The Analytic Hierarchy Process (AHP) for Decision Making

Decision Making involves setting priorities and the AHP is the methodology for doing that.

Real Life Problems Exhibit:

Strong Pressures and Weakened Resources

Complex Issues - Sometimes There are No “Right” Answers

Vested Interests

Conflicting Values

Most Decision Problems are Multicriteria

- Maximize profits
- Satisfy customer demands
- Maximize employee satisfaction
- Satisfy shareholders
- Minimize costs of production
- Satisfy government regulations
- Minimize taxes
- Maximize bonuses

General Observations on Decision Making

- Logical thinking is broad but its validity is localized to the area of study.
- Logical thinking is verbal and qualitative.
- The mind can't deal with the effects of intangible factors in a precise way.
- The mind can't deal with too many factors at the same time.
- We need to scroll our understanding by providing judgments about the parts.
- We also need a method of synthesis that puts together the pieces in a valid way.
- We need to do benefit, cost, opportunity, and risk analyses.
- Prediction is critical to decision making. A decision must survive the turbulence of the environment which one needs to anticipate.
- Group participation is necessary and must be made possible and easy.
- It is also necessary to allow for all sorts of dependencies and feedback among the elements of the decision.

Fundamental Requirements of a Decision Theory

If one were to regard all decision techniques as offspring of a holistic decision approach, then they should at least:

- Yield the same answer for the best decision
- Structure a decision problem as thoroughly as required by the complexity involved and measure its intangibles
- Allow for dependence and feedback
- Include judgments of different decision makers and weight them by their importance
- Be capable of accurate prediction
- Allow for the optimal allocation of resources

Why Do We Make Decisions?

All that we ever think or do is predicated on the fact that we are alive. The dead do not know anything. It appears that the purpose of life is continuity by reproducing its kind. Thus to the best of our knowledge the most general purpose of everything we do is survival.

We have intelligence, wisdom and creativity to assist life to survive. The meaning of life as it is served by these three is to stimulate our consciousness and increase our awareness to expand the meaning, quality and value of our lives in the future. Decision-making uses intelligence, wisdom and creativity to help us transit from the past and present to the future. We make decisions because we want to be right, satisfy many needs, and get the greatest advantage, whatever that may be, out of life.

We all make decisions all the time consciously and unconsciously. Of course most decisions are made on the spur of the moment and are not subject to our best thinking efforts because we do not know enough at the time to work them out that fast. What interests us are the serious, complex and long term decisions that are worthwhile to work out. We make decisions for the purpose of persuasion and agreement, to design a best strategy for action and for coming out right and best as things turn out. For decisions to come out right we need to consider the effects on our value system of the implications of that decision.

The values that drive us to make a decision may be some or all of the following: economic, social, political, psychological, religious-scientific-ideological and technical. The last may be cultural, behavioral, legal, medical, physical, historical, archeological, and so on, depending on the nature of the particular decision.

A decision may be too costly to make and its outcome may be uncertain. Evaluating a decision requires that we consider both its *benefits* and its *costs* as if it has already been made. We also need to consider both the possible *opportunities* that it could give rise to and the r*isks* or the likelihood that it may not work out in the face of hazards and uncertainties of the future. In many situations these four aspects of evaluation have different importance. Their importance must be assessed in terms of the values mentioned above and how important these values themselves are for the given decision.

Who Makes Decisions?

Decisions are made by individuals or by groups, by children and by adults, by people and by other forms of life. Individuals and groups may cooperate to make a decision or may find themselves in a conflict situation that requires the assistance of a mediator. There are scientific methods for dealing with individual judgments working together cooperatively in a group and for conflict situations.

What Kinds of Decisions Are There?

Simple and complex, individual and group, cooperation and conflict, short- and long-term, rationally worked out or involving the use of force, worked out sequentially or as a whole.

When Do We Make Decisions?

Every aspect of our life may be regarded as part of a decision process that is subject to internal or external influences. Thus decision-making is an on-going process.

Where Do We Make Decisions?

In a reflective, informed and organized atmosphere and also in a battlefield or an emergency and spontaneously. Thus we need to be trained in making decisions so when we are in a decision- making situation, we can move forward without becoming paralyzed.

How Do We Make Decisions?

There are several wide-ranging ways for making a decision. The crudest and most primitive is by using chance, such as flipping an unbiased coin, because all the alternatives seem equally likely due to the absence of adequate information about each of them. A more rational way of making decisions is to count the number of advantages and disadvantages and make or not make that decision depending on which is greater.

The same approach can be used when there are several alternatives to consider. In between this approach and that of tossing a coin, one can simply make a list of factors and reflect on them. Sometimes decisions are made by a committee because it is thought that there is safety in having several people evaluate a decision, but they can all be uninformed or when informed they can lack objectivity.

Still it is better to have several people involved because they can raise more questions than a single individual. Some decisions are avoided by taking a long time to consider them in which case it is hoped that they would no longer need to be made. A more rational and scientific approach to making decisions is by prioritization and the making of tradeoffs.

Scientific decision-making has a richness in it due to the complex structures it offers the decision-maker to relate and prioritize the elements of the decision. It enables him to construct or piece together the elements into a whole and perceive possibilities that may not have been there in the beginning. Most importantly scientific decisions enable one to make compromises and tradeoffs among the parts because they are not equally important.

Order, Proportionality and Ratio Scales

- All order, whether in the physical world or in human thinking, involves proportionality among the parts, establishing harmony and synchrony among them. Proportionality means that there is a ratio relation among the parts. Thus, to study order or to create order, we must use ratio scales to capture and synthesize the relations inherent in that order. The question is how?

Decision Making in Complex Environments

Decision making today is a science. People have hard decisions to make and they need help because many lives may be involved, the survival of the business depends on making the right decision, or because future success and diversification must survive competition and surprises presented by the future.

We have a scientific way to make decisions that is practical, and both biologically and mathematically correct because it parallels our inborn talent to make pairwise comparisons among the elements being considered. Its correctness as a scientific theory has been tested in numerous predictions in business, in economic forecasting and in predicting the outcomes of sporting events, and to the successful outcome of political and social conflicts.

The Analytic Hierarchy Process is the way to make decisions implemented by its powerful and user-friendly computer software, Expert Choice and Team Expert Choice.

I am here to show you what the Analytic Hierarchy Process is, and how it works in practice by applying it to many examples.

Decision Making

We need to prioritize both tangible and intangible criteria:

- In most decisions, intangibles such as
  - political factors and
  - social factors

  take precedence over tangibles such as
  - economic factors and
  - technical factors
- It is not the precision of measurement on a particular factor that determines the validity of a decision, but the importance we attach to the factors involved.
- How do we assign importance to all the factors and synthesize this diverse information to make the best decision?

Aren't Numbers Numbers?

An elderly couple looking for a town to which they might retire found Summerland, in Santa Barbara County, California, where a sign post read:

Summerland

Population		3001
Feet Above Sea Level		208
Established		1870
	Total	5079

"Let's settle here where there is a sense of humor," said the wife; and they did.

Knowledge Is Not In The Numbers

Isabel Garuti is an environmental researcher whose father-in-law is a master chef in Santiago, Chile. He owns a well known Italian restaurant called Valerio. He is recognized as the best cook in Santiago. Isabel had eaten a favorite dish risotto ai funghi, rice with mushrooms, many times and loved it so much that she wanted to learn to cook it herself for her husband, Valerio's son, Claudio. So she armed herself with a pencil and paper, went to the restaurant and begged Valerio to spell out the details of the recipe in an easy way for her. He said it was very easy. When he revealed how much was needed for each ingredient, he said you use a little of this and a handful of that. When it is O.K. it is O.K. and it smells good. No matter how hard she tried to translate his comments to numbers, neither she nor he could do it. She could not replicate his dish. Valerio knew what he knew well. It was registered in his mind, this could not be written down and communicated to someone else. An unintelligent observer would claim that he did not know how to cook, because if he did, he should be able to communicate it to others. But he could and is one of the best.

Valerio <u>can</u> say, “Put more of this than of that, don’t stir so much,” and so on. That is how he cooks his meals - by following his instincts, not formalized logically and precisely. The question is: how does he synthesize what he knows?

Knowing Less, Understanding More

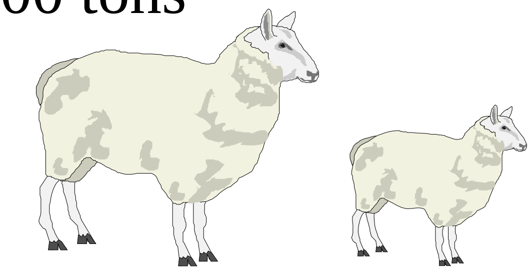
You don’t need to know every-thing to get to the answer.

Expert after expert missed the revolutionary significance of what Darwin had collected. Darwin, who knew less, somehow understood more.

Do Numbers Have an Objective Meaning?

Butter: 1, 2,…, 10 lbs.; 1,2,…, 100 tons

Sheep: 2 sheep (1 big, 1 little)



Temperature: 30 degrees Fahrenheit to New Yorker, Kenyan, Eskimo

Since we deal with **Non-Unique Scales** such as [lbs., kgs], [yds, meters], [Fahr., Celsius] and such scales cannot be combined, we need the idea of **PRIORITY**.

PRIORITY becomes an abstract unit valid across all scales.

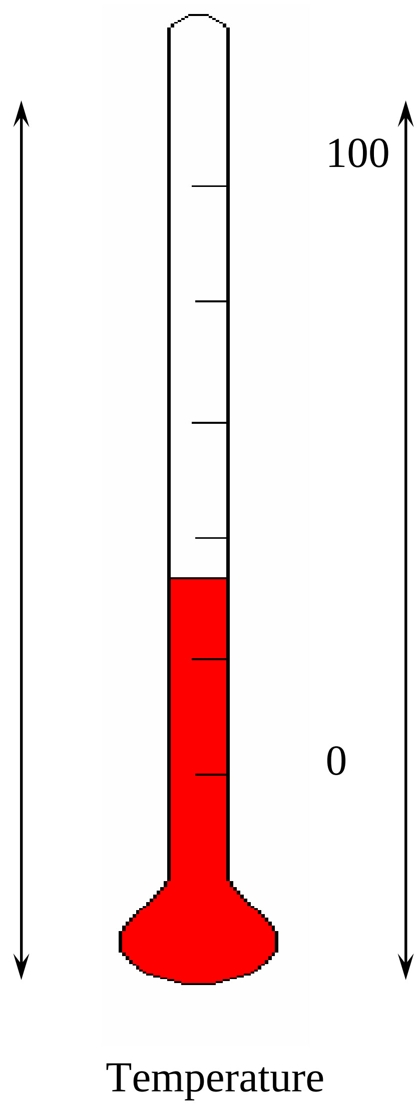
A priority scale based on preference is the AHP way to standardize non-unique scales in order to combine multiple criteria.

Nonmonotonic Relative Nature of Absolute Scales

Good for
preserving food

Bad for
preserving food

Good for
preserving food



Bad for
comfort

Good for
comfort

Bad for
comfort

OBJECTIVITY!?

Bias in upbring: objectivity is agreed upon subjectivity. We interpret and shape the world in our own image. We pass it along as fact. In the end it is all obsoleted by the next generation.

Logic breaks down: Russell-Whitehead Principia; Gödel's Undecidability Proof.

Intuition breaks down: circle around earth; milk and coffee.

How do we manage?

Making a Decision

Widget B is cheaper than Widget A

Widget A is better than Widget B

Which Widget would you choose?

Basic Decision Problem

Criteria: Low Cost > Operating Cost > Style

Car:	A	B	B
	V	V	V
Alternatives:	B	A	A

Suppose the criteria are preferred in the order shown and the cars are preferred as shown for each criterion. Which car should be chosen? It is desirable to know the strengths of preferences for tradeoffs.

Background on AHP

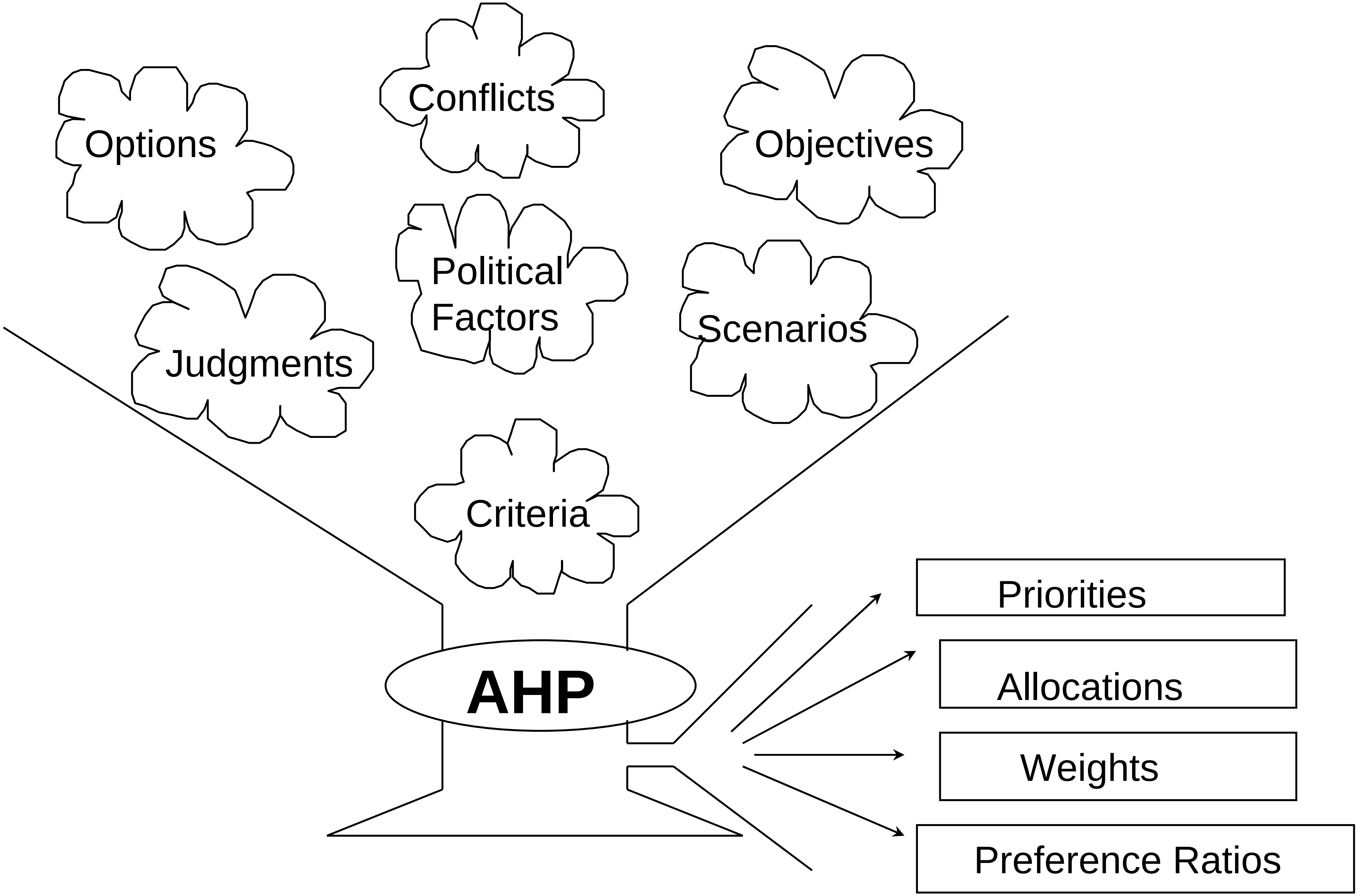
To understand the world we assume that:

We can describe it

We can define relations between its parts and

We can apply judgment to relate the parts according to

a goal or purpose that we have in mind.


Options
Conflicts
Objectives
Political Factors
Judgments
Scenarios
Criteria
AHP
Priorities
Allocations
Weights
Preference Ratios

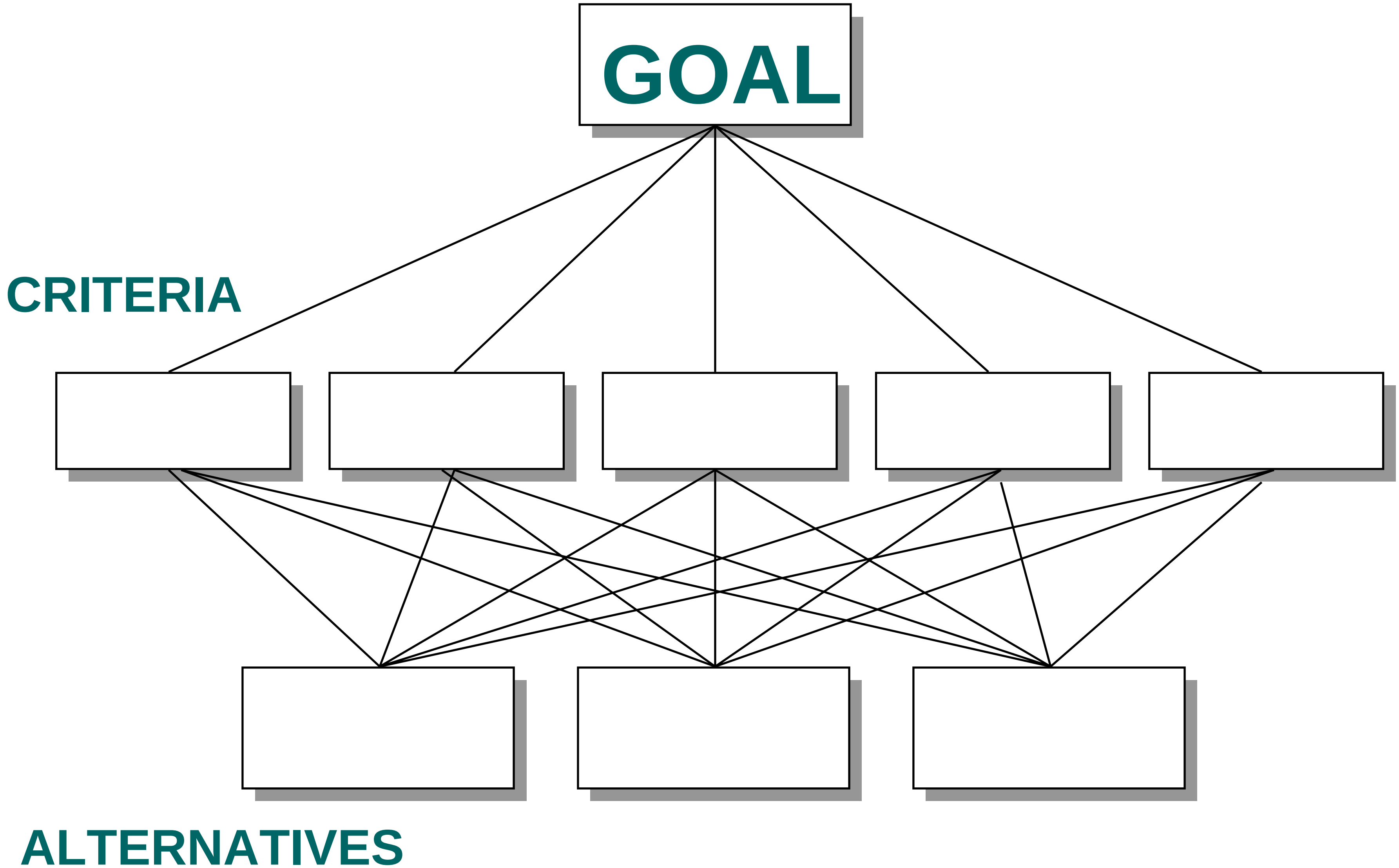
Decision Making is a process that leads one to:

- Understand and define the problem as completely as possible.
- Structure a problem as a hierarchy or as a system with dependence loops.
- Elicit judgments that reflect ideas, feelings and emotions.
- Represent those judgments with meaningful numbers.
- Synthesize Results
- Analyze sensitivity to changes in judgments.

TYPES OF DECISIONS

There are at least three types of decisions

- Choosing a single best alternative. This includes Zero/One, Yes/No types of decisions. If there are several alternatives, one looks for the best one among them.
- Ranking alternatives on an ordinal or interval scale without regard to proportionality among them. It is difficult to use this information to allocate resources.
- Ranking alternatives proportionately on a ratio scale This makes it possible to allocate resources and choose a best portfolio. This is the Analytic Hierarchy Process approach and includes the others.


GOAL
CRITERIA
ALTERNATIVES

Power of Hierarchic Thinking

A hierarchy is an efficient way to organize complex systems. It is efficient both structurally, for representing a system, and functionally, for controlling and passing information down the system.

Unstructured problems are best grappled with in the systematic framework of a hierarchy or a feedback network.

Relative Measurement

In relative measurement a preference judgment is expressed on each pair of elements with respect to a common property they share.

In practice this means that a pair of elements in a level of the hierarchy are compared with respect to parent elements to which they relate in the level above.

Relative Measurement cont.

If, for example, we are comparing two apples according to weight we ask:

- Which apple is bigger?
- How much bigger is the larger than the smaller apple?
  Use the smaller as the unit and estimate how many more times bigger is the larger one.
- The apples must be relatively close (homogeneous) if we hope to make an accurate estimate.

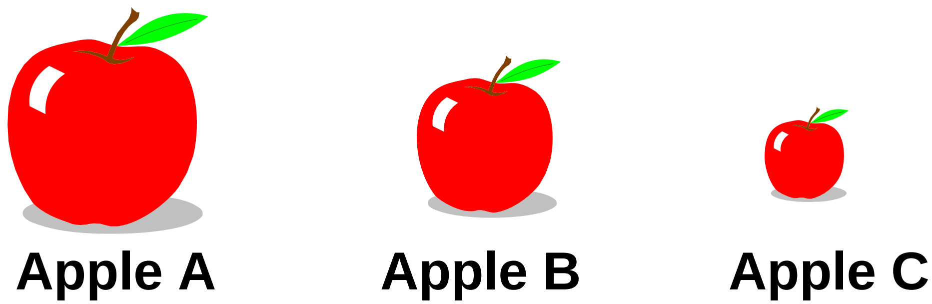
Relative Measurement cont.

- The Smaller apple then has the reciprocal value when compared with the larger one. There is no way to escape this sort of reciprocal comparison when developing judgments

- If the elements being compared are not all homogeneous, they are placed into homogeneous groups of gradually increasing relative sizes (homogeneous clusters of homogeneous elements). Judgments are made on the elements in one group of small elements, and a "pivot" element is borrowed and placed in the next larger group and its elements are compared. This use of pivot elements enables one to successively merge the measurements of all the elements. Thus homogeneity serves to enhance the accuracy of measurement.

Comparison Matrix

Given: Three apples of different sizes.



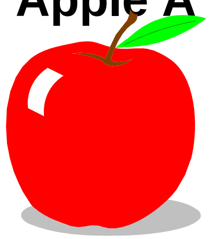
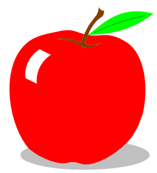
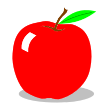
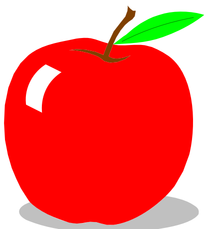
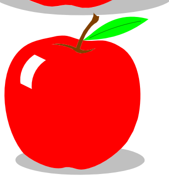
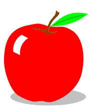
We Assess Their Relative Sizes By Forming Ratios

Size Comparison	Apple A	Apple B	Apple C
Apple A	S_1/S_1	S_1/S_2	S_1/S_3
Apple B	S_2 / S_1	S_2 / S_2	S_2 / S_3
Apple C	S_3 / S_1	S_3 / S_2	S_3 / S_3

Pairwise Comparisons

Size

- Apple A
- Apple B
- Apple C

Size Comparison	Apple A 	Apple B 	Apple C 	Resulting Priority Eigenvector	Relative Size of Apple
 Apple A	1	2	6	6/10	A
 Apple B	1/2	1	3	3/10	B
 Apple C	1/6	1/3	1	1/10	C

When the judgments are consistent, as they are here, any normalized column gives the priorities.

Consistency

In this example Apple B is 3 times larger than Apple C. We can obtain this value directly from the comparisons of Apple A with Apples B & C as 6/2 = 3. But if we were to use judgment we may have guessed it as 4. In that case we would have been inconsistent.

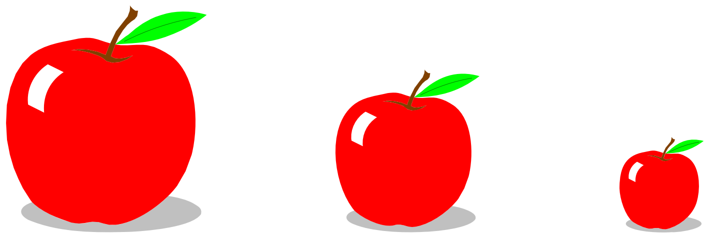
Now guessing it as 4 is not as bad as guessing it as 5 or more. The farther we are from the true value the more inconsistent we are. The AHP provides a theory for checking the inconsistency throughout the matrix and allowing a certain level of overall inconsistency but not more.

Consistency cont.

- Consistency itself is a necessary condition for a better understanding of relations in the world but it is not sufficient. For example we could judge all three of the apples to be the same size and we would be perfectly consistent, but very wrong.

- We also need to improve our validity by using redundant information.

- It is fortunate that the mind is not programmed to be always consistent. Otherwise, it could not integrate new information by changing old relations.



Consistency cont.

With the AHP We Can Allow Some Inconsistency

Because the world of experience is vast and we deal with it in pieces according to whatever goals concern us at the time, our judgments can never be perfectly precise.

It may be impossible to make a consistent set of judgments on some pieces that make them fit exactly with another consistent set of judgments on other related pieces. So we may neither be able to be perfectly consistent nor want to be.

We must allow for a modicum of inconsistency.

Consistency cont.

How Much Inconsistency to Tolerate?

- Inconsistency arises from the need for redundancy.
- Redundancy improves the validity of the information about the real world.
- Inconsistency is important for modifying our consistent understanding, but it must not be too large to make information seem chaotic.
- Yet inconsistency cannot be negligible; otherwise, we would be like robots unable to change our minds.
- Mathematically the measurement of consistency should allow for inconsistency of no more than one order of magnitude smaller than consistency. Thus, an inconsistency of no more than 10% can be tolerated.
- This would allow variations in the measurement of the elements being compared without destroying their identity.
- As a result the number of elements compared must be small, i.e. seven plus or minus two. Being homogeneous they would then each receive about ten to 15 percent of the total relative value in the vector of priorities.
- A small inconsistency would change that value by a small amount and their true relative value would still be sufficiently large to preserve that value.
- Note that if the number of elements in a comparison is large, for example 100, each would receive a 1% relative value and the small inconsistency of 1% in its measurement would change its value to 2% which is far from its true value of 1%.

Comparison of Intangibles

The same procedure as we use for size can be used to compare things with intangible properties. For example, we could also compare the apples for:

- TASTE
- AROMA
- RIPENESS

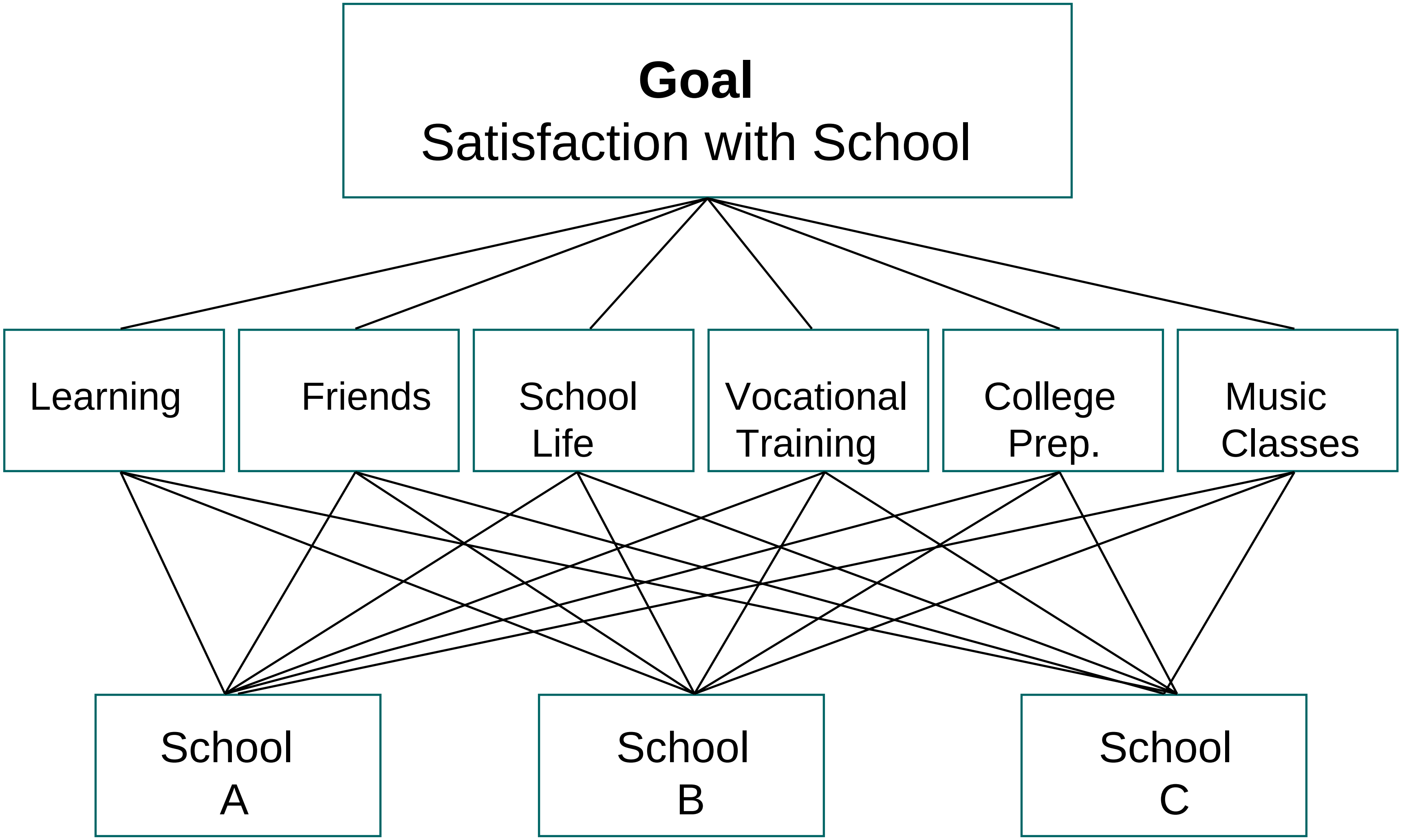
The Analytic Hierarchy Process (AHP) is the Method of Prioritization

- AHP captures priorities from paired comparison judgments of the
- elements of the decision with respect to each of their parent criteria.

- Paired comparison judgments can be arranged in a matrix.

- Priorities are derived from the matrix as its principal eigenvector,
- which defines a ratio scale. Thus, the eigenvector is an intrinsic
- concept of a correct prioritization process. It also allows for the
- measurement of inconsistency in judgment.

- Priorities derived this way satisfy the property of a ratio scale
- just like pounds and yards do.


Goal
Satisfaction with School
Learning
Friends
School Life
Vocational Training
College Prep.
Music Classes
School A
School B
School C

Scale For Pairwise Comparisons

Equal importance

Moderate importance of one over another

Strong or essential importance

Very strong or demonstrated importance

Extreme importance

Fundamental Scale of Absolute Numbers for Pairwise Comparisons

1	Equal importance
3	Moderate importance of one over another
5	Strong or essential importance
7	Very strong or demonstrated importance
9	Extreme importance
2,4,6,8	Intermediate values
	Use Reciprocals for Inverse Comparisons

Scale For Comparing Two Close Elements

1.1	Very Slight
1.3	Moderate
1.5	Strong
1.7	Very Strong
1.9	Extreme

A more effective way for comparing elements that are very close is to include them in the comparison of a larger set with which they are homogeneous and allow for their difference when comparing them with other elements. The answers are invariably very good.

Which Drink is Consumed More in the U.S.?

Drink Consumption in the U.S.	Coffee	Wine	Tea	Beer	Sodas	Milk	Water
Coffee	1	9	5	2	1	1	1/2
Wine		1	1/3	1/9	1/9	1/9	1/9
Tea			1	1/3	1/4	1/3	1/9
Beer				1	1/2	1	1/3
Sodas					1	2	1/2
Milk						1	1/3
Water							1

The derived scale based on the judgments in the matrix is:

Coffee	Wine	Tea	Beer	Sodas	Milk	Water
.177	.019	.042	.116	.190	.129	.327

with a consistency ratio of .022.

The actual consumption (from statistical sources) is:

.180	.010	.040	.120	.180	.140	.330

Estimating which Food has more Protein

Food Consumption in the U.S.	A	B	C	D	E	F	G
A: Steak	1	9	9	6	4	5	1
B: Potatoes		1	1	1/2	1/4	1/3	1/4
C: Apples			1	1/3	1/3	1/5	1/9
D: Soybean				1	1/2	1	1/6
E: Whole Wheat Bread					1	3	1/3
F: Tasty Cake						1	1/5
G: Fish							1

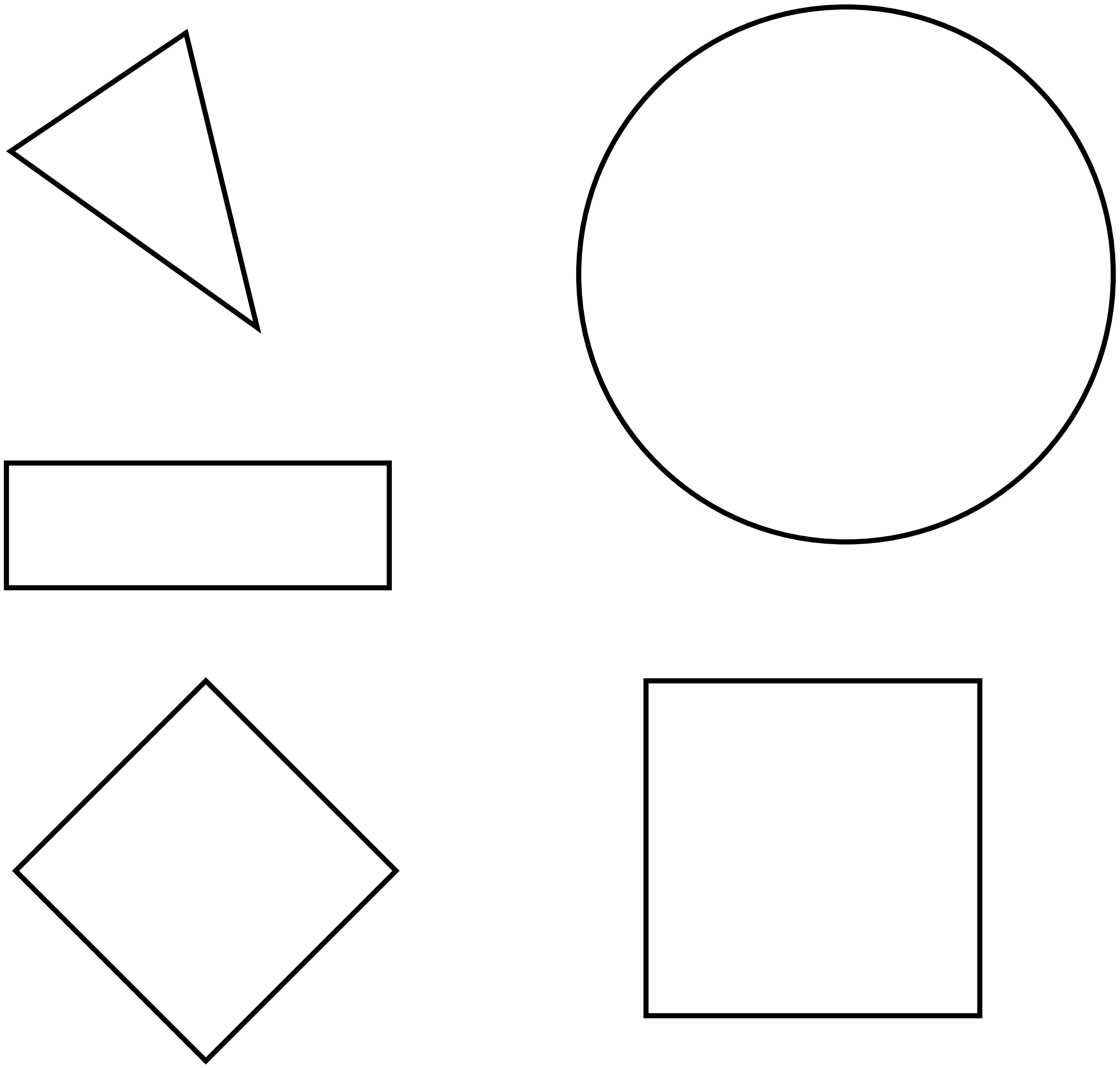
The resulting derived scale and the actual values are shown below:

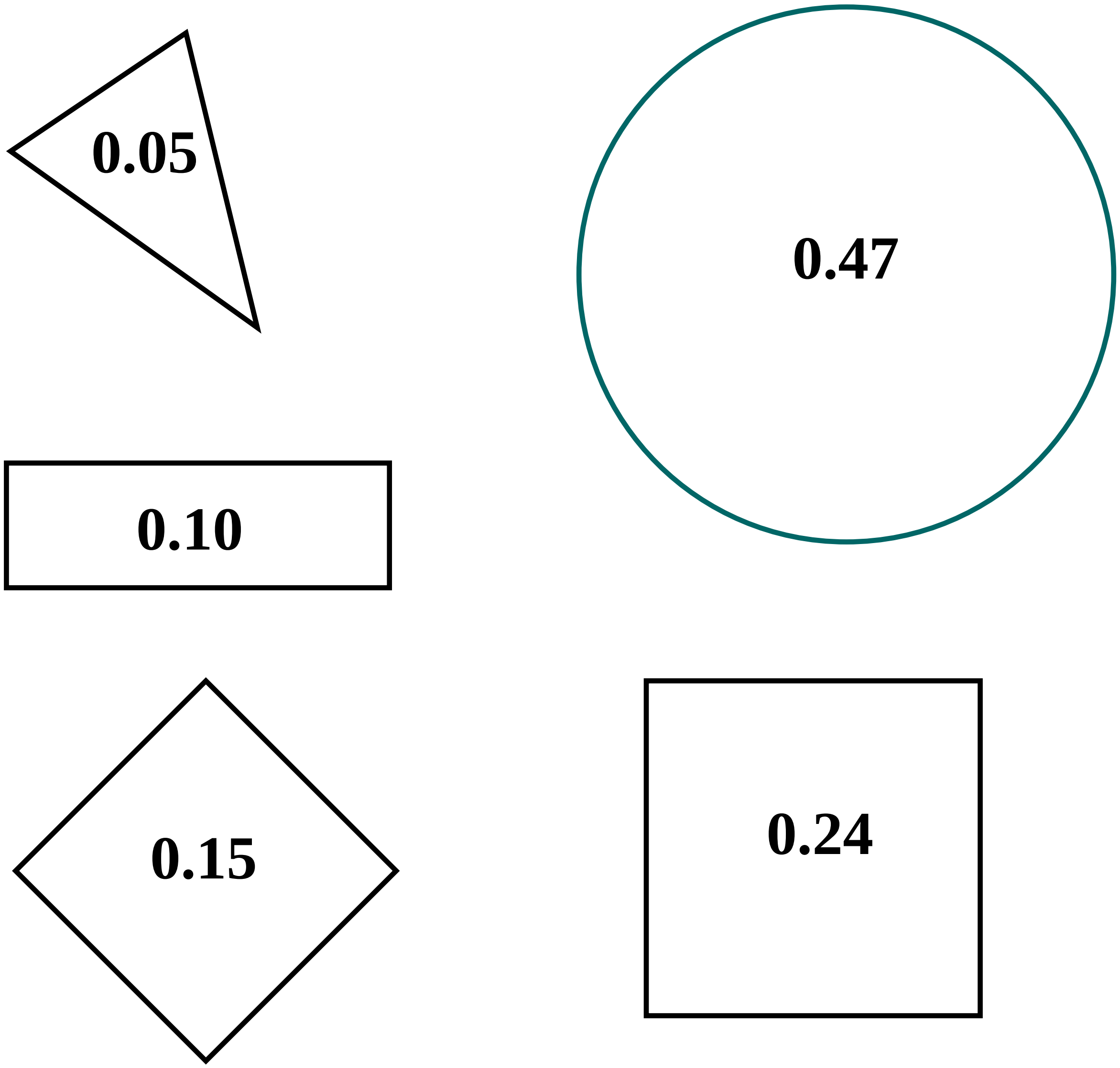
	Steak	Potatoes	Apples	Soybean	W. Bread	T. Cake	Fish
Derived	.345	.031	.030	.065	.124	.078	.328
Actual	.370	.040	.000	.070	.110	.090	.320

(Derived scale has a consistency ratio of .028.)

Fundamental Scale of Absolute Numbers Correspondence with Feelings

Low	1	Equal	(low low)
	2	Between	(medium low)
	3	Moderate	(high low)
Medium	4	Between	(low medium)
	5	Strong	(medium medium)
	6	Between	(high medium)
High	7	Very strong	(low high)
	8	Between	(medium high)
	9	Extreme	(high high)




0.05
0.47
0.10
0.15
0.24

School Selection

	L	F	SL	VT	CP	MC	Weights
Learning	1	4	3	1	3	4	.32
Friends	1/4	1	7	3	1/5	1	.14
School Life	1/3	1/7	1	1/5	1/5	1/6	.03
Vocational Trng.	1	1/3	5	1	1	1/3	.13
College Prep.	1/3	5	5	1	1	3	.24
Music Classes	1/4	1	6	3	1/3	1	.14

Comparison of Schools with Respect to the Six Characteristics

	Learning A	B	C	Priorities
A	1	1/3	1/2	.16
B	3	1	3	.59
C	2	1/3	1	.25

	Friends A	B	C	Priorities
A	1	1	1	.33
B	1	1	1	.33
C	1	1	1	.33

	School Life A	B	C	Priorities
A	1	5	1	.45
B	1/5	1	1/5	.09
C	1	5	1	.46

	Vocational Trng. A	B	C	Priorities
A	1	9	7	.77
B	1/9	1	1/5	.05
C	1/7	5	1	.17

	College Prep. A	B	C	Priorities
A	1	1/2	1	.25
B	2	1	2	.50
C	1	1/2	1	.25

	Music Classes A	B	C	Priorities
A	1	6	4	.69
B	1/6	1	1/3	.09
C	1/4	3	1	.22

Two Criteria Measured in Dollars, Sums and Relative sums

200	150	350	.269
300	50	350	.269
500	100	600	.462

200/1000	150/300		.269
300/1000	50/300	sums ♑	.269
500/1000	100/300		.462

Multiply first column by its relative importance 1000/1300 and second column by 300/1300 add and then normalize to get the relative amounts. The importance of the criteria is essential for synthesizing relative values of pure numbers.

Composition and Synthesis

Impacts of School on Criteria

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	**Composite Impact of Schools**
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

The School Example Revisited Composition & Synthesis:

Impacts of Schools on Criteria

Distributive Mode

(Normalization: Dividing each entry by the total in its column)

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	Composite Impact of Schools
A	.16	.33	.45	.77	.25	.69	.37
B	.59	.33	.09	.05	.50	.09	.38
C	.25	.33	.46	.17	.25	.22	.25

The Distributive mode is useful when the uniqueness of an alternative affects its rank. The number of copies of each alternative also affects the share each receives in allocating a resource. In planning, the scenarios considered must be comprehensive and hence their priorities depend on how many there are. This mode is essential for ranking criteria and sub-criteria, and when there is dependence.

Ideal Mode

(Dividing each entry by the maximum value in its column)

	.32 L	.14 F	.03 SL	.13 VT	.24 CP	.14 MC	Composite Impact of Schools	Normal-ized
A	.27	1	.98	1	.50	1	.65	.34
B	1	1	.20	.07	.50	.13	.73	.39
C	.42	1	1	.22	.50	.32	.50	.27

The Ideal mode is useful in choosing a best alternative regardless of how many other similar alternatives there are.

Rank Preservation & Reversal

The Use of Standards

Two decision problems can have the identical structure and identical judgments, but with different names for the criteria and alternatives, yet one would insist on preserving rank in one (buying the best computer) and on allowing rank to change in the other (buying the best tie) when copies are added.

Rank preservation is made by using standards. Rank reversal can take place when we choose the best alternative in a group without reference to standards.

Absolute Measurement

In absolute measurement the properties of an element are compared or “rated” against a standard.

In this method an element is compared against an ideal property; i.e. a “memory” of that property. Generally, only the final alternatives of choice are measured absolutely.

For example, students applying for admission are rated on grades, letters of recommendation and standardized test scores. A student’s final rating is the weighted sum of the ratings on the various criteria.

Evaluating Employees for Raises

GOAL

- Dependability (0.075)
  - Outstanding (0.48) .48/.48 = 1
  - Very Good (0.28) .28/.48 = .58
  - Good (0.16) .16/.48 = .33
  - Below Avg. (0.05) .05/.48 = .10
  - Unsatisfactory (0.03) .03/.48 = .06
- Education (0.200)
  - Doctorate (0.59) .59/.59 =1
  - Masters (0.25).25/.59 =.43
  - Bachelor (0.11) etc.
  - High School (0.05)
- Experience (0.048)
  - >15 years (0.61)
  - 6-15 years (0.25)
  - 3-5 years (0.10)
  - 1-2 years (0.04)
- Quality (0.360)
  - Excellent (0.64)
  - Very Good (0.21)
  - Good (0.11)
  - Poor (0.04)
- Attitude (0.082)
  - Enthused (0.63)
  - Above Avg. (0.23)
  - Average (0.10)
  - Negative (0.04)
- Leadership (0.235)
  - Outstanding (0.54)
  - Above Avg. (0.23)
  - Average (0.14)
  - Below Avg. (0.06)
  - Unsatisfactory (0.03)

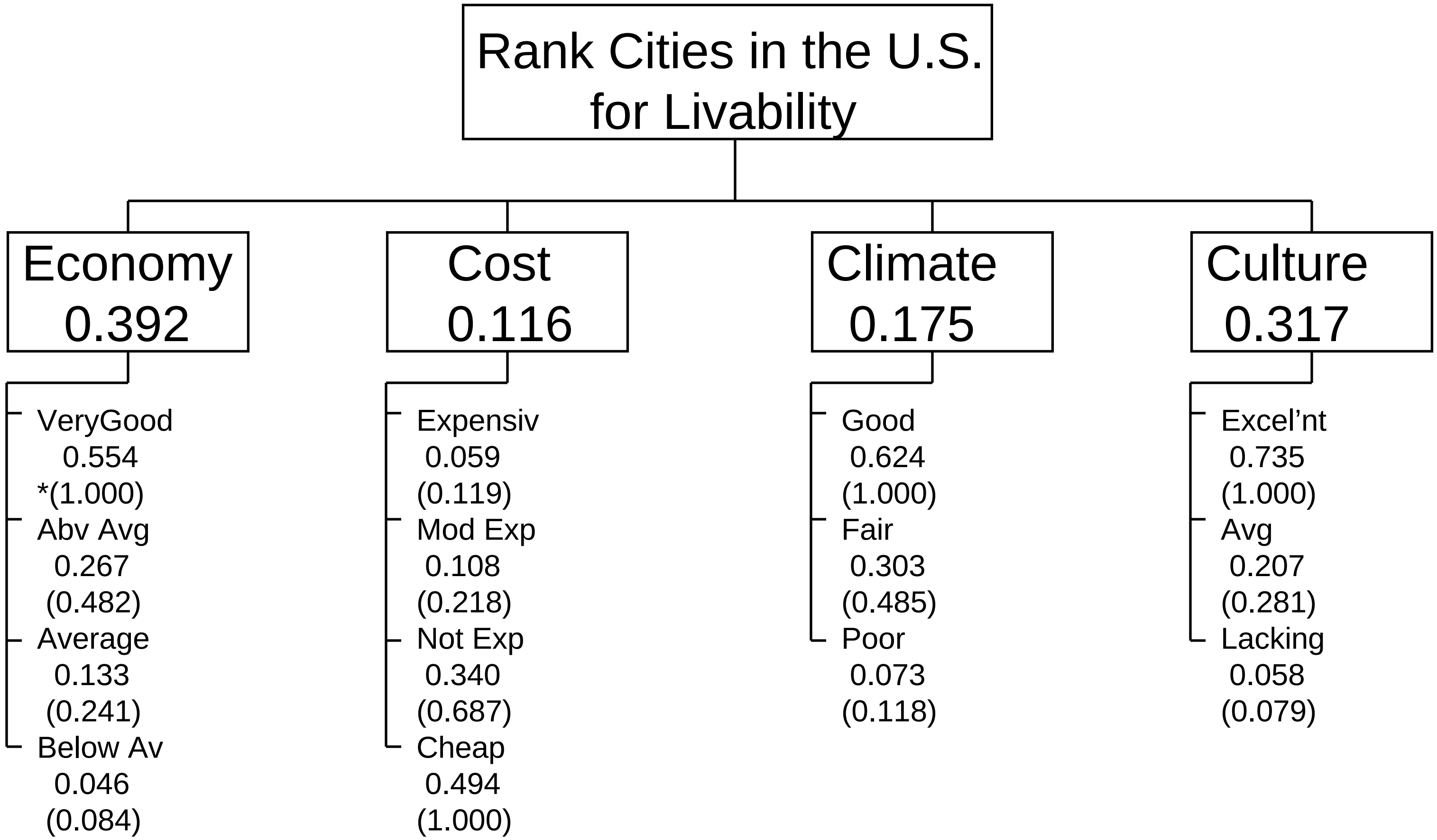
Final Step in Absolute Measurement

Rate each employee for dependability, education, experience, quality of work, attitude toward job, and leadership abilities.

	Dependability 0.0746	Education 0.2004	Experience 0.0482	Quality 0.3604	Attitude 0.0816	Leadership 0.2348	Total	Normalized
Peters, T.	Outstand	Doctorate	>15 years	Excellent	Enthused	Outstand	1.000	0.153
Hayat, F.	Outstand	Masters	>15 years	Excellent	Enthused	Abv. Avg.	0.752	0.115
Becker, L.	Outstand	Masters	>15 years	V. Good	Enthused	Outstand	0.641	0.098
Adams, V.	Outstand	Bachelor	6-15 years	Excellent	Abv. Avg.	Average	0.580	0.089
Kesselman, S.	Good	Bachelor	1-2 years	Excellent	Enthused	Average	0.564	0.086
Kelly, S.	Good	Bachelor	3-5 years	Excellent	Average	Average	0.517	0.079
Joseph, M.	Blw Avg.	Hi School	3-5 years	Excellent	Average	Average	0.467	0.071
Tobias, K.	Outstand	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.466	0.071
Washington, S.	V. Good	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.435	0.066
O'Shea, K.	Outstand	Hi School	>15 years	V. Good	Enthused	Average	0.397	0.061
Williams, E.	Outstand	Masters	1-2 years	V. Good	Abv. Avg.	Average	0.368	0.056
Golden, B.	V. Good	Bachelor	.15 years	V. Good	Average	Abv. Avg.	0.354	0.054

The total score is the sum of the weighted scores of the ratings. The money for raises is allocated according to the normalized total score. In practice different jobs need different hierarchies.

Most Livable Cities



*Numbers in parenthesis are "ideal" priorities, obtained by dividing each local priority by the highest priority in the group.

Most Livable Cities cont.

Cities rated showing verbal intensities

1	Boston	ABV AVG	EXPENSIVE	POOR	EXCEL'NT	0.541
2	Chicago	AVERAGE	EXPENSIVE	POOR	EXCEL'NT	0.446
3	Dallas	ABV AVG	MOD EXP	FAIR	AVG	0.388
4	Denver	AVERAGE	MOD EXP	FAIR	AVG	0.294
5	Los Angeles	AVERAGE	MOD EXP	FAIR	AVG	0.294
6	Miami	AVB AVG	MOD EXP	GOOD	AVG	0.478
7	New York	ABV AVG	EXPENSIV	FAIR	EXCEL'NT	0.605
8	Philadelphia	AVERAGE	NOT EXP	GOOD	EXCEL'NT	0.666
9	San Fran	ABV AVG	EXPENSIV	GOOD	EXCEL'NT	0.695
10	Seattle	AVERAGE	NOT EXP	POOR	AVG	0.284

Cities sorted by priority with ideal city added.

1	Ideal City	VERYGOOD	CHEAP	GOOD	EXCEL'NT	1.000
2	San Fran	ABV AVG	EXPENSIV	GOOD	EXCEL'NT	0.695
3	Philadelphia	AVERAGE	NOT EXP	GOOD	EXCEL'NT	0.666
4	New York	ABV AVG	EXPENSIV	FAIR	EXCEL'NT	0.605
5	Boston	ABV AVG	EXPENSIV	POOR	EXCEL'NT	0.541
6	Miami	ABV AVG	MOD EXP	GOOD	AVG	0.478
7	Chicago	AVERAGE	EXPENSIV	POOR	EXCEL'NT	0.446
8	Dallas	ABV AVG	MOD EXP	FAIR	AVG	0.388
9	Denver	AVERAGE	MOD EXP	FAIR	AVG	0.294
10	Los Angeles	AVERAGE	MOD EXP	FAIR	AVG	0.294
11	Seattle	AVERAGE	NOT EXP	POOR	AVG	0.284

Cities rated showing corresponding numerical intensities.

1	Ideal City	VERYGOOD	CHEAP	GOOD	EXCEL'NT	1.000
2	San Fran	ABV AVG	EXPENSIV	GOOD	EXCEL'NT	0.695
3	Philadelphia	AVERAGE	NOT EXP	GOOD	EXCEL'NT	0.666
4	New York	ABV AVG	EXPENSIV	FAIR	EXCEL'NT	0.605
5	Boston	ABV AVG	EXPENSIV	POOR	EXCEL'NT	0.541
6	Miami	ABV AVG	MOD EXP	GOOD	AVG	0.478
7	Chicago	AVERAGE	EXPENSIV	POOR	EXCEL'NT	0.446
8	Dallas	ABV AVG	MOD EXP	FAIR	AVG	0.388
9	Denver	AVERAGE	MOD EXP	FAIR	AVG	0.294
10	Los Angeles	AVERAGE	MOD EXP	FAIR	AVG	0.294
11	Seattle	AVERAGE	NOT EXP	POOR	AVG	0.284

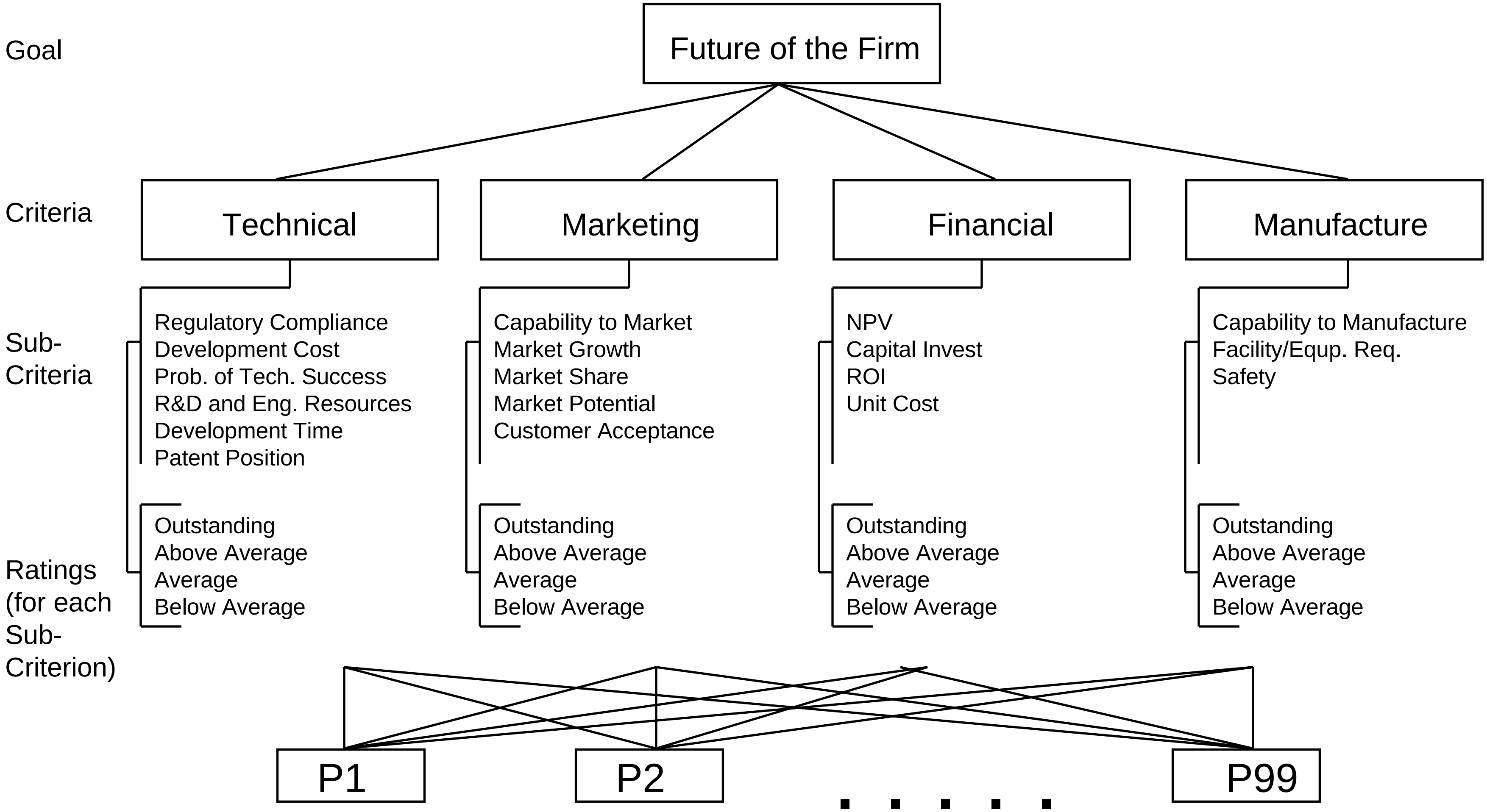
Numerical Intensities; Ideal city gets full priority for each criterion

1	Ideal City	0.3922	0.1156	0.1749	0.3173	1.000
2	San Fran	0.1889	0.0137	0.1749	0.3173	0.695
3	Philadelphia	0.0945	0.0795	0.1749	0.3173	0.666
4	New York	0.1889	0.0137	0.0849	0.3173	0.605
5	Boston	0.1889	0.0137	0.0206	0.3173	0.541
6	Miami	0.1889	0.0252	0.1749	0.0892	0.478
7	Chicago	0.0945	0.0137	0.0206	0.3173	0.446
8	Dallas	0.1889	0.0252	0.0849	0.0892	0.388
9	Denver	0.0945	0.0252	0.0849	0.0892	0.294
10	Los Angeles	0.0945	0.0252	0.0849	0.0892	0.294
11	Seattle	0.0945	0.0795	0.0206	0.0892	0.284

Benchmark Measurement

Instead of using intensities, we can compare all the alternatives with respect to well known alternatives called benchmarks that are different and range from the best to the worst for each criterion. For example, with respect to dependability we can put three well known individuals who are respectively, extremely dependable, moderately dependable and undependable. With respect to leadership we may use five such individuals and so on. We then pairwise compare each individual with these benchmarks to obtain a priority. Here again, in the end we can use the distributive or ideal modes. The benchmarks are compared only once. However, new judgments are needed for each alternative when it is compared with them. For more work, one obtains greater accuracy in the final priorities. This process is known as "Benchmark Measurement".

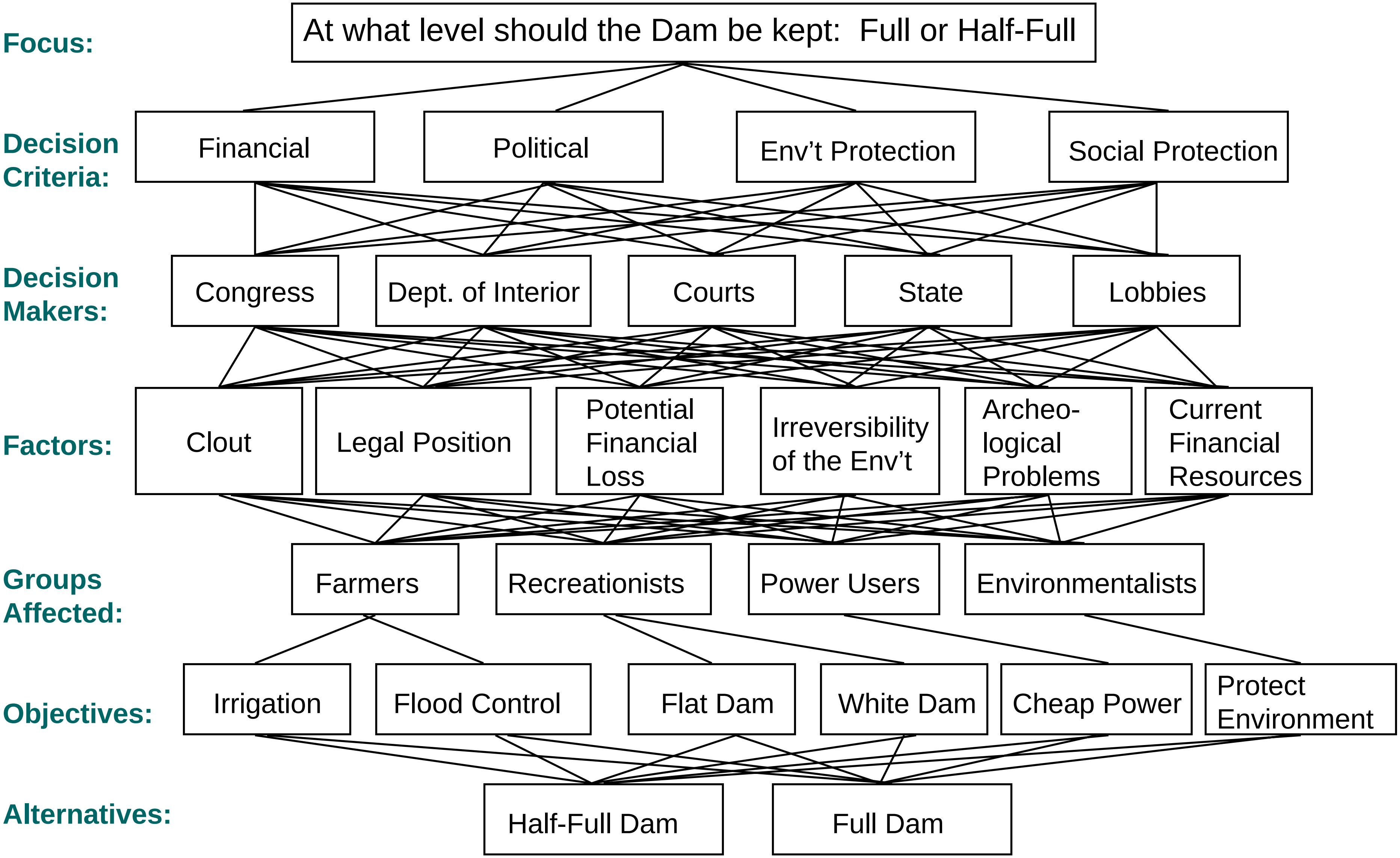
AHP Hierarchy for R&D Project Selection

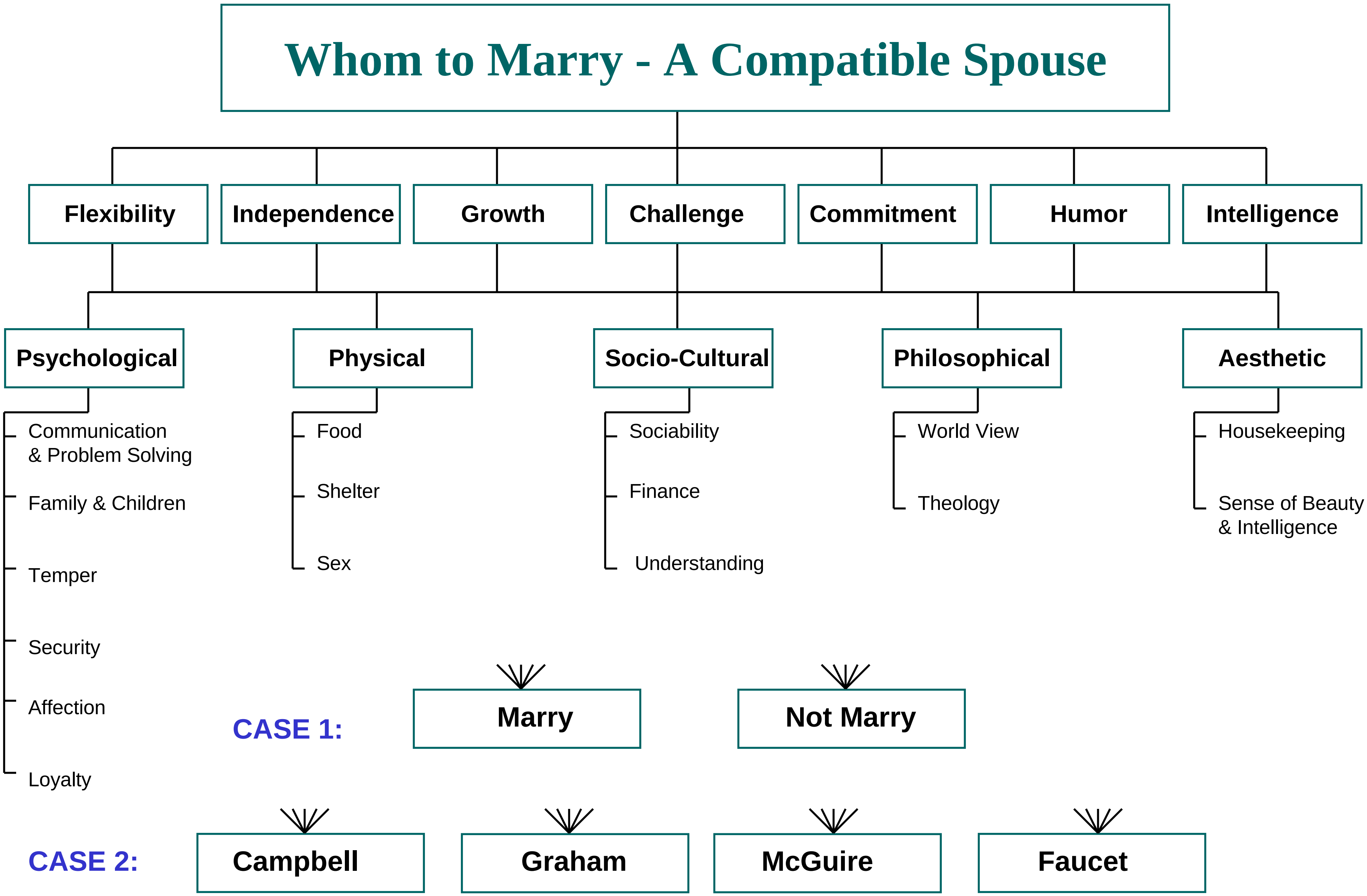


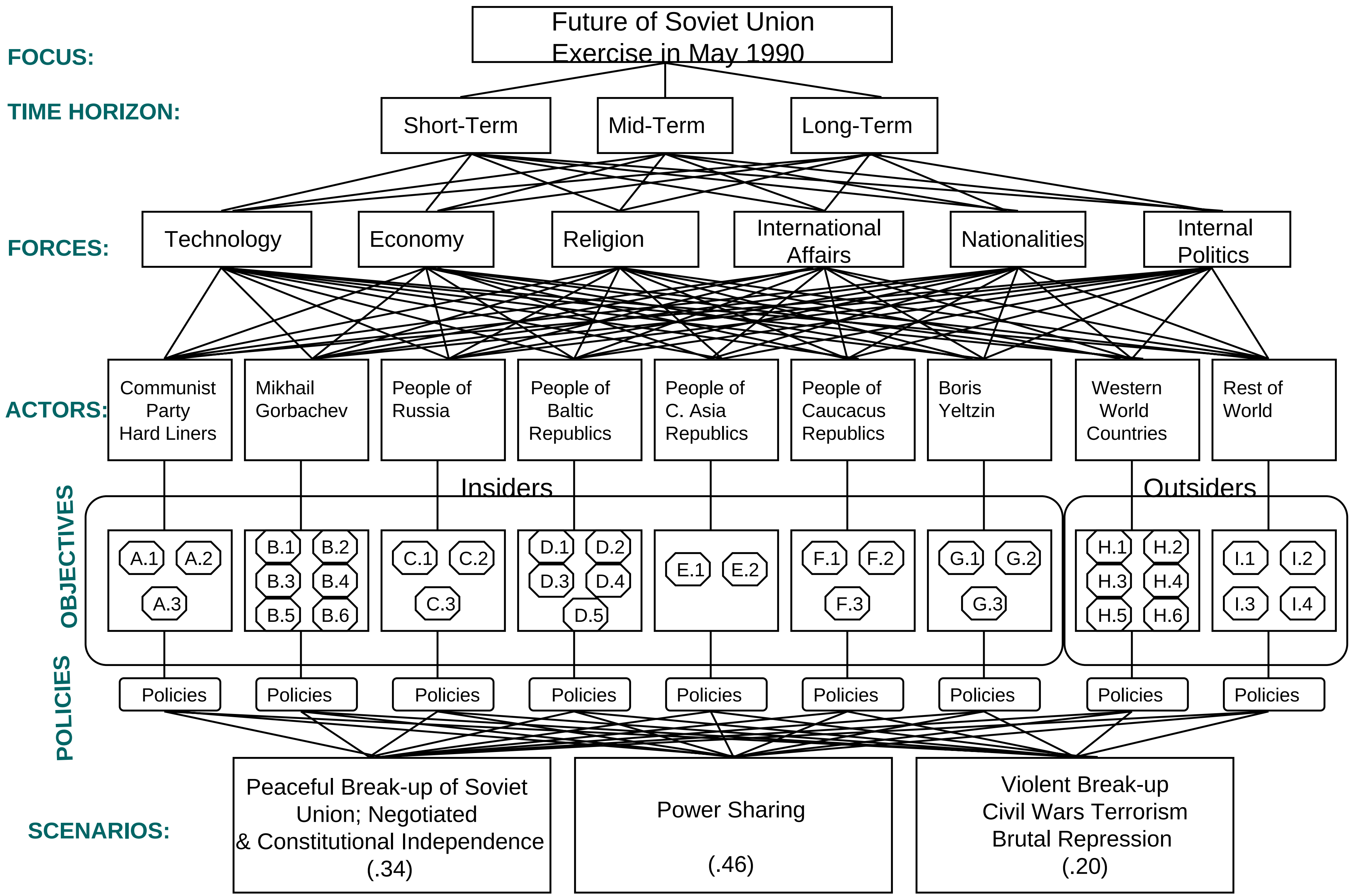
This approach for R&D project selection has been and is currently being used by a hypothetical firm, Novatech, Inc., which manufacturers and sells a line of fertilizers.

(see Golden, G.L. (eds), Analytic Hierarchy Process - Applications and studies, 1989, Springer-Verlag. p. 82-99.)

A Complete Hierarchy to Level of Objectives




Whom to Marry - A Compatible Spouse
Flexibility
Independence
Growth
Challenge
Commitment
Humor
Intelligence
Psychological
Physical
Socio-Cultural
Philosophical
Aesthetic
Communication & Problem Solving
Family & Children
Temper
Security
Affection
Loyalty
Food
Shelter
Sex
Sociability
Finance
Understanding
World View
Theology
Housekeeping
Sense of Beauty & Intelligence
CASE 1:
Marry
Not Marry
CASE 2:
Campbell
Graham
McGuire
Faucet


FOCUS:
TIME HORIZON:
FORCES:
ACTORS:
OBJECTIVES
POLICIES
SCENARIOS:
Future of Soviet Union
Exercise in May 1990
Short-Term
Mid-Term
Long-Term
Technology
Economy
Religion
International Affairs
Nationalities
Internal Politics
Communist Party Hard Liners
Mikhail Gorbachev
People of Russia
People of Baltic Republics
People of C. Asia Republics
People of Caucacus Republics
Boris Yeltzin
Western World Countries
Rest of World
Insiders
Outsiders
A.1 A.2 A.3
B.1 B.2 B.3 B.4 B.5 B.6
C.1 C.2 C.3
D.1 D.2 D.3 D.4 D.5
E.1 E.2
F.1 F.2 F.3
G.1 G.2 G.3
H.1 H.2 H.3 H.4 H.5 H.6
I.1 I.2 I.3 I.4
Policies
Peaceful Break-up of Soviet Union; Negotiated & Constitutional Independence (.34)
Power Sharing (.46)
Violent Break-up Civil Wars Terrorism Brutal Repression (.20)

Should U.S. Sanction China? (Feb. 26, 1995)

BENEFITS

Protect rights and maintain high Incentive to make and sell products in China (0.696)	Rule of Law Bring China to responsible free-trading 0.206)	Help trade deficit with China (0.098)
Yes .80	Yes .60	Yes .50
No .20	No .40	No .50

Yes 0.729 **No 0.271**

COSTS

$ Billion Tariffs make Chinese products more expensive (0.094)	Retaliation (0.280)	Being locked out of big infrastructure buying: power stations, airports (0.626)
Yes .70	Yes .90	Yes .75
No .30	No .10	No .25

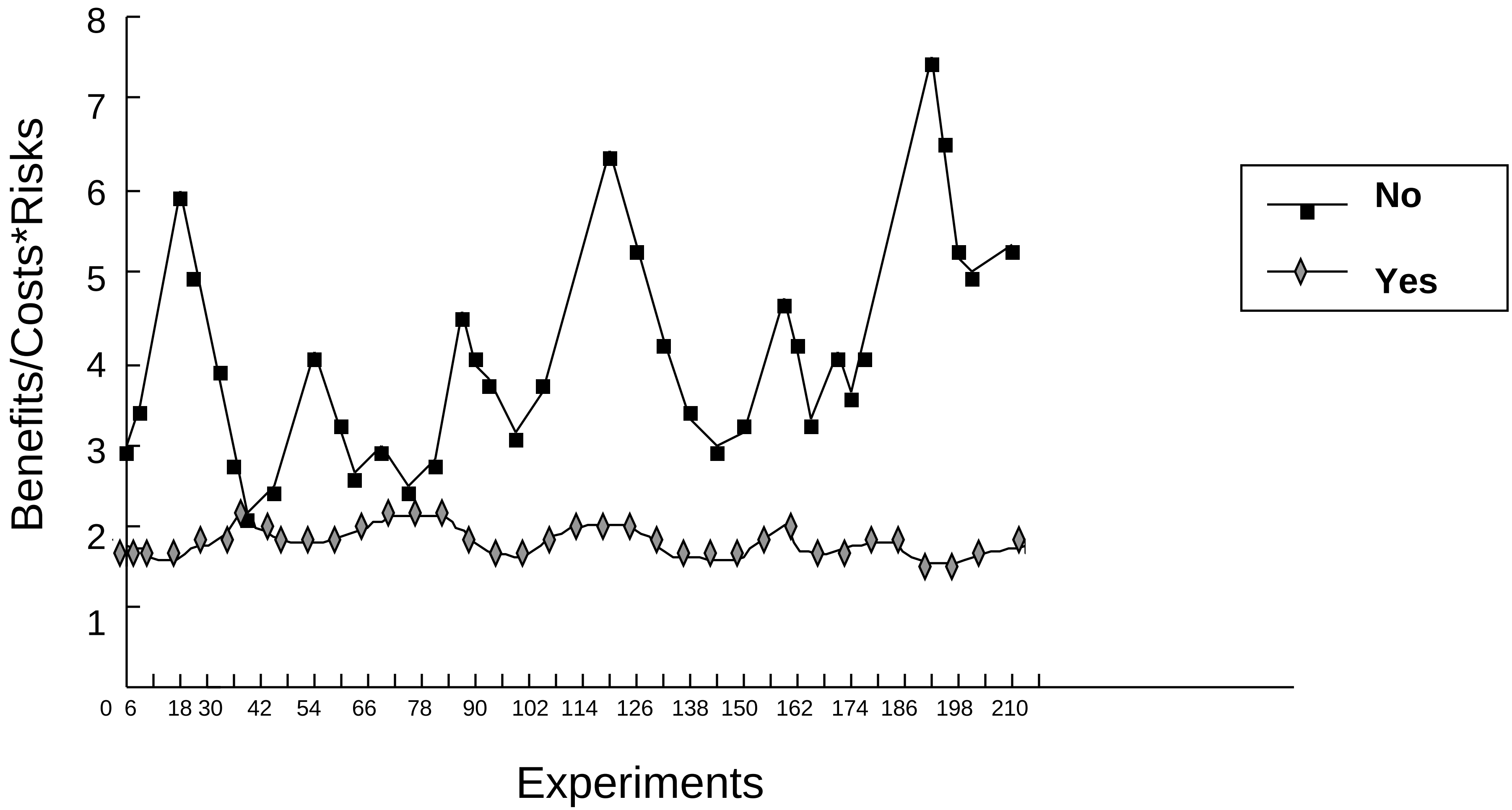
Yes 0.787 **No 0.213**

RISKS

Long Term negative competition (0.683)	Effect on human rights and other issues (0.200)	Harder to justify China joining WTO (0.117)
Yes .70	Yes .30	Yes .50
No .30	No .70	No .50

Yes 0.597 **No 0.403**

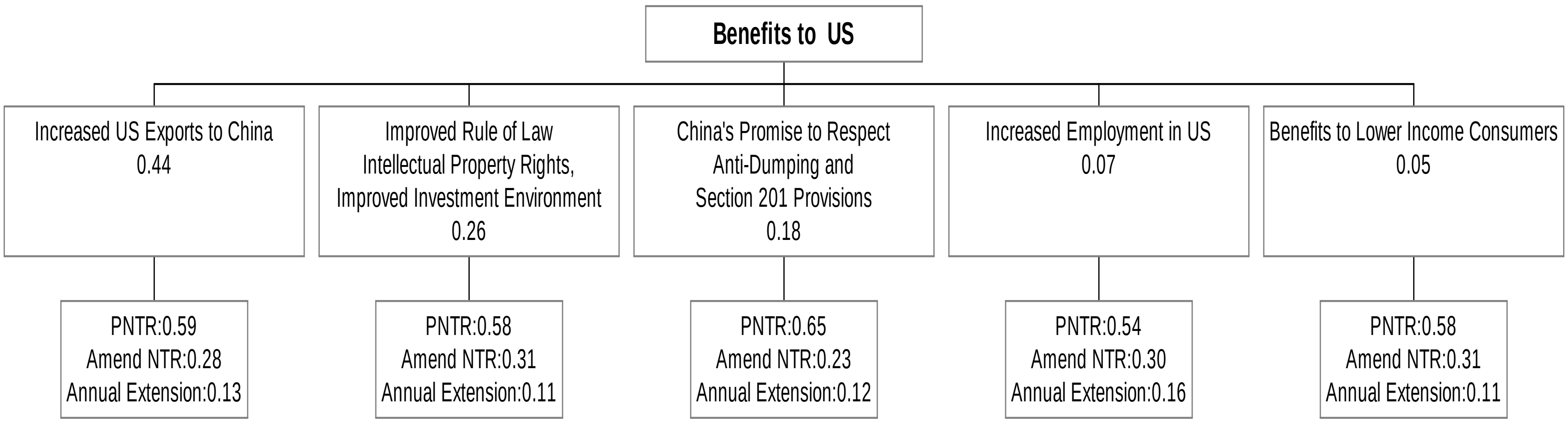
Result: $\dfrac{\text{Benefits}}{\text{Costs x Risks}}$; **YES** $\dfrac{.729}{.787 \times .597} = 1.55$ **NO** $\dfrac{.271}{.213 \times .403} = \boxed{3.16}$



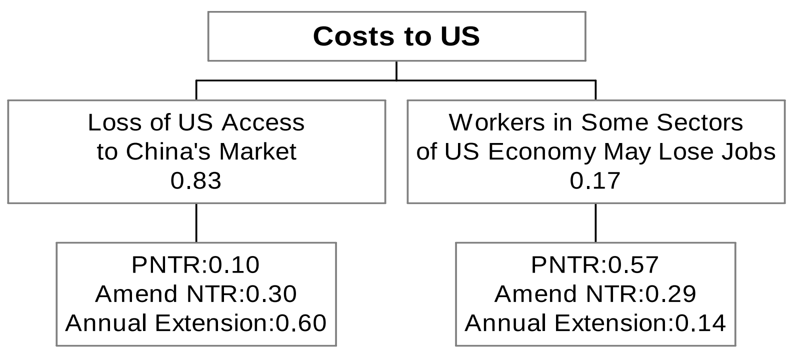
8
7
6
5
4
3
2
1
Benefits/Costs*Risks
0 6 18 30 42 54 66 78 90 102 114 126 138 150 162 174 186 198 210
Experiments
No
Yes

The Decision by the US Congress on China's Permanent Normal Trade Relations Status

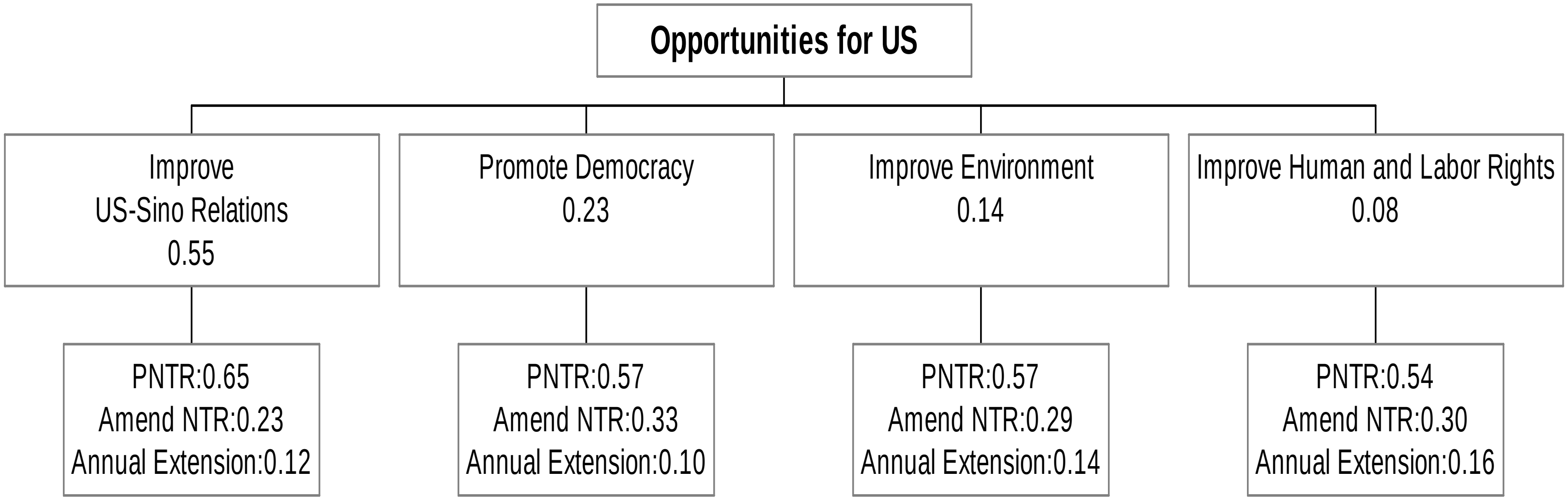
The Four Decision Hierarchies for Benefits, Costs, Opportunities, and Risks



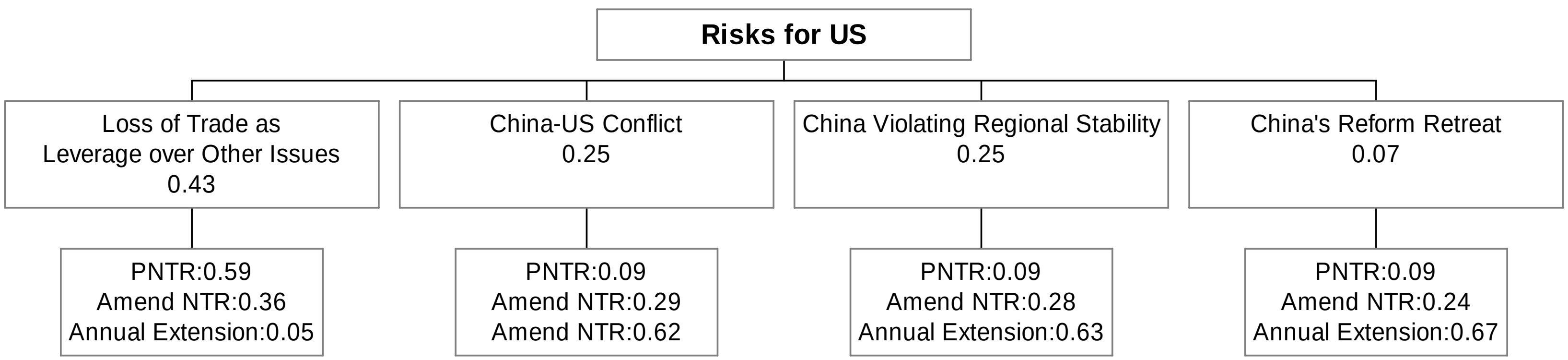
Benefits Synthesis: PNTR: 0.60 Amend NTR: 0.28 Annual Extension: 0.12



Costs Synthesis: PNTR: 0.58 Amend NTR: 0.23 Annual Extension: 0.19



Opportunities Synthesis: PNTR: 0.61 Amend NTR: 0.27 Annual Extension: 0.12



Risks Synthesis: PNTR: 0.42 Amend NTR: 0.17 Annual Extension: 0.41

Hierarchy for Ratings Benefits, Costs, Opportunities, and Risks

Factors for Evaluating the Decision

Economic: 0.56
-Growth (0.33)
-Equity (0.67)

Security: 0.32
-Regional Security (0.09)
-Non-Proliferation (0.24)
-Threat to US (0.67)

Political:0.12
-Domestic Constituencies (0.80)
-American Values (0.20)

Priority Ratings for the Importance of Benefits, Costs, Opportunities, and Risks

		Benefits	**Costs**	**Opportunities**	**Risks**
Economic (0.56)	Growth (0.19)	High	Very Low	Medium	Very Low
	Equity (0.37)	Medium	High	Low	Low
Security (0.32)	Regional (0.03)	Low	Medium	Medium	High
	Non-Proliferation (0.08)	Medium	Medium	High	High
	Threat to US (0.21)	High	Very High	High	Very High
Political (0.12)	Constituencies (0.1)	High	Very High	Medium	High
	American Values (0.02)	Very Low	Low	Low	Medium
Priorities		**0.25**	**0.31**	**0.20**	**0.24**

Very High (0.42), High (0.26), Medium (0.16), Low (0.1), Very Low (0.06)

Significance of the Three Alternatives

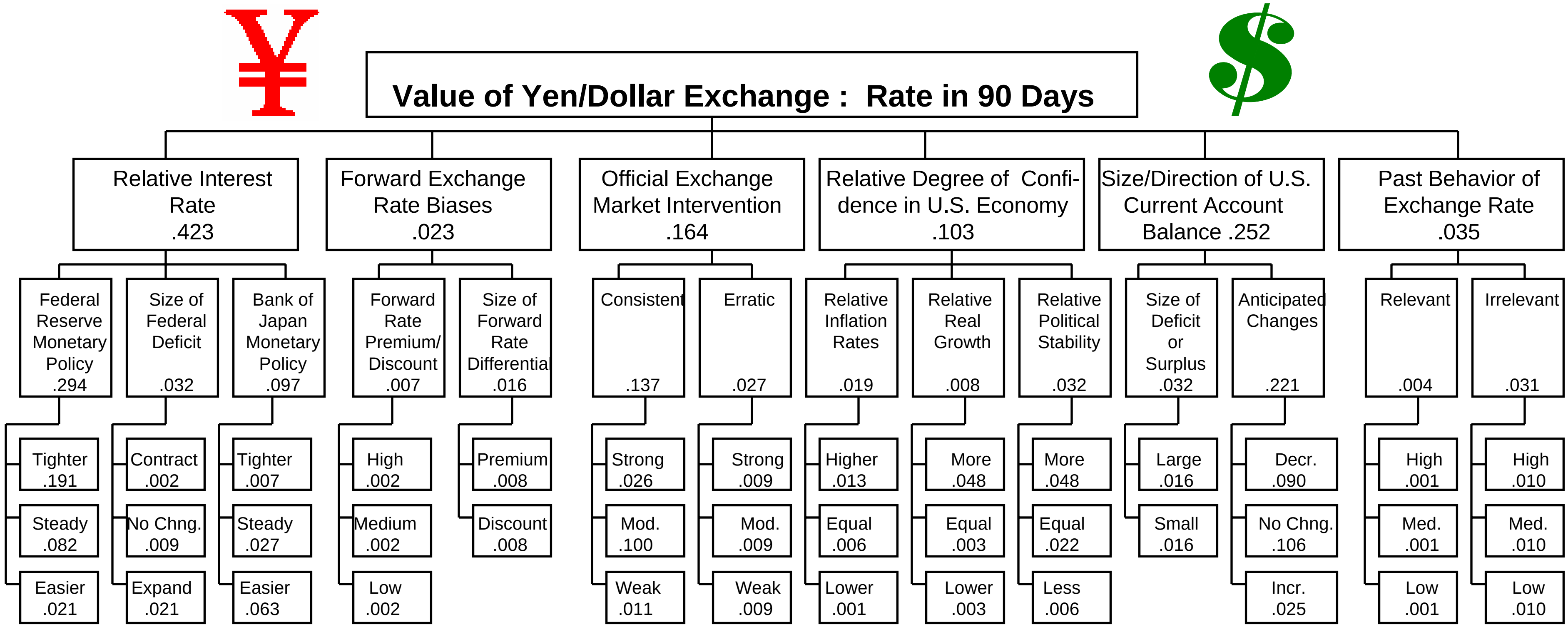
	Priorities	Overall Importance of Criteria	PNTR	Amend NTR	Annual Extension
Benefits to US (0.25)	Increased US Exports to China (0.44)	Increased US Exports to China (0.110)	0.59	0.28	0.13
	Improved Rule of Law (0.26)	Improved Rule of Law (0.065)	0.58	0.31	0.11
	China's Promise to Respect US Provisions (0.18)	China's Promise to Respect US Provisions (0.045)	0.65	0.23	0.12
	Increased Employment in US (0.07)	Increased Employment in US (0.018)	0.54	0.30	0.16
	Benefits to Lower Income Consumer (0.05)	Benefits to Lower Income Consumer (0.013)	0.58	0.31	0.11
Costs to US (0.31)	Loss of US Access to China's Market (0.83)	Loss of US Access to China's Market (0.258)	0.67	0.22	0.11
	Workers in Some Sectors of US Economy May Lose Jobs (0.17)	Workers in Some Sectors of US Economy May Lose Jobs (0.053)	0.14	0.28	0.58
Opportunities for US (0.20)	Improve US-Sino Relations (0.55)	Improve US-Sino Relations (0.110)	0.65	0.23	0.12
	Promote Democracy (0.23)	Promote Democracy (0.046)	0.57	0.33	0.10
	Improve Environment (0.14)	Improve Environment (0.028)	0.57	0.29	0.14
	Improve Human and Labor Rights (0.08)	Improve Human and Labor Rights (0.016)	0.54	0.30	0.16
Risks for US (0.24)	Loss of Trade as Leverage over Other Issues (0.43)	Loss of Trade as Leverage over Other Issues (0.100)	0.07	0.11	0.82
	US-China Conflict (0.25)	US-China Conflict (0.060)	0.69	0.21	0.10
	China Violating Regional Stability (0.25)	China Violating Regional Stability (0.060)	0.68	0.22	0.10
	China's Reform Retreat (0.07)	China's Reform Retreat (0.017)	0.66	0.25	0.09
Final Analysis			**0.55**	**0.24**	**0.21**

Sensitivity Analyses

All possible combinations of increasing and decreasing Benefits, Costs, Opportunities, and Risks by 5% were made. It was found that the ranks of the alternatives remained the same and their priorities did not change up and down by more than 4 %.

Then the the criteria weights where perturbed up and down for the fixed derived priorities of the benefits, costs, opportunities, and risks. It was found that the ranks of the alternative remained stable except when the two criteria “Workers in Some Sectors of US Economy May Lose Jobs” and “Loss of Trade as Leverage over Other Issues” were either each increased by more than 4 times its existing priority or if their priorities were jointly increased by 3 times their existing values. Both of these perturbations seem extreme and unlikely.

Our conclusion was that it is in the best interest of the United States to grant China PNTR status.

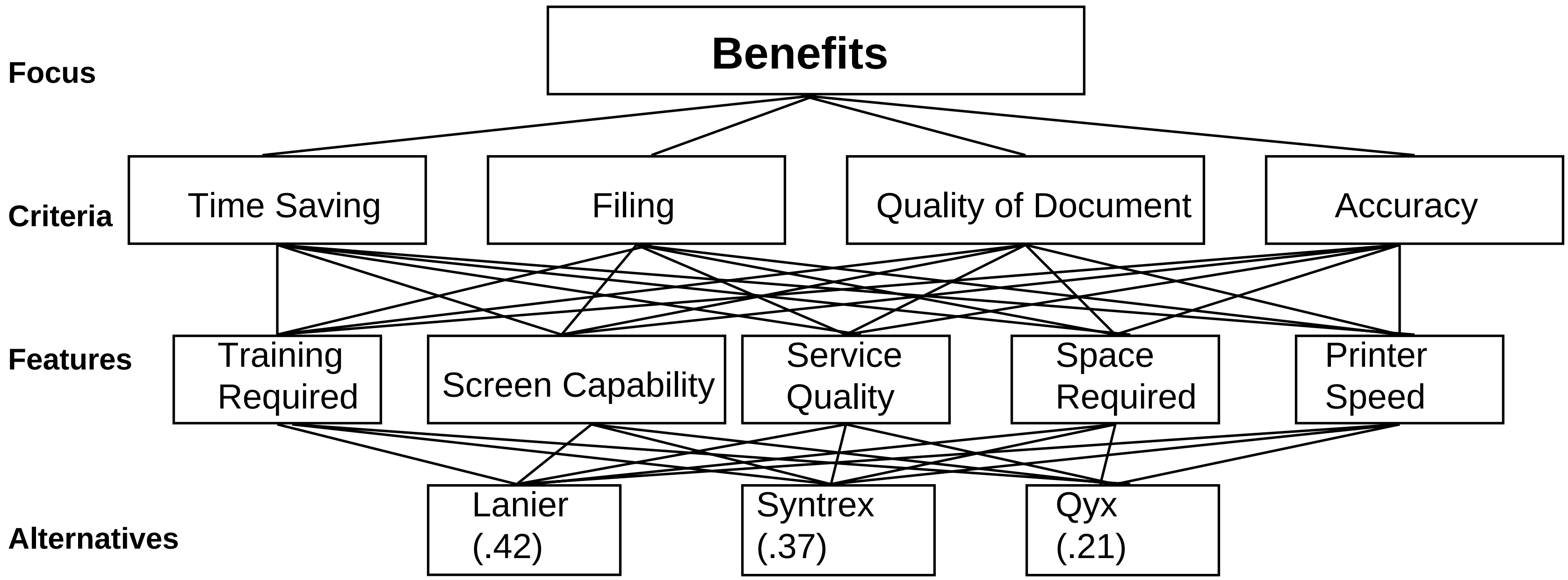


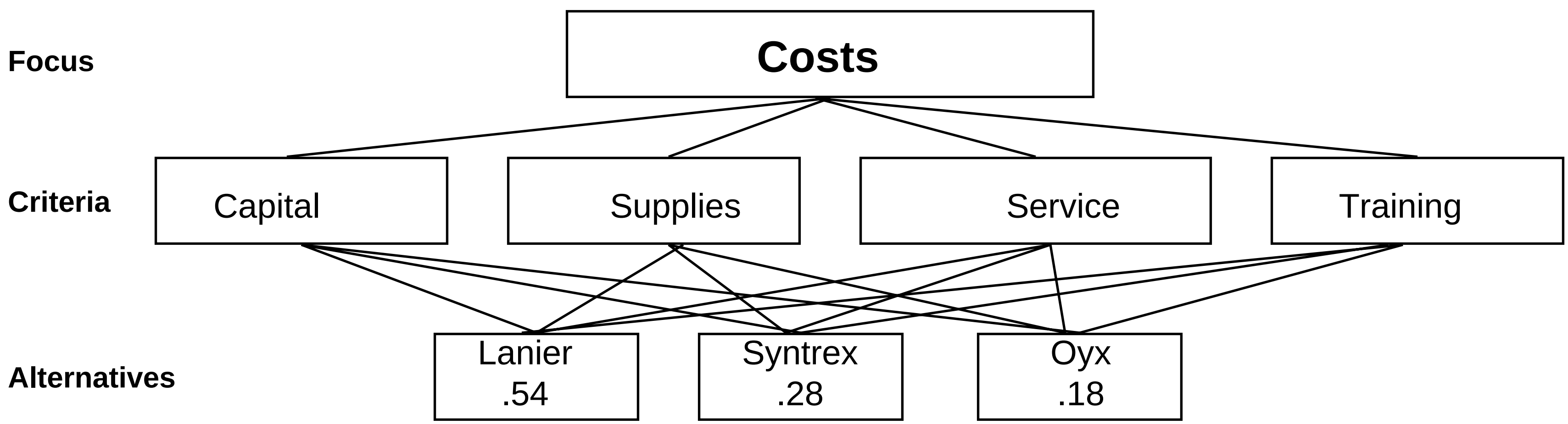
Probable Impact of Each Fourth Level Factor

119.99 and below	119.99-134.11	134.11-148.23	148.23-162.35	162.35 and above
Sharp Decline 0.1330	Moderate Decline 0.2940	No Change 0.2640	Moderate Increase 0.2280	Sharp Increase 0.0820

Expected Value is 139.90 yen/$

Best Word Processing Equipment





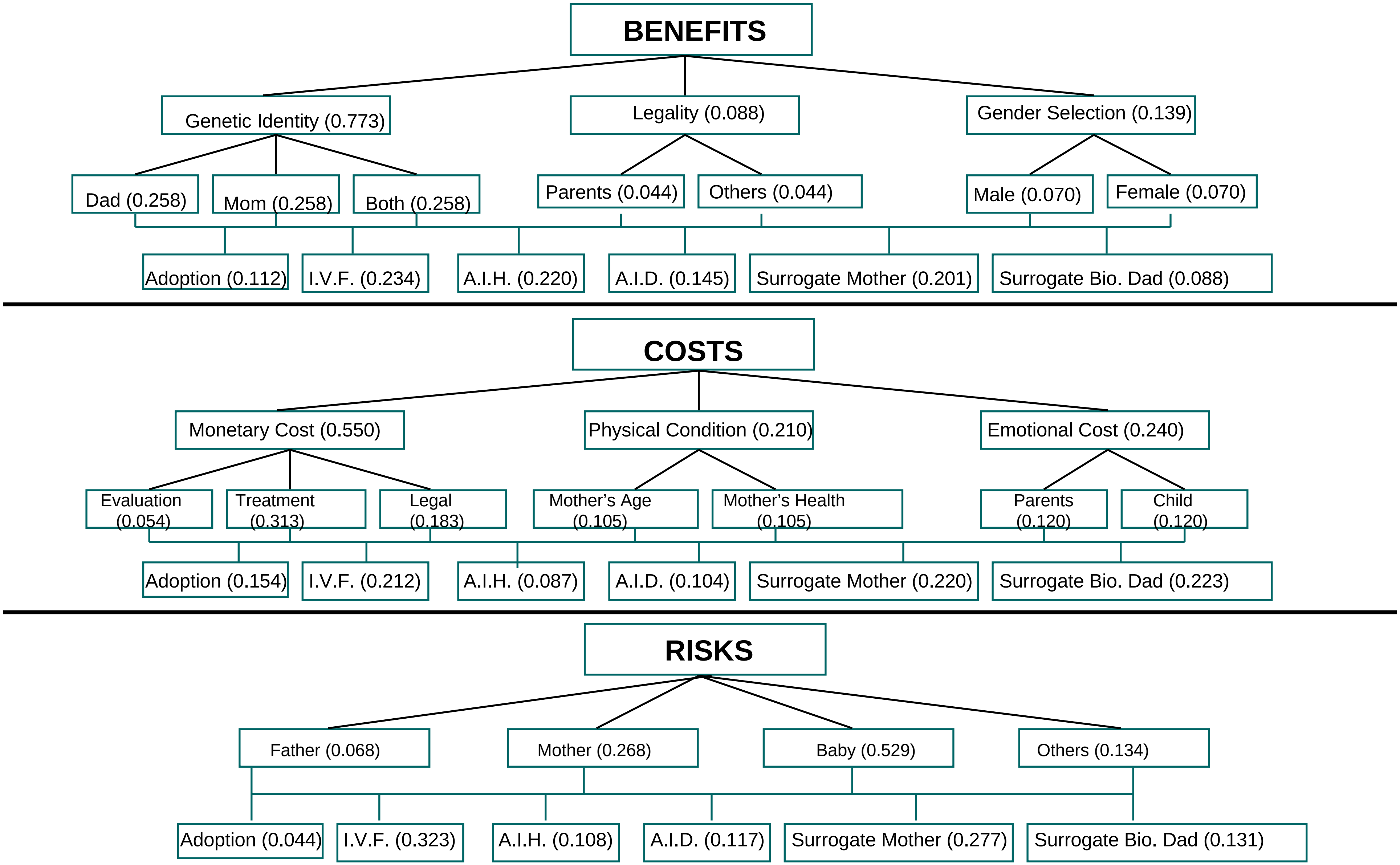
Best Word Processing Equipment Cont.

Benefit/Cost Preference Ratios

Lanier	Syntrex	Qyx
$\frac{.42}{.54} = 0.78$	$\frac{.37}{.28} = 1.32$	$\frac{.21}{.18} = 1.17$

Best Alternative

If We Can’t Have Our Own Child, What Should We Do?



Our Own Child cont.

A.I.D.:	artificial insemination using donor sperm
A.I.H.:	artificial insemination using husband’s sperm
Adoption:	adoption
I.V.F.:	in vitro fertilization
Sur. B.F.:	surrogate mother using her egg & biological father’s sperm
Sur. Mom:	surrogate mother using biological egg & sperm

	Benefits	**Costs**	**Risks**	**C*R**	**B/C*R**	**%**
Adoption	0.112	0.154	0.044	0.007	16.471	26.920
IVF	0.234	0.212	0.323	0.069	3.095	5.060
AIH	0.220	0.087	0.108	0.009	23.404	38.260 ←
AID	0.145	0.104	0.117	0.012	11.885	19.460
Sur. Mom	0.201	0.220	0.277	0.061	3.301	5.400
Sur. B.F.	0.088	0.223	0.131	0.029	3.014	4.920

Top Down & Bottom Up Evaluation Process

In a top down evaluation process a best choice is made on the basis of the forces and knowledge available at the top. Given the goals and values, find the best alternative. To do that, one distributes the forces downward from the most general to the most particular. One determines the relative strength with which an alternative satisfies a condition or possesses and attribute.

In a bottom up process, we have a given set of alternatives with common aggregate properties or attributes. We establish priorities for the alternatives in terms of the attributes to determine which one contributes more to the importance of that attribute, then for the attributes in terms of higher aggregate attributes and so on up to the goal. It is essential to recognize that the attributes inherit their significance from the alternatives from which they are derived. The influence of the alternatives is transmitted upwards if we think of weighting as taking place from the bottom up.

In a top down process the number and type of alternatives is open and one attempts to find a best choice from what can be made available at the time. In the bottom up process the alternatives are known, generally exhaustive and one wants to choose a best one among them. None of them may be greatly desired. Nevertheless, one must find the best one among them.

How Do You Know the Hierarchy is the Right One?

The outcome of a decision depends on the faithfulness with which the structure represents the underlying complexity of the problem.

How Large Should a Hierarchy Be?

LARGE enough to capture your major concerns.

SMALL enough to remain sensitive to change in what is important.

Some people formulate small hierarchies and go out and act on what they learn.

Other people formulate elaborate hierarchies and find it impossible to act.

Three Principles In Forming a Hierarchy

1 - From more general and less controllable to more specific and controllable.

2 - Can you use the elements in a level to compare the elements in the level immediately below?

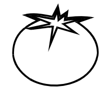
3 - A hierarchy should be rich enough to represent your problem, but simple enough to reflect sensitivity.

Clustering & Comparison

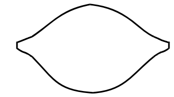
Volume


.07

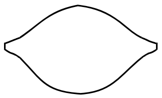
Unripe Cherry Tomato


.28

Small Green Tomato


.65

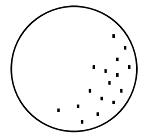
Large Lime


.08

Large Lime

$\frac{.08}{.08} = 1$

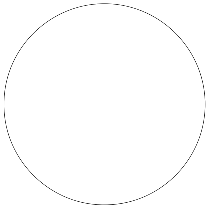
.65x1=.65


.22

Unripe Grapefruit

$\frac{.22}{.08} = 2.75$

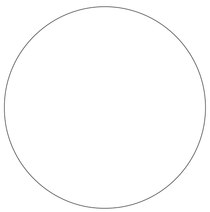
.65x2.75=1.79


.70

Honeydew

$\frac{.70}{.08} = 8.75$

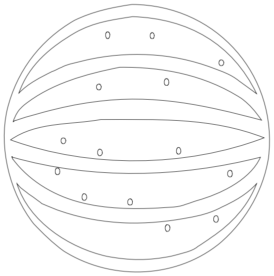
.65x8.75=5.69


.10

Honeydew

$\frac{.10}{.10} = 1$

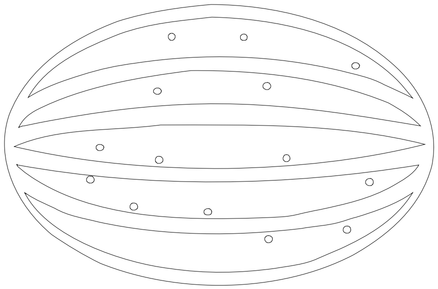
5.69x1=5.69


.30

Sugar Baby Watermelon

$\frac{.30}{.10} = 3$

5.69x3=17.07


.60

Oblong Watermelon

$\frac{.60}{.10} = 6$

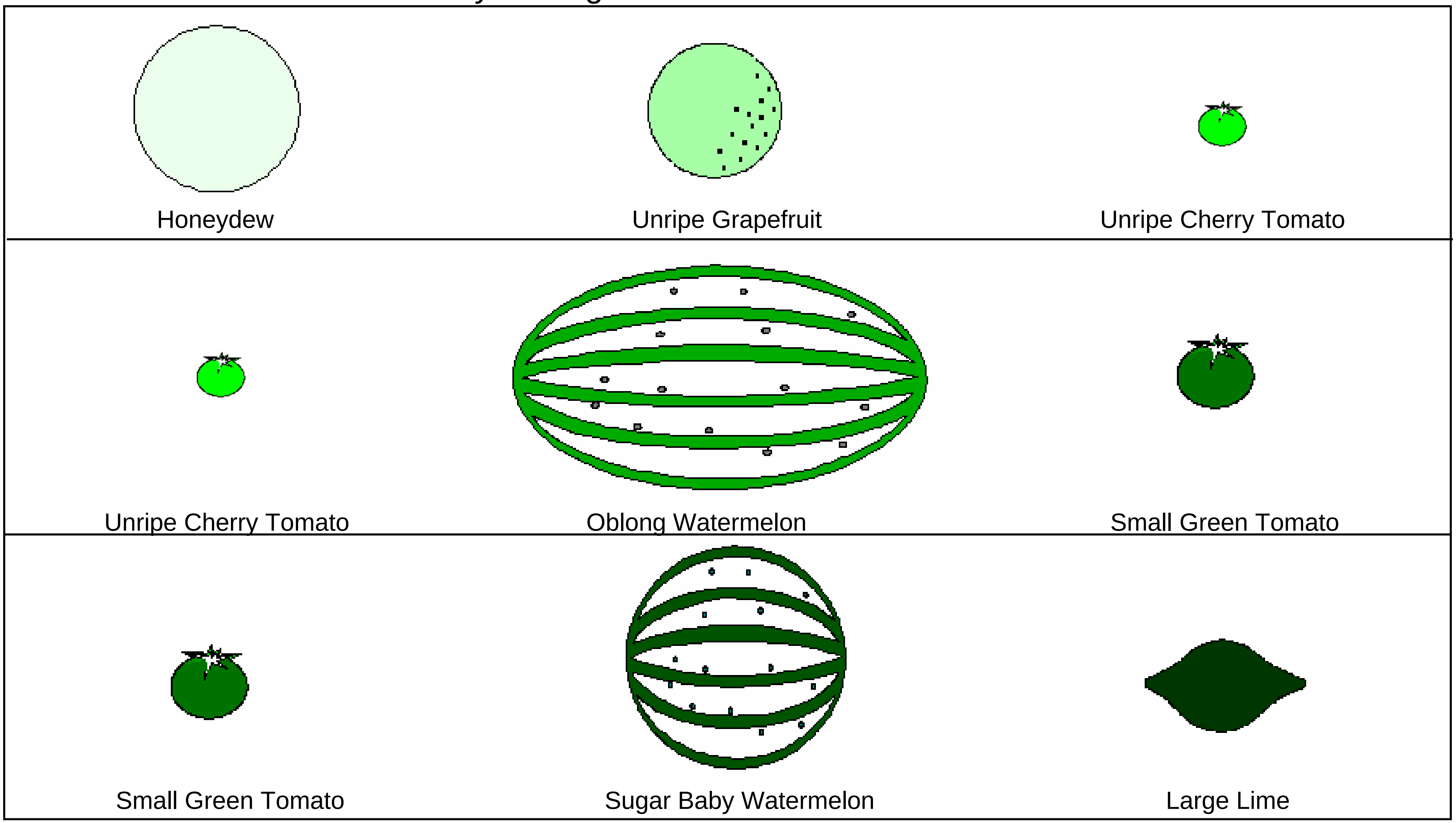
5.69x6=34.14

This means that $\frac{34.14}{.07} = 487.7$ unripe cherry tomatoes are equal to the oblong watermelon.

Clustering & Comparison

Color

How intensely more green is X than Y relative to its size?



Group Decision Making and the Geometric Mean

Suppose two people compare two apples and provide the judgments for the larger over the smaller, 4 and 3 respectively. So the judgments about the smaller relative to the larger are 1/4 and 1/3.

<u>Arithmetic mean</u>

$$4 + 3 = 7$$

$$1/7 \neq 1/4 + 1/3 = 7/12$$

<u>Geometric mean</u>

$$\sqrt{4 \times 3} = 3.46$$

$$1/\sqrt{4 \times 3} = \sqrt{1/4 \times 1/3} = 1/\sqrt{4 \times 3} = 1/3.46$$

That the Geometric Mean is the unique way to combine group judgments is a theorem in mathematics.

Where Utility Theory is Hard to Use

1 - It is based on lottery comparisons and gambles.

2 - It does not have a unique way to construct its utility functions.

3 - It derives interval scale numbers which can't be added, multiplied, or used to allocate resources.

4- It can't handle dependence and feedback because of interval scales, and thus abdicates the most important aspect of decision making to Bayes probability theory.

5- It has unresolved paradoxes. Maurice Allais even won the Nobel Prize for finding an important paradox!

6- It is not adaptable to combining group judgments or capturing decision maker power in a scientific way.

7- It needs much training to apply - it is unnatural.

Five Guides for Priorities in Decision making

1 - Establish the goal for measuring influence and worth of all the decisions you make: overall satisfaction.
2 - Determine the global objectives that serve this goal.
3 - Perform BOCR(benefits, costs, opportunities, risks) evaluation of a particular decision. These have the controlling criteria of the analysis.
4- Structure a hierarchy or a network under each BOCR criterion.
5- Provide judgments, derive priorities to make tradeoffs and synthesize the final outcome for the alternatives of the decision.