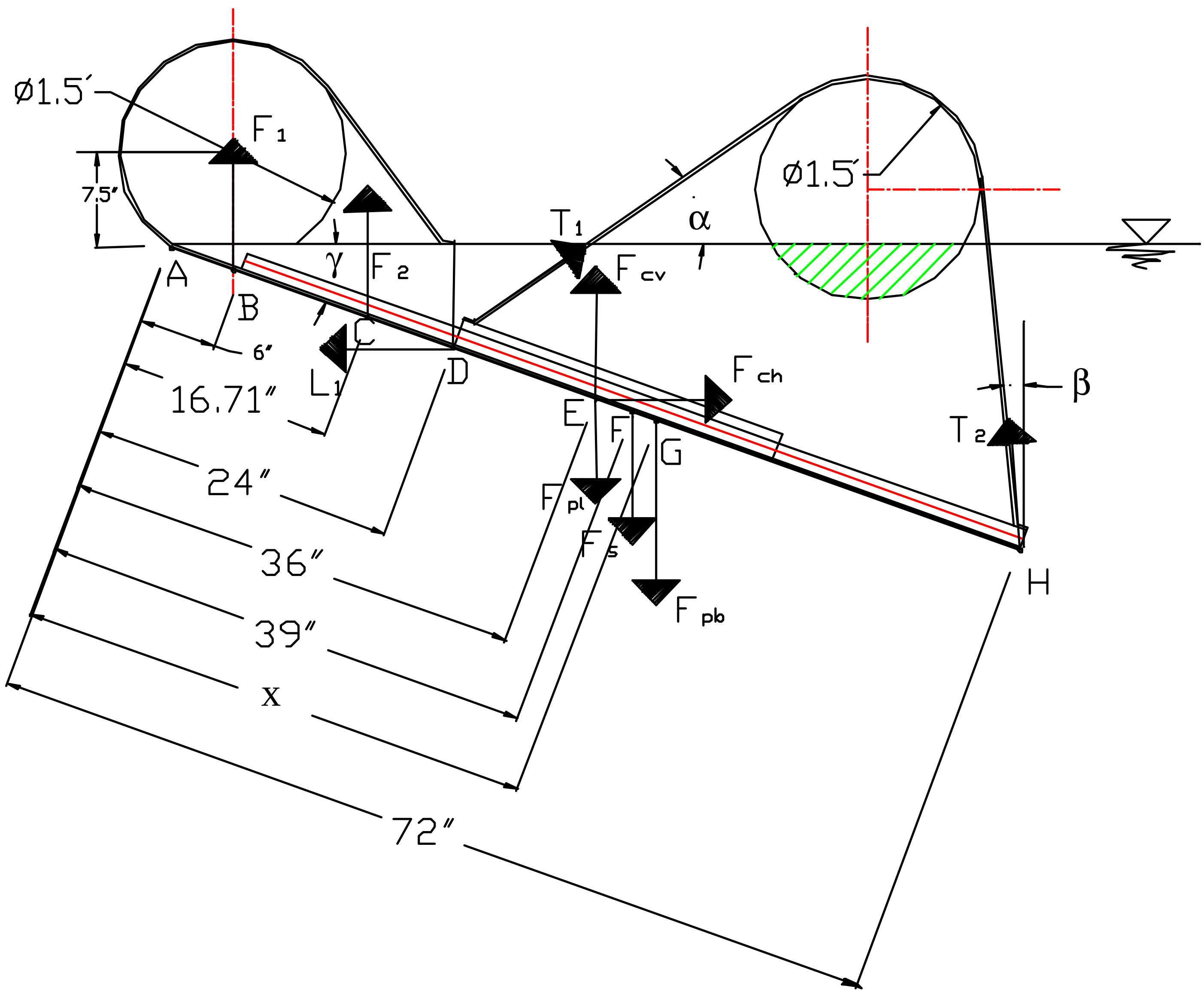


Figure 1

## *Assumptions, Design Highlights*

1. The Ramp Boom will be treated as a rigid plate in the fore and aft dimensions due to the reinforcement provided by the 66" long aluminum angle stiffeners and the triangular shaped syntactic foam fwd. flotation wedges.

2. The forward and aft pontoons are to be inflated during deployment and deflated upon recovery. Their weight is insignificant with respect to the other Ramp Boom components.

3. The syntactic foam fwd. floatation wedge is to be installed in 30" wide sections with a 12" wide wedge-shaped gap between each section. A free-flooding covered space is to be contained in the 12" wide gap between the 30" wide syntactic foam wedges. This will permit the Ramp Boom to be folded for storage. The free flooding covered-spaces will not be open at the forward edge of the Ramp Boom so that no oil can pass through the fwd. pontoon.

4. The Ramp Boom surface will be fabricated from Poly-fabric reinforced Polyurethane sheet with a density of 36 $oz/yd^2 = 0.25\ {}^{\#}/ft^2$ .

5. The lift coefficient, $C_L$ , is calculated for $20^o$ ramp boom inclination angle by using computational fluid dynamics methods. Since this angle corresponds to the midpoint of the range of inclination angles of interest and since the range is narrow ( $20^o \pm 5^o$ ), the lift coefficient found for $20^o$ will be taken to be constant over the interval. That is, the dependency of $C_L$ on the ramp angle is neglected.

6. The aluminum reinforcing angles used to stiffen the Polyurethane sheet used for the construction of the Ramp Boom are to be placed on the top of the sheet and will be held in place with rivets. The angle size chosen for this design is 1"x1"x1/4". The alloy with the best strength and corrosion resistance for this application is 6061-T6. The length of each stiffener is 66" and they are spaced 6" apart. To further aid in stiffening the Ramp Boom in the fore and aft directions, these reinforcing angles will be placed starting at the aft-most end and running 66" forward. This will place them under the syntactic foam block, which will reinforce the area of the Ramp Boom just aft of the block.

## Force Analysis of Plate A-H[(*)]

$F_1 = 0$ since the equilibrium position of the forward pontoon under 3 knots of current is desired to be non-submerged.

## Calculation of $F_2$

First the buoyancy force of the syntactic foam block will be calculated in an assumed condition of total submersion in sea water for a unit width of 1 ft. ($\rho_{\text{sea water}} = 64^{\#}/\text{ft}^3$ , $\rho_{\text{foam block}} = 30^{\#}/\text{ft}^3$ )

$F_{\text{Block}}$ = (Volume of Block) – (Volume of Seawater Displaced)

$$F_{\text{Block}} = \left[\frac{1}{2}(24'')^2 \sin g \cos g \,(1\,ft)\left(\frac{1\,ft^2}{144 in^2}\right)\left(64^{\#}/ft^3\right)\right] - \left[\frac{1}{2}(24'')^2 \sin g \cos g \,(1\,ft)\left(\frac{1\,ft^2}{144 in^2}\right)\left(30^{\#}/ft^3\right)\right]$$

$$F_{\text{Block}} = \left[(64-30)^{\#}/ft^3\right]\left[\frac{1}{2}(24'')^2 \sin g \cos g \,(1\,ft)\left(\frac{1\,ft^2}{144 in^2}\right)\right]$$

Averaging this buoyant force over a 42” overall width to account for a 30” wide block section and a 12” wide free-flooded section.

$$F_2 = \frac{30''\,F_{\text{Block}} + 12''\,F_{(12''\text{ wide free flooded section})}}{42''} \qquad \text{where } F_{(12''\text{ wide free flooded section})} = 0$$

$$F_2 = \frac{30''}{42''}\left[(64-30)^{\#}/\,ft^3\right]\left[\frac{1}{2}(24'')^2 \sin g \cos g \,(1\,ft)\left(\frac{1\,ft^2}{144 in^2}\right)\right]$$

$$F_2 = 48.57\left[\sin g \cos g\right]$$

*(*) for application points of the forces, refer to figure 1.*

**Calculation of $F_{cv}$ and $F_{ch}$**

The equation for the normal lift on a fixed inclined plate submerged in a fluid, moving with a velocity of 3 knots is :

$F_c = (1/2).\rho.\ V^2\ .\ C_L\ .\ A$ where $\rho = (64^{\#}/ft^3) / (32.2 ft/sec^2) = 1.99$ slugs/ft

$V = 3$ kts $= 5$ ft/sec

$C_L = 0.7$ for a $20^0$ angle

$A = 1' \times 6' = 6\ ft^2$

$F_c = (1/2)[(64^{\#}/ft^3)/(32.2ft/sec^2)](5ft/sec)^2(0.7)(6ft^2)$

$F_c = 104.35^{\#}$ Normal to the plane of the Ramp Boom

Breaking this normal force into its vertical and horizontal components for a unit Ramp Boom width of 1 ft yields :

$F_{cv} = 104.35^{\#} (\cos\gamma)$ Vertical lift component

$F_{ch} = 104.35^{\#} (\sin\gamma)$ Horizontal lift component

Summing the horizontal forces from the Ramp Boom free-body diagram;

$L_1$ = Tensile force required to restrain the boom / unit length

$L_1 = F_{ch} = 104.35^{\#} (\sin\gamma)$

**Calculation of $F_{pl}$**

The density of the Polyurethane sheet to be used is 36 $oz/yd^2$ .

$= (36\ oz)(0.0625^{\#}/oz) / (yd^2)(9ft^2/yd^2) = 0.25^{\#}/ft^2$

$F_{pl} = (0.25^{\#}/ft^2)(6ft^2) = 1.5^{\#}$ per unit width in air.

Note : It will be assumed that the in-water weight will be equal to the in-air weight for the Polyurethane sheet.

**Calculation of $F_s$**

$F_{s'}$ = The in-air weight of the pair of stiffeners used in each unit (12") width of the Ramp Boom
$F_{s'} = (2)(0.51^{\#}/\text{linear ft})[(66")/(12"/\text{ft})] = 5.6^{\#}$

The volume of the 2 aluminum stiffeners ;
$= (5.6^{\#}) / (0.0966^{\#}/\text{in}^3)(1728\text{in}^3/\text{ft}^3) = 0.034 \text{ ft}^3$

The volume of the saltwater displaced by the aluminum stiffeners ;
$= (64^{\#}/\text{ft}^3)(0.034\text{ft}^3) = 2.2^{\#}$

$F_s$ = The actual in-water weight of the aluminum stiffeners, and;
$F_s = F_{s'} - 2.2^{\#} = 5.6^{\#} - 2.2^{\#} = 3.4^{\#}$
Note : This weight is for a *pair* of aluminum stiffeners.

**Calculation of $F_b$**

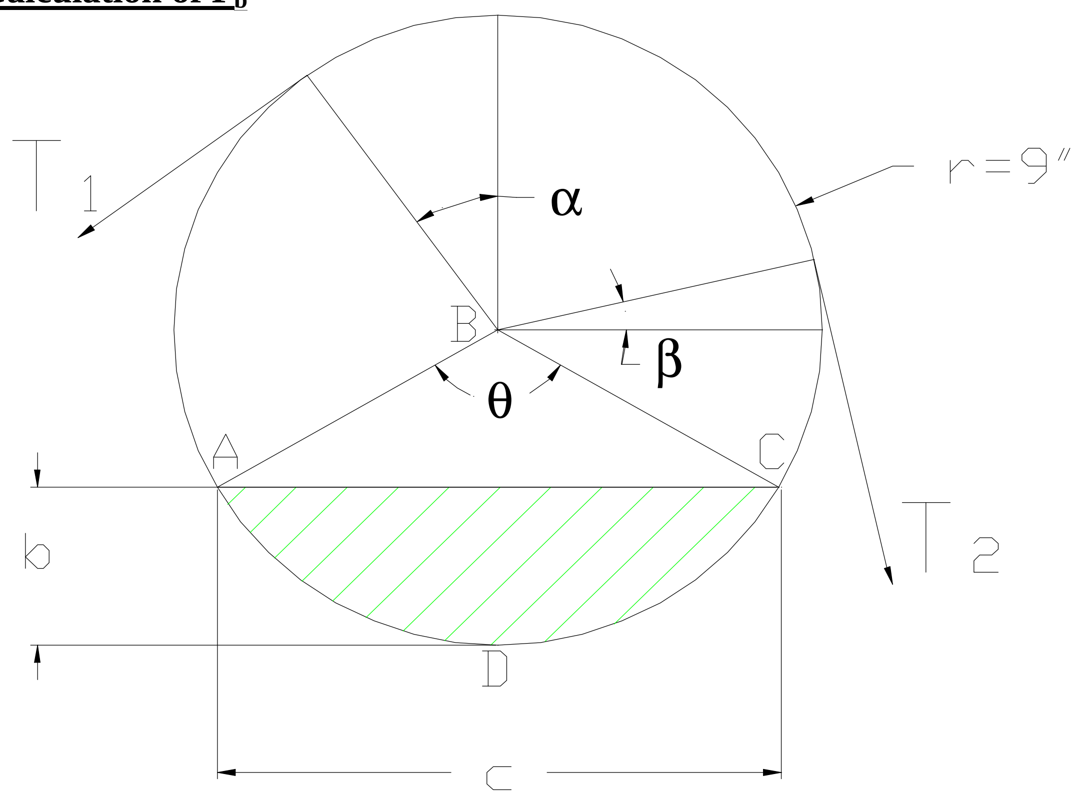


**Figure 2**

From geometry, refer to figure 2,

$$c = 2\sqrt{2rb - b^2} = 2r\sin\left(\frac{\theta}{2}\right)$$

$A' = Area$ of the submerged sector ABCD including $\Delta$ABC

$$\mathrm{A}' = \pi \mathrm{r}^2 \frac{\theta}{360} \qquad \theta : \text{in degrees}$$

A = Area of segment ADC $= A_{ABCD} - A_{\Delta ABC}$

$$A = \pi r^2 \frac{\theta}{360} - \frac{1}{2}c(9-b)$$

combining these equations to find $\mathrm{F_b}$, with r = 9"

$$F_b = \left[81\pi\left(\frac{\theta}{360}\right) - \frac{1}{2}c(9-b)\right]\left(64^{\#} / ft^3\right)\left(\frac{1 ft^2}{144 in^2}\right)$$

where $\rho_{\text{sea water}} = 64^{\#} / ft^3$

From vertical force equilibrium of after pontoon, substituting the 1st eqn

$$T_1 \sin\alpha + T_2\cos\beta = \left(\frac{64}{144}\right)\left[9\pi\left(\frac{\sin^{-1}\left(\sqrt{18b-b^2}/9\right)}{20}\right) - (9-b)\sqrt{18b-b^2}\right]$$

From horizontal force equilibrium of after pontoon

$$T_1\cos\alpha - T_2\sin\beta = 0$$

## Calculation of $F_{pb}$

Combining the necessary equations obtained so far (refer to figure 1), the vertical force equilibrium of the ramp boom gives,

$$48.57\sin\gamma\cos\gamma + T_1\sin\alpha + T_2\cos\beta + 104.35\cos\gamma - F_{pb} - 1.5 - 3.4 = 0$$

Taking the moment of the forces about point A, with some simplification

$$F_{pb} = \frac{-354.086\sin\gamma\cos\gamma + 1252.2\cos\gamma + 48T_2\cos\beta + 24T_1\cos\alpha\tan\gamma - 72T_2\sin\beta\tan\gamma + 1252.2\sin\gamma\tan\gamma - 69}{x-24}$$

## Problem Definition

By use of the equations obtained so far, the minimization problem can now be defined as follows:

Find $\overset{r}{X}^{T} = \{ x \; g \; a \; b \; T_1 \; T_2 \; b \}$

which minimizes;

$$F_{pb}\left(\overset{r}{X}\right) = \left\{ \frac{-354.086 \sin g \cos g + 1252.2 \cos g + 48 T_2 \cos b + 24 T_1 \cos a \tan g - 72 T_2 \sin b \tan g + 1252.2 \sin g \tan g - 69}{x - 24} \right\}$$

subject to;

$$T_1 \sin a + T_2 \cos b = \left(\frac{64}{144}\right)\left[ 9p \left( \frac{\sin^{-1}\left(\sqrt{18b - b^2} / 9\right)}{20} \right) - (9 - b)\sqrt{18b - b^2} \right]$$

$$T_1 \cos a - T_2 \sin b = 0$$

$$\frac{24 \sin g + 9 - b + 9 \cos a}{\tan a} + (\tan b)(72 \sin g + 9 \sin b + 9 - b) + 9(\sin a + \cos b) - 48 \cos g = 0$$

$$g + b = 25^{o}$$

$$b \leq 9$$

$$b \leq 10^{o}$$

$$24 \cos g + \frac{24 \sin g + 9 \cos a + 9 - b}{\tan a} + 9 \sin a \leq 72 \cos g - 9$$

$$x \leq 24 \cos g + \frac{24 \sin g + 9 \cos a + 9 - b}{\tan a} + 9 \sin a$$

$$\overset{r}{X} \geq \overset{r}{0}$$

Note : All quantities that are specified in the assumptions section and some dimensions of the system as given in figure 1 are taken as pre-assigned parameters of the minimization problem. The problem is defined accordingly.

In the problem definition as stated above, the last two equality constraints and the inequality constraints require further explanation.

The first of the last two equality constraints is a geometric constraint. It is derived such that starting from a reference line (water surface is chosen for that purpose), the same elevation should be reached by following two different paths along the ramp boom system to arrive a final point (center of aft pontoon is chosen for that purpose). A careful investigation of the equation yields to the conclusion that, it implies nothing but treatment of the system as a rigid body. That is the distance between any two points on a rigid body is fixed.

The second of the last two equality constraints arise from the static equilibrium of the ramp boom system. In the static case the lift force due to the water current no longer exists. This will cause an unbalanced moment in the clockwise direction. Hence, it is evident by intuition that the system will come to a new equilibrium position where $\beta=0$ (see figure 1). Also both of the pontoon will be submerged in order to satisfy the vertical force equilibrium. The ramp inclination angle ($\gamma$) is required to be in the range $15^o$-$25^o$. This range of $\gamma$ is known to correspond to the maximum amount of oil collected by this ramp boom system. For further information refer to (Fang, 2000). Therefore with this constraint the upper limit of $\gamma$ is set to $25^o$ which is achieved at the static case where $\beta=0$. (see Appendix 2)

The first inequality constraint sets the upper limit of the depth of submersion of the after pontoon (b) as 9 inches, which is equal to the radius of the pontoon. This is a necessity arising from the need of reserve buoyancy that should be kept on after pontoon considering the static position.

The second inequality constraint, in conjunction with the last equality constraint sets the lower limit of $\gamma$ as $15^o$. The third inequality constraint makes sure that the center line of the after pontoon always remain on top of the ramp boom plate. The fourth inequality constraint ensures that the center of gravity of the ballast weight remains between the centers of the two pontoons. This is required as far as the moment stability of the system is concerned.

The last inequality constraint is the non-negativity condition imposed on the design variables which is evident from the physics of the problem.

## Classification of the Problem

The problem just stated above belongs to the following categories:
It is a constrained optimization problem.
It is a parameter optimization problem.
It is a non-optimal control problem.
It is a nonlinear optimization problem.
It is a real-valued optimization problem.
It is a deterministic optimization problem.
It is a non-separable optimization problem.
It is a single objective optimization problem.

## Solution

This problem can be solved by use of any of the well-known nonlinear constrained optimization techniques. The "solver" algorithm which is built in Microsoft Excel is used for that purpose. The algorithm uses generalized reduced gradient (GRG2) method to optimize constrained nonlinear problems.

The design variables are each assigned a cell in the Excel worksheet. The constraints are separated into two parts such that the right-hand sides and left-hand sides of each inequality (or equality) are assigned to a different cell. Finally the objective function is assigned to the last cell. Then the problem is defined to "solver" by means of relating the cells accordingly to express the constraints. The target cell is chosen as the one containing the objective function. The cells that contained design variables are chosen to be the ones to be changed to minimize the target cell. The "solver" has been run several times for different starting points and the objective function is minimized. The results are summarized below. (see App 3)

| design variables | alpha | beta | gamma | x | T1 | T2 | b |
|---|---|---|---|---|---|---|---|
| valid starting point | 25 | 15 | 40 | 100 | 15 | 60 | 10 |
| valid starting point | 30 | 5 | 20 | 100 | 2.5 | 25 | 8 |
| valid starting point | 45 | 25 | 30 | 25 | 50 | 2 | 0 |
| valid starting point | 5 | 5 | 3 | 50 | 0.5 | 100 | 15 |
| valid starting point | 1E-07 | 1E-07 | 1E-07 | 25 | 1E-07 | 1E-07 | 1E-07 |
| | | | | | | | |
| optimum point | 37.331 | 10 | 15 | 35.887 | 1 | 4.579 | 1.6362 |
| optimum weight | 113.15 | | | | | | |

**Table 1**

As it can be seen from table 1, there is a wide spectrum of starting points which in turn leads to the same optimum solution. The solution has converged to a finite value of 113.15#/ft of additional ballast weight to be added to the ramp boom system in order to keep the system in equilibrium under 3 knots of water current. This result is satisfactory and consistent with the expectations.

For starting points that are far away from the optimum solution, the "solver" failed to find a solution. Such points are indicated below.

| design variables | alpha | beta | gamma | x | T1 | T2 | b |
|---|---|---|---|---|---|---|---|
| invalid starting point | 100000 | 100000 | 100000 | 100000 | 100000 | 100000 | 100000 |
| invalid starting point | 50 | 100 | 100 | 100 | 100 | 100 | 100000 |

**Table 2**

Also for starting points that included a value of "x" that is smaller than 24", a feasible solution could not be found by the "solver". Due to the characteristic of the problem, when "x" is assigned a value that is smaller than 24", the algorithm searched through the direction of decreasing "x", which in turn resulted in failure to find a feasible solution. As it can be seen from table 1, the optimum value of "x" is 35.887" from the tip of the ramp boom.

The effect of pre-assigned parameters on the solution is also observed. The objective function and the constraints are modified such that they are expressed as functions of design variables and pre-assigned variables. The "solver" has been run in the same manner as explained above but changing one pre-assigned parameter at a time. Every pre-assigned parameter is changed to ±5% of its original value. The results are tabulated in Appendix 1. It can be seen from the table that none of the pre-assigned parameters has a significant effect on the optimum value when it is subject to perturbations. Physically this conclusion implies that if any of the dimensions or weights that is taken as a pre-assigned parameter appears to be different than its original value due to manufacturing faults, it will not contribute to a significant change in the system's behavior. For example if the radius of the after pontoon happens to be 8.55" (5% smaller than the original value), it corresponds to only a 3" shift in the position of the ballast weight, with the ballast weight itself remaining approximately constant. (App. 1) Such small deviations can be tolerated in this system, so that it is not sensitive.

# Conclusion

In this study, a mathematical programming technique is used to optimize a practical engineering problem. The minimum weight that should be added to a ramp oil boom system is found by using a nonlinear constrained optimization technique, namely, generalized reduced gradient method.

The problem is first formulated by means of force analysis on the ramp boom system and the constraints are set due to geometric dependencies. Behavior as well as side constraints are set at this stage. Finally, the "solver" algorithm which was available as an Excel application has been run to solve the problem.

As a result, the minimum ballast weight that should be added to the ramp boom system was found out to be $113.15^{\#}$/ft and this result agreed well with the expectations. The corresponding values of the design variables also appeared to be reasonable. Physically it is possible to build such a system. The effect of pre-assigned parameters has also been investigated. Some specific dimensions on the ramp boom, the center of gravity of various components and the weight of some of the components were set as pre-assigned parameters at the beginning of the study. These, if they had difference between their assigned and the actual values, could have effected the system performance and hence the optimum solution. For that purpose, the pre-assigned parameters are changed by 5% of their original values, one at a time and the corresponding optimum solution is found for each case. At the end, it is found that none of the pre-assigned parameters significantly effected the system's performance and hence the optimum value when they are subject to changes. This proved the stability and insensitivity of the system against manufacturing defects and faults.

# Appendix 1

| alpha | beta | gamma | x | T1 | T2 | b | weight | t | u | v | y | z | r | Fpl | Fs |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 37.3313 | 10 | 15 | 35.8872 | 1 | 4.579041 | 1.63616 | 113.153 | 16.71 | 24 | 36 | 39 | 72 | 9 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 35.9769 | 1 | 4.579041 | 1.63616 | 113.153 | 17.545 | 24 | 36 | 39 | 72 | 9 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 35.7975 | 1 | 4.579041 | 1.63616 | 113.153 | 15.874 | 24 | 36 | 39 | 72 | 9 | 1.5 | 3.4 |
| 38.9902 | 10 | 15 | 34.3788 | 1 | 4.476022 | 1.61901 | 114.319 | 16.71 | 25.2 | 36 | 39 | 72 | 9 | 1.5 | 3.4 |
| 35.7557 | 10 | 15 | 37.3892 | 1 | 4.673331 | 1.65152 | 112.04 | 16.71 | 22.8 | 36 | 39 | 72 | 9 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 37.5819 | 1 | 4.579041 | 1.63616 | 113.151 | 16.71 | 24 | 37.8 | 39 | 72 | 9 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 34.1925 | 1 | 4.579041 | 1.63616 | 113.153 | 16.71 | 24 | 34.2 | 39 | 72 | 9 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 35.8286 | 1 | 4.579041 | 1.63616 | 113.153 | 16.71 | 24 | 36 | 40.95 | 72 | 9 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 35.9458 | 1 | 4.579041 | 1.63616 | 113.153 | 16.71 | 24 | 36 | 37.05 | 72 | 9 | 1.5 | 3.4 |
| 34.1864 | 10 | 15 | 36.0882 | 1 | 4.763735 | 1.66591 | 113.290 | 16.71 | 24 | 36 | 39 | 75.6 | 9 | 1.5 | 3.4 |
| 41.0726 | 10 | 15 | 35.6820 | 1 | 4.341405 | 1.59606 | 112.969 | 16.71 | 24 | 36 | 39 | 68.4 | 9 | 1.5 | 3.4 |
| 39.3201 | 10 | 15 | 35.8478 | 1 | 4.455083 | 1.58748 | 113.058 | 16.71 | 24 | 36 | 39 | 72 | 9.4 | 1.5 | 3.4 |
| 35.3736 | 10 | 15 | 35.9244 | 1 | 4.69567 | 1.68604 | 113.240 | 16.71 | 24 | 36 | 39 | 72 | 8.5 | 1.5 | 3.4 |
| 37.3313 | 10 | 15 | 35.8871 | 1 | 4.579041 | 1.66168 | 113.078 | 16.71 | 24 | 36 | 39 | 72 | 9 | 1.6 | 3.4 |
| 37.3313 | 10 | 15 | 35.8873 | 1 | 4.579041 | 1.63616 | 113.228 | 16.71 | 24 | 36 | 39 | 72 | 9 | 1.4 | 3.4 |
| 37.3313 | 10 | 15 | 35.8825 | 1 | 4.579041 | 1.63616 | 112.983 | 16.71 | 24 | 36 | 39 | 72 | 9 | 1.5 | 3.57 |
| 37.3313 | 10 | 15 | 35.8919 | 1 | 4.579041 | 1.63616 | 113.323 | 16.71 | 24 | 36 | 39 | 72 | 9 | 1.5 | 3.23 |

# Appendix 2

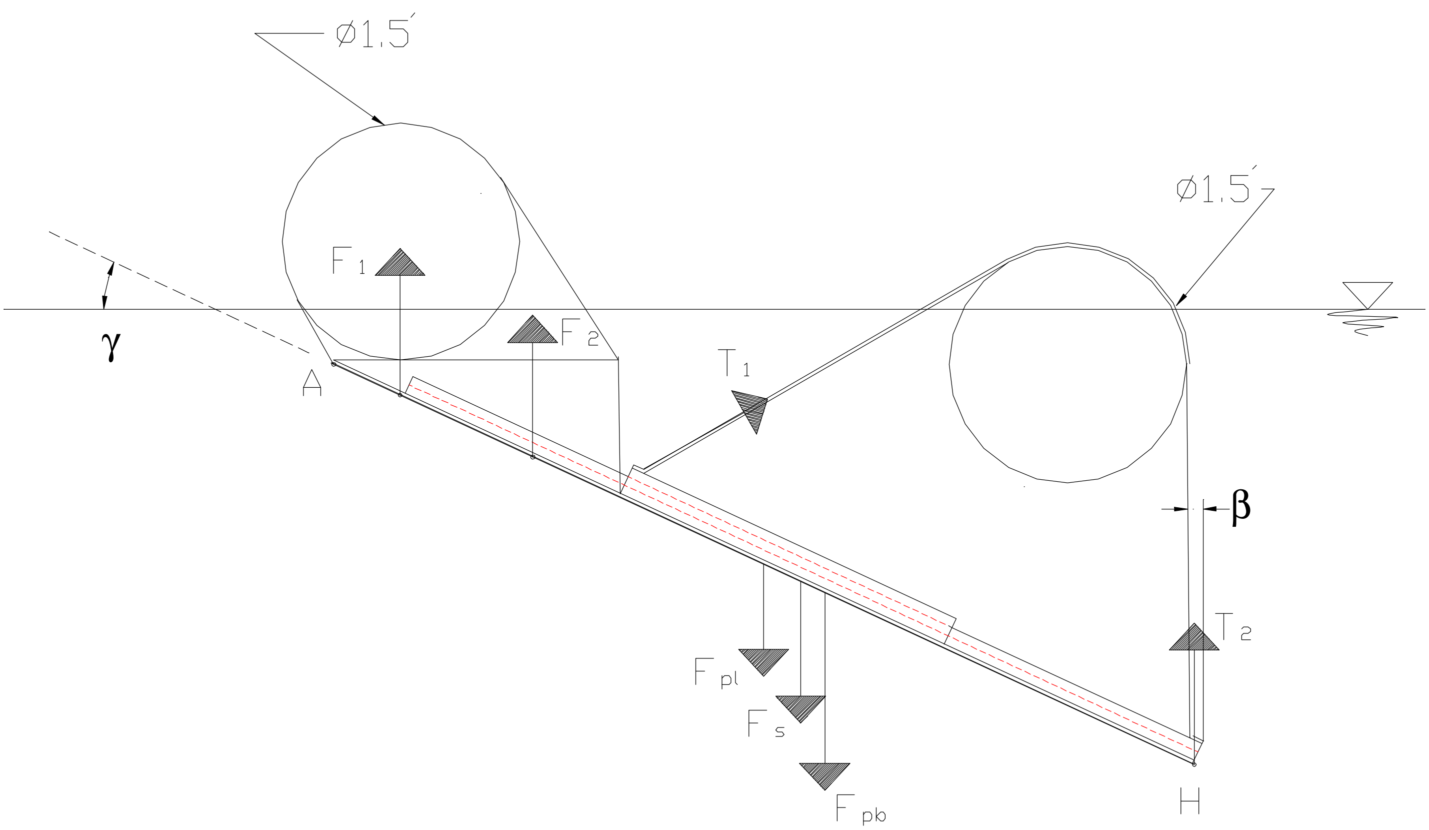


**Figure 3**

# Appendix 3

## Answer Report

Target Cell (Min)

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $M$2 | weight | 79.43835299 | 113.1531155 |

Adjustable Cells

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $A$2 | alpha | 40 | 37.33131556 |
| $B$2 | beta | 10 | 10 |
| $C$2 | gamma | 50 | 15 |
| $D$2 | x | 100 | 35.88725067 |
| $E$2 | T1 | 50 | 1 |
| $F$2 | T2 | 100 | 4.57904121 |
| $G$2 | b | 9 | 1.636167939 |

Constraints

| Cell | Name | Cell Value | Formula | Status | Slack |
|---|---|---|---|---|---|
| $D$2 | x | 35.88725067 | $D$2<=$N$2 | Not Binding | 19.93627884 |
| $H$2 | T1cosa-T2sinbe | -8.37674E-09 | $H$2=0 | Binding | 0 |
| $I$2 | T1sina+T2cosbe | 5.115898369 | $I$2=$J$2 | Not Binding | 0 |
| $K$2 | gamma+beta | 25 | $K$2=25 | Binding | 0 |
| $L$2 | 4-left | 2.25769E-08 | $L$2=0 | Binding | 0 |
| $M$2 | weight | 113.1531155 | $M$2=$O$2 | Binding | 0 |
| $N$2 | aftpont position | 55.82352952 | $N$2<=72*COS($C$2*PI()/180)-9 | Not Binding | 4.723129974 |
| $B$2 | beta | 10 | $B$2<=10 | Binding | 0 |
| $A$2 | alpha | 37.33131556 | $A$2>=0 | Not Binding | 37.33131556 |
| $B$2 | beta | 10 | $B$2>=0 | Not Binding | 10 |
| $C$2 | gamma | 15 | $C$2>=0 | Not Binding | 15 |
| $D$2 | x | 35.88725067 | $D$2>=0 | Not Binding | 35.88725067 |
| $E$2 | T1 | 1 | $E$2>=0 | Not Binding | 1 |
| $F$2 | T2 | 4.57904121 | $F$2>=0 | Not Binding | 4.57904121 |
| $G$2 | b | 1.636167939 | $G$2>=0 | Not Binding | 1.636167939 |
| $E$2 | T1 | 1 | $E$2>=1 | Binding | 0 |
| $F$2 | T2 | 4.57904121 | $F$2>=1 | Not Binding | 3.57904121 |
| $G$2 | b | 1.636167939 | $G$2<=9 | Not Binding | 7.363832061 |

# Appendix 3 (contd.)

## Sensitivity Report

Adjustable Cells

| Cell | Name | Final Value | Reduced Gradient |
|---|---|---|---|
| $A$2 | alpha | 37.33131556 | 0 |
| $B$2 | beta | 10 | -0.707944809 |
| $C$2 | gamma | 15 | 0 |
| $D$2 | x | 35.88725067 | 0 |
| $E$2 | T1 | 1 | 5.194694837 |
| $F$2 | T2 | 4.57904121 | 0 |
| $G$2 | b | 1.636167939 | 0 |

Constraints

| Cell | Name | Final Value | Lagrange Multiplier |
|---|---|---|---|
| $D$2 | x | 35.88725067 | 0 |
| $H$2 | T1cosa-T2sinbe | -8.37674E-09 | -5.758632363 |
| $I$2 | T1sina+T2cosbe | 5.115898369 | -0.015402287 |
| $K$2 | gamma+beta | 25 | 0.216997662 |
| $L$2 | 4-left | 2.25769E-08 | 0.047623824 |
| $M$2 | weight | 113.1531155 | 1 |
| $N$2 | aftpont position | 55.82352952 | 0 |

## Limits Report

| Cell | Target Name | Value |
|---|---|---|
| $M$2 | weight | 113.1531155 |

| Cell | Adjustable Name | Value | Lower Limit | Target Result | Upper Limit | Target Result |
|---|---|---|---|---|---|---|
| $A$2 | alpha | 37.33131556 | 37.33131556 | 113.1531155 | 37.33131556 | 113.1531155 |
| $B$2 | beta | 10 | 10 | 113.1531155 | 10 | 113.1531155 |
| $C$2 | gamma | 15 | 15 | 113.1531155 | 15 | 113.1531155 |
| $D$2 | x | 35.88725067 | 35.88725067 | 113.1531155 | 35.88725067 | 113.1531155 |
| $E$2 | T1 | 1 | 1 | 113.1531155 | 1 | 113.1531155 |
| $F$2 | T2 | 4.57904121 | 4.57904121 | 113.1531155 | 4.57904121 | 113.1531155 |
| $G$2 | b | 1.636167939 | 1.636167939 | 113.1531155 | 1.636167939 | 113.1531155 |