A Second Order Differential Importance Measure for Reliability and Risk Applications

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Abstract: The Differential Importance Measure (DIM) is a first-order sensitivity measure that ranks the parameters of the risk model according to the fraction of total change in the risk that is due to a small change in the parameters' values, taken one at a time. However, the DIM does not account for the effects of interactions among components. In this paper, a second-order extension of the DIM, named DIM^{II}, is proposed for accounting of the interactions of pairs of components when evaluating the change in system performance due to changes of the reliability parameters of the components. A numerical application is presented in which the informative contents of DIM and DIM^{II} are compared.

Keywords: Differential Importance Measure, Joint Importance, second order sensitivity measure.

1. INTRODUCTION

A limitation of the Importance Measures (IM) [1-3] currently used in reliability and risk analysis is that they rank only individual components or basic events whereas they are not directly applicable to combinations or groups of components or basic events [2]. In practice different basic events may, for example, represent different modes of failure or unavailability of a single component and in order to determine the importance of such component one has to consider all the related basic events as a group. Furthermore, many risk-informed applications deal with evaluating the risk change associated to changes in the plant technical specifications (surveillance and/or test frequencies, etc): such changes may indeed impact a group of components. To partially overcome this limitation, recently, the Differential Importance Measure, DIM, has been introduced for use in risk-informed decision making [3]. The DIM is a first-order sensitivity measure that ranks the parameters of the risk model according to the fraction of the total change in the risk that is due to a small change in the parameters' values, taken one at a time. The DIM bears an important property of additivity: the DIM of a group of components or basic events is the sum of the DIMs of the single components or basic events of the group. However, since DIM considers risk changes due to small changes of the parameters' values, it does not account for interactions among components.

The need for IMs capable of considering combinations of components arises also when planning a budget-constrained improvement in the reliability of a system design for example by replacing one of its components with a better-performing one, or by inspecting and maintaining it more frequently. Due to the budget constraints, the improvement may need to be accompanied by the sacrifice of the performance of another, less important component. The interactions of these coupled changes to system design must be accounted for when assessing the importance of the system components. To this aim, second order sensitivity measures such as the Joint Reliability Importance (JRI) and Joint Failure Importance (JFI) measures have been introduced [4, 5].

In this paper, a second-order extension of the DIM, named DIM^{II}, is proposed for accounting of the interactions of pairs of components when evaluating the change in system performance due to changes of the reliability parameters of the components. The extension aims at supplementing the first-order information provided by DIM with the second-order information provided by JRI and JFI. Obviously, the need of resorting to information on second-order effects depends on the magnitude of the changes of the parameters values and on the non linearity of the system.

2. EVALUATING THE CHANGE IN THE SYSTEM PERFORMANCE

We consider a system of n components. Let O be a generic measure of the system performance (e.g unreliability, unavailability, risk, etc., depending on the application at hand). The performance O is a function of the components' unavailabilities (or failure probabilities) q_i, i=1, 2, …, n, i.e. $O=g_q(q_1, q_2, \ldots, q_n)$. A change in system performance due to arbitrary changes in the values of the q_i, i=1, 2, n can be expanded in McLaurin series as:

$$\Delta O = \sum_{i=1}^{n} \frac{\partial O}{\partial q_i} \Delta q_i + \sum_{i=1}^{n} \sum_{h>i=1}^{n} \frac{\partial^2 O}{\partial q_i \partial q_h} \Delta q_i \Delta q_h + \sum_{i=1}^{n} \sum_{h>i=1}^{n} \sum_{k>h=1}^{n} \frac{\partial^3 O}{\partial q_i \partial q_h \partial q_k} \Delta q_i \Delta q_h \Delta q_k + \ldots \qquad (1)$$

Using the rare event approximation, the risk measure O can be written in terms of the probabilities of the n_{cs} minimal cutsets:

$$O \approx \sum_{j=1}^{n_{cs}} M_j \qquad (2)$$

where M_j is the probability of the j-th cutset. Then, alternatively, the change in O due to generic changes of the parameters Δq_i, i=1, 2, …, n is [6]:

$$\Delta O = \sum_{i=1}^{n} S_i \Delta q_i + \sum_{i=1}^{n} \sum_{h>i=1}^{n} S_{ih} \Delta q_i \Delta q_h + \sum_{i=1}^{n} \sum_{h>i=1}^{n} \sum_{k>h=1}^{n} S_{ihk} \Delta q_i \Delta q_h \Delta q_k + \\ + \ldots + \sum_{i=1}^{n} \sum_{h>i=1}^{n} \ldots \sum_{r>s}^{n} S_{ih\ldots r} \Delta q_i \Delta q_h \ldots \Delta q_r \qquad (3)$$

where $S_i = \sum_{j=1}^{n_{cs}} \frac{\partial M_j}{\partial q_i}$, $S_{ih} = \sum_{j=1}^{n_{cs}} \frac{\partial^2 M_j}{\partial q_i \partial q_h}$, and so on. Eq. (1) reduces to eq. (3) if the rare event approximation of eq. (2) holds. The right-hand part of eq. (3) contains as many terms as the largest number of components in any minimal cutset. The quantities S_i, S_{ih}, $S_{ih\ldots r}$ can be straightforwardly calculated as follows [6]: S_i is the sum of the contributions to O in eq. (2) of the minimal cutsets containing element i, with its unavailability set to 1; S_{ih} is the sum of the contributions to O of the minimal cutset containing elements i and h with their unavailabilities set to 1, $S_{ih\ldots r}$ is the sum of the contributions of the minimal cutset containing elements i, h,

…, r with their unavailabilities set to 1. Note from eq. (3) that the interaction terms S_{ih}, $S_{ih\ldots r}$ assume a value of zero if the components i and h, i, h, … and r, respectively do not appear together in one of the minimal cutsets. Thus, for example when the rare event approximation holds, for groups of components belonging to different blocks in series only the first-order terms in eq. (3) contribute to ΔO since they do not appear together in any minimal cutset. On the contrary, for components in parallel logic, contributions from the higher-order terms in eq. (3) are expected, since the components always appear together in a minimal cutset.

3. FIRST-ORDER IMPORTANCE MEASURES: BIRNBAUM AND DIM

The Marginal Reliability Importance (MRI) (often referred to as the Birnbaum IM) of component i is defined with respect to its unavailability q_i as [1, 4]:

$$MRI(i) = \frac{\partial O}{\partial q_i} \tag{4}$$

According to the MRI, components for which a variation in unavailability results in the largest variation of the system performance have the highest importance.

The MRI applies when the components' unavailabilities or failure probabilities q_i, i=1, 2, …, n are known explicitly. However, the quantities q_i are often expressed in terms of additional reliability parameters x_k, k=1, 2,…, n_p such as failure and repair rates, maintenance and inspection frequencies, etc: in turn, the system performance O can be expressed in terms of the parameters x_k, i.e. $O = g_x(x_1, x_2, ..., x_{n_p})$. Furthermore, the MRI applies to single components. However, the changes may affect a number of components at the same time. For example, a change in a maintenance frequency will affect the unavailabilities of all of the components that undergo that particular maintenance policy.

Recently, the Differential Importance Measure (DIM) has been introduced to quantify the importance of the parameters x_k entering the system performance model [3]. DIM considers the total variation of the output function O due to a small variation of its parameters, taken one at a time. If the variation of the parameter is small enough, the variation of O is the total differential dO:

$$dO = \sum_{k=1}^{n_p} \frac{\partial O}{\partial x_k} \cdot dx_k \tag{5}$$

The DIM of the parameter x_l, $DIM(x_l)$, is defined as the fraction of the total change in O which pertains to the change in the parameter x_l:

$$DIM(x_l) = \frac{dO_{x_l}}{dO} = \frac{\frac{\partial O}{\partial x_l} \cdot dx_l}{\sum_{k=1}^{n_p} \frac{\partial O}{\partial x_k} \cdot dx_k} \tag{6}$$

The DIM is additive in the sense that the DIM of a subset of parameters x_r, x_s, .., x_t, is [3]: DIM($x_r \cup x_s \cup \ldots \cup x_t$)= DIM($x_r$)+ DIM($x_s$)+...+ DIM($x_t$).

The DIM can be useful in risk-informed applications involving the quantification of risk changes in O due to proposed changes of a plant technical specification, e.g. a surveillance/test/maintenance frequency. Being a first order local sensitivity measure, the DIM can be used to forecast a finite change ΔO due to any change in the parameters' values only provided that these latter changes are small enough to be used in (6). Only in this case, in fact, the higher-order contributions to ΔO in eqs. (1) and (3), which describe the interactions due to simultaneous change in pairs of parameters, triplets, etc. can be neglected. The effects of these interactions are illustrated in the next Section with reference to pairs of components.

4. JOINT FAILURE AND RELIABILITY IMPORTANCES

To evaluate quantitatively the interaction between components, the concepts of Joint Failure Importance (JFI) and of Joint Reliability Importance (JRI) of pairs of components have been introduced as an extension to the single-component MRI [4, 5]. JFI is introduced when the considered system performance O is a measure of the system loss (i.e. unreliability, unavailability, risk, etc.) and it is expressed in terms of the components' unavailabilities q_i, i=1, 2, …, n. JRI refers to the case in which O is a measure of the system gain (i.e. reliability, availability, etc.) and is expressed in terms of the components' availabilities $p_i=1-q_i$. JFI and JRI for components i and h are defined as:

$$JFI(i,h)=\frac{\partial^2 O}{\partial q_i \partial q_h}; \qquad JRI(i,h)=\frac{\partial^2 O}{\partial p_i \partial p_h}=-JFI(i,h) \tag{7}$$

An interesting property of the joint importance measures is the possibility of determining the sign of JFI(i,h) and JRI(i,h) based on the relative logical position of components i and h within the system. In particular [4]:

JFI(i,h) ≥ 0 (JRI(i,h) ≤ 0) for components in parallel

JFI(i,h) ≤ 0 (JRI(i,h) ≥ 0) for components in series

More generally:

JFI(i,h) ≥ 0 (JRI(i,h) ≤ 0) if components i and h appear together in at least one minimal cut-set but not in any minimal path-set.

JFI(i,h) ≤ 0 (JRI(i,h) ≥ 0) if components i and h appear together in at least one minimal path-set but not in any minimal cut-set.

Joint importance measures are useful to quantify the interactions of components with respect to the system performance. Awareness of such interactions among components is useful when the analysts are interested in evaluating the effects on the system of modifications regarding two components or, in a more general sense, two parameters (e.g. failure rates, maintenance periods, etc). Indeed, when planning a modification of a reliability parameter of a component towards a better performance (e.g. replacing it with a better-performing one, inspecting or maintaining it more frequently) one is often forced, by budget constraints, to sacrifice the performance of another.

5. SECOND ORDER DIFFERENTIAL IMPORTANCE MEASURE

For finite changes in the components unavailabilities Δq_i, it may be relevant to evaluate also the second order contribution to ΔO. With reference to components i and h, the variation of O, ΔO_{ih}, due to the variations of the parameters Δq_i and Δq_h is:

$$\begin{aligned} \Delta O_{ih} &= \frac{\partial O}{\partial q_i}\Delta q_i + \frac{\partial O}{\partial q_h}\Delta q_h + \frac{\partial^2 O}{\partial q_i \partial q_h}\Delta q_i \Delta q_h = \\ & MRI(i)\Delta q_i + MRI(h)\Delta q_h + JFI(i,h)\Delta q_i \Delta q_h \end{aligned} \quad (8)$$

A second order DIM, DIM^{II} can thus be defined as:

$$DIM^{II}(i,h) = \frac{\Delta O_{ih}}{\Delta O^{II}} \quad (9)$$

Note that, as stated in Section 2, when the rare event approximation can be used in the system modeling, one can neglect the computation of the JFI of pairs of components if the they do not belong to the same minimal cutset.

6. NUMERICAL EXAMPLE

6.1. Comparing the information of DIM and DIM^{II}

Consider the system of Figure 1. As system performance O we consider its limit unavailability. The components' unavailabilities are q_1=q_2 =10^{-3}, q_3=q_4=q<<1. In this numerical example we will compare the informative content of the measures DIM and DIM^{II} when assessing the effect on the system performance O of changes in the components' unavailabilities of pairs of components.

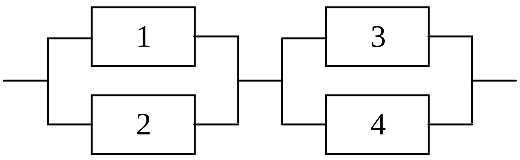

Figure 1. System reliability block diagram

Consider at first the behavior of DIM(1)=DIM(2) and DIM(3)=DIM(4) as functions of the parameter q=q_3=q_4 in the interval ($5\cdot10^{-4}$, $2\cdot10^{-3}$) shown in Figure 2. The change in the parameters' values is Δq_i=$10^{-3}\cdot q_i$, for i=1, 2, 3, 4. For q=10^{-3} (i.e. q_1=q_2=q_3=q_4), the four components have the same DIM, due to the symmetry of the system. Then, as expected, in the cases $q \neq q_1$=q_2 the most unavailable components are the most important according to the DIM. Indeed, DIM(1)=DIM(2) > DIM(3)=DIM(4) for q_1=q_2=10^{-3}> q=q_3=q_4 and DIM(1)=DIM(2) < DIM(3)=DIM(4) for q_1=q_2=10^{-3}<q= q_3=q_4.

From the additivity property [3], the first order DIMs for the pairs of components are:

$$\text{DIM}(1,2) = \text{DIM}(1) + \text{DIM}(2) = 2\cdot\text{DIM}(1) \quad (10)$$

DIM(3,4) = DIM(3) + DIM(4) = 2·DIM(3)

DIM(1,3) = DIM(1) + DIM(3) = DIM(2) + DIM(4) = DIM(2,4)

The behavior of the DIMs for the above pairs of parameters in the system is reported in Figure 3 as functions of q. Note that for $q=10^{-3}=q_1=q_2$, all measures DIM(1,2), DIM(1,3), DIM(2,4), are equal, due to the symmetry of the system and to the fact that the DIM considers the variation of one parameter value at a time. Still, when varying the values of the unavailabilities of two components simultaneously, one would expect a difference between the case of a pair of components in parallel, say (1, 2), and the case of a pair of components in series, say (1, 3). This difference can be traced by considering second-order interactions among the components, i.e. the DIM^{II}. In the cases $q \neq q_1=q_2$, the measures DIM(1,2)=2· DIM(1) and DIM(3,4)=2· DIM(3) duplicate the behavior of DIM(1)=DIM(2) and DIM(3)=DIM(4). Instead, DIM(1,3) =DIM(2,4) is independent on q. Indeed,

$$DIM(1,3) = \frac{DIM(1) + DIM(3)}{\sum_{j=1}^{n} DIM(j)} = \frac{DIM(1) + DIM(3)}{2 \cdot (DIM(1) + DIM(3))} = 0.5 \tag{11}$$

Let us now compute the second-order sensitivity coefficients (eq. (7)):

$$JFI(1,2) = \frac{\partial^2 O}{\partial q_1 \partial q_2} = 1 - q_3 q_4 = 1 - q^2 \qquad JFI(1,3) = -q_2 q_4 = -10^{-3} q$$

$$JFI(3,4) = 1 - q_1 q_2 = 1 - 10^{-6} \qquad JFI(2,4) = -q_1 q_2 = -10^{-3} q$$

As expected [4], JFI<0 for the components 1 and 3 in series and JFI>0 for the components 1 and 2 and 3 and 4 in parallel. Furthermore, as anticipated in Section 2, the absolute value of the JFI(1, 3) of components 1 and 3 in series is smaller than that of the two parallel pairs (1,2) and (3, 4). Indeed the values of the unavailabilities q_i are such that the rare event approximation in eq. (2) certainly holds. This would suggest that, actually, one could neglect the contribution corresponding to the interaction term of the couple of components in series.

Figure 3 also reports the measures $DIM^{II}(1, 2)$, $DIM^{II}(1, 3)$ and $DIM^{II}(3, 4)$ (symbols ◊, dots and *, respectively). Due to the small values of the variation of the parameters considered ($\Delta q_i = 10^{-3} \cdot q_i$ i=1, 2, 3, 4), the measures DIM^{II} do not differ appreciably from the DIM, since the contribution to the total change in the output performance O of the second-order terms are negligible. In the same Figure the DIM and DIM^{II} measures are reported for pairs of components in correspondence of larger values of the relative parameters' change, $\Delta q_i/q_i$, i=1, 2, 3, 4. As expected, the values of DIM and DIM^{II} differ progressively when higher values of $\Delta q_i/q_i$ are considered. As a general observation, DIM^{II} differs significantly from DIM for the pair (1,3) of components in series logic, whereas DIM^{II} reproduces the behaviour of DIM for the pairs (1,2) and (3,4) of components in parallel logic. This behaviour could seem unexpected since, as above stated, the interaction term JFI of components in series logic is negligible, whereas that of the components in parallel logic is large. In words, this fact can be explained as follows. In practice, due to the values of JFI(1,3)≈0 and JFI(1,2)≈1 (and JFI(3,4)≈1) we can write for $DIM^{II}(1,3)$ and $DIM^{II}(1,2)$:

$$DIM^{II}(1,3) \cong \frac{\frac{\partial O}{\partial q_1}\Delta q_1 + \frac{\partial O}{\partial q_3}\Delta q_3}{\Delta O^{II}}; \qquad DIM^{II}(1,2) \cong \frac{\frac{\partial O}{\partial q_1}\Delta q_1 + \frac{\partial O}{\partial q_2}\Delta q_2 + \Delta q_1 \Delta q_2}{\Delta O^{II}} \tag{12}$$

Due to the small value of the interaction term JFI(1,3)≈0, the numerator of $DIM^{II}(1,3)$ is equal to that of the DIM(1,3) for any value of $\Delta q_i/q_i$, whereas the denominator, which accounts for all of the JFIs, progressively increases for increasing values of $\Delta q_i/q_i$. As a consequence of this fact, the value of $DIM^{II}(1,3)$ is progressively shifted downwards for increasing values of $\Delta q_i/q_i$. Instead, as for the pair (1,2), both the numerator and the denominator of eq. (12) change their values from those of the numerator and the denominator of DIM(1,2), but the change is such that the ratio is approximately independent on Δq_i.

Let us first consider the case $q=10^{-3}=q_1=q_2$. While still DIM(1,2)=DIM(1,3)= DIM(3,4)= 0.5 by construction, $DIM^{II}(1,2) = DIM^{II}(3,4) > DIM^{II}(1,3)$ (Table 1). The ranking produced by the measure DIM^{II} suggests that increasing simultaneously the unavailabilities of the pairs of components in parallel logic (1,2) or (3,4) has a greater impact on the system unavailability than the same action performed on the pairs of components in series (1,3). This result is physically reasonable. An increase in unavailability of two components has more effect on the system unavailability if performed on components on the same node (i.e. in parallel) rather than on components on different nodes (i.e. in series). Indeed, with reference to the values of the ΔO_{ih}^{II} reported in Table 2, in the former situation the change in components unavailabilities is more critical since it impacts components on the same node, thus creating a system bottleneck. Instead, the latter situation is less critical for the system unavailability since the increase in components unavailability is shared by the two nodes. Table 1 also reports the values of DIM and DIM^{II} corresponding to $q=9\cdot10^{-4}$ and $q=1.2\cdot10^{-3}$, for the case $\Delta q_i/q_i=0.5$. As for the case $q=9\cdot10^{-4}$, DIM(1, 2)>DIM(1, 3)>DIM(3, 4), whereas $DIM^{II}(1, 2)>DIM^{II}(3, 4)>DIM^{II}(1, 3)$. Again, DIM considers the contribution to ΔO arising from a change in the unavailability of one of the components at a time. Therefore, the pair (1,2) results the most important according to this measure, since the two components 1 and 2, have the largest values of the first-order DIM (Figure 2), being more unavailable than components 3 and 4 ($q_1=q_2=10^{-3}>q_3=q_4=9\cdot10^{-4}$). The ranking provided by DIM^{II} is different and it reflects again that an increase in the unavailabilities of two components has more effect on the system unavailability if performed on components on the same node rather than on components on different nodes (refer also to the values of ΔO_{ih}^{II} reported in Table 2). This leads to the ranking inversion between the pairs (3,4) and (1,3). Similar considerations apply to the case of $q=1.2\cdot10^{-3}$.

Table 1. Values of DIM and DIM^{II} for the pairs of components (1,2), (1,3) and (3,4) for different values of q and $\Delta q_i/q_i$, i=1, 2, …, n

	$\Delta q_i/q_i$				DIM			DIM^{II}		
q	i=1	i=2	i=3	i=4	(1,2)	(1,3)	(3,4)	(1,2)	(1,3)	(3,4)
10^{-3}	0.5	0.5	0.5	0.5	0.50	0.50	0.50	0.50	0.40	0.50
$9\cdot10^{-4}$	0.5	0.5	0.5	0.5	0.55	0.50	0.45	0.55	0.40	0.45
$1.2\cdot10^{-3}$	0.5	0.5	0.5	0.5	0.41	0.50	0.59	0.41	0.40	0.59
10^{-3}	-0.5	-0.5	-0.5	-0.5	0.50	0.50	0.50	0.50	0.67	0.50
10^{-3}	0.5	-0.5	-0.5	+0.5	0.50	0.50	0.50	0.50	0.00	0.50

Table 2. Values of ΔO_{ih} and ΔO_{ih}^{II} for the pairs of components (i, h), for different values of q and $\Delta q_i/q_i$, i=1, 2, …, n

	$\Delta q_i/q_i$				ΔO_{ih}			ΔO_{ih}^{II}		
q	i=1	i=2	i=3	i=4	(1,2)	(1,3)	(3,4)	(1,2)	(1,3)	(3,4)
10^{-3}	0.5	0.5	0.5	0.5	10^{-6}	10^{-6}	10^{-6}	$1.25 \cdot 10^{-6}$	10^{-6}	$1.25 \cdot 10^{-6}$
$9 \cdot 10^{-4}$	0.5	0.5	0.5	0.5	10^{-6}	$9.05 \cdot 10^{-7}$	$8.10 \cdot 10^{-7}$	$1.25 \cdot 10^{-6}$	$9.05 \cdot 10^{-7}$	$1.01 \cdot 10^{-6}$
$1.2 \cdot 10^{-3}$	0.5	0.5	0.5	0.5	10^{-6}	$1.22 \cdot 10^{-6}$	$1.44 \cdot 10^{-6}$	$1.25 \cdot 10^{-6}$	$1.22 \cdot 10^{-6}$	$1.80 \cdot 10^{-6}$
10^{-3}	-0.5	-0.5	-0.5	-0.5	-10^{-6}	-10^{-6}	-10^{-6}	$-7.50 \cdot 10^{-7}$	-10^{-6}	$-7.50 \cdot 10^{-7}$
10^{-3}	0.5	-0.5	-0.5	+0.5	0	0	0	$-2.50 \cdot 10^{-7}$	0	$-2.50 \cdot 10^{-7}$

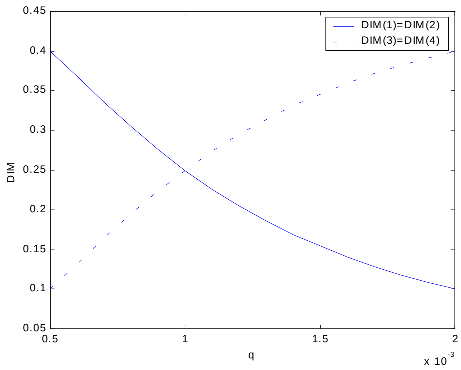

Figure 2. Values of DIM(1)=DIM(2) and DIM(3)=DIM(4) for different values of q, $\Delta q_i = 10^{-3} q_i$

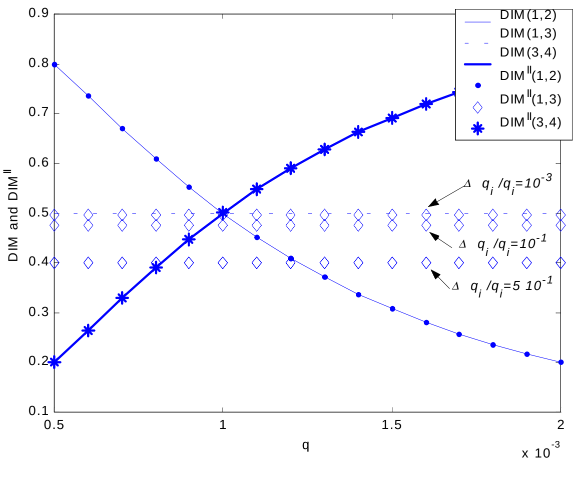

Figure 3. Values of DIM and DIM^{II} for the pairs of components (1,2), (1,3)=(2,4) and (3,4) for different values of q. Case of $\Delta q_i = 10^{-3} q_i$, i=1, 2, …, n

6.2. Use of DIM^{II} in risk-informed applications

In risk-informed applications, the information provided by DIM^{II} is handled by a decision-maker in different ways, depending on his/her goals.

Consider firstly the case of an analyst interested in reducing the costs associated to the system operation by replacing two components with two less expensive, but also less performing, ones or by extending their maintenance frequencies. For example, with reference to the case of $q=10^{-3}=q_1=q_2$ and $\Delta q_i/q_i=0.5$, the performance of the pair of components (1, 3) can be sacrificed if required by budget constraints with minor consequences on the system unavailability, contrary to the case of acting on the pairs of components (1, 2) or (3, 4) in parallel logic, as $DIM^{II}(1,2) = DIM^{II}(3,4) > DIM^{II}(1,3)$. Note that this conclusion can be inferred only on the basis of the ranking produced by the second-order measure DIM^{II}, as DIM(1, 2)=DIM(1, 3)=DIM(2 ,4).

Consider now the case in which the analyst is interested in identifying the pairs of components to be improved to get the largest improvement in system performance. In the case of $q=10^{-3}=q_1=q_2$ and $\Delta q_i/q_i= - 0.5$, i=1, 2, 3, 4 the values of the DIM^{II} (reported in Table 1) suggest that the improvement efforts should be devoted to the pair of components (1, 3) in series, characterized by the largest value of DIM^{II}, and thus leading to the largest reduction in system unavailability (see also the values of ΔO_{ih}^{II} in Table 2). Again, this result is obvious from the physical viewpoint: the improvement has more beneficial effects on the system availability if performed on components on different nodes (i.e. in series) rather than on components on the same node (i.e. in parallel). Indeed, in the latter case, the improvement would be less effective due to the presence of the other non-improved node in series, which remains an unvaried system bottleneck.

Another situation that can occur in risk-informed decision-making arises from the fact that, in practice, the analyst has often to cope with a constrained budget that might forbid spending resources on two components of a pair. Thus, in this case the final decision of the analyst must be a trade-off between improving the availability of a component while worsening that of another, still with the goal of attaining the largest improvement in the system availability. In this case, the analyst is looking at changes in the components unavailabilities Δq_i and Δq_h with opposite signs (i.e. if $\Delta q_i>0$ then $\Delta q_h<0$ and viceversa) and, thus, it is preferable to act on pairs of components with JFI>0. In this case, if we refer again to the case $q=q_3=q_4= 10^{-3}=q_1=q_2$, for the generic pair ($i$, h), the net contribution of the first-order terms of eq. (1) equals zero since MRI(i)=MRI(h) and $\Delta q_i =-\Delta q_h$ and the system output variation ΔO_{ih}^{II} becomes:

$$\Delta O_{ih}^{II} = \frac{\partial^2 O}{\partial q_i \partial q_h} \Delta q_i \Delta q_h = JFI(i,h) \Delta q_i \Delta q_h \leq 0 \tag{13}$$

The values of the DIM(i, h) and DIM^{II}(i, h) and of the corresponding ΔO_{ih} and ΔO_{ih}^{II} for the pairs (i, h)=(1, 2), (1, 3) and (2, 4) are reported in Table 1 and Table 2 respectively with reference to the case $q=q_3=q_4= 10^{-3}=q_1=q_2$, $\Delta q_i/q_i=0.5$, i=1, 2, 3, 4. Evidently, those pairs of components with JFI>0, i.e. (1, 2) and (3, 4), are characterized by negative contributions ΔO_{ih}^{II}, corresponding to an increase in system unavailability. Indeed, if the unavailabilities of two components in parallel logic are changed in opposite directions, then, since the components with the lowest unavailability determines the unavailability of the pair, the overall system unavailability decreases. On the contrary, if the unavailabilities of two components belonging to different nodes in series are changed in opposite directions, then due to the weak interactions among the components (JFI≈0 in eq. (25)) the system unavailability remains basically unchanged.

CONCLUSIONS

This paper considers the differential importance measure, DIM, and the Joint Failure Importance measure, JFI, recently introduced in literature. The DIM is a first-order sensitivity measure that ranks the parameters of the risk model according to the fraction of the total change in the risk that is due to a small change in the parameters' values, taken one at a time, and, by construction, it does not account for second-order interactions among components. Instead, the JFI measure is a second order sensitivity measure, which considers the interactions of coupled changes to system design.

In this paper, a second-order extension of the DIM, named DIM^{II}, is proposed for accounting of the interactions of pairs of components when evaluating the change in system performance due to changes of the reliability parameters of the components. The extension aims at supplementing the first-order information provided by DIM with the second-order information provided by JRI and JFI.

A numerical application is presented in which the informative contents of DIM and DIM^{II} are compared. The results confirm that in certain cases when second-order interactions among components are accounted for, the importance ranking of the components may differ from those produced by a first-order sensitivity measure. Obviously, the need of resorting to information on second-order effects depends on the magnitude of the changes of the parameters values and on the non linearity of the system.

It is shown in the paper that in some applications it is possible to determine a priori whether the interaction term in DIM^{II} can be neglected even for large changes in the parameters, thus avoiding the computation of the JRI and JFI measures for all of the possible pairs of components. In particular, second-order interactions among components are negligible if the components do not appear together in the same minimal cutset. Furthermore, guidelines for the use of DIM^{II} in risk-informed decision-making are provided for different cases.

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