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Statistical Model Updating and Validation Applied to Nonlinear Transient Structural Dynamics

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ABSTRACT

This paper presents an example of a statistics-based model updating and validation philosophy applied to a nonlinear transient structural dynamics problem. The problem being analyzed is the response of a steel/polymer foam assembly during a drop test. The objective of the simulation is to accurately predict the acceleration response of the cylinder with the appropriate statistical distribution. Model validation is performed to ensure that the predictions agree with the experimental measurements to within an acceptable limit. A systematic approach to the model validation and updating problem is followed. The approach begins with the definition of the validation criteria, including the features of interest and the metrics to be used in comparing the features. Next, the mean values of the simulation parameters are updated, followed by an updating of the parameter covariance values to more accurately predict the distribution of the features of interest. An assessment of the predictive accuracy of the simulation using the final parameter estimates concludes the demonstration of statistical model validation and updating.

INTRODUCTION

The purpose of the research described in this paper is to define a theoretical framework under which a) the statistical accuracy of nonlinear structural dynamics simulation predictions can be assessed with respect to experimental results, and b) the simulations can be improved to more accurately predict the experimental results. The motivation for including statistical analysis in this effort is driven by the desire to: a) study the effects of variable environmental and experimental conditions; b) represent the response of a population of units with a single model; and c) include outlier behavior in predictive analysis - i.e. units whose behavior falls “in the tails” of the distributions.

A few definitions will be made to facilitate consistent discussion throughout the paper. *Validation* is the evaluation of the accuracy of a computational prediction with respect to experimental data. A *feature* is a parameter identified from the physical response that represents the important characteristics of the signal. It may or may not have any particular physical significance. A *feature metric* is a mathematical measure used to compare the values of two or more features (or to compare sample statistics derived from the features). *Simulation parameters* are the user-controlled input values for the computational simulation. Finally, *model updating* is the process of systematically perturbing the simulation parameters to obtain a more accurate model as defined by the metric.

It should be noted that we make a distinction between model validation and model updating as being separate yet coupled actions. Validation has as its emphasis the assessment of the accuracy of the simulation with respect to the experiment, whereas updating is the actual process of

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improving the simulation. They are separate steps, but we have found that it is difficult to discuss the assessment issues without also discussing how to improve the simulation. Thus, the two are coupled.

Model updating can be dangerous if not properly applied. To analyze this issue, we first ask the question: “When is it appropriate to ‘tune’ model parameters?” There are basically three types of parameters that are appropriate to tune: a) variable testing conditions (e.g. angle of impact, preloads of joints, thermal variability); b) unit-to-unit variability (e.g. material properties, geometry, friction); c) unknown or unmodeled mechanics (e.g. complex interfaces). Just tuning the parameters because it makes the simulation prediction “look more like the experimental measurement” is not a legitimate approach, because then we have no guarantee that the perturbations are physically meaningful or motivated by the desire to improve the modeling procedure.

It should be remembered that the goal of updating is to provide good advice to the analysts, not try to replace their brains. Engineering judgment and system-specific knowledge will always be inherent in the model validation and updating process. We must not adjust parameters blindly. Careful selection of which parameters are updated as well as candidate values for the parameters is crucial. Finally, it must be remembered that the ability to “tune” a model is never a substitute for understanding the underlying mechanics.

The traditional approach to model validation and updating typically uses Fourier-based quantities (resonant frequencies, mode shapes, etc.) and the corresponding eigensolution of the simulation model. [1] However, when the response is significantly nonlinear and/or only very short time histories are available, the assumptions underpinning Fourier analysis can be violated. Some of the relevant sources of nonlinearity which can disrupt Fourier analysis are: a) nonlinear material response (e.g. elasto-plastic, hyperelastic, etc.); b) intermittent contact between components; and c) coulomb friction or other energy loss mechanisms that cannot be easily decoupled via modal analysis. Thus, to perform model updating on nonlinear transient system response simulations, we need features that are not constrained by assumptions of Fourier analysis, and we want features that are still meaningful from a physical standpoint. For example, in the field of shock response analysis, peak acceleration and peak arrival time are often the signal features of highest interest in the simulation prediction.

Once the features of interest are selected, we still need a quantitative metric to assess the difference in these features between the simulation and the experiment. Such a quantitative metric provides an objective function for an optimization over the candidate values of the simulation parameters. Furthermore we often have situations where different signal features compete for importance in the prediction. A quantitative metric provides a systematic procedure for assessing the composite quality of the prediction over several features. For example, consider the experimental result and the two simulation predictions shown in Figure 1. Simulation A predicts the peak acceleration of the signal more accurately, whereas simulation B predicts the peak arrival time of the signal more accurately. Which of these two simulations is “better?” The answer can only be determined by defining a quantitative metric to compare multiple features (with different units, magnitudes, etc.) in a generalized manner.

To ensure that the computational model predicts the full range of behavior of the component, it is important to include the experimental variability in the computational model, which requires an understanding of the sources of experimental variability. If all significant sources of variability are included in the computational simulation, then updating the statistical distribution of the simulation parameters should accurately predict the distribution of the experimental features.

THEORETICAL BACKGROUND

The proposed 3-step process for statistical validation of nonlinear structural dynamics models is summarized in Figure 2.

Step 1: Define Validation Criteria

The first step in the process is the definition of validation criteria. This step includes defining the signal feature of interest, defining a metric to implement the feature comparison, and defining independent criteria to assess the predictive accuracy of the updated model. Definition of the signal feature of interest is quite application specific. In the field of linear vibrations, quantities derived from the Fourier transform of the signal are most often the features of interest, including the frequency response function, the modal frequency, and the mode shape. For a shock-response application (which is the example used in this paper), the peak acceleration, peak arrival time, and shock response spectrum may be the significant features of interest. The features extracted from the simulated response will be represented by the vector quantity F_S, whereas the features extracted from the experimentally measured data will be represented by the vector quantity F_E.

There are three metrics that will be defined here for potential use, depending upon the amount of information generated by the analysis of the features. To compare the sample means of the two feature vectors, a simple Euclidean distance may be used:

$$M = \sqrt{\left(m(F_S) - m(F_E)\right)^T \left(m(F_S) - m(F_E)\right)} \qquad (1)$$

To compare the sample mean of the simulation feature vector with the sample mean and sample covariance of the experimental feature vector, the Mahalanobis distance may be used [2]:

$$M = \sqrt{\left(m(F_S) - m(F_E)\right)^T \Sigma^{-1}(F_E)\left(m(F_S) - m(F_E)\right)} \qquad (2)$$

By inspection of Eq. (2), it is seen that the Mahalanobis distance is simply the multivariate version of the standard normal statistic

$$Z = \frac{x - m}{s} \qquad (3)$$

In the further case where both the sample mean and sample covariance are known for both the simulation features and the experimental features, the Kullback-Leibler (KL) relative entropy function may be used [3]:

$$M = \frac{1}{2}\left(Trace\left(S(F_S)S^{-1}(F_E)\right) - N_p - \log\left(\frac{\det\left(S(F_S)\right)}{\det\left(S(F_E)\right)}\right)\right) + \frac{1}{2}\left(\mu(F_S) - \mu(F_E)\right)^T S^{-1}(F_E)\left(\mu(F_S) - \mu(F_E)\right) \qquad (4)$$

The KL metric assesses agreement between both the means and covariances of the features. So assuming that the features are multivariate normal, the KL measure gives an overall assessment of how well the full feature distributions match up between simulation and experiment. The method is theoretically extendable to additional statistical moments of the simulation parameters, provided that an appropriate metric is defined.

The final component of Step 1 is the definition of independent criteria to assess the predictive accuracy of the updated model. This can be accomplished using features other than the ones used for the updating process or comparison of the time histories directly. If the validation experiment has been conducted under various test conditions, it is best to use one set of test conditions for the model updating and then another set for the independent validation.

Step 2: Update Parameter Mean Values

The second step of the statistical model updating strategy is the parametric updating of the means of the model parameters to minimize the metric of distance between the deterministic simulation features and the statistical experimental features. The process for this step is shown in Figure 3. The Mahalanobis distance is used as the metric in this step. Prior to implementing the update, the parameters to be updated must be carefully selected. The parameters that we update must meet two conditions: a) they must be known to be uncertain, and b) the metric of interest (in this case Mahalanobis) must be sensitive to them.

Referring to the discussion of appropriate update parameters in the introduction, we see that some of the parameters will not be updated, some will be updated for mean values only, and some will be updated for both mean and covariance values, depending upon the situation. For example, parameters that do not have significant uncertainty (e.g. elastic modulus of steel) or are measured (e.g. input loads) would not be updated at all. Other simulation parameters, such as the properties of a particular lot of material, may be unique to each test article, but expected to be constant from test to test. Thus, only the mean values are udpated. Still other simulation parameters, such as the test-specific conditions (e.g. impact angles and preloads), may be expected to vary from test to test. Thus, we would want to update the covariances of these parameters to correctly predict the statistical distribution of experimental results. The sensitivity of the metric to each potential updating parameter must be assessed to ensure that perturbing the parameter can help to converge the metric.

Once the parameter set for the update has been determined, the response surface of the metric as a function of the simulation parameters must be mapped. [4] When there are several parameters of interest, the response surface can have a high dimension so that the number of simulation runs (i.e. function evaluations) to map the response surface is prohibitively high. In such a case it is recommended that an efficient sampling technique be utilized. The response surface allows computationally efficient evaluation of gradients for a sensitivity-based optimization approach. After the response surface has been calculated, the mean values of the parameters are updated by minimizing the Mahalanobis metric via a quasi-Newton technique.

Step 3: Update Parameter Covariance Values

After the simulation parameter mean values are in agreement with the experimental predictions, the covariances of certain simulation parameters are updated to minimize the KL metric between the simulation feature distribution and the experimental feature distribution, as shown in Figure 4. This process begins with the selection of an initial covariance matrix for these simulation parameters, based on engineering judgment and system-specific knowledge of what reasonable values for these parameters should be. For this research, multivariate normal distributions are used, so that the distributions are uniquely determined by a mean vector and a covariance matrix. [5]

Once the initial parameter distributions are selected, the statistics on the simulation parameters must be propagated through the computational simulation to find the corresponding statistics on the simulation features. A brute-force Monte Carlo simulation is one approach. Another approach is some directed-sampling technique such as bootstrap [6] or Latin Hypercube [7]. The KL metric may then be computed and checked for convergence. A quasi-Newton scheme is once again employed to minimize the metric, using the response surface from Step 2 to compute needed sensitivity values.

EXPERIMENT AND SIMULATION

The demonstration application is a shock test that features a component characterized by a nonlinear, visco-elastic material behavior. An illustration of this setup is provided in Figure 5.

The analysis program used for these calculations is HKS/Abaqus-Explicit, a general-purpose package for finite element modeling of nonlinear structural dynamics [8]. It features an explicit time integration algorithm, which is convenient when dealing with nonlinear material behavior, potential sources of impact or contact, and high frequency excitations. It can be observed from Figure 5 that the main two components (steel impactor and foam layer) are assembled on a high-strength bolt that is screwed into a threaded insert in the carriage. The design intent was to make the structure behave axisymetrically, but in reality very slight angles created significantly 3D behavior. Both finite element meshes are shown in Figure 6 Another important parameter is the amount of preload applied by the bolt used to hold this assembly together. The torque applied was not measured during testing and it may have varied from test to test.

During the actual test, the carriage that weights 955 lb (433 kg) is dropped from various heights and impacts a rigid floor. The input acceleration is measured on the top surface of the carriage and three output accelerations are measured on top of the steel impactor that weights 24 lb (11 kg). Figure 7 illustrates the test setup and instrumentation. This impact test is repeated several times to collect multiple data sets from which the repeatability of the experiment can be assessed. Typical accelerations measured during the impact tests are depicted in Figure 8. It can be seen that the peak acceleration is over 1000g on top of the impact cylinder causing large deformations in the foam layer. The variation in peak acceleration between measurement channels shown in Figure 6 suggests that a non-zero angle of impact is involved, making it necessary to model this system with a 3D discretization. The experimental configurations are summarized in Table 1.

Table 1. Number of Observations for Each Drop Test Condition

	Low Velocity Impact (13in./0.3m Drop)	High Velocity Impact (155in./4m Drop)
Thin Foam (0.25in./6.3mm)	10 Tests	5 Tests
Thick Foam (0.50in./12.6mm)	10 Tests	5 Tests

VALIDATION AND UPDATING OF SIMULATION PARAMETERS

The 3-step statistical validation process described in the theory section will now be applied to the drop-test experiment. To illustrate the nonlinear statistical updating procedure, the data sets will be limited to the 10 drops with the thin foam sample at the low-velocity impact (approx 100 in/sec).

Step 1: Define Validation Criteria

The first step in the model validation and updating process is to define the validation criteria. To start with we will define the signal features of interest. Because this is a shock response test, our features of interest will be the peak acceleration and peak arrival time of each channel. For simplicity of visualization, we will select a feature vector of dimension 2 for this example. Thus, the feature vector of interest is composed of the peak accelerations from the first two channels, written as

$$F = \left[Pk_1 \quad Pk_2 \right]^T \tag{1}$$

The metric of interest for the update of the parameter means will be the Mahalanobis distance, as defined in Eq. (2), and for the update of the parameter covariances the metric will be the KL criteria as defined in Eq. (4). The success of the updating procedure will be determined by visual assessment of the improvement in the simulated time history.

Visualization of the feature vector is accomplished by extracting an observation of the feature vector from each of the 10 experimental records, then plotting feature #1 (channel 1 peak acceleration) vs. feature #2 (channel 2 peak acceleration). Figure 9 shows a close-up of the variability in the peak acceleration magnitudes for channel 1 across all 10 experimental records (left plot). The peak value from each of these records forms the abscissa value for the right-side plot of Figure 9, and the ordinate values are drawn from the corresponding record of channel 2. Also shown in this figure is the approximate 95% elliptical confidence interval for this data set. [5]

Steps 2 and 3: Update Simulation Parameter Means and Covariances

The next step in the test/analysis correlation process is to select which parameters in the model should be modeled statistically. This is best accomplished using a sensitivity analysis of the desired features to the possible sources of variability. The possible sources of variability considered are: measurement noise floor, foam material properties, input acceleration level (drop-test impulse), bolt preload, and impact angle. (It is assumed that any computational sources of variability will be the same for each simulation.) The measured noise floor may be excluded as a significant source of variability by considering the closeup of the first experimental observation for channel 1 shown in Figure 10. The noise (the measured acceleration prior to the impact) ranges from about 1g to about 4g, which is less than 0.3% of the 1260g peak magnitude. The foam material properties may be inaccurate, but should not vary during the test. Thus, they may affect the accuracy of the mean simulation prediction, but should not affect the covariance. The input acceleration level is a definite source of variability, but because the input accelerations have been measured this parameter will not be subject to updating. (However, the variability in the input signal will be considered during the probabilistic simulations.) The variations in bolt preload and impact angle are also unknowns in the simulation. Thus, there are five candidate parameters that meet the criteria of being known sources of variability: two material property scaling parameters for the foam, two angles of impact (to describe arbitrary out-of-plane rotations) and the bolt preload.

To further narrow the parameters for updating, we must consider the sensitivity of the Mahalanobis metric to each of these parameters. The value of the Mahalanobis metric for variations in one of the impact angles and the bolt preload are shown in Figure 11. (Each of the multiple lines on these plots represents a value of the second impact angle.) In the case of the impact angle, it is clear that the Mahalanobis metric has a minimum value between 0.5 and 1.0 degrees. In the case of the bolt preload, it is clear that the metric is converging to the minimum as the bolt preload increases. The two material property-scaling parameters show similar sensitivities. Thus, all five of the candidate parameters demonstrate sensitivity and should be retained in the updating process. The mean values of these five simulation parameters are varied simultaneously to minimize the Mahalanobis metric. Next, initial covariance values are selected for the two impact angles and the bolt preload, and the covariance between these three parameters is updated.

Results of Updating and Assessment of Independent Metric

Example results of the statistical model updating and validation are shown in Figure 12. In the left-hand plot, the scatter of the features is shown for the experiment, the pre-update (nominal) simulation, and the post-update simulation. It can be seen that the post-update simulation reproduces both the mean and the covariance of the experimental features more accurately than does the pre-update simulation. The independent validation metric is shown on the right, and shows clear improvement in the accuracy of the predicted time history between the pre-update and post-update simulations.

CONCLUSION

This paper presents a procedure for performing statistical model validation and updating for nonlinear structural dynamics simulations. The approach represents a paradigm that is unconstrained by the assumptions of Fourier analysis and can be implemented non-invasively for any linear or nonlinear computational mechanics code. The approach allows the experimental and simulated features of interest to be put into a visual context for intuitive interpretation of the validation and updating results. The 2-step updating process first updates the means of the simulation parameters, then the covariance matrix of the simulation parameters, to minimize appropriate distance metrics. A sample validation and updating analysis is shown for a shock-response test with a nonlinear foam material.

ACKNOWLEDGMENTS

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FIGURES

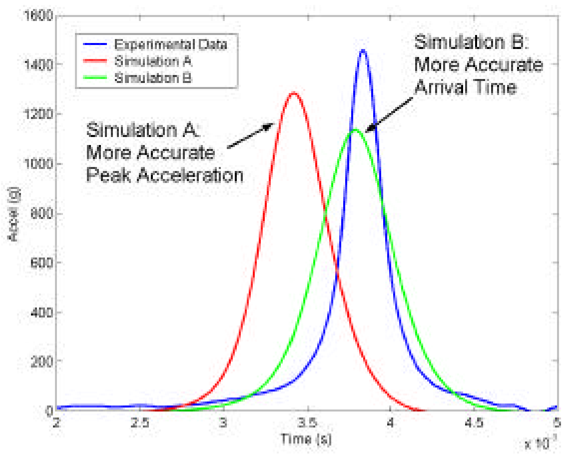

Figure 1: Competing Accuracy of Features Between Two Simulations

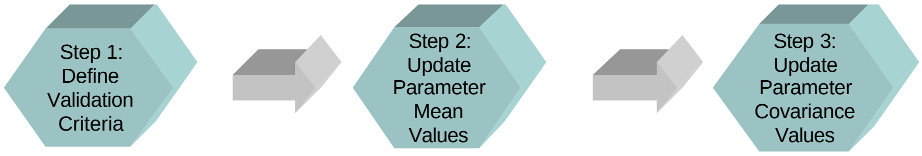

Figure 2: Illustration of Overall Statistical Nonlinear Model Validation Process

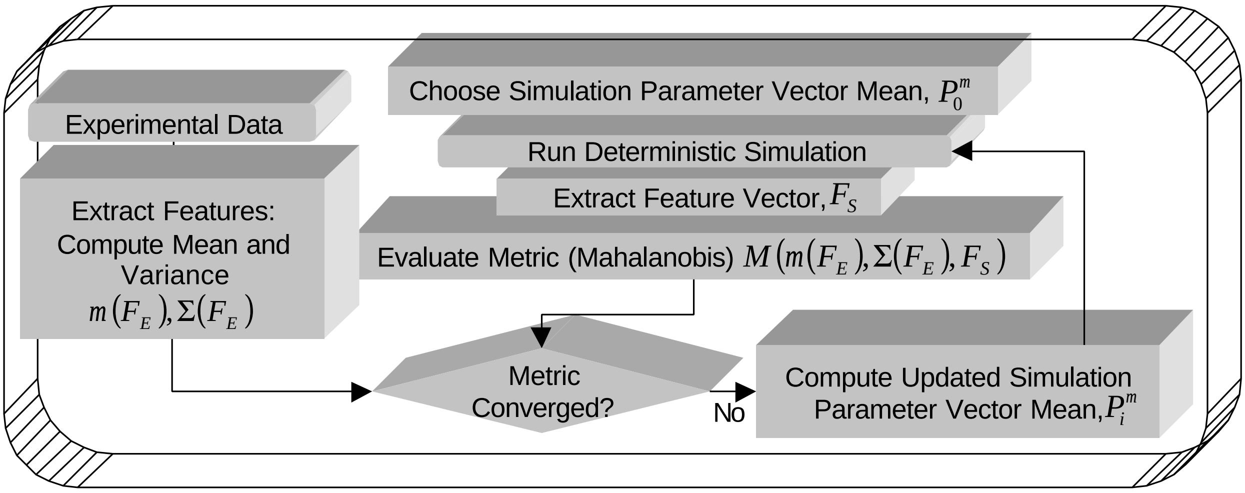

Figure 3: Illustration of Step 2 in Statistical Model Updating Procedure

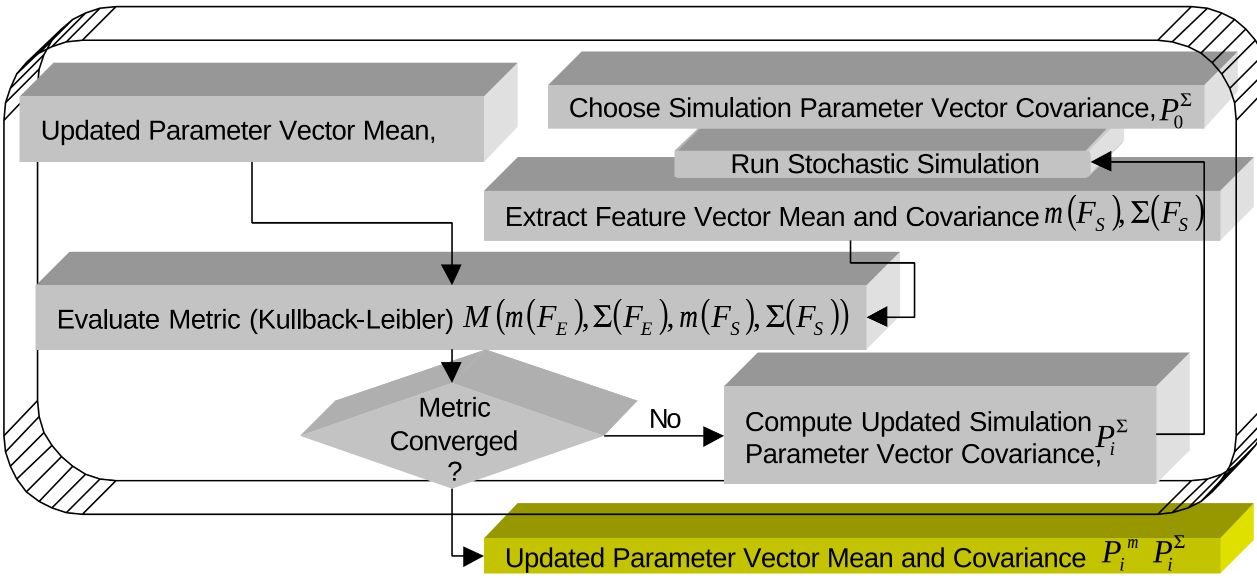

Figure 4: Illustration of Step 3 in Statistical Model Updating Procedure

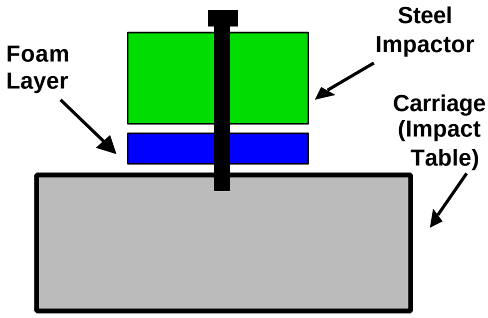

Figure 5: Schematic of Drop Test Assembly

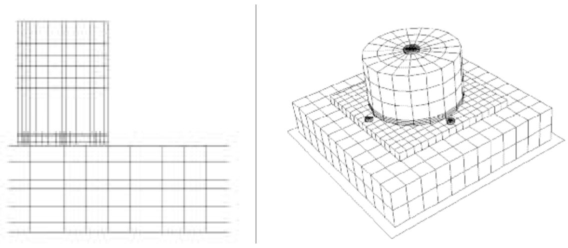

Figure 6: Axisymmetric and 3D Finite Element Models of Drop Test Assembly

Figure 7: Photographs of the Drop Test Experimental Configuration

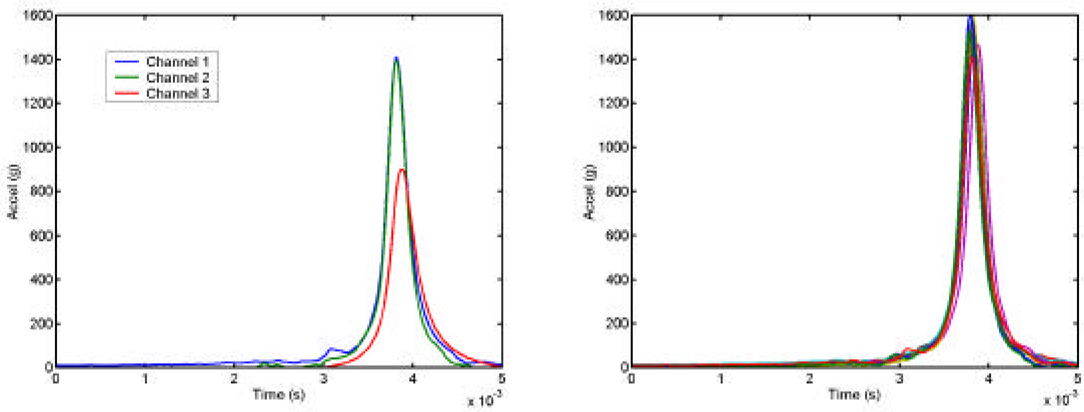

Figure 8: Example Time History for Low Velocity, Thin Sample
Left: Response of 3 Channels for Single Drop, Right: Channel 1 Response for 10 Drops

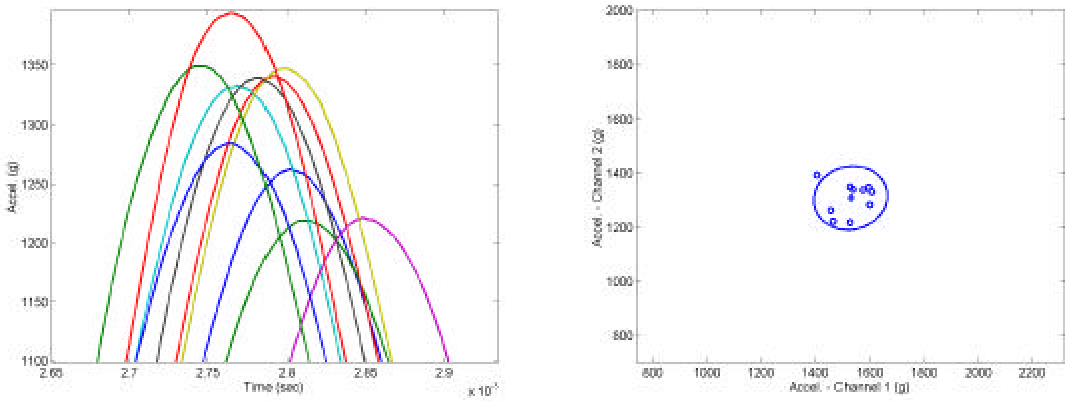

Figure 9: Variability in Peak Response Magnitude for Channel 1 (left) and Corresponding Feature Plot (right)

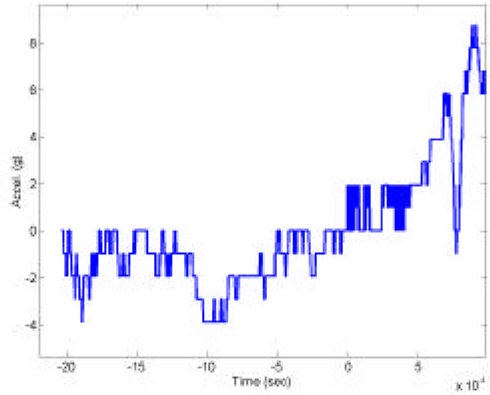

Figure 10: Sample Noise Floor for Drop Test

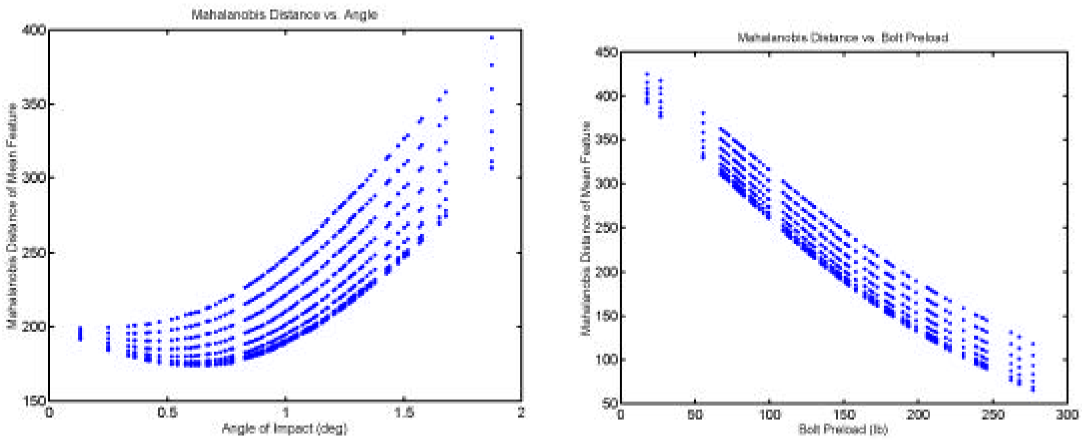

Figure 11: Sensitivity of Mahalanobis Metric to Impact Angle (left) and Bolt Preload (right)

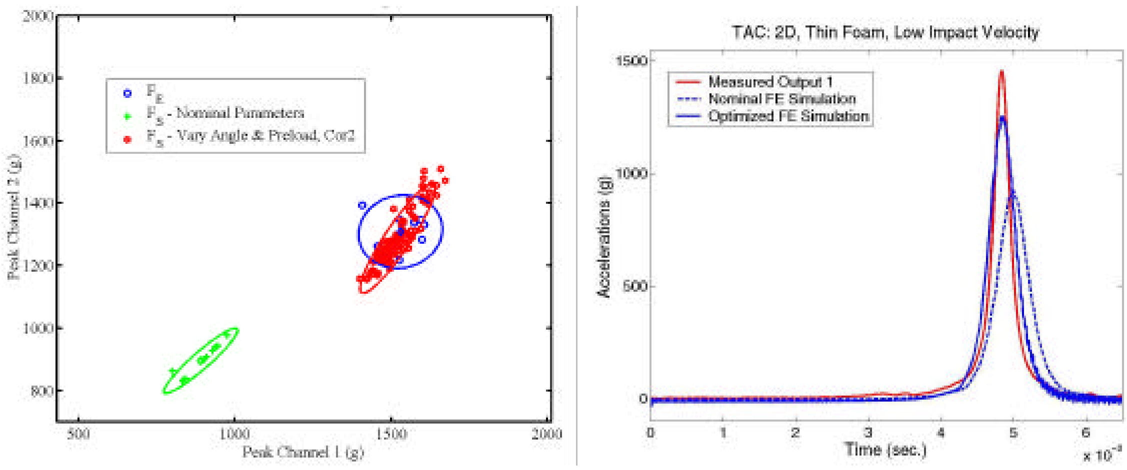

Figure 12: Effect of Parameter Updating on Distribution of Simulation Features and Time History