MSHA'S

SIMPLE TECHNIQUE FOR

PREDICTING THE

STRESS DISTRIBUTION

IN A MINE PANEL

by

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ABSTRACT

This report describes a unique approach for predicting stress distribution in coal mine pillars. It presents an equation for calculating the stress field in a coal seam adjacent to an excavation. The equation was derived from elastic theory concerning flat elliptical cracks. It is feasible to perceive an excavation in a coal seam as a crack in an infinite mass. The stress zone of influence at the crack tip is the same zone of influence that extends into a coal pillar - this has been verified by two-dimensional numerical models. The only parameters necessary for the equation to function are the thickness of the overburden and the width of the opening.

The elastic stress equation describes the stress influence zone for one side of an entry. Entries have two sides, therefore, two zones of influence. If a coal seam has several entries, it is necessary to superposition the zones of influence, from each entry, in order to accurately predict the stress distribution across the entire mine panel. Using this superposition technique, three-dimensional analysis may be possible.

The report also describes a technique to simulate yielding. Wilson's equations will predict yield behavior of the pillar edges. The elastic stress equation and superpositioning techniques will estimate the stress distribution in the elastic core of the pillars.

Computer software will use all the techniques to provide a tool for quickly assessing the stability of a room and pillar design. The code is already functioning in Lotus 1-2-3®. The software should be useful to both amateur and experienced numerical modelers.

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1.0 INTRODUCTION

People in the coal industry occasionally need to know how a room and pillar design will affect ground stability. Numerical modeling is effective in predicting stresses, however, it is slow and requires a certain amount of technical prose to operate. A useful tool would be a computer program that quickly and easily determines the stability of a mine layout. MSHA Technical Support is developing software to accomplish this goal. A significant portion of the development requires creating an algorithm to predict the effect that room and pillar geometry has on pillar stress.

This report presents a technique that predicts the stress distribution, amount of yielding and thus, the stability in a series of pillars. The pillars can be different shapes and sizes. The first part of the paper introduces a curious equation for estimating stresses in a coal seam. The basis of the equation is from elastic theory concerning flat elliptical cracks. It is feasible to perceive an excavation in a coal seam as a crack in an infinite mass. The stress zone of influence at the crack tip is the same zone of influence that extends into a coal rib. The only parameters necessary for elastic analysis are entry width and seam depth. The equation compares well to the results of numerical modeling.

The second topic involves procedures for superpositioning the stress disturbance induced by each and every entry. A mine panel contains a thin gridwork of tunnels. Each entry or crosscut has an additive effect on the stress distribution in the pillar supports. To obtain the stress distribution in a mine panel, it is necessary to superposition the stress zone of influence for each excavation. The pillar equation and superposition technique permits instantaneous, two-dimensional elastic analysis of mine panels. Three-dimensional analysis may be possible.

The final section describes a procedure for simulating pillar yield. An assumption is made that coal behaves in an elasto-plastic manner. A technique is introduced to simulate pillar yield. The yield method is empirical in its present form. A more complicated "elastic" solution is available - it can be incorporated into the software at a later date. A zone of influence describes the area affected by the pillar yielding. As the pillar yields, this zone of influence increases. Several formulas are available that describe the stress distribution in a yielding coal seam. Most of these formulas are empirical in derivation. Any one of these equations can be used in the yield analysis - it is simply a matter of substituting the equation.

Computer software will incorporate all techniques described in this paper. The software already operates as a macro in Lotus 1-

2-3® spreadsheet. Hewlett Packard markets a handheld computer that contains Lotus 1-2-3 as its operating system. It should be possible to operate the software in this handheld. This would be convenient for performing analysis in the field.

Rock Mechanics is not an exact science, especially in coal-bearing strata. Numerical techniques that predict ground stability must make many assumptions to effectively model rock behavior. This report describes techniques that predict stress distributions which correlate well to that predicted by two-dimensional numerical modeling. The MSHA technique is extremely fast and simple to use.

2.0 DEVELOPMENT OF THE PILLAR EQUATION

For years MSHA Technical Support has been discussing a method for simulating pillar yield. In 1987 G. Karabin and L. Lauritzen wrote an in-house report describing a routine to progressively yield pillars by comparing the stress on the pillar edge to the strength of the edge. The tributary area method predicts the stress and Wilson's equation determines the strength of the edge. As the pillar yields, the insupportable load is transferred to adjacent pillars. This paper expands these techniques by incorporating an equation that predicts the stress distribution in the elastic core and defines the dynamic zone of influence caused by the yielding coal. A superpositioning technique makes everything work to correlate well with numerical modeling predictions.

The MSHA Elastic Stress Equation (MESE) was an accidental discovery.[1] A project was underway to better understand how a virgin stress field is altered by the presence of an entry. Two-dimensional numerical modeling using FLAC® demonstrates how an excavation alters the stress distribution in the model.[2] To

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[1]This paper includes several pillar equations of different names. In order to avoid confusion it was decided to name the new equation the MSHA Elastic Stress Equation (MESE)

[2]FLAC Fast Lagrangian Analysis of Continua is a commercial two-dimensional modeling software package marketed by Itasca

speed up the analysis, FLAC® output was loaded directly into Lotus 1-2-3® spreadsheet software in hopes of constructing a mathematical relationship for pillar stress distribution. Halfway through the FLAC® investigations, a request came for a technical review of two reports written by Karl Zipf ([1,2])*. After reviewing the reports and playing around with a particular equation, a new equation emerged.

A section in one of Zipf's reports referred to an analytic solution for stresses surrounding a crack. According to Zipf

> "Jaeger and Cook ([3]) provide certain analytic solutions essential for checking boundary element method programs. The solutions are for an elliptic crack of length 2c subject to a stress P at infinity perpendicular to this crack plane (Fig. 1). In more practical terms, this problem corresponds to an infinitely wide (into the page) slot of length 2c in an infinite medium (i.e., a longwall panel of width 2c and an infinite length."

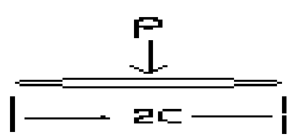

Figure 1
Elliptical Crack of Length 2c

The vertical stress distribution along the edge of this opening is given by:

$$\sigma_{yy} = P \coth(\epsilon)$$
$$where \quad \epsilon = \cosh^{-1}(\frac{x}{c}) \qquad \mathbf{1}$$

σ_{yy} = The vertical stress into the rib adjacent to the crack
P = the stress at infinity perpendicular to the crack
x = the distance into the rib, and
c = half the crack width

This equation applies along the x axis for x/c > 1

Normal convergence across this opening is given by

$$v = \frac{4(1-\sigma_{yy}^{2})}{E} P\sqrt{(c^2-x^2)} \qquad \mathbf{2}$$

where
v = vertical convergence, and
E = Young's modulus of the roof

There is a software package named DERIVE® [1] that can analyze and simplify many complex mathematical equations. A very interesting thing happened while playing around with equation (1) using DERIVE. After assigning values to the variables P and c (i.e., P=1000 and c=50), a query was made for DERIVE to generate an equation of σ_{yy} in terms of x. DERIVE responded with the equation

$$\sigma_{yy} = \frac{1000x}{\sqrt{(x^2-2500)}} \qquad \mathbf{3}$$

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[1]Soft Warehouse Inc., 3660 Waialae Ave. Suite 304, Honolulu, Hawaii 96816-3236

It was immediately obvious the relationship was

$$s_{yy} = \frac{Px}{\sqrt{(x^2 - c^2)}} \qquad \mathbf{4}$$

Equation (4) implies the ability to predict rib stresses with only knowledge of the seam depth and entry width. This equation is given the name (MESE) MSHA Elastic Stress Equation.[1] To test the validity of the equation, FLAC® will analyze cracks of various widths and seam depths. A comparison will be made to the prediction of the MESE. Figure 2 is the result of the first comparison. In this model, the FLAC elements are 5 ft. square (i.e., seam height = 5 ft.)[2]. Note; the only discrepancy between the MESE and the FLAC data is in the element immediately adjacent to the entry (i.e., 2.5 ft. into the rib). The question arises, is this discrepancy due to the MESE or is it a limitation in FLAC® due to edge effects? To study this problem, it is necessary to increase resolution by reducing element size of the FLAC® model. Figure 3 (1-ft. elements) demonstrates that with smaller elements, the only difference is at the edge element - stresses match well at 2.5 ft. into the rib. This indicates that it is FLAC's limitations that account for the discrepancy.

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[1]To avoid confusion with the many equations mentioned in this report, a decision was made to give a name to the pillar stress equation

[2]Since the element size is 5 ft., this implies that the crack thickness is 5ft. which is the average coal seam thickness, therefore, the name entry will be substituted for crack.

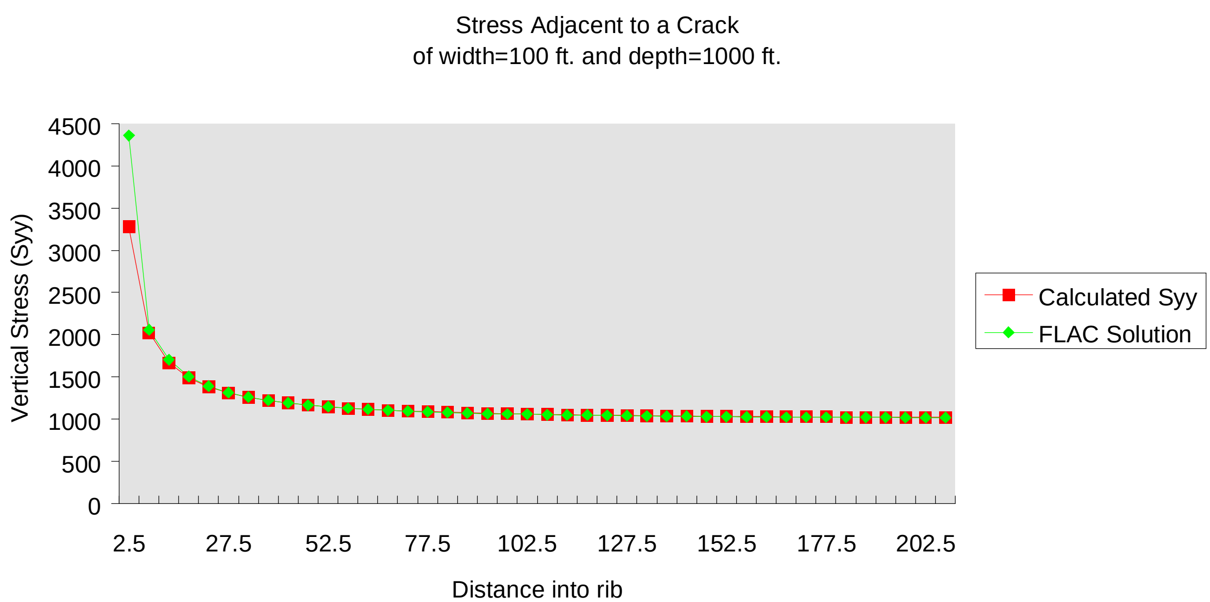

Figure 2
Crack with 5 ft. elements

Additional studies are essential to compare the effects from varying entry width and depth of seam. Figures 2 through 6 are graphs of stress versus distance into the rib for different models. Each figure corresponds to a different model in which either the variable "P" or "c" is unique.[1] Table 1 displays the values of "P" and "c" used to generate the figure. The graphs demonstrate the accuracy of the pillar formula.

TABLE 1

Figure Number	Variable P (psi)	Variable c (feet)
2	1000	50
3	1000	100
4	1000	25
5	500	25
6	250	25

[1]Seam depth is directly related to the force "P" used in the equation. The variable "c" is one half the entry width.

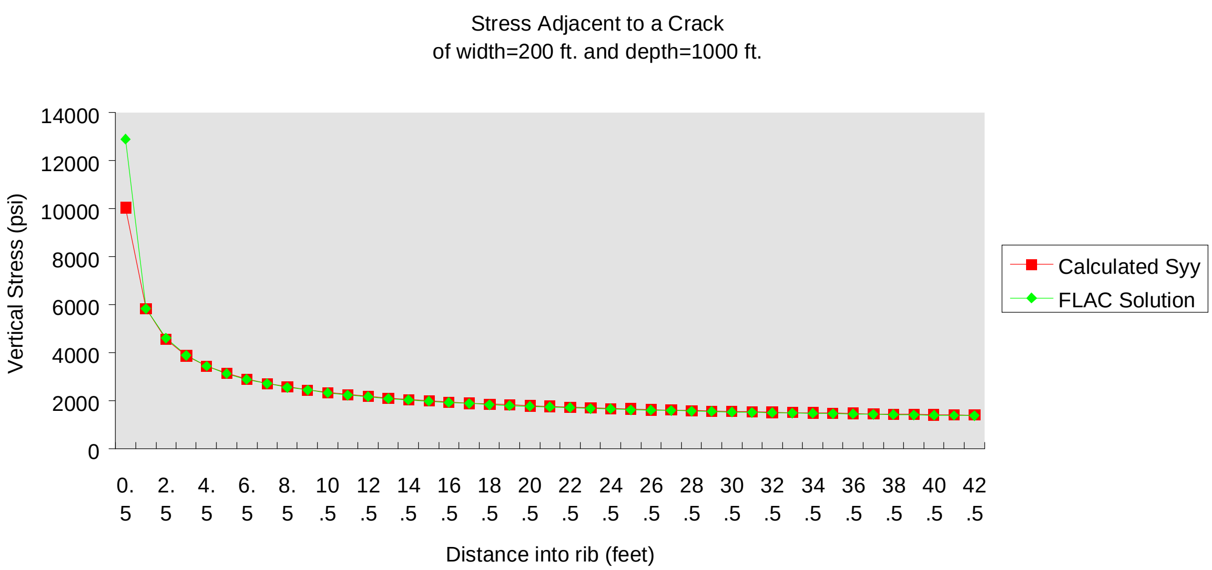

Figure 3
Entry with 1 ft. elements and crack width = 200 ft.

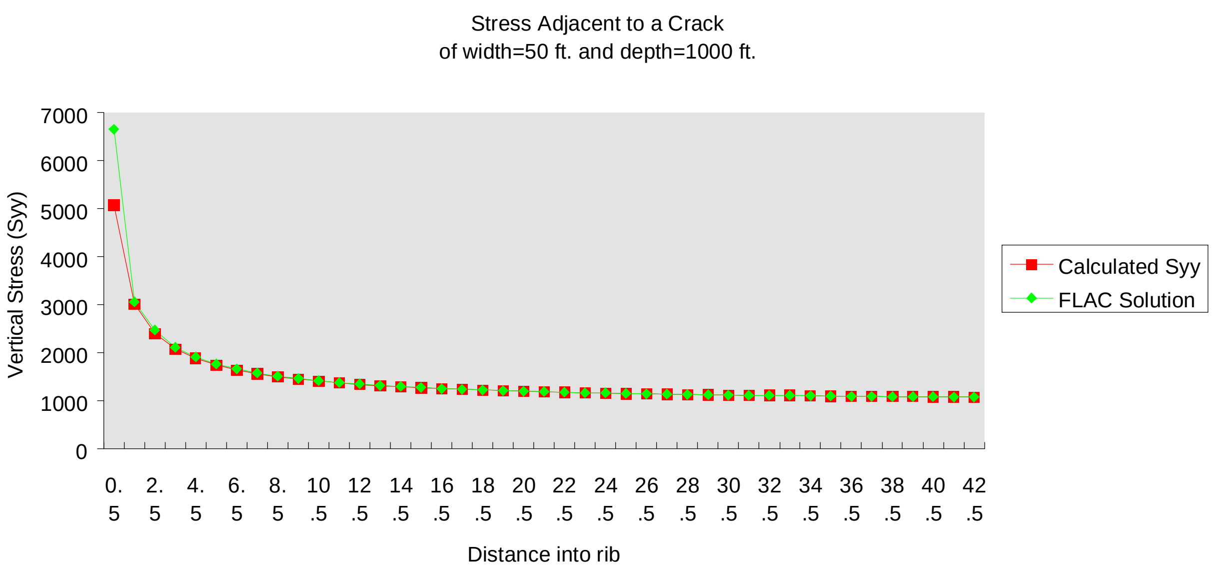

Figure 4
Entry with 1 ft. elements and crack width = 50 ft.

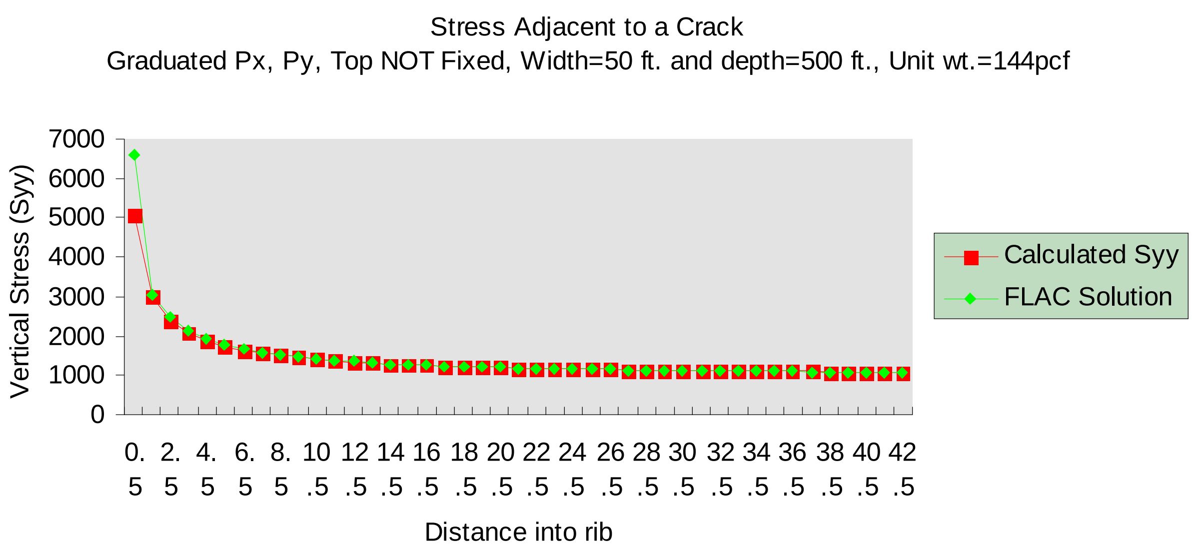

Figure 5
Entry width = 50 ft. and seam depth = 500 ft.

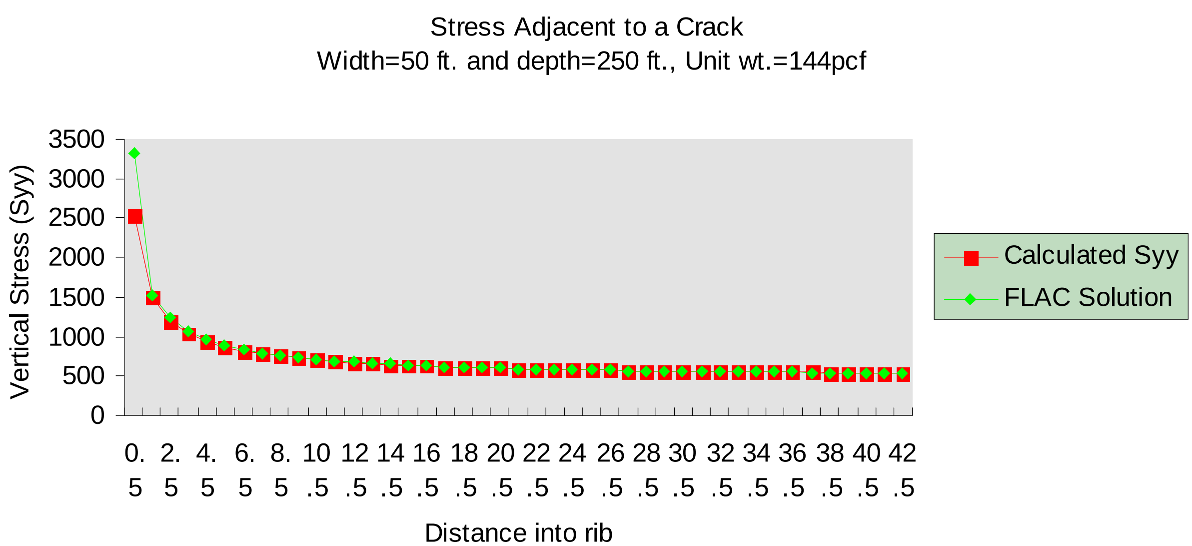

Figure 6
Entry width = 50 ft. and seam depth = 250 ft

The original formula of Jaeger and Cook stipulates that the force is at infinity and the model is assumed homogeneous. This is not a real-life situation. In reality, the loading force is a gradient and the rock is far from homogeneous. Two additional models will investigate the effect on stress distribution caused by these differences.

Figure 7 is a graph from a model in which the coal's modulus is reduced by 50 percent - there is a slight difference between the FLAC solution and the pillar equation. It should be easy to compensate for this deviation. In figure 8, a gradient stress field is added to the model to simulate the effects of gravity. The results compare well with the pillar equation. All in all, it appears the pillar equation will effectively model mine pillars.

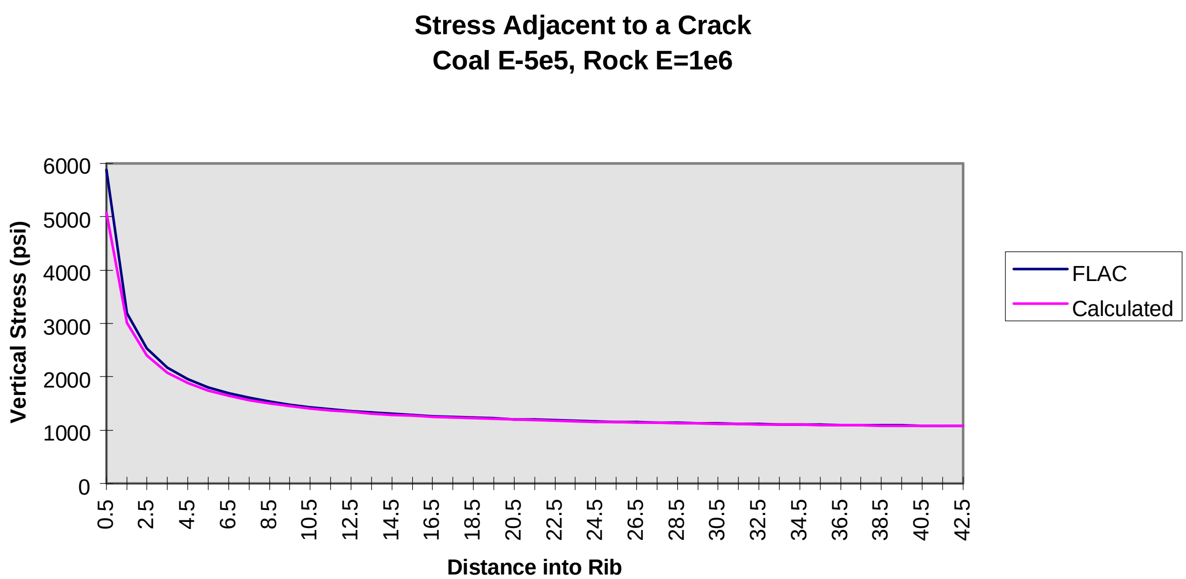

Figure 7
Coal Modulus 50 Percent of Surrounding Strata

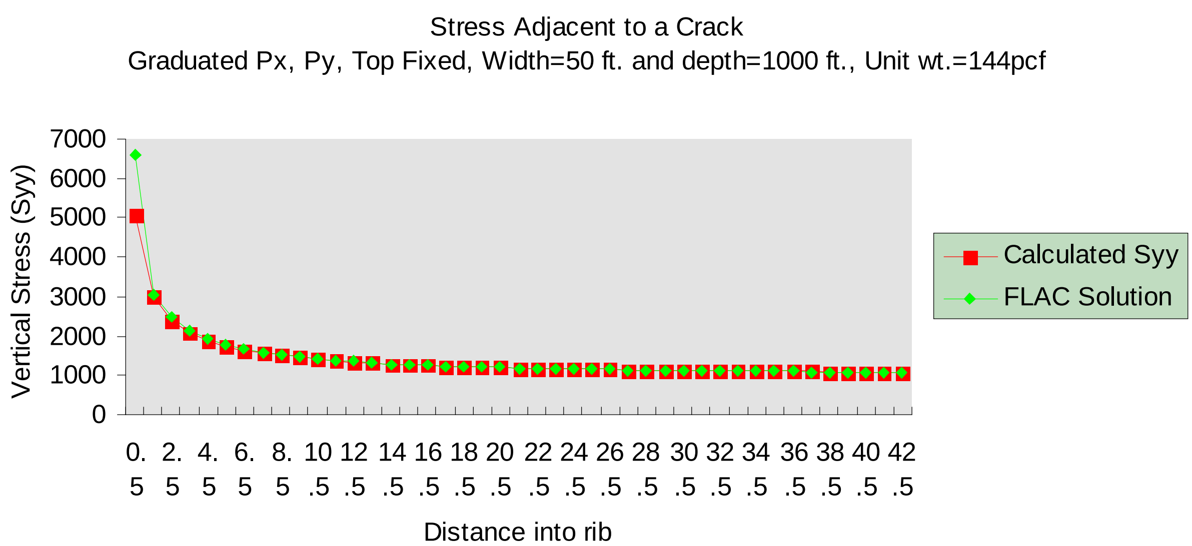

Figure 8

Graduated Stresses Added to Simulate Gravity Loading

3.0 DETERMINATION OF PRINCIPAL STRESSES WITHIN THE PILLARS

Figure 9 is a graph of σ_{yy} and σ_{xx} as predicted by FLAC. It can be seen that the σ_{xx} curve has the same shape as the σ_{yy}; however, it is a fraction less in amplitude. This fractional amount can be approximated by the expression:

$$s_{xx} = g\, s_{yy} \qquad \textbf{5}$$

where

σ_{xx} = horizontal stress
σ_{yy} = vertical stress
γ = Poisson's ratio

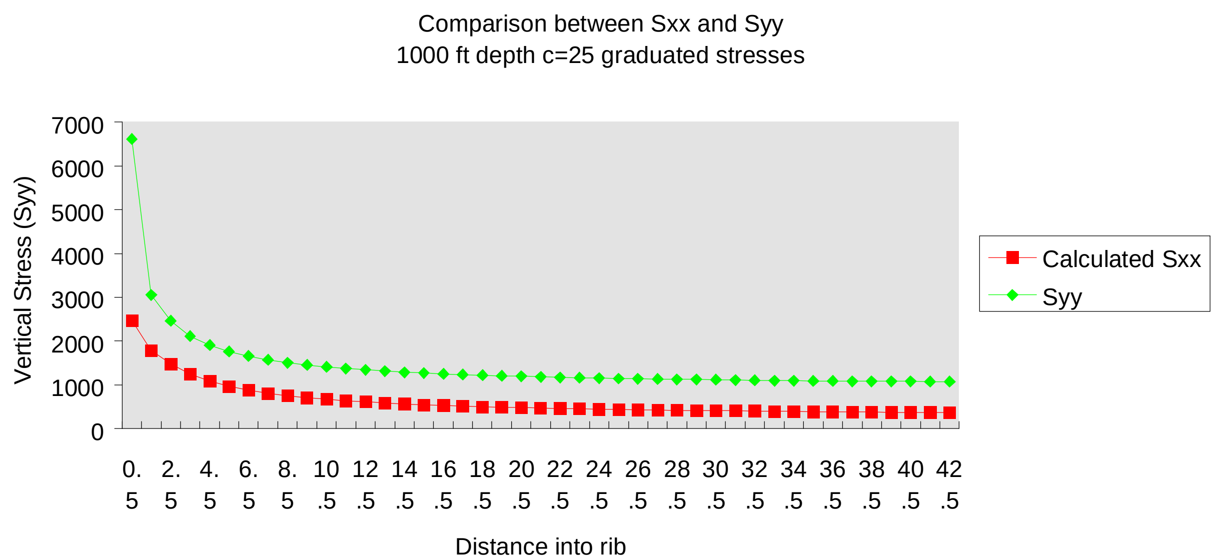

Figure 9
Comparison of the Shape of σ_{vv} and σ_{xx}

4.0 SUPERPOSITION OF EQUATION TO SIMULATE PANELS

The MESE formula successfully predicts the stress distribution for a single excavation. This section will discuss methods for determining the additive effect from several closely spaced excavations such as encountered in room and pillar mining.

The section 4.1 describes modeling a single pillar subjected to the effects of two entries. Section 4.2 expands the technique to account for the combined effects from all entries that make up the mine panel. Using superposition, it is possible to combine the stress influence zone from all entries onto each and every pillar in the model. Section 4.3 discusses special treatment for small pillars.

4.1 Stresses on a Single Pillar

There are three distinct σ_{yy} forces at work on a single pillar sandwiched between two entries; (i) the in situ stress due to gravitational loading, (ii) the stress induced from the presence of the left entry, (iii) the stress due to the presence of the right entry (refer to Fig. 10).

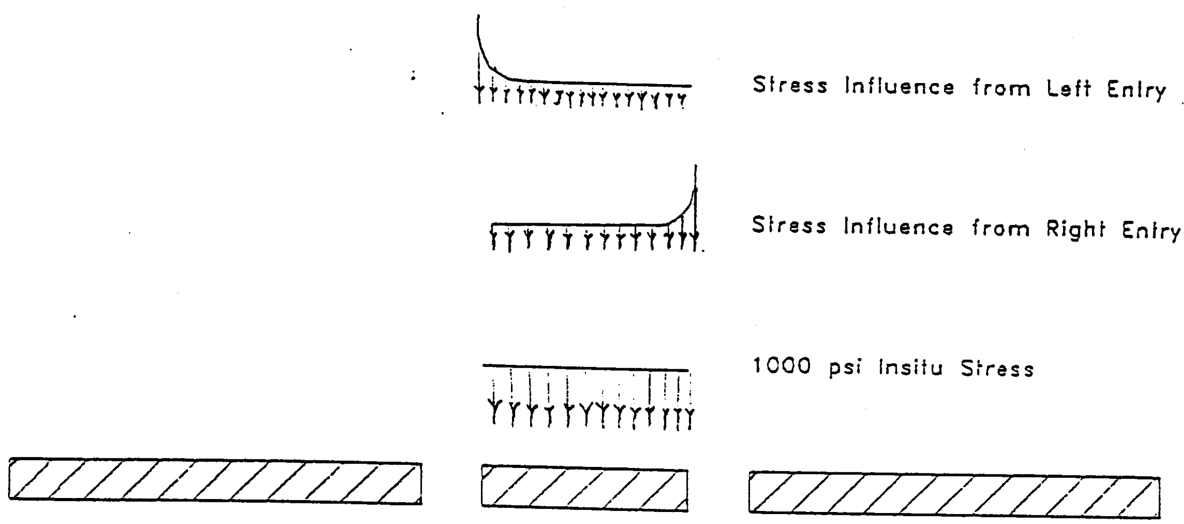

Figure 10
Superposition of Stresses on a Pillar
Influenced by Two Adjacent Entries

Because the in situ (gravity) stress is considered separate from the entry induced stress, the pillar equation (Eq 4) takes a slightly different form. The formula remains similar to the original; however, it is now necessary to subtract the in situ gravity stress P as shown below

$$s_{yy} = \frac{Px}{\sqrt{(x^2 - c^2)}} - P \qquad \textbf{6}$$

Where σ_{yy} = The additional stress attributed to the mine opening, and

P = The in situ gravitational stress (a product of the depth and the unit weight)

Figure 11 plots the superimposed stress distribution on a single pillar.[1]

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[1] The superposition technique used on some of the earliest work (i.e., figures 11 and 15) wasn't correct. The latest superposition technique correlates well with the FLAC solution

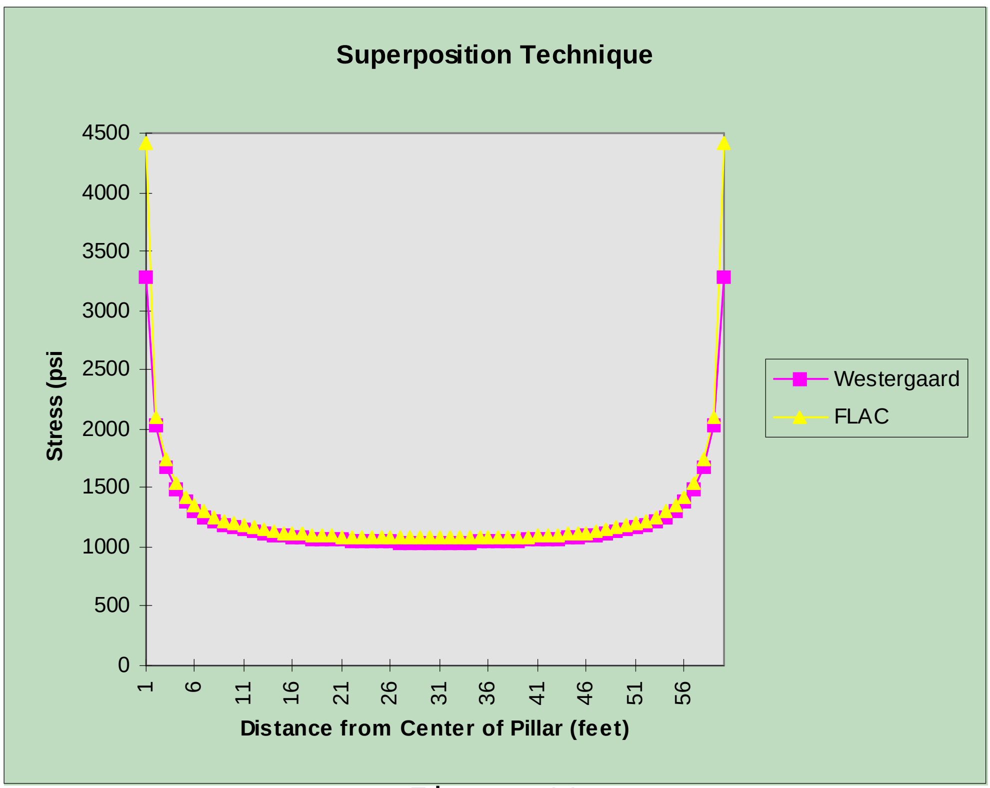

Figure 11
Stress Superpositioning to Predict Pillar loading

4.2 Stress Superposition on a Series of Pillars

The stress on a series of pillars will be a combination of all the stresses induced by all the entries. Each entry will affect the stress on every pillar in the panel up to the point where the influence zone is far enough away from the entry as to produce a negligible result. Later in the report (Sec. 6.2) will demonstrate this zone to equal the distance 4c. Each entry will affect stress distributions in both right and left directions across the entire panel. By superpositioning all the stress influence zones, it should be possible to accurately predict the stress distribution in the mine panel. Figure 12 demonstrates that superposition accurately predicts stress distribution over a mine panel.

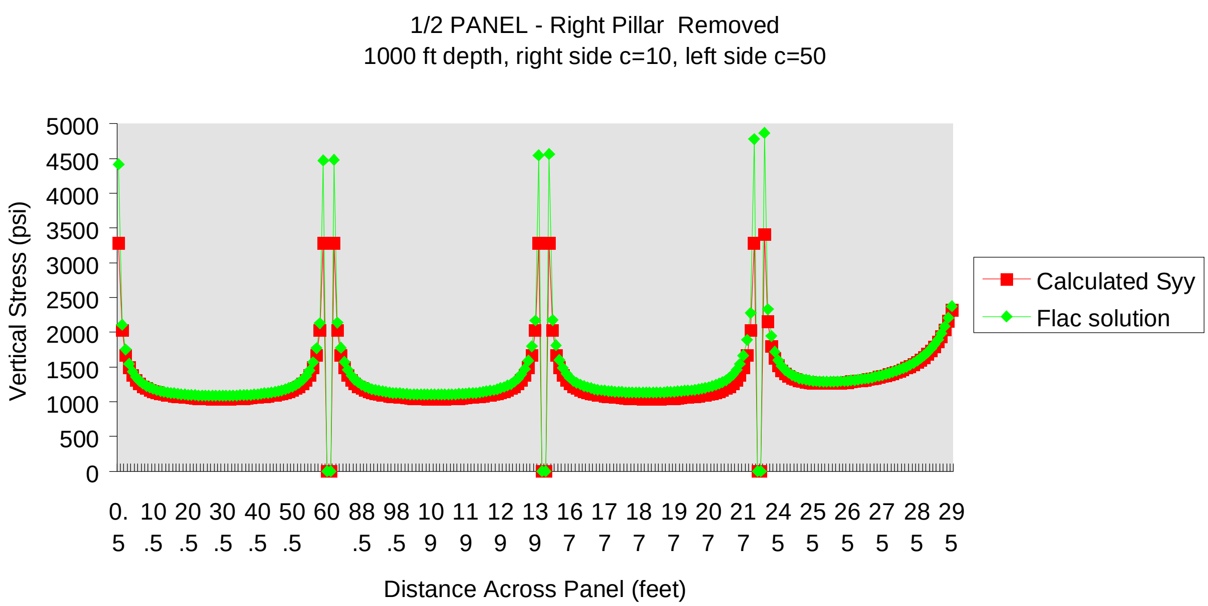

Figure 12
Stress Superpositioning for a Series of Pillars

4.3 Stress Superposition for Small Pillars

Small pillars require additional superposition treatment. A small pillar is any that has a width less than the width of the adjacent entries (i.e., 2c in equation 4). Figure 13 is a stress distribution comparison between FLAC and the standard superposition method. Notice the variance in stress predictions for the 5-ft-wide pillars. There is a small amount of stress that overrides these pillars. This is the cause of the variance. Therefore, it is necessary to average the rideover stress and then redistribute it back onto the pillar.

The rideover stress on one side of the pillar will determine the additional stress to add to that particular edge. The average rideover stress is assigned to variable P in equation 4 to determine the additional edge load. Figure 14 demonstrates how the superposition technique works on 5-ft. pillars. Figure 15 illustrates the method on a panel of 20-ft.-wide pillars; the center pillar is 5-ft. wide

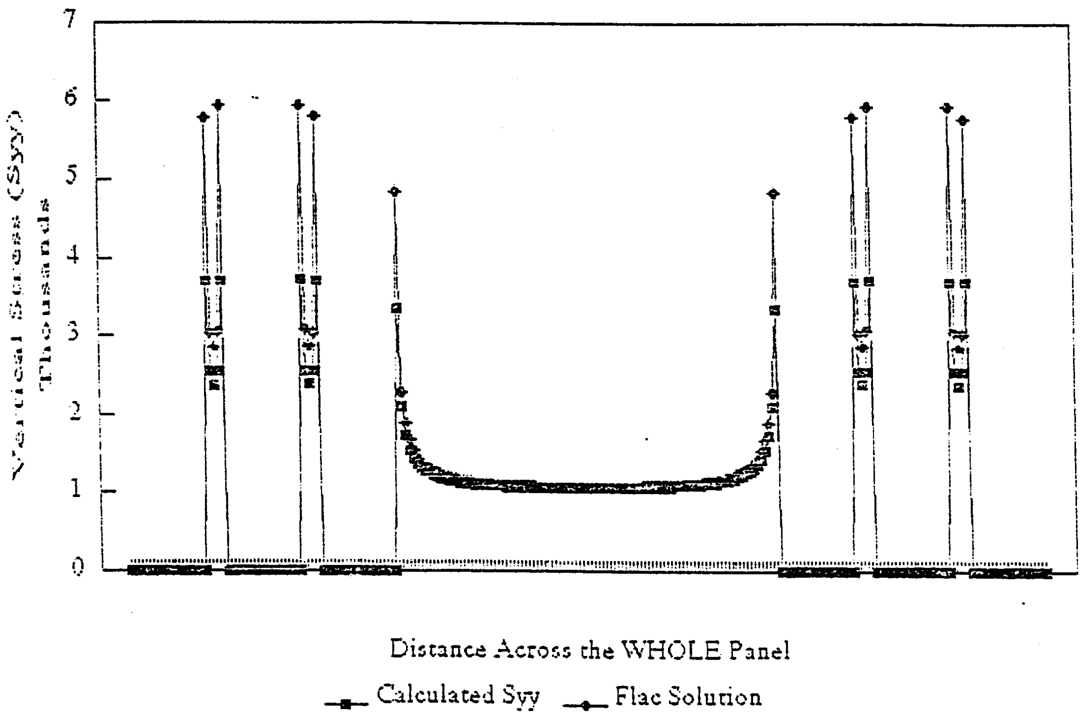

Figure 13

Panel with Small Pillars (Standard Superposition Method)

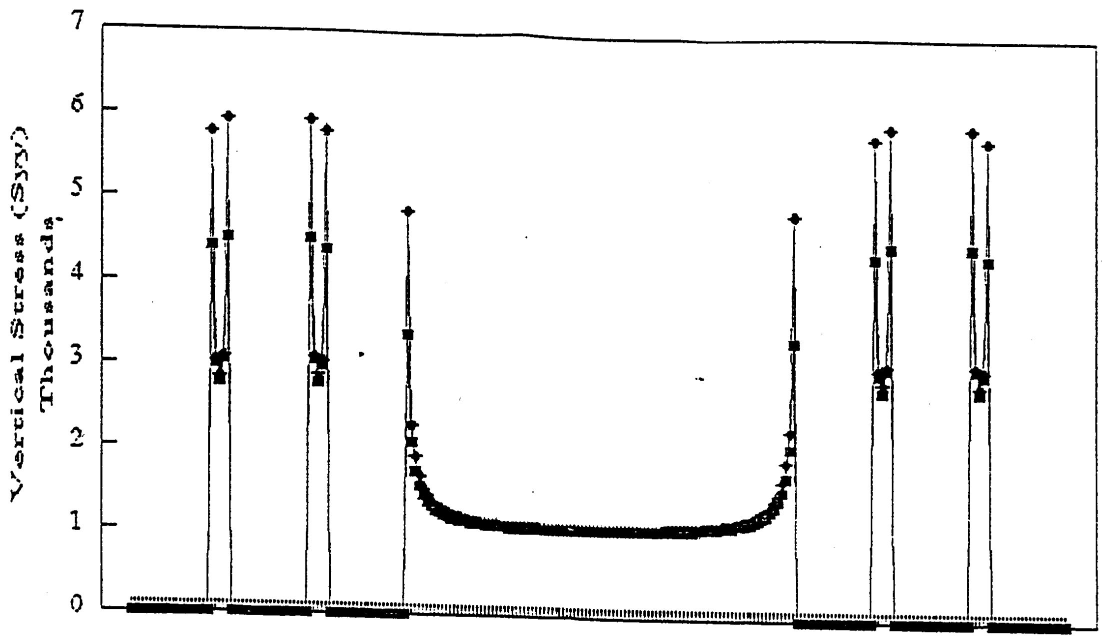

Figure 14
Panel with small pillar design
Includes four-5ft wide pillars and a 100 ft wide center pillar

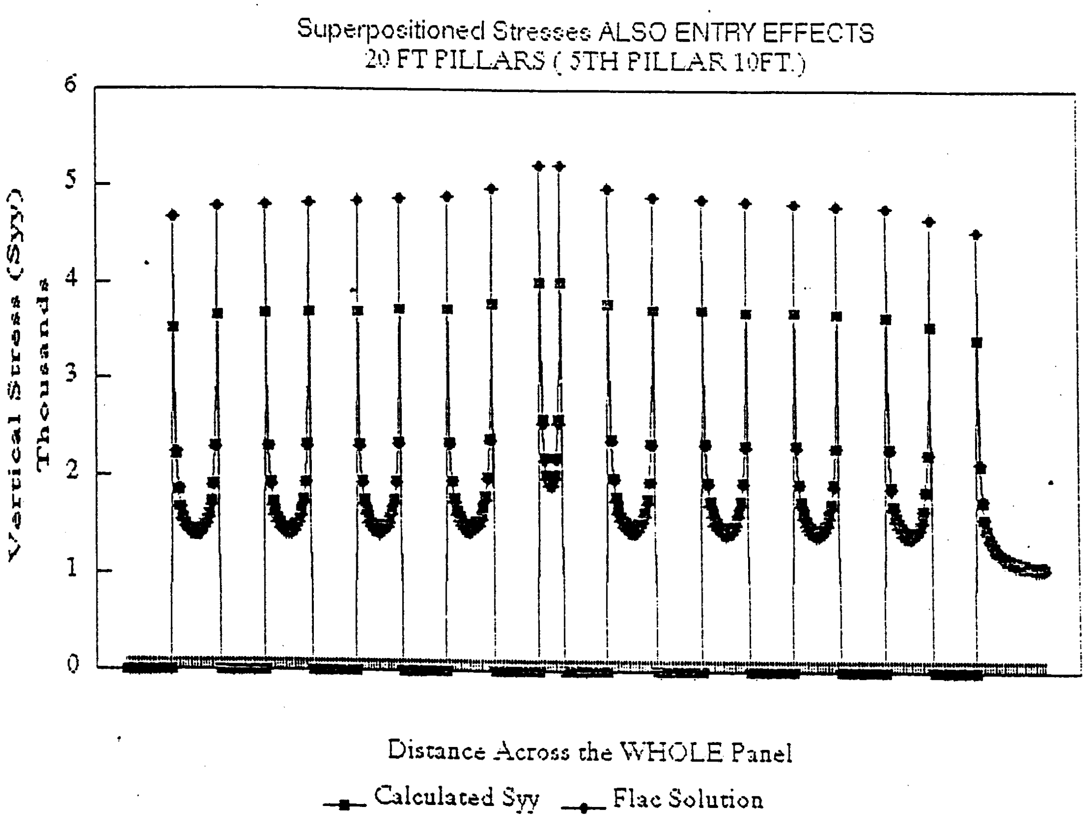

Figure 15
Panel with 20 ft. wide pillars
Center pillar is 5 ft. wide

5.0 SIMULATING YIELDED PILLARS

An assumption is made that pillar behavior is more elasto-plastic then elastic - the pillar yields. It is possible to incorporate failure criteria into the MSHA software. Jaeger provides Griffith failure criteria for flat elliptical cracks. This criteria may be perfectly suited for the pillar equation and superposition method. If so, it will be added at some future date. In the meantime, it should be possible to estimate pillar yield using pillar strength formulas.

The first section below will demonstrate that superposition can model the irregular pillar loading caused by an adjacent yielding pillar. Later sections will incorporate pillar strength equations into the yielding process in an attempt to simulate the yielding behavior.

In the most critical situation, the pillar would completely yield and not offer any vertical support to the roof. While this situation is unlikely, it is easy to model and will demonstrate how the superposition techniques predict the pillar loading. Following this, an improved method is shown that permits the yielded pillar to retain a residual roof support.

The center pillar (i.e., pillar #5) in the model will completely yield and transfer its load to the remaining pillars.[1] Simulating complete pillar yield is simply a matter of removing the pillar, thus creating a very large opening - it is assumed that the yielded pillar does not support any load. This new opening will be the combined widths of yielded pillar #5 plus the two adjacent entries (i.e., 20+60+20= 100 ft). The large opening will cause the superposition stress to be considerably higher on the ribs adjacent to the yielded pillar. Figure 16 is the stress profile for pillar #4 which is next to the completely yielded pillar #5. The superimposed MESE corresponds well to the FLAC solution. Figure 17 is a stress comparison of the right half of the panel - due to symmetry, the left half should be the same. Notice the fifth pillar completely yields, offering no support to the roof. Superposition accounts for the close comparison between the pillar formula and FLAC®.

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[1]Remember that there are ten pillars in the FLAC model. Because of symmetry, it is only necessary to plot half the model

Figure 16
Fourth Pillar Adjacent to Yielded Fifth Pillar
Note Increased Load on Right Side

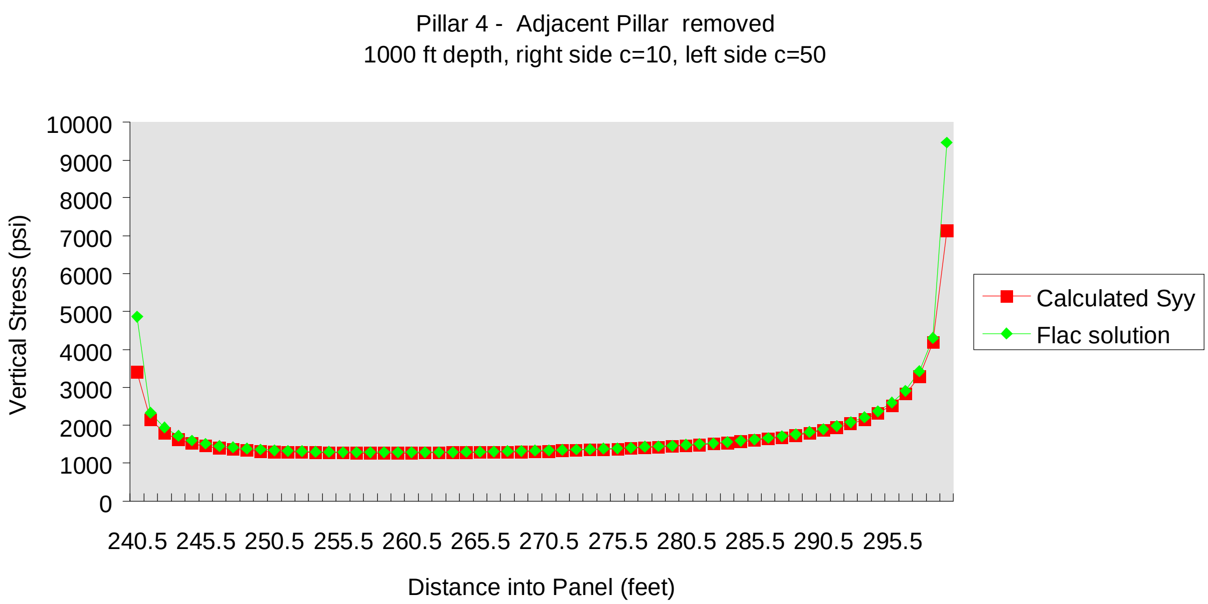
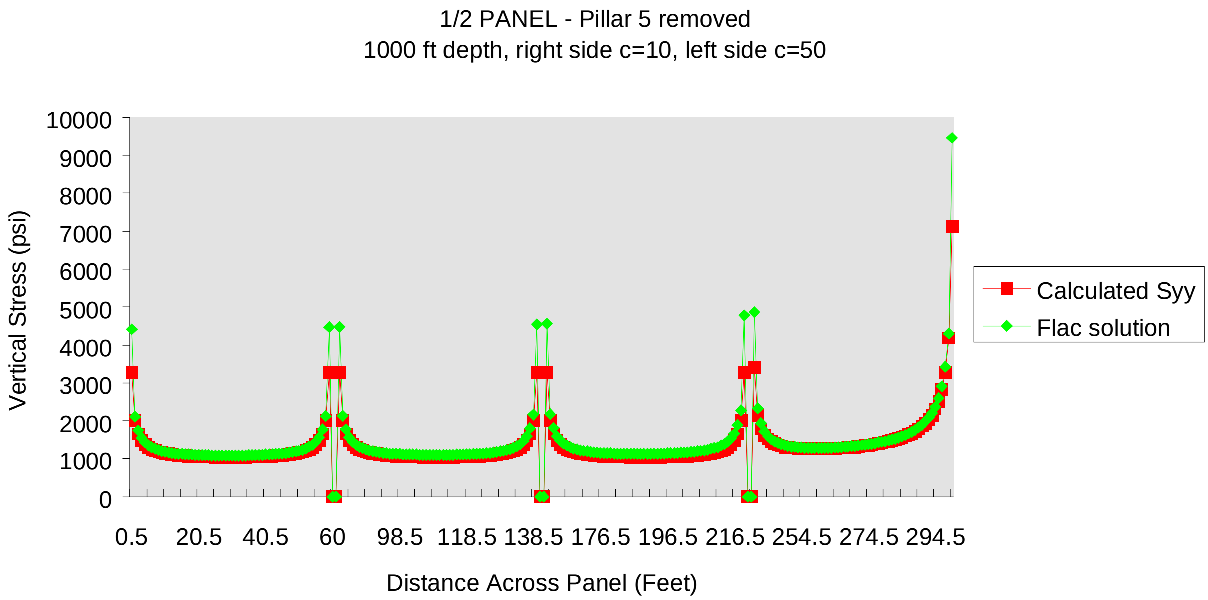


Figure 17
Half Section of Panel
Fifth Pillar Yielded



6.0 SIMULATING PROGRESSIVELY YIELDING PILLARS

The following paragraphs discuss an approach to simulate progressively yielding pillars. A comparison is made between the strength of a pillar section and the stress on that section. If the stress exceeds the strength, the pillars yields. There are several different pillar strength equations. It will be possible to choose the equation that best matches the in situ conditions at the mine. The pillar strength equations are, for the most part, empirically derived. There are elasticity based failure equations that can be added at a future date.

The examples shown compare well to the pillar strength formula predictions. However, it is possible to achieve any stability condition simply by changing the value of a variable in the formulas. Numerical modeling, in coal-bearing strata, is not an exact science - you have to massage the input to match the stress-strain behavior observed at the test site. Once the behavior is matched, the modeling is an effective tool for predicting stability.

6.1 Pillar Strength Equations

Because of the high stress on the very edge of the pillar, an assumption is made that the pillar edge will fail and this failure will propagate in towards the center of the pillar. The propagation will continue until the frictional resistance of the broken coal is sufficient to support the vertical load of the roof. There are several pillar strength equations that predict the residual load bearing capacity of broken coal.

The amount of load the pillar can support will increase with confinement (i.e., distance into the rib). To determine the residual load bearing capacity of the yielded edge, Wilson(5) studied numerical modeling combined with Mohr/Coulomb failure criteria. Mark(4) derived this capacity by integrating various popular empirical pillar strength formulas.

The following are some of the more popular empirical formulas for determining the residual load bearing capacity of a yielded pillar edge. These equations approximate the residual load bearing capacity as a function of distance from the rib into the pillar core.

The following equations are from Mark.[4]

>Holland-Gaddy/Hustrulid-Swanson	$\sigma_v = 2.65\ S_1(x/h)^{1/2}$	(6)
>Obert-Duvall/Wang	$\sigma_v = S_1(0.78 + 1.32\ x/h)$	(7)
>Bieniawski	$\sigma_v = S_1(0.64 + 2.16\ x/h)$	(8)
>Wilson	$\sigma_v = k\ p'(2x/h + 1)^{k-1}$	(9)

where

σ_v = residual load capacity

S_1 = unconfined compressive strength

x = distance into rib

h = height of pillar

k = triaxial stress factor
= $[1+\sin(\phi)]/[1-\sin(\phi)]$
(ϕ = 30 - 37 degrees)

p' = unconfined compressive strength of failed coal at pillar edge (14 psi)

As an initial attempt to model yielding, these empirical formulas will predict the failure stress as a function of depth into the pillar.

6.2 Zone of Influence

Saint Venant's rule suggests that an excavation only affects the stress distribution in the immediate area surrounding it. Figures 18 and 19 verify this fact. Figure 18 is taken from a book on elasticity (source unknown) concerning radial and tangential stress and Saint Venant's effect. Figure 19 is a plot of the (MESE) MSHA Elastic Stress Equation (4). Both figures suggest that at a distance 4c from the excavation, the stress distribution is unaffected by the presence of the excavation. Also note in Figure 19, the apparent effects of variables P and C in the MSHA equation.

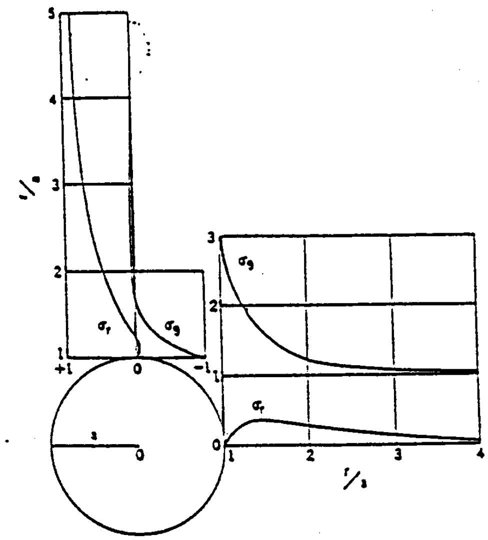


Figure 18
Stress concentrations along axes of symmetry for circular openings subjected to a uniaxial stress field.

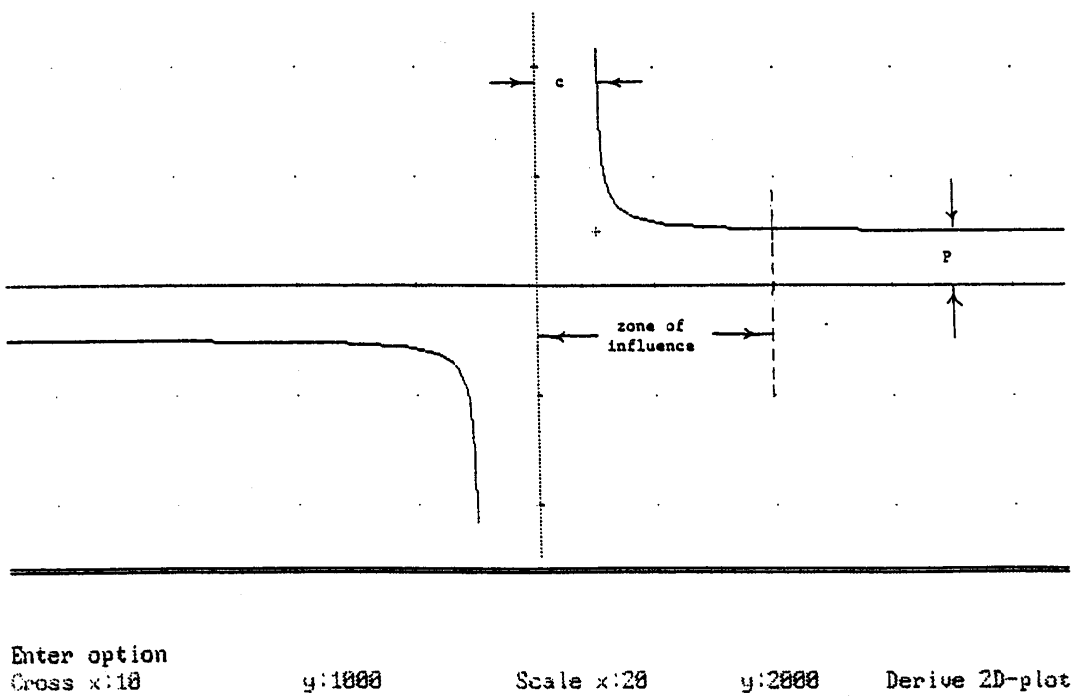


Figure 19
Stress distribution predicted by MSHA pillar formula. At a distance 4c the excavation has a minimal effect on stress distribution.

6.3 Yielding Technique

The method of yielding incorporates the MESE, superpositioning techniques, a pillar strength equation, and the dynamic zone of influence to predict the final stress distribution in a series of pillars. The pillar strength equation will determine the stress distribution in the broken pillar edges. The MESE will indicate the stress distribution in the elastic core. The zone of influence will control the area in which conservation of energy is to be applied.

If the pillar completely yields, there is no elastic core, and the pillar strength equation predicts the stress distribution over the entire pillar. The variable c in equation (4) controls the size of the zone of influence (refer to sec. 6.2). In a situation of total yield, the variable c extends from the center of the entry to the pillar center.

The failure analysis consists of comparing the strength of the pillar to the stress applied on the pillar. The pillar width is divided into sections. The failure analysis begins on the outer-most section and works its way into the center. If the analysis predicts the section to be unstable, the section is allowed to yield. The amount of stress that the yielded portion can no longer support is transferred onto the next pillar section. The failure analysis will then proceed on the next section. The process will continue inward until the failure analysis predicts stability. If it is not possible to achieve stability, the pillar will not accept any more load than that predicted by the pillar strength equation.

The MESE predicts the stress in the elastic core. As the pillar edge yields, the core becomes smaller and receives increased loading. To reflect this effect, the variable c in the equation (4) increases from the pillar edge to the new yield-elastic boundary. This results in an increase in stress on the elastic core as if the fracture section were mined away. However, the fractured pillar edge has a residual load bearing capacity that must be taken into consideration. To insure conservation of energy, it is necessary to subtract this residual load from the load on the elastic core.

A dynamic zone of influence will dictate the area in which conservation of energy is to apply. This zone is equal to 4c (refer sec. 6.2), where c is equal to the distance from the center of the entry to the yield-elastic boundary. In the yielding process, the variable c increases; therefore, the zone of influence will also increase.

The three-dimensional plot of the empirical pillar strength formula will take on the shape of a pyramid. Similarly, the three-dimensional plot of the MESE would take on the shape of an inverted pyramidal depression. To determine the stress acting at any particular section of the pillar would involve complex calculus and is impractical. It is much easier to analyze a thin vertical slice (unity in thickness) through the pillar center. The shape of this slice is similar to the shape of the two-dimensional plot in figure 11.

The yielding process involves dividing the pillar into many equal segments. The analysis begins on the outermost section and works its way into the core. The process can be divided into the following steps; i) determine the value of c (i.e., the distance from the entry center to the pillar edge or yield-elastic boundary), ii) determine the zone of influence caused by the presence of the entry, iii) determine the total in situ load in the zone of influence, iv) determine the value of the variable P by dividing the total in situ load by the zone of influence, v) use the MESE and superpositioning techniques to determine the load on the segment, vi) use a pillar strength equation to determine the strength of the segment, vii) if the load is greater then the strength of the segment, the pillar section is assumed yielded and the analysis continues, viii) choose the next segment, and repeat the analysis on this segment beginning at step i. This process will continue until either stability is achieved or the yield zone reaches the center of the pillar (i.e., the entire pillar yields).

The following is an example of the yielding process. A more complete example exists in Appendix A. The assumption is made that the pillar is wide enough so that stress superposition can be ignored. Wilson's equation is chosen to provide the residual strength characteristics of the yield zone. To determine the load or strength over the entire section, it is necessary to take the integral of either equation for the limits defined by the edges of the section. Since comparison is made between two integrals, the limits of the integral can be in feet instead of inches.

PILLAR YIELD METHOD INCORPORATING STRESS INFLUENCE ZONE"

MSHA Stress Formula

$$\frac{Px}{\sqrt{x^2 - c^2}} \qquad \mathbf{7}$$

Initial conditions
C = 10
P = 1000

Wilson Strength Formula

$$KT \left[\frac{2x}{H} + 1 \right]^{K-1} \qquad \mathbf{8}$$

Initial conditions
H = 5
K = 3
T= 14

EXAMPLE

The section 0-5 ft. into the pillar has yielded
Determine the stability on the next section 5 - 10 ft into the pillar
C = C + 5
C = 15

AREA_OF_INFLUENCE = 4 x C = 60 (9)

TOTAL_ELASTIC_LOAD = P x AREA_OF_INFLUENCE (10)
= 60000

Subtract Wilson's residual strength of yielded edge from TOTAL_ELASTIC_LOAD

WILSON'S EQUATION

$$K\,T\left[\frac{2\,x}{H}+1\right]^{K-1} \qquad (11)$$

To determine the amount of residual strength for the section

$$\int_{0}^{5} K\,T\left[\frac{2\,x}{H}+1\right]^{K-1} dx \qquad (12)$$

Subtract residual load from the Total Elastic load

$$TOTAL\_ELASTIC\_LOAD = TOTAL\_ELASTIC\_LOAD - \int_{0}^{5} K\,T\left[\frac{2\,x}{H}+1\right]^{K-1} dx$$

$$TOTAL\_ELASTIC\_LOAD = 60000 - \int_{0}^{5} K\,T\left[\frac{2\,x}{H}+1\right]^{K-1} dx$$

= 59090

Determine New Value for P

$$P = \frac{TOTAL\_ELASTIC\_LOAD}{} \qquad (13)$$

AREA_OF_INFLUENCE

= 984

Determine MSHA Stress on 5 - 10 ft. Section

MSHA EQUATION

$$\frac{P\,x}{\sqrt{(x^2 - C^2)}} \qquad (14)$$

MSHA Load on 5 - 10 ft. Section
Remember that c = 15

$$\text{SECTION\_STRESS} = \int_{15}^{20} \frac{P\,x}{\sqrt{(x^2 - C^2)}}\,dx \qquad (15)$$

= 13028

Determine Wilson Strength of 5-10 ft. Section

$$\text{WILSON\_STRENGTH} = \int_{5}^{10} K\,T \left[\frac{2\,x}{H} + 1\right]^{K-1} dx$$

= 3430

SECTION_STRESS is > WILSON_STRENGTH, Therefore Section Yields

ANALYSIS WILL CONTINUE ON THE NEXT SECTION (i.e. 10 - 15 ft)

This technique will continue until either stability is achieved or the yield zone extends to the Pillar center

NOTE: The value for the integral in equation (15)

> results in the formation of a complex conjugate due to the square root of zero which can be attributed to solution of the lower limit. However, results are satisfactory if the value of the integral at the lower limit is assumed to equal zero.

It's unnecessary to perform the yielding analysis more than once. Once the yield range is known, analysis will continue on the elastic core by including the increased load due to superposition. The software will not use the integral of the MSHA pillar stress equation.

6.3.1 Griffith Failure Criteria for the surface of a flat elliptical crack[(3)p277]

Jaeger describes failure criteria for the surface surrounding a flat elliptical crack. It should be possible to incorporate this yield criteria into the model. This would permit a more precise yield model.

7.0 PREDICTING ROOF STRESSES

Using equation (2), it is possible to determine the convergence of the entry roof. Using the beam or plate theory and the knowledge of roof convergence, it should be possible to predict the stresses in the immediate roof.

8.0 COMPUTER SOFTWARE

The first version of the computer software will operate in Lotus1-2-3®. The elastic version of the code is already functioning - most of the plots are taken from Lotus. Hewlett Packard markets a handheld calculator that uses Lotus 1-2-3 as its operating system. This code should be able to operate from within the calculator. Another version of the code will be written in C language. This code is nearly half complete.

9.0 CONCLUSION

The MSHA elastic stress equation (MESE) accurately predicts the stress distribution in the elastic core of a coal pillar. Using superposition techniques, it is possible to estimate the stress distribution throughout an entire mine panel. Superposition permits analysis of more irregular pillar geometries and yielding processes. Small pillars require additional superposition measures to account for rideover stress. Pillar strength can be estimated using any of the popular empirical formulas. If the load on the pillar exceeds the pillar strength, the pillar edge yields thus transferring a portion of its load to adjacent pillars. The pillar strength will then be reevaluated and the process continues until a stable condition exists.

Computer software incorporates all these techniques to predict panel stability. The software is quick and easy to use. It provides the layman with a tool to estimate mine design stability. It also could assist the experienced modeler as a tool to "hone in" on a design before final modeling, using more sophisticated software.
The yield portion is not the most elegant solution, however, it does produce satisfactory results. Incorporation of the Griffith failure criteria for flat elliptical cracks should improve the validity of the failure analysis considerably. Rock mechanics is

not an exact science, especially in coal-bearing strata. Numerical analysis is only a tool for estimation design considerations. The results from any numerical modeling software should not be taken for its face value. Any unusual pillar design should be tried on an experimental basis.

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APPENDIX A

APPENDIX A EXAMPLE OF THE PILLAR YIELDING PROCESS

Initial Parameters

FIRST SEGMENT

MSHA EQUATION
P = 1000 psi
C = 10 (20 ft. entry)
segment width = 5 ft.

WILSON'S STRENGTH
p' = 14 psi
H = 5
K = 3
segment width = 5 ft.

PILLAR YIELD METHOD INCORPORATING STRESS INFLUENCE ZONE"
ORIGINAL CONDITIONS

MSHA Stress Formula
C = 10
P = 1000

Wilson Strength Formula
H = 5
K = 3
let T = P'
T = 14

EXAMPLE
The section 0-5 ft. into the pillar has yielded. Determine the stability on the next section (5 - 10 ft into the pillar)
C = C + 5
C = 15

AREA_OF_INFLUENCE := 4 C = 60

TOTAL_ELASTIC_LOAD := P AREA_OF_INFLUENCE = 60000

Subtract Wilson's residual strength of yielded edge from TOTAL_ELASTIC_LOAD

WILSON'S EQUATION

$$K\,T\left[\frac{2\,x}{H} + 1\right]^{K - 1}$$

To determine the amount of residual strength for the section

$$\int_0^5 K\,T\left[\frac{2\,x}{H} + 1\right]^{K - 1} dx$$

Subtract residual load from the Total Elastic load

$$TOTAL\_ELASTIC\_LOAD := TOTAL\_ELASTIC\_LOAD - \int_0^5 K\,T\left[\frac{2\,x}{H} + 1\right]^{K - 1} dx$$

$$TOTAL\_ELASTIC\_LOAD := 60000 - \int_0^5 K\,T\left[\frac{2\,x}{H} + 1\right]^{K - 1} dx$$

$$= 59090$$

Determine New Value for P

$$P := \frac{TOTAL\_ELASTIC\_LOAD}{AREA\_OF\_INFLUENCE}$$

$$= 985$$

Determine MSHA Stress on 5 - 10 ft. Section

MSHA EQUATION

$$\frac{Px}{\sqrt{x^2 - C^2}}$$

MSHA Load on 5 - 10 ft. Section
Remember that c = 15

$$SECTION\_STRESS = \int_{15}^{20} \frac{P\,x}{\sqrt{(x^2 - C^2)}}\,dx$$

$$= 13028$$

Determine Wilson Strength of 5-10 ft. Section

$$WILSON\_STRENGTH := \int_{5}^{10} K\,T\left[\frac{2\,x}{H} + 1\right]^{K-1} dx$$

$$= 3430$$

SECTION_STRESS is > WILSON_STRENGTH, Therefore Section Yields

ANALYSIS WILL CONTINUE ON THE NEXT SECTION (i.e. 10 - 15 ft)

This technique will continue until either stability is achieved or the yield zone extends to the Pillar center

NOTE

The value for the integral

$$51: \int_{15}^{20} \frac{P\, x}{\sqrt{(x^2 - C^2)}}\, dx$$

results in the formation of a complex conjugate if you take into consideration the square root of zero. However, results are satisfactory if the value of the integral at the lower limit is assumed to equal zero.

ANALYZE NEXT SECTION (10 - 15 FT)

C := 20

AREA_OF_INFLUENCE := 4 C = 80

TOTAL_ELASTIC_LOAD := 1000 AREA_OF_INFLUENCE

= 80000

Subtract Wilson's residual strength

$$\text{OTAL\_ELASTIC\_LOAD} := \text{TOTAL\_ELASTIC\_LOAD} - \int_{0}^{10} K\, T \left[\frac{2\, x}{H} + 1\right]^{K - 1} dx$$

$$\text{TOTAL\_ELASTIC\_LOAD} := 80000 - \int_{0}^{10} K\, T \left[\frac{2\, x}{H} + 1\right]^{K - 1} dx$$

= 75660

Determine new value for P

$$P := \frac{\text{TOTAL\_ELASTIC\_LOAD}}{\text{AREA\_OF\_INFLUENCE}}$$

$= 945.75$

Determine MSHA stress on 10 - 15 ft. section

$$SECTION\_STRESS := \int_{20}^{25} \frac{P\ x}{\sqrt{(x^2 - C^2)}}\ dx$$

$= 14186$

Determine Wilson's strength for the 10 - 15 ft. section

$$WILSON\_STRENGTH := \int_{10}^{15} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$= 7630$

Section_stress is > Section_strength therefore, the SECTION YIELDS

Analyze NEXT Section (15 - 20 ft)

C := 25

AREA_OF_INFLUENCE := 4 C = 100

TOTAL_ELASTIC_LOAD := 1000 AREA_OF_INFLUENCE

= 100000

Subtract Wilson residual strength

$$TOTAL\_ELASTIC\_LOAD := TOTAL\_ELASTIC\_LOAD - \int_{0}^{10} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$$\text{TOTAL\_ELASTIC\_LOAD} := 10^{5} - \int_{0} K\, T \left[\frac{2\,x}{H} + 1\right]^{K - 1} dx$$

$$= 88030$$

Determine New Value for P

$$P := \frac{TOTAL\_ELASTIC\_LOAD}{AREA\_OF\_INFLUENCE}$$

$$= 880$$

Determine MSHA stress on Section 15-20 ft

$$SECTION\_STRESS := \int_{25}^{30} \frac{P\ x}{\sqrt{(x^2 - C^2)}}$$

$$= 14598$$

Determine Wilson's Strength for Section 15-20 ft.

$$WILSON\_STRENGTH := \int_{15}^{20} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$$= 13510$$

Section_stress is > Section_strength therefore, the section yields.

REDUCE SECTION WIDTH FROM 5 ft. TO 2 ft.

<u>Analyze Next Section (20 - 22 ft.)</u>

$C := 30$

$AREA\_OF\_INFLUENCE := 4\ C = 120$

$TOTAL\_ELASTIC\_LOAD := 1000\ AREA\_OF\_INFLUENCE$

$$= 120000$$

Subtract Wilson's strength

$$TOTAL\_ELASTIC\_LOAD := TOTAL\_ELASTIC\_LOAD - \int^{20} K\ T \left[\frac{2\ x}{} + 1\right]^{K - 1} dx$$

60

0 H

$$TOTAL\_ELASTIC\_LOAD := 1.2\ 10^{5} - \int_{0}^{20} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$$= 94520$$

Determine New value for P

$$P := \frac{TOTAL\_ELASTIC\_LOAD}{AREA\_OF\_INFLUENCE}$$

$$= 787.666$$

Determine MSHA Stress on Section 20 - 22 ft

$$SECTION\_STRESS := \int_{30}^{32} \frac{P\ x}{\sqrt{(x^{2} - C^{2})}} dx$$

$$= 8771.08$$

Determine Wilson Strength for Section 20 - 22 ft

$$WILSON\_STRENGTH := \int_{20}^{22} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$$= 7426.72$$

Section_stress is > Section_strength, therefore the section yields.

Analyze Next Section 22 - 24 ft.

$C := 32$

$AREA\_ON\_INFLUENCE := 4\ C = 128$

$TOTAL\_ELASTIC\_LOAD := 1000\ AREA\_OF\_INFLUENCE$

$= 128000$

Subtract Wilson's Residual Strength

$$TOTAL\_ELASTIC\_LOAD := TOTAL\_ELASTIC\_LOAD - \int_{0}^{22} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$$TOTAL\_ELASTIC\_LOAD := 1.2\ 10^{5} - \int_{0}^{22} K\ T \left[\frac{2\ x}{H} + 1\right]^{K - 1} dx$$

$= 87093$

Determine new value for P

$$P := \frac{TOTAL\_ELASTIC\_LOAD}{AREA\_OF\_INFLUENCE}$$

$= 725.777$

Determine MSHA stress on Section 22 - 24 ft.

$$SECTION\_STRESS := \int_{32}^{34} \frac{P\ x}{\sqrt{(x^{2} - C^{2})}} dx$$

$= 8338.54$

Determine Wilson Strength of Section 22 - 24 ft

$$\int^{24} \left[\ 2\ x \qquad \right]^{K - 1}$$

$$\text{WILSON\_STRENGTH} := \int_{22} K\ T \left[\frac{\quad}{H} + 1\right] dx$$

$$= 8743.84$$

The WILSON_STRENGTH IS > SECTION_STRESS therefore, the section is stable. This implies that the yield zone is 22 ft into the pillar. Wilson has an equation that predicts the yield-elastic depth into pillar.

Q = density x overburden thickness

$$Q := \frac{170}{144} 1000$$

$$= 1180.55$$

"@ H=5, T=14, K=3"

$$y = \frac{H}{2} \left[\left(\frac{Q}{T}\right)^{1 / (K - 1)} - 1\right]$$

$$y = 20.4572$$

This is close to the zone predicted above

Wilson also predicts the stress at the yield-elastic boundary as

$\sigma = Kq$ on the yield side of the yield-elastic boundary

$$\sigma = K\ Q$$

$$\sigma = 3541.66$$

$$K\ T \left[\frac{2\ 22}{H} + 1\right]^{K - 1}$$

$$= 4033.68$$

This is close.

Wilson predict the stress on the elastic side of the yield-elastic boundary as

$$\sigma' = KQ + \text{Uniaxial compressive strength}$$

The MSHA equation predicts the stress is

$C := 32$

$$\frac{P\ 32.25}{\sqrt{(32.25^2 - C^2)}}$$

$= 5840.18$

This implies the uniaxial compressive strength is

$= 5840 - 4033.68$

$= 1806.32$

This is a little high but OK. Actually the MSHA formula predicts the vertical stress at the extreme edge of the elastic boundary is infinity - which seems correct if you assume the edge is a sharp corner.