

Cosmic Calibration - Statistical Modeling for Dark Energy

UCSC

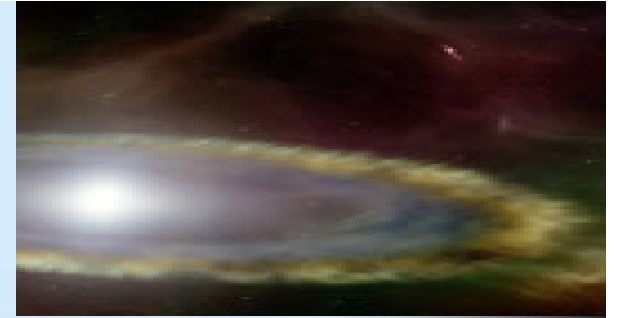
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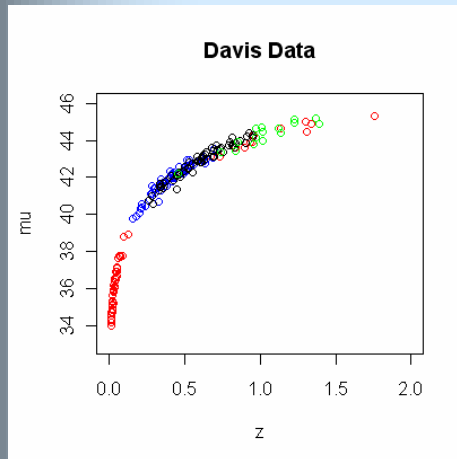
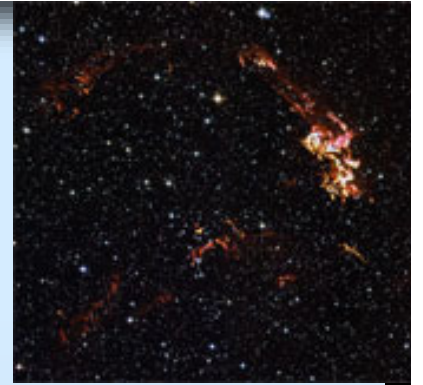
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Overview

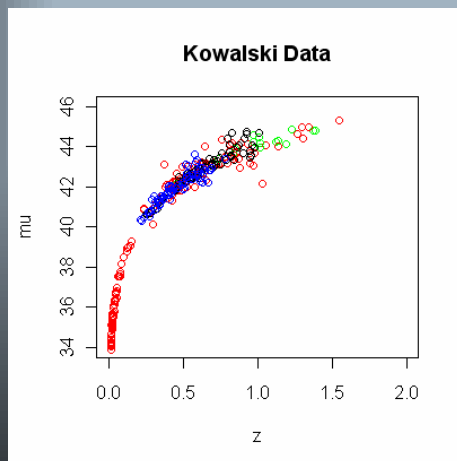


- The Universe is expanding (1920s)
- Observations have been made that the Universe is expanding at an accelerating pace (1998)
- Ordinary matter would mean the Universe is decelerating
- Dark energy is the unknown driver of this acceleration
- Dark energy has an equation of state relating its pressure to density; this equation is our focus to understand more about the nature of dark energy
- This equation of state can be measured by studying the luminosity distance-redshift relation for supernovae
- In this study, we employ supernovae data, including measurement errors, to determine whether the equation of state is constant or not
- Our current method is based on Bayesian analysis of a differential equation and modeling $w(z)$ directly, where $w(z)$ is the equation of state parameter.

Datasets of Interest



The data we receive has a redshift (z) value for each supernova and a value for μ (observed distance modulus) and a standard deviation for the observational error of μ . These are summary statistics for each supernovae that have come from complex fitting algorithms of weeks worth of observational data.



The Davis data -192 supernovae (SNe Ia)
The Kowalski data – 307 supernovae (SNe Ia)

The four colors mark different observers of the supernovae. Certain astronomers focus on particular values of z to collect supernova data.

Likelihood Equation

- The main parameter of interest is $w(z)$
- Three other unknown parameters also have to be estimated H_0 , Ω_m , and σ
- The main equation of interest is a transformation:

$$T(z, H_0, \Omega_m) = 25 + 5 \log_{10} \left(\frac{c(1+z)}{H_0} \int_0^z \left(\Omega_m (1+s)^3 + (1-\Omega_m)(1+s)^3 e^{3 \int_0^s \frac{w(u)}{1+u} du} \right)^{-0.5} ds \right)$$

- To be able to use this equation we will need to specify a form for $w(u)$.
- This leads to the following likelihood equation:

$$L(\sigma, w_0, H_0, \Omega_m) \propto \left(\frac{1}{\tau_i \sigma} \right)^n e^{-\frac{1}{2} \sum_{i=1}^n \left(\frac{\mu_i - T(z_i, H_0, \Omega_m)}{\tau_i \sigma} \right)^2}$$

Model 1: $w(u) = w_0$

Priors:

Prior sensitivity was examined and thus far it seems that the prior does not change the outcome of the estimations; we are also using rather non-informative priors with large spread:

$$\pi(w_0) \sim U(-25, 2)$$

$$\pi(H_0) \sim N(73, 3.2)$$

$$\pi(\Omega_m) \sim N(0.266, 0.04)$$

$$\pi(\sigma) \sim IG(2.01, 1)$$

Posteriors: (these are obtained through MCMC Metropolis-Hastings steps)

Dataset	w_0	H_0	Ω_m	σ
Davis	(-1.43, -0.93)	(65.09, 67.79)	(0.227, 0.346)	(0.94, 1.10)
Kowalski	(-1.37, -0.89)	(69.50, 71.56)	(0.238, 0.357)	(0.95, 1.08)

Model 2: $w(u) = \alpha + \beta u$

Priors:

$$\pi(\alpha) \sim U(-25, 2)$$

$$\pi(\beta) \sim U(-10, 10)$$

$$\pi(H_0) \sim N(73, 3.2)$$

$$\pi(\Omega_m) \sim N(0.266, 0.04)$$

$$\pi(\sigma) \sim IG(2.01, 1)$$

Posteriors:

Dataset	α	β	H_0	Ω_m	σ
Davis	(-1.54, -0.77)	(-2.26, 1.59)	(64.9, 67.8)	(0.22, 0.35)	(0.93, 1.11)
Kowalski	(-1.53, -0.97)	(-0.52, 1.98)	(69.9, 72.1)	(0.21, 0.35)	(0.95, 1.08)

Conclusions



Is $w(u) = -1$?

- We cannot conclusively say that $w(u) = -1$
- But currently both Model 1 and Model 2 support this hypothesis
- The fitters being used to compile the two datasets are producing different results for H_0
- Also all methods presented here have been tested with simulated datasets and correct results have been obtained



Future Work

- More work needs to be done in explaining the role of H_0 and the differences in the two datasets
- We are in the process of fitting a Gaussian Process to $w(u)$ instead of explicitly specifying its parametric form
- We also have found trends in the standard deviations for the measurements of μ from different observers that will be examined
- The cosmologists would like to know where more observations (on the z axis) are needed to shrink uncertainty