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APPLYING INFORMATION-GAP REASONING TO THE PREDICTIVE ACCURACY ASSESSMENT OF TRANSIENT DYNAMICS SIMULATIONS

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ABSTRACT

An alternative to the theory of probability is applied to the problem of assessing the robustness, to parametric sources of uncertainty, of the correlation between measurements and computer simulations. The uncertainty analysis relies on the theory of information-gap, which models the clustering of uncertain events in families of nested sets instead of assuming a probability structure. The system investigated is the propagation of a transient impact through a layer of hyper-elastic material. The two sources of non-linearity are (1) the softening of the constitutive law representing the hyper-elastic material and (2) the contact dynamics at the interface between metallic and crushable materials. Information-gap models of uncertainty are developed to represent uncertainty in the knowledge of the model’s parameters and in the form of the model itself. Although computationally expensive, it is demonstrated that information-gap reasoning can greatly enhance our understanding of a system when the theory of probability cannot be applied due to insufficient information.

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1. INTRODUCTION

Relying on numerical simulations, as opposed to field measurements, to analyze the structural response of complex systems requires that the predictive accuracy of the models be assessed. This activity is generally known as "model validation" [1]. Model validation requires the comparison of model predictions with test measurements at several points of the design / operational space. For example, numerical models of flutter must be validated for various combinations of fluid velocity and wing angle-of-attack. Because validation experiments become expensive when the system investigated is complex, only a few data sets are generally available. This lack of adequate representation of the design / operational space makes it questionable whether statistical models of predictive accuracy can be developed.

In this work, we focus on one aspect of model validation that consists in assessing the robustness of a decision to uncertainty. In this context, "decision" refers to assessing the accuracy of predictions and verifying that the accuracy is adequate for the purpose intended. Likewise, "uncertainty" can represent experimental variability, variability of the model's parameters but also inappropriate modeling rules in regions of the design / operational space where experiments are not available.

An alternative to the theory of probability is applied to the problem of assessing the robustness of model predictions to sources of uncertainty. The analysis technique is based on the theory of information-gap, which models the clustering of uncertain events in embedded convex sets instead of assuming a probability structure [2]. Unlike other theories developed to represent uncertainty, information-gap does not assume probability density functions (which the theory of probability does) or membership functions (which fuzzy logic does). It is therefore appropriate in cases where limited data sets are available. The main disadvantage of information-gap is that the efficiency of sampling techniques cannot be exploited because no probability structure is assumed. Instead, the robustness of a decision with respect to uncertainty is studied by solving a sequence of optimization problems, which becomes computationally expensive as the number of decision and uncertainty variables increases. The concepts are illustrated with the propagation of a transient impact through a layer of hyper-elastic material [3]. The numerical model includes a softening of the hyper-elastic material's constitutive law and contact dynamics at the interface between metallic and crushable materials. Although computationally expensive, it is demonstrated that the information-gap reasoning can greatly enhance our understanding of a moderately complex system when the theory of probability cannot be applied.

2. UNCERTAINTY AND ITS EFFECT ON COMMON MODELING ACTIVITIES

Uncertainty plays a central role in many activities of modeling and simulation. Clearly, conceptual ambiguity and numerical ambiguity are the result of imprecision or lack of information, which both translate into uncertainty. Epistemic uncertainty occurs in modeling activities when the laws that govern the evolution of a system are not known with absolute certainty. In fact, it may be stated that "***Uncertainty, rather than being an accident of the scientific method, is rather its very nature***." (Adapted from a quote of Andrea Saltelli in Reference [4].) We start by discussing the effect of uncertainty on common modeling activities. The discussion also introduces the notations used throughout this paper.

2.1 Modeling

The numerical model we seek to develop using the finite element method (or any other modeling technique) provides a non-linear mapping between the output features y and the model's decision variables q. The output features generally represent the physical quantities predicted by the model, such as mean stress, peak acceleration, resonant frequencies, etc. The decision variables represent modeling parameters such as those of the constitutive law, design parameters such as the thickness of a shell thickness, or numerical parameters such as friction coefficients. The model is simply denoted by the equation $y=M(q)$.

The selection of q entails both the identification of the model's parameter values as well as the choice between conceptually distinct classes of models. A model specified by q is validated when it can be asserted with confidence that q accurately represents the physical properties of the system throughout the design domain.

Note that the variables q are not restricted to numerical parameters or design variables. Included in the set of decision variables q can be variables that might represent a choice between different modeling rules. For example, one of the variables q_k could represent a choice between an axi-symmetric model or a three dimensional model, if both are consistent with the intended application. This allows for a broad definition of the decision variables q.

2.2 Uncertainty

In addition to the decision variables q, we deal with uncertain variables u represented by a model of uncertainty denoted by $U(u^0;a)$:

$$y = M(q;u), \quad u \in U\left(u^0;a\right) \tag{1}$$

The unknown u may represent a damping mechanism that we are not aware of, a coefficient of strain-rate dependency, a non-linear stiffness parameter, etc. We may have information about the uncertain variables u, however this information may be quite fragmentary. We may not even know the identity of some of these uncertain variables. Or, we may be unsure whether a given variable should be categorized as a decision variable q or an uncertain variable u. This expresses the fact that the actual sources of uncertainty and their influence on the performance indicators y are incompletely known.

The model of uncertainty $U(u^0;a)$ possesses three distinct attributes. First, a theory must be selected to represent the uncertainty mathematically. In most engineering sciences, probability is the theory of choice. In this case, the uncertainty model $U(u^0;a)$ describes the frequency of occurrence of particular events *u*. Other models of uncertainty might include the theory of plausibility and belief of Dempster and Shaffer [5], intervals or fuzzy logic [6].

The second attribute of the uncertainty model $U(u^0;a)$ is u^0, which represents everything that might be known about the quantities *u*. The third attribute is the parameter *a*, which represents the degree of uncertainty. In the context of a probabilistic model of uncertainty, for example, $U(u^0;a)$ represents the set of possible values of a random variable *u*, knowing that a measurement of the variable has provided the value u^0 with an uncertainty level equal to *a*. The uncertainty level *a* could represent the accuracy of the measurement system.

2.3 Decision-making

The main algorithmic difficulty of decision-making is to propagate the uncertainty *u* through the model $y=M(q;u)$ and through a decision-making criterion denoted by $R(q;u)$. The criterion $R(q;u)$ generally depends on the results *y* of the finite element analysis. Without loss of generality, we will assume that the decision is accepted—or the performance of the system is deemed acceptable—if the following inequality is satisfied:

$$R(q;u) \le R_C \tag{2}$$

where R_C represents a user-defined threshold of performance.

For example, the decision-making criterion $R(q;u)$ could represent the assessment of whether or not a structural component can survive an applied load. The decision takes the form of an inequality such as $y<y_{MAX}$ where *y* represents the peak stress experienced by the component and y_{MAX} represents a user-defined, maximum allowable level such as the material's yield stress. With the notation previously introduced, $R(q;u)$ defines the peak stress response, $R(q;u)=max(y)$, while the threshold R_C represents the allowable stress, $R_C=y_{MAX}$.

The numerical model $y=M(q;u)$ provides the value of the peak stress *y* for a given design or material *q*. The uncertainty *u* might represent the lack-of-knowledge about the loading that the structural component will actually experience once the system is deployed in the field.

2.4 Robustness or the Assessment of Immunity to Uncertainty

One of the main difficulties in decision-making is that uncertainty can make decisions ambiguous. In the problem of system identification, for example, more than one model generally fit the data equally well. One model may be better than the others but we cannot know which. Ambiguity originates from the fact that the uncertain variables *u* interact with the decision variables *q* and thereby preserve the potential for decision ambiguity.

However, if a decision is reached that happens to be highly insensitive to variations of the uncertain variables, then the validity of this decision is established with confidence. By establishing the ***immunity*** of the decision variables *q* to the uncertain variables *u*, we ameliorate the interaction between the latter and the former and thereby reduce the ambiguity in the decision-making process. In the context of model validation, for example, this "factoring out" of the uncertain variables strengthens the confidence in the validity of the model throughout the design domain. The assessment of immunity of a decision to uncertainty is referred to as ***robustness*** in the remainder.

3. SCOPE OF THIS WORK

This work addresses the problem of calibrating a numerical model to make it reproduce the test data to a given level of accuracy. In light of the above discussion, uncertainty must be accounted for and the robustness of the calibration to that uncertainty must be established. That is, we wish to provide an answer to the question: ***How much uncertainty can be tolerated without having to change the model?*** If the robustness to uncertainty is great, then the ambiguity is low, since the data could vary greatly without inducing a different model.

Our analysis departs from the state-of-the-art in finite element model updating [7] in two main ways. First, non-linear finite element models are developed to represent fast, transient events. Some of the materials involved exhibit a softening behavior and they cannot be represented with a linear constitutive relation. In addition, the dynamical phenomenon of interest is high-frequency wave propagation. It occurs in less than one millisecond and cannot be represented using a truncated basis of low-frequency mode shapes.

Second, due to our fragmentary information and incomplete understanding of the processes involved, uncertainty cannot be represented with the theory of probability. Not being able to rely on probabilities implies that uncertainty propagation cannot take advantage of efficient sampling techniques. Therefore, the computational cost of the method proposed for non-probabilistic uncertainty propagation should not come as a surprise.

4. INFORMATION-GAP MODELING OF UNCERTAINTY

Models of information-gap are briefly introduced to represent a generic uncertainty. A mathematical formulation is then proposed to investigate the robustness of a decision to uncertainty. The application to test-analysis correlation is discussed in sections 5 and 7.

4.1 Information-gap Modeling

We start by briefly explaining how an information-gap model (IGM) of uncertainty is constructed. An IGM is simply a collection of nested sets of uncertain events. The "size" of these sets is controlled by the horizon-of-uncertainty parameter a. The sets, denoted $U(u^0;a)$, are nested so that $a<a'$ means that $U(u^0;a)$ is included in $U(u^0;a')$. In other words, the range of uncertain events increases as the uncertainty parameter a increases. For example, describing a random variable X with an IGM could consist of establishing nested intervals within which X varies around a nominal value denoted by X^0:

$$U(X^0;a) = \left\{ X \;\middle|\; \left| X - X^0 \right| \le a \right\} \quad a > 0 \tag{3}$$

Other examples of IGM are provided in References [2, 8]. The important point is that no probability structure is assumed. Instead, information-gap theory hypothesizes the ***structure*** of the uncertainty space and dictates how uncertain events cluster around one another, but no measure functions are posited.

It is important to realize that an IGM is much less restrictive than postulating the frequency of occurrence of uncertain events in terms of a probability density function or postulating a possibility of occurrence in terms of a family of fuzzy membership functions. In fact, an IMG includes all possible representations of uncertainty—that is, all possible probability density or membership functions—consistent with the constraint introduced by the horizon-of-uncertainty parameter a. The theory of information-gap is therefore well suited to the cases of extreme uncertainty or cases where no evidence is available to suggest the choice of a particular probability structure.

4.2 Robustness to Uncertainty

In section 2.3, decision-making has symbolically been denoted "$R(q;u)<R_C$" where $R(q;u)$ denotes the performance or decision-making criterion and R_C represents a user-defined threshold of performance. The value of the performance level R_C is not chosen a priori. As in all information-gap analyses, the performance level R_C is embroiled in a basic trade-off and its value is chosen in light of the resolution of that trade-off.

The basic decision function of information-gap decision theory is the robustness $a*$. The robustness of decision q is the greatest value of the uncertainty parameter a at which the performance predicted by the model is never worse than R_C. The robustness is therefore formally defined as an optimization problem:

$$a^* = \underset{a \ge 0}{\mathbf{Argmax}} \quad \underset{u \in U(u^O;a)}{\mathbf{max}} \quad \left\{ R(q;u) \;\middle|\; R(q;u) \le R_C \right\} \tag{4}$$

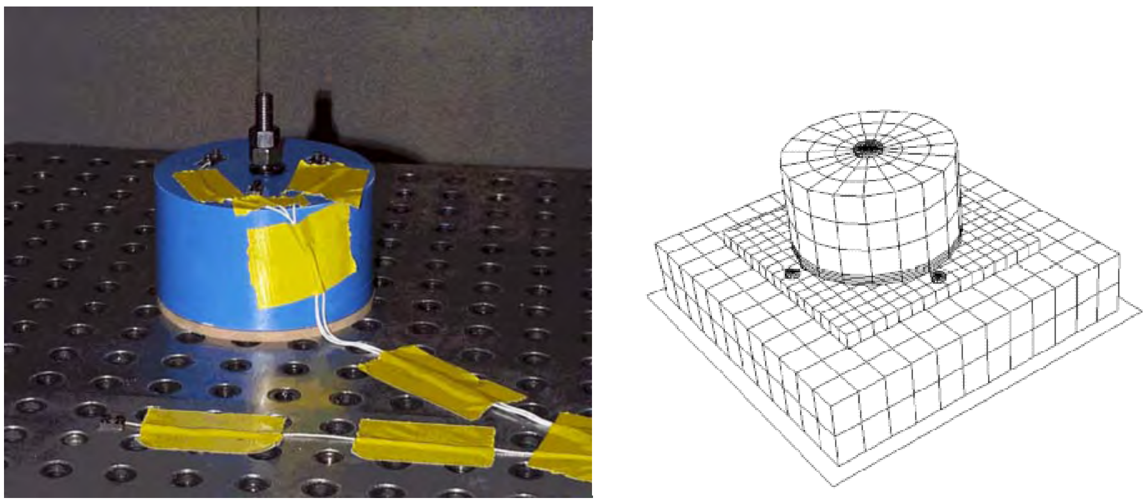
The significance of the robustness function is that it assesses the degree of variation of the uncertain u that does not jeopardize the performance of the system. If the robustness of decision q is large, then the performance of the corresponding model is immune to variations of the unknown quantities u. On the other hand, if $a*$ is small, then even very small fluctuations of the uncertain quantities u endanger the performance. At low robustness, decisions based on the model are likely to be questionable due to the influence of variability and modeling error.

5. SHOCK PROPAGATION THROUGH A HYPER-ELASTIC MATERIAL

The application of interest is a high-frequency shock performed in the Summer of 1999 at Los Alamos National Laboratory. The experiment was designed to study the propagation of a shock wave through a non-linear, visco-elastic material. References [3, 9] provide details about the experiment, the data collected and the statistical effect and parameter calibration analyses. Testing and modeling activities are briefly summarized below for completeness.

5.1 Impact Test Setup

The test consists of dropping from various heights a carriage (drop table) on which are mounted a layer of hyper-elastic material and a 10.77 kg steel cylinder. Upon impact on a concrete floor, a shock wave is generated that propagates through the hyper-elastic material. The heavy steel cylinder compresses the hyper-elastic pad and causes elastic and plastic strains during a few milliseconds. A photograph of the setup is shown in Figure 1-a.



(a) Photograph of the drop test setup. *(b) Illustration of the finite element mesh.*
Figure 1. Drop test experiment and instrumentation.

Four acceleration measurements were collected during each test. One acceleration signal was measured on the top surface of the carriage and three acceleration signals were measured on top of the steel cylinder. The former is referred to as the "input" acceleration signal and the latter are referred to as the "output" acceleration signals at sensors 1, 2 and 3.

The features y that we would like the numerical model to reproduce are defined as the peak acceleration at sensor 2 and the corresponding time-of-arrival, that is, the time it takes the shock wave to travel from the drop table to sensor 2 on top of the steel cylinder. These two features are denoted as *PAC* and *TOA*, respectively. The impulse is so short in time—and the shape of the pulse can be reproduced by a half-sine wave—that matching these two features is sufficient to capture the response's energy content.

5.2 Finite Element Modeling

Figure 1-b illustrates the finite element model developed for numerical simulation. The analysis program used is HKS/Abaqus®-Explicit, a general-purpose package for finite element modeling of non-linear structural dynamics [10]. It features an explicit time

integration algorithm, which is convenient when dealing with non-linear material behavior, contact dynamics and high frequency excitation. The model shown in Figure 1-b defines 963 nodes, 544 volume elements and two contact pairs located at the cylinder/pad interface and the pad/carriage interface. It yields a total of 2,889 degrees of freedom composed of structural translations in the three directions and Lagrange multipliers defined for handling the contact constraints. An analysis running on a typical single-processor workstation is executed in approximately 10 minutes of CPU time.

The finite element simulation had to be parameterized in an effort to capture the material variability, the experimental variability and other sources of uncertainty. Because it was observed during the physical experiments that the drop table did not always hit the floor perfectly horizontally, two tilt angles t_1 and t_2 were introduced in the numerical simulation. Another source of variability was the torque P_B applied to the tightening bolts that held the assembly together on the carriage. A small scaling variation s_I of the measured impulse was also allowed to account for potential sensor calibration errors and other systematic bias introduced by data decimating and filtering. Other input parameters are not included in this analysis because previous work has demonstrated that these additional parameters do not explain the observed variability [3, 9].

In summary, the physical experiments as well as the finite element model define a non-linear mapping $y=M(q;u)$ between the four decision variables $q=(t_1;t_2;P_B;s_I)$ and two output features $y=(PAC;TOA)$. To clarify a potential notation ambiguity, it is emphasized that the quadruplet $(t_1;t_2;P_B;s_I)$ that has been denoted by q could also be denoted by the symbol u. The distinction between the two notations comes from the type of uncertainty model adopted.

5.3 Test-analysis Correlation

The mapping $y=M(q;u)$ can either be evaluated experimentally by performing a test, which is expensive and time-consuming, or it can be simulated in a few minutes with the finite element model. To take advantage, however, of the modeling capability, we must assess the predictive accuracy of the model and decide, among many possible choices of models, which one provides the best performance.

Each possible model is defined by a particular choice of the decision variable quadruplet $q=(t_1;t_2;P_B;s_I)$. Clearly, what makes this problem non-trivial is the fact that the quadruplet q is unknown and that this uncertainty cannot be modeled probabilistically.

To solve the problem, the concepts developed in section 4 are applied to the correlation between test measurements and model predictions. In this context, the performance criterion $R(q;u)$ is defined in terms of the following test-analysis correlation metric:

$$R(q;u) = e(q)^T W_{ee}^{-1} e(q) + \left(q - q^0\right)^T W_{qq}^{-1} \left(q - q^0\right) \qquad (5)$$

where $e(q)=y^{Test}-y(q)$ represent the distance between measured and predicted $(PAC;TOA)$ features and q^0 denotes the nominal value of the quadruplet $(t_1;t_2;P_B;s_I)$. The matrices W_{ee} and

W_{qq} are weighting coefficients. Note that the variables *u* that convey the uncertainty do not appear explicitly in equation (5); they are defined in sections 6 and 7.

Decision-making consists in identifying the models—that is, selecting the variables $(t_1;t_2;P_B;s_I)$ of the finite element simulation—that provide the best possible correlation *R(q;u)* between measurements and predictions, given the sources of uncertainty. The threshold of acceptable performance R_C then represents the maximum acceptable prediction error, for example, R_C=20%. A model is "good enough" or "acceptable" if it satisfies the inequality (2). In other words, R_C specifies the level of ***satisficing*** for calibration of the model. While the fidelity function *R(q;u)* depends upon the uncertain or unknown quantities *u*, it will turn out that this is not an impediment to the analysis. On the contrary, it is the prime mover of the information-gap robustness analysis discussed in section 7.

6. IMPLEMENTATION OF THE ROBUSTNESS ANALYSIS

During the physical experiments, the values of the four variables $(t_1;t_2;P_B;s_I)$ were neither controlled nor measured, making it difficult to represent their variability with a probability distribution. It was nevertheless assessed that the uncontrolled variation of these quantities during the tests caused a significant variability of the measured acceleration signals and their features *PAC* and *TOA*. While $(t_1;t_2;P_B;s_I)$ were considered controllable decision variables *q* in section 5, we now regard them as unknown and uncontrollable entities *u*. This lack-of-knowledge is represented by a model of information-gap.

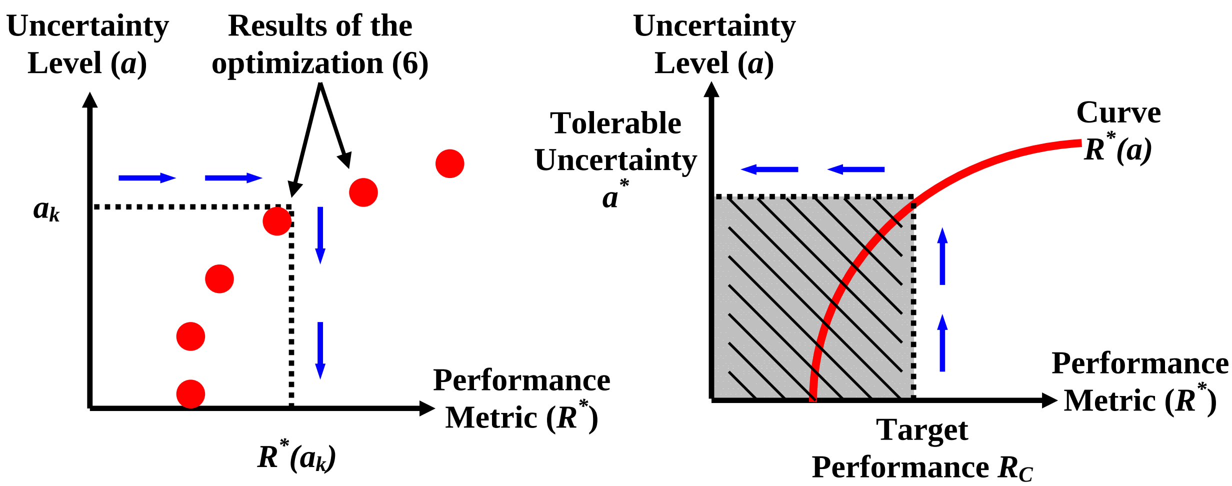
The worst case would be to state that the variation of parameters $(t_1;t_2;P_B;s_I)$ is unknown within specific bounds dictated by physical constraints. The results of this analysis are presented in section 7. To discuss the implementation of a robustness analysis, suffices to say that the unknown $u=(t_1;t_2;P_B;s_I)$ belongs to an information-gap model represented by a family of nested sets $U(u_0;a)$ where $a>0$. As explained in section 4, a generic robustness analysis estimates the horizon of uncertainty a^* that guarantees an acceptable performance. Here, we seek to identify the models that provide a test-analysis correlation metric *R(q;u)* smaller than the acceptance threshold of R_C=20%.

The decision-making procedure is conceptually illustrated in Figure 2. Figure 2-a shows that, at each horizon of uncertainty a_k, an optimization problem must be solved that provides the worst possible performance $R^*(a_k)$ within the set $U(u^0;a_k)$:

$$R^*(a_k) = \max_{u \in U(u^o;a_k)} R(q;u) \qquad (6)$$

Focusing on the worst possible test-analysis correlation metric $R^*(a_k)$ at each level of uncertainty a_k provides an assessment of the ***adverse*** effect of uncertainty on performance. Decisions—or, in our case, models—that feature a high robustness a^* are preferred because they minimize the potentially adverse effect of uncertainty.

For the application, the optimization searches for the model that yields the worst possible test-analysis correlation metric $R(q;u)$ at each uncertainty level. The uncertainty levels a_k are pre-defined by the user, depending on how many optimization problems can be solved given the available computational resource. A gradient-based BFGS algorithm is wrapped around the HKS/Abaqus® finite element package through a Matlab™ interface. This implementation is feasible because the algorithm optimizes only four variables and requires a relatively small number of finite element calculations to converge, typically less than 60. To solve a more complicated optimization problem, it may become advantageous to replace the finite element model by a trained, fast-running surrogate as suggested in Reference [1].



(a) Info-gap robustness calculation. *(b) Inference of the allowable uncertainty.*

Figure 2. Conceptual information-gap analysis.

The sequence of points $\{a_k;R^*(a_k)\}$ of Figure 2-a is then used to approximate the continuous performance curve $R^*(a)$ shown in Figure 2-b. In figure 2-a, information flows from the vertical axis (a) to the horizontal axis (R^*). Decision-making in Figure 2-b reverses the flow of information. For the target performance R_C, the tolerable horizon of uncertainty a^* is obtained by reading the performance curve $R^*(a)$. The shaded area in Figure 2-b represents the acceptable operating region. If the analyst is only interested in estimating a^*, reconstructing the entire curve $R^*(a)$ is not necessary and efficient search strategies can be devised that take advantage of its proven monotonic form [11].

7. RESULTS OF THE ROBUSTNESS ANALYSIS

As stated previously, the analysis assumes that no evidence is available to suggest that a particular probability structure might be appropriate to represent the uncertain parameters $(t_1;t_2;P_B;s_I)$. Furthermore, the analysts are not willing to make assumptions. In this context, all that can be stated is that the parameters t_1, t_2, P_B and s_I vary within intervals. The upper and lower bounds of these intervals are derived from physical observation. For example, the preload P_B is always positive and it cannot exceed a limit determined by taking measurements on the system after testing was completed. Similarly, the drop table's guiding system imposes physical constraints on the angles t_1 and t_2 and it is determined that no more than a 2-degree tilt could possibly have occurred. Note that such information is extremely sparse compared to probability information. Nevertheless, it can be modeled with models of information-gap.

7.1 Studying the Sensitivity of Robustness to Each Individual Variable

The analysis starts with the definition of four information-gap uncertainty levels a_1-a_4, one for each uncertainty variable t_1, t_2, P_B and s_I. The robustness analysis focuses on one variable u_k at a time, with the other three kept constant and equal to their nominal values. This approach lets us study the effect of each uncertainty variable independently from the others. The performance criterion $R(q;u)$ is the test-analysis correlation metric defined in equation (5). The intervals of uncertainty $U(u_k^0;a_k)$ are defined as follows:

$$U(u_k^0;a_k) = \left\{ u_k \;\middle|\; a_k u_k^- \le u_k - u_k^0 \le a_k u_k^+ \right\} \quad a_k > 0 \tag{7}$$

where the lower and upper limits u_k^- and u_k^+ represent the minimum and maximum bounds that the parameter u_k cannot exceed. The symbol u_k denotes one of the four unknowns $(t_1;t_2;P_B;s_I)$ and u_k^0 represents an assumed nominal value. The definition (7) implies that the uncertainty parameter a_k is scaled between zero and one. Thus, the value a_k=1 represents a total lack-of-knowledge. Table 1 defines the nominal values u_k^0 and bounds u_k^- and u_k^+.

Table 1. Definition of the input parameters of the finite element model.

Symbol	Definition	Nominal Value u_k^0	Lower Bounds u_k^-	Upper Bounds u_k^+	Units
t_1	First tilt angle	0.50	0.00	2.00	Degree
t_2	Second tilt angle	0.50	0.00	2.00	Degree
P_B	Bolt preload	1.72	0.00	3.45	MPa
s_I	Input scaling	1.00	0.90	1.10	Unit-less

Figure 3 shows the four partial robustness curves a_k-versus-$R^*(a_k)$, from which the optimal solutions a_k^* can be read directly. The reason why the worst test-analysis correlation $R^*(a_k)$ increases when the uncertainty level a_k increases is because $R^*(a_k)$ is the solution of a maximization problem (6). For larger horizon-of-uncertainty parameters a_k, the search space $U(u_k^0;a_k)$ also becomes larger and the solution $R^*(a_k)$ can only increase.

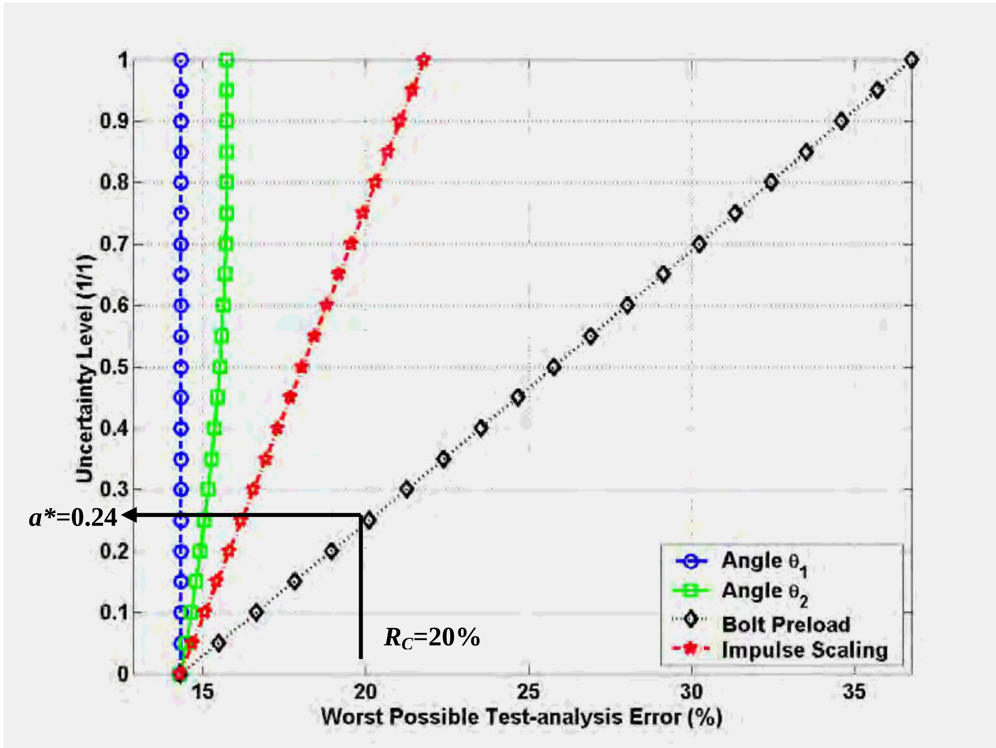


Figure 3. Partial robustness versus information-gap uncertainty.

It can be observed that the most adverse effects of uncertainty are associated with the parameters P_B (bolt preload) and s_I (impulse scaling). Uncertainty in the preload P_B has the potential to deteriorate the original 14.5% error (at *a=0)* to more than 37% (at *a=1).* Uncertainty in the tilt angles, on the other hand, produces little-to-no deterioration of the correlation. From this analysis, we learn that it is not critical during an experiment to attempt to control or measure the tilt angles. It is also learned that no more than a*=24% uncertainty can be tolerated in the knowledge of the bolt preload P_B to guarantee that the test-analysis correlation error remains acceptable, that is, less than R_C=20%.

7.2 Studying the Robustness-opportunity Trade-off for Cost-benefit Analysis

The results presented in Figure 3 cannot be used to evaluate the robustness function, as shown schematically in Figure 2, because each of the four uncertain variables is studied separately. A second analysis is therefore performed where the unknowns *u* are calibrated simultaneously as they should be in a rigorous robustness analysis. At each uncertainty level, the following two constrained optimization problems are solved:

$$R^*(a_k) = \max_{u \in U(u^o; a_k)} R(q;u) \qquad R^*(b_k) = \min_{u \in U(u^o; a_k)} R(q;u) \qquad (8)$$

On the left of equation (8), searching for the worst test-analysis correlation error at each uncertainty level provides the robustness function *a**. On the right of equation (8), searching for the best possible error provides what is referred to in Reference [2] as the ***opportunity*** function *b**. Opportunity assesses the potentially ***beneficial*** effect of uncertainty. Whereas one always attempts to maximize the robustness *a** of a decision, the opportunity *b** is a quantity that is minimized because it provides the least uncertainty that guarantees a given performance. In both cases, the uncertainty is represented by an IGM where each unknown varies within an interval whose size depends on the horizon-of-uncertainty parameter a_k:

$$U(u^0;a_k) = \left\{ u = (t_1;t_2;P_B;s_I) \;\middle|\; a_k u^- \le u - u^0 \le a_k u^+ \right\}, \quad a_k > 0 \tag{9}$$

At any given uncertainty level a_k or b_k, the intervals defined by the family of nested sets provide the constraints for the optimization problems (8). Figure 4 illustrates the results of the robustness and opportunity analyses. It is emphasized that each point in Figure 4 is the result of an optimization problem.

Cost-benefit analysis stems from discussing the trade-off between the robustness *a** and opportunity *b** functions. The first information conveyed in Figure 4 is that no more than *a**=17% uncertainty must be tolerated to guarantee an acceptable performance, meaning that the finite element simulation predicts the physical test with less than R_C=20% error. Controlling the uncertainty to no more than 17% during the physical experiments and the development of the numerical model surely comes at a significant cost. For example, this constraint can be translated into a requirement of the accuracy needed to measure the torque applied by the tightening bolts so that the estimation of preload is known within 17%.

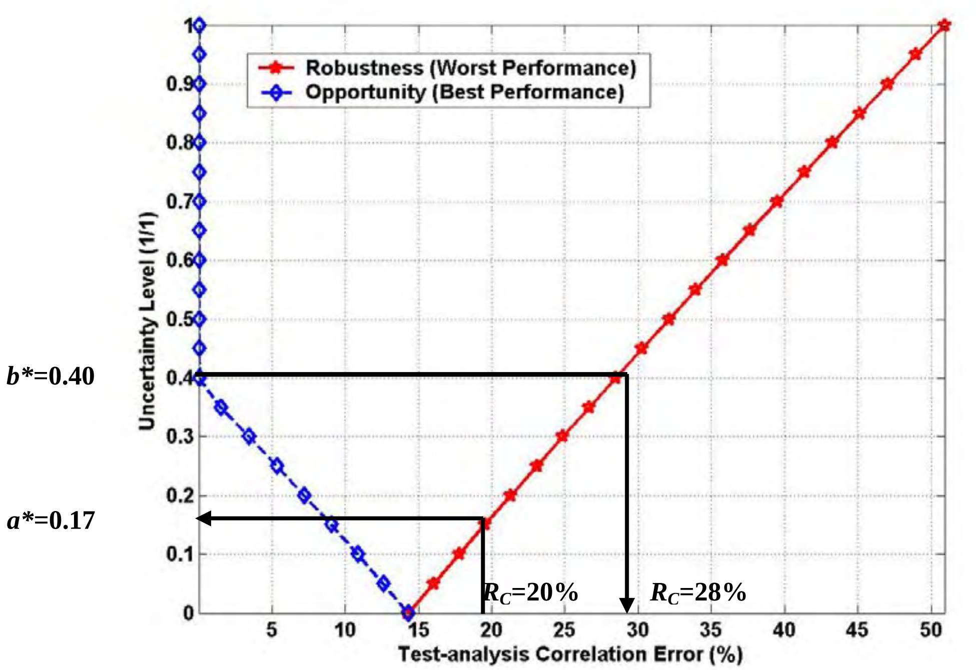


Figure 4. Robustness and opportunity versus information-gap uncertainty.

Figure 4 also shows that, at the 17% uncertainty level, the simulation would still provide, in the best possible case, at least 8% test-analysis correlation error. The 8% error corresponds to 17% uncertainty on the opportunity curve. Hence, Figure 4 conveys information regarding the potential benefits of the uncertainty. For example, one could ask what it would take to obtain a finite element model that provides the possibility of producing a "perfect" prediction of the physical test (that is, less than 1% error). The answer is, again, provided by the opportunity function: b^*=40%. On the other hand, 40% uncertainty only guarantees R_C=28% test-analysis correlation error. Weighting the cost of controlling no more than 17% uncertainty (with the benefit of guaranteeing no more than R_C=20% error) against the lesser cost of allowing up to 40% uncertainty (with the risk of deteriorating the test-analysis correlation to 28% error) is the essence of decision-making.

CONCLUSION

The quantification and propagation of uncertainty through a linear model has been the subject of extensive studies in many sciences, especially in the context of probability theory. The relationship between uncertainty and non-linear dynamics is not well understood, the main reason being the difficulty in analyzing non-linear systems.

This publication illustrates how information-gap models can be defined and analyzed for studying the propagation of uncertainty through a non-linear finite element simulation without relying on probabilities. We concentrate on establishing the robustness-to-uncertainty of a numerical model in reproducing the experimental measurements. This work demonstrates that model assessment under uncertainty can be solved without relying on the theory of probability. Analysts are therefore offered a practical alternative for situations where only sparse data are available or only limited testing is possible.

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