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# A SIMULATION PROCESS TO ENHANCE THE BOUNDARY ELEMENT METHOD OF ANALYSIS

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**Abstract**. The Boundary Element Method of Numerical Modeling has become an increasingly popular rock mechanics - mine design tool. It has been used by the Roof Control Division, Pittsburgh Safety and Health Technology Center, MSHA, to aid in the resolution of complex ground control problems since 1985. This paper will present an overview of an evolving simulation process and details of methods utilized to generate necessary coal, rock and gob input properties. Also included is a description of a Deterioration Index System that is used to link model output to observed mine conditions and two case studies to illustrate that process.

## Introduction

A primary function of the Roof Control Division, Pittsburgh Safety and Health Technology Center, is to provide technical assistance to MSHA and the mining industry in the resolution of complex roof control problems. In order to evaluate mining systems not easily treated by simplified empirical or analytical methods, Boundary Element Numerical Modeling was initiated in 1984 and expanded in 1987 with acquisition of the BESOL (1) system. The ability of the 3-D Boundary Element Method to model large mine areas with complex geometries has led to the successful simulation of actual mine conditions and the identification of potential solutions to ground control problems in over 20 mines throughout the country. The technique has been applied to a variety of mining scenarios including longwall and room and pillar operations utilizing both conventional and yield pillar configurations. The influence of vertical and horizontal stress has been modeled to simulate underground conditions ranging from deteriorating roof and persistent falls to areas of squeezing ground and complete pillar failure.

In the process of developing numerical models for the various mining operations analyzed during the last ten years, a systematic simulation methodology has evolved. Techniques to estimate coal, rock and gob backfill properties have been established, and a Deterioration Index was developed to quantify in-mine roof, floor and pillar behavior to assist in calibrating model parameters and evaluating potential mine design alternatives. This paper will present a brief description of the BESOL system, an overview of the simulation process used and details of methods used to construct models and estimate rock mechanics parameters. A discussion of the Deterioration Index system and details of two case studies, which typify actual mine simulations and illustrate techniques used to evaluate ground conditions and proposed mining options, are also included.

## Besol System Description

BESOL is a system of computer programs for solving rock mechanics problems based on the boundary element displacement discontinuity method of analysis. The 3-dimensional MS221 version (yielding and multiple seam capability) was acquired from Crouch Research, Inc., and has been used by the Roof Control Division to evaluate complex mining systems since 1987. BESOL is complete with graphic pre- and post-processors that greatly simplify model construction and output data interpretation.

Figure 1 presents a generalized boundary element model that illustrates a tabular seam or ore body surrounded by a homogenous, isotropic linearly elastic rock mass. Input data includes elastic rock mass properties and rock strength criteria, seam properties and backfill or artificial support characteristics. A definition of the seam plane(s), detailed geometry of the excavation, mining depth, seam height and a complete 3-dimensional in-situ-stress state of the model are also required. Output capabilities include stress, strain and displacement computations within user selected areas (both on and off the seam plane), Failure Index (roof and floor safety factors) calculations in the rock mass and Energy Release estimates in yielding areas.

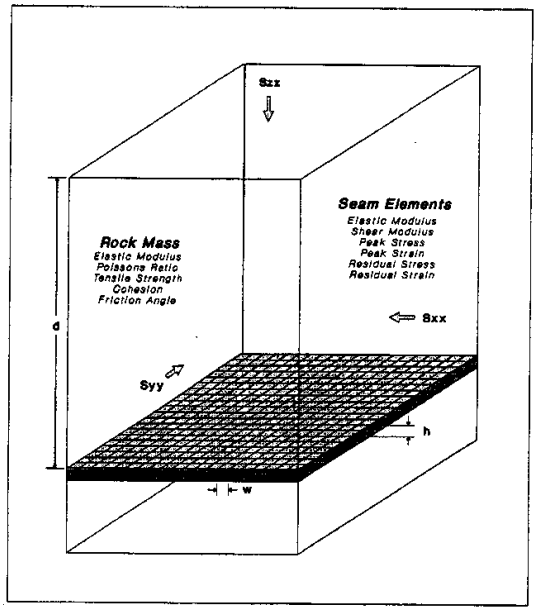


Figure 1. Generalized BESOL Boundary Element Model.

## Simulation Process

Figure 2 outlines an eight-step process used during the simulation of underground mining systems. While the process is specifically directed to numerical modeling applications, it can also be used in conjunction with empirical or analytical methods. Mine conditions are categorized in a number of locations where differing pillar sizes, panel configurations and overburden levels are found. The Deterioration Index System to be discussed later in the paper aids in the description of in-mine ground conditions. Coal, rock and gob properties must subsequently be established consistent with the requirements of a particular numerical method. Models are then constructed to simulate conditions in the areas observed underground. The results of each simulation must be closely examined to ensure that they correlate with observed conditions. If reasonable correlations cannot be made, the model must be recalibrated (material properties adjusted) and the process repeated. It should be noted that relating the output of numerical models (stress, convergence, etc.) to observed conditions is often

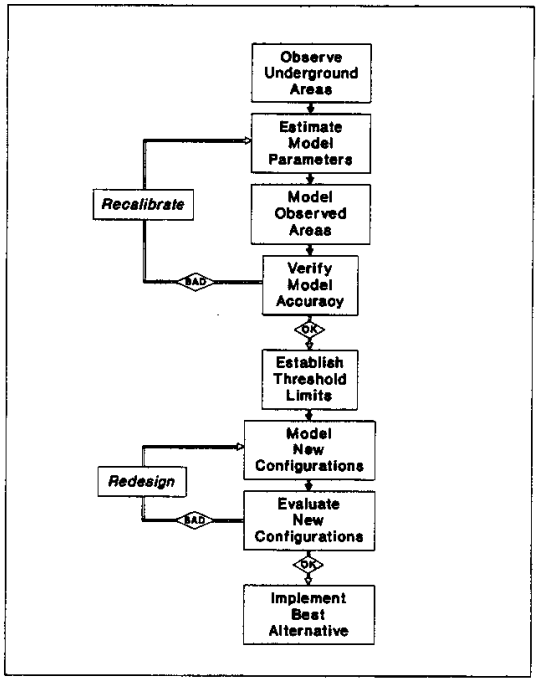


Figure 2. Simulation Process Flow Chart.

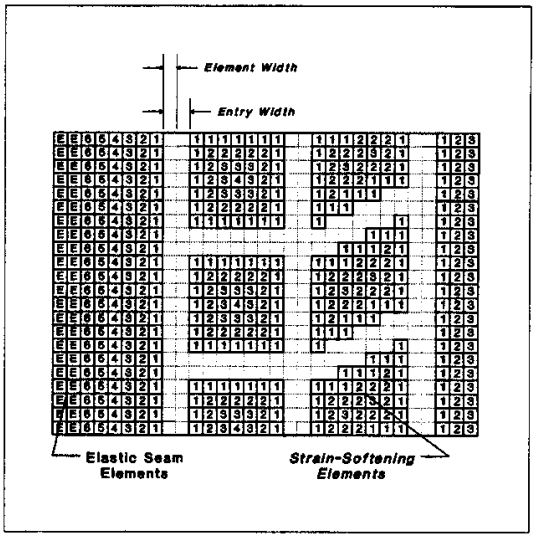


Figure 3. Model Elements and Strain-Softening Locations.

difficult given the complexities of the underground environment. The use of regression techniques to relate model results to actual conditions can simplify that task.

Once the accuracy of the model is verified, threshold limits delineating acceptable and unacceptable mining conditions must be established in order to evaluate the effectiveness of proposed design alternatives. Generally, several alternatives are modeled under the conditions expected at the mine location where the design will be implemented and those configurations are evaluated relative to the threshold limits established. The best alternative is then identified (either meeting the threshold criteria or providing the most favorable conditions) and cautiously implemented. The level of confidence in achieving a successful design is directly proportional to the breadth of the evaluation and the degree of correlation noted in the model verification process.

### Mining Geometry and Initial Stress

An essential element in the simulation process is creating a model grid that duplicates the in-mine geometry. The seam must be broken into elements of a size that allow the entry, crosscut and pillar dimensions to be accurately reproduced. Seam elements must be small enough to model details of the mine geometry and produce discernable differences in performance, yet large enough to allow broad areas of the mine to be included in the simulation.

As a general rule, setting the element size at one half the entry width (Figure 3) has provided acceptable results in most coal mining applications. A 3.0 m element width (for a 6.0 m wide entry/crosscut configuration) enables a large area (540 m x 810 m) to be modeled and yet provides the stress and convergence detail needed to effectively evaluate conditions. Both larger (1 entry width) and smaller (1/4 entry width) element sizes have been used out of necessity in specific applications. However, their use is limited to scenarios where detail (large elements) or influence area (small elements) are not critical.

Initial stress conditions on the rock mass, in the absence of known high horizontal stress fields, have generally been assumed as follows:

**Szz (vertical) = 24.9 kPa per meter of depth**
**Sxx (x-horizontal) = 50% of the vertical stress**
**Syy (y-horizontal) = 50% of the vertical stress**

These values have resulted in effective simulations of in-mine conditions in the vast majority of cases modeled - even on occasions when the influence of horizontal stress was suspected. It has been rare where high horizontal stress was found to actually control mine conditions and high horizontal stress values are only used when clear evidence of their existence and magnitude is available.

### Rock Properties

The rock mass properties needed for Boundary Element models are minimal since the assumption of a linearly elastic material is made. BESOL requires only estimates of the Modulus of Elasticity and Poisson's ratio of the rock. The Roof Control Division uses a weighted average technique (2) to calculate the rock mass Modulus of Elasticity. As many borehole logs as possible located over areas to be modeled are examined and the percentages of the various rock types (i.e., shale, sandstone, coal, etc.) in each core are identified. Those values are averaged, multiplied by the Modulus of Elasticity of each rock type to calculate composite portions and then summed to

estimate the rock mass Modulus of Elasticity. Ideally, individual strata moduli are established by site-specific tests. If that data is not available, then published data for local mine roof strata or typical rock properties must be used. It should be noted that published data for particular rock types vary widely and some judgement is needed in selecting appropriate values.

A similar weighted average process is recommended for the calculation of Poisson's ratio. Again, the use of site-specific data would be ideal, but estimates based on published data are generally used. Poisson's ratio ranging from 0.20 to 0.25 have been acceptable in the analyses made to date.

As noted previously, the BESOL system contains a Failure Index (safety factor) calculation to evaluate the rock strength/stress ratios using either a Mohr-Coulomb or Hoek and Brown failure criteria. To date, only the Mohr-Coulomb technique has been used which requires input of cohesion, friction angle and tensile strength of the rock (roof or floor) material. Since the analysis of the rock structure is completely elastic, exact properties (although desirable) are not required. The Failure Index analysis is treated in a relative manner (higher Failure Indices indicate a more stable condition) and the following parameters, which describe typical coal mine shale roof strata, have provided reasonable results:

**tensile strength = 6.9 MPa**
**cohesion = 5.5 MPa**
**friction angle = 25 degrees**

The Failure Index has been successfully used to indicate high roof and floor stress locations and the effect of mining changes to relieve those stresses. Coupling it with stress and convergence data provides a more complete picture of mine stability that can be correlated to observed conditions.

### Coal Properties

Establishing representative coal properties for a Boundary Element analysis is the most critical step in model formulation. A yielding seam capability is necessary to accurately simulate the complex underground environment where localized coal failure results in the redistribution and concentration of stress in adjacent areas. The strain-softening (3) approach has been identified as a reasonable method of describing coal seam behavior. While that concept has been widely discussed, little specific information is available concerning the actual construction of a strain-softening model.

The Roof Control Division has established a technique (2) to make a first approximation of the stress and strain values needed to describe the strain-softening characteristics of a specific coal seam. As generalized in Figure 4, peak and residual (post peak) stress and strain levels are required for seam elements located at various distances from a mined area. BESOL allows up to six user-defined elements; each characterized by three stress-strain values. Model elements located further than six elements away from a free face are treated as linearly elastic (Figure 3).

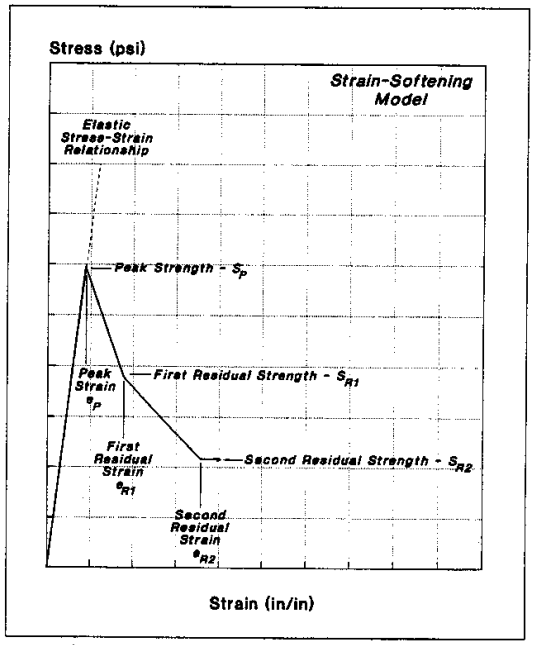


Figure 4. General Strain-Softening Characteristics.

**Peak Coal Strength** values are estimated at the center of each of the six yielding seam elements by the following equation:

$$S_p(i) = S_i * (0.78 + 1.74\ x/h) \quad (1)$$

where:

$S_p(i)$ = Peak strength of element, MPa
$S_i$ = In situ coal strength, MPa
x = Distance from el. center to free face, m
h = Seam height, m

The above equation was based on the derivations of Mark and Iannacchione (4) for estimating the stress gradient in the yield zone of several empirical pillar design formulas, and represents an average of the Bieniawski and Obert-Duvall methods. The in-situ coal strength is usually based on uniaxial compression tests of samples acquired from the mine, although published data has also been used when site-specific data was not available. Strength reduction factors of 1/5 for 5.1 cm cubes and 1/4 for 7.6 cm cubes have been used to estimate in situ strength from test data and have generally provided acceptable results. Since the seam is considered to behave elastically until peak stress is reached, the total strain at that level is simply:

$$e_p(i) = S_p(i) / E \quad (2)$$

where:

$e_p(i)$ = Strain at peak strength of element, m/m
$S_p(i)$ = Peak strength of element, MPa
E = Coal Seam Modulus of Elasticity, MPa

**Residual (post peak) Seam Stress** and strain values are approximated by the following relationships:

$$S_{R1}(i) = (0.1385 * \ln(x) + 0.578)\, S_p(i) \quad (3)$$
$$e_{R1}(i) = 2 * e_p(i) \quad (4)$$
$$S_{R2}(i) = (0.2254 * \ln(x) + 0.268)\, S_p(i) \quad (5)$$
$$e_{R2}(i) = 4 * e_p(i) \quad (6)$$

where:

$S_{R1}(i)$ = First residual stress level, MPa
$e_{R1}(i)$ = Strain at first residual stress level, m/m
$S_{R2}(i)$ = Second residual stress level, MPa
$e_{R2}(i)$ = Strain at second residual stress level, m/m

x = Distance from el. center to free face, m

The BESOL system also requires estimates of the seam Shear Modulus (G) and similar shear stress-strain characteristics for the six yieldable elements described above. This geotechnical data is rarely available and estimates (using the previously described procedure) based on a Shear Modulus equal to 1/2 to 1/3 of the Elastic Modulus have been used.

It must again be emphasized that while the methodology described above has been successfully used to estimate coal strain softening properties, the properties generated are only a first approximation that must be verified for accuracy. Although in-situ measurements have generally validated properties assigned to near excavation locations, peak and residual stress levels deeper than 6.1 m into a pillar or solid coal (where yielding rarely occurs) are largely unverified. Further, the procedure has been applied only to a limited number of coal seams, none of which experienced "bump" problems. The application of this technique to "bump coal" is not recommended as the strength increase due to confinement would likely exceed that predicted by the peak stress equations.

## Gob Properties

When numerical models contain large mined areas such as longwall or pillar line gobs, some mechanism must be employed to simulate caving and stress relief associated with those areas. Without it, the full weight of the overburden would be transferred to adjacent areas and result in a significant overestimation of abutment loads. The actual stress relief process is complex and is comprised of caving, bulking and subsequent compaction of the gob material. While a number of investigators, recently Pappas and Mark (5), have evaluated the behavior of gob material, little published data exists regarding the simulation of caving in three-dimensional Boundary Element numerical models.

The BESOL system provides a fill material that has been used to absorb a portion of the gob loads and provides a measure of stress relief associated with caving. The stress-strain relationship for the fill material is based on the work of Salamon (6) and is of the form:

$$\sigma_n = a * e_n / (b - e_n) \quad (7)$$

where:

$e_n$ = normal strain of the fill element
b = limiting value of normal strain
a = stress to compress fill 1/2 of b

For a first approximation, values for the necessary equation constants have been estimated as:

**a = 0.69 MPa**
**b = 0.50 m/m**

Fill material defined by the above parameters was tested in a number of general scenarios and the resultant model abutment loads compared reasonably well to those predicted by the inverse square decay function used by Mark (7) in the ALPS methodology. Backfill of this type has been placed in gob areas during the BESOL simulation of nine mines that have been successfully evaluated. As with the other material properties discussed in this paper, the suitability of gob backfill based on the above or any other parameters must be verified.

## Deterioration Indices and Analysis

As mentioned previously, the most critical phase of the simulation process is verifying model accuracy through correlation with actual underground conditions. To aid in that exercise, a set of Deterioration Indices (Tables 1-3) were established to quantify pillar, roof and floor behavior. Observed sites are assigned a numerical rating on a scale of 0 - 5 (0 being the best condition and 5 the most severe) in each of the three categories. The Deterioration Index levels are reasonably well defined to minimize subjectivity of observations and promote consistency in ratings from site to site.

**Pillar Deterioration Index (PDI)** establishes observable sloughing levels that can be directly related to numerical model projections (Table 1). A rating of 1.5 would indicate corner crushing for a distance equal to 1-element width (usually 1/2-entry width) in the boundary element model.

Table 1. Pillar Deterioration Index (PDI).

| *Pillar Deterioration Index* | |
|---|---|
| 0 | Virtually No Sloughing |
| 1.0 | Corner Sloughing |
| 2.0 | Light Perimeter Sloughing |
| *2.5* | *Onset of Pillar Stability Concerns* |
| 3.0 | Significant Perimeter Sloughing |
| *3.5* | *Supplemental Support Required* |
| 4.0 | Severe Perimeter Sloughing |
| 5.0 | Complete Pillar Failure |

A rating of 2 indicates some perimeter sloughing, but to a depth less than 1-element width. This would correspond to a model indicating yielding of some, but not all, of the perimeter seam elements. At the 2.5 level, sloughing would be severe enough to cause concern over the stability of the area. A PDI of 3.5 would represent a situation where sloughing caused widening of the entry to a point that supplemental support (cribs or posts) was required to narrow the roadway. A corresponding model would

indicate yielding of all perimeter elements and elevated pillar core stresses observed. A model response equivalent to a level 4 would indicate deeper pillar yielding and core stresses approaching the maximum capacity while a level of 5 would correspond to total pillar yielding and elevated convergence.

**Roof Deterioration Index (RDI)** defines a rating scale to quantify the condition of the roof strata in observed areas (Table 2). Unlike the PDI, however, roof deterioration cannot be directly correlated to model output. The levels were established to correspond to progressively more significant observable phenomena ranging from roof flaking or sloughing (level 1) to widespread and massive roof falls (level 5). The severity of each feature can be identified within a 1-point band. For instance, areas with only a hint of roof cutters would be rated at 1.6 while those containing many severe cutters (a situation causing roof stability concerns) would receive a 2.5 rating. A roof deterioration index of 3.5 would correspond to conditions where supplemental support was required to maintain stability.

Table 2. Roof Deterioration Index (RDI).

| *Roof Deterioration Index* | |
|---|---|
| 0 | Virtually No Deterioration |
| 1.0 | Flaking or Spalling |
| 2.0 | Cutter Roof |
| *2.5* | *Onset of Roof Stability Concerns* |
| 3.0 | Broken Roof |
| *3.5* | *Supplemental Support Required* |
| 4.0 | Significant Roof Falls |
| 5.0 | Widespread & Massive Roof Falls |

**Floor Deterioration Index (FDI)** provides a measure of mine floor stability relative to fracturing and the level of heave experienced (Table 3). Like the RDI, this index cannot be directly correlated to the model output, and the established levels represent progressively more serious floor conditions. An FDI of 2.5 was set to represent the occurrence of heave which causes concern over floor stability while a level of 3.5 relates to a condition that impedes passage and would require grading to maintain an active travelway.

Table 3. Floor Deterioration Index (FDI).

| *Floor Deterioration Index* | |
|---|---|
| 0 | Virtually No Deterioration |
| 1.0 | Sporadic Cracks |
| 2.0 | Consistent Localized Cracks |
| *2.5* | *Onset of Floor Stability Concerns* |
| 3.0 | Widespread Cracks & Obvious Heave |
| *3.5* | *Travel Impeded - Grading Required* |
| 4.0 | Significant Floor Displacement |
| 5.0 | Complete Entry Closure |

The Deterioration Indices have been effectively used to describe in-mine ground conditions. While simulation output such as stress and convergence can often be directly related to observed conditions, many instances arise where the combined influence of a number of factors affects ground behavior. To better establish those relationships and provide an effective means of evaluating potential design alternatives, a multiple linear regression can be used to relate model output to observed (Deterioration Index) conditions. Equations are generated of the form:

$$DI = f(\text{Stress, Convergence, F.I.}) \quad (8)$$

and corresponding correlation coefficients are calculated. Once the model accuracy is verified by comparing predicted to observed pillar yielding, examining the regression correlation coefficients and using the regression equations to back calculate Deterioration Indices for the observed (modeled) areas, design alternatives can be modeled and expected conditions predicted.

The Deterioration Index - Regression Equation technique has proved to be a viable method of verifying numerical model accuracy and evaluating the potential of design alternatives provided relatively consistent mining conditions exist. When changing roof, pillar or floor strengths are encountered, the useability of the regression technique can be greatly reduced. Further, the relationships established are based on strata reaction at a particular mine, and only those observed (which are limited by current mine design and environment) can be included in the data base. This is a particular concern when the use of yield pillars as an alternative configuration is considered, but no complete pillar yielding is evident at the mine.

The Roof Control Division is currently exploring the use of normalizing parameters in the regression analysis to alleviate those difficulties. Factors such as in-situ coal strength and seam height (for the PDI), a roof rock rating such as the CMRR (8) for the RDI and a floor characterization number (for the FDI) are being evaluated to determine their usefulness in the regression analysis to buffer the variations found within a given mine and also between mines. If successful, the resultant technique could enhance individual mine analyses and allow the experience of many mines to be utilized.

## Case Study: Mine A

An investigation was made at an Eastern Kentucky coal mine to determine the cause of a roof fall and deteriorating ground conditions encountered on a full pillaring section. The mine is located in the Hazard No. 4 seam with a mining height of 0.8 - 1.0 meters. Figure 5 presents an illustration of a portion of the 1 Left Mains that was developed as a 5-entry system on 15.2 m x 18.3 m centers with 6.1 m wide entries and crosscuts. Panels were driven to the right and retreated as the Mains were advanced (13 panels in all). Following development of the Mains (and panels) to the property boundary, retreating of those pillars was initiated. As Figure 5 illustrates, a roof fall occurred one crosscut outby the pillar line as the 18th row of blocks was being extracted. Cover at the face was about 245 m but ranged from 145 m near the mouth of the section (some 730 m outby) to over 290 m roughly 100 meters inby and to the right of the fall. The immediate roof strata consisted of a laminated shale, 4.6 m thick, and was over-lain by a

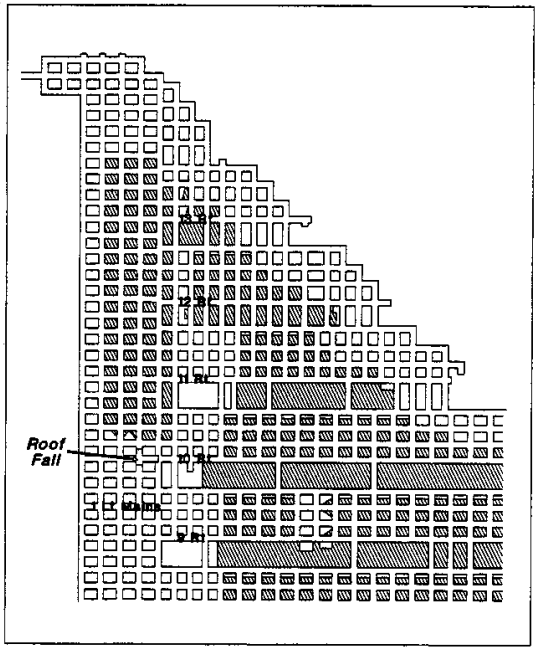


Figure 5. Mine A: Partial Mine Map of Pillaring Section - Roof Fall Area.

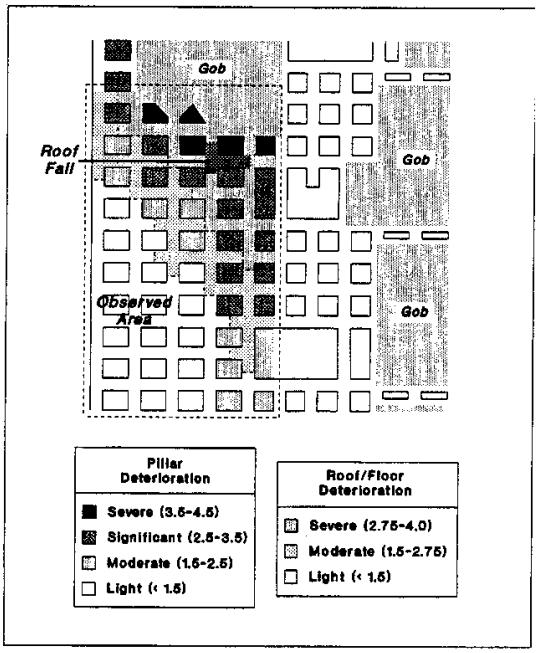


Figure 6. Mine A: Observations on Pillaring Section - Roof Fall Area.

6.1 m thick sandstone layer. Various shale and often significant sandstone members also comprised the majority of the remaining overburden to the surface. Roof support was provided by 1.2 m long fully grouted bolts installed on a 1.2 m x 1.2 m pattern throughout the Mains.

Visual observations were made throughout the 1 Left Mains to characterize ground conditions under various depths of cover and degrees of gob influence. Significant deterioration (heavy pillar sloughing, cutters and broken roof zones) was noted in the face area and conditions were most severe in the immediate vicinity of the roof fall. Outby the face, conditions gradually improved although the right side of the Mains consistently showed heavier deterioration than the left side. The most significant conditions noted in the outby area corresponded to zones of heavier cover, suggesting that overburden depth and the adjacent gobbed areas contributed to the deteriorating conditions. Detailed Deterioration Index ratings were made throughout the observed areas to quantify the roof, floor and pillar behavior. Higher PDI, RDI and FDI levels corresponded to more severe deterioration, which was observed in the face area and along the right side of the Mains. Figure 6 presents a composite Deterioration Index drawing of conditions observed at and just outby the face, illustrating the concentration of deterioration in the vicinity of the roof fall and along the right side of the section.

Three BESOL models were subsequently created to simulate conditions in the areas observed during the underground investigation. The first model (covering the area shown in Figure 5) was used to simulate mining at the time of the roof fall, and also at inby and proposed outby face positions where cover was approximately 245 m. Additional models were constructed of the outby areas (3-Right - 200 m cover and 1-Right - 145 m cover) to provide model verification under significantly differing conditions. Table 4 summarizes the various stress, geometric and

Table 4. Mine A: BESOL Input Data.

**General Input Data**

Seam Height ... 0.8 m
Mining Height ... 1.0 m
Mining Depth ... 120 m - 275 m
Entry Width ... 6.1 m
Horizontal stress ... 1/2 vertical
Seam element size ... 3.0 m

**Material Properties**

Rock elastic modulus ... 8687 MPa
Rock poissons ratio ... 0.21
Rock tensile strength ... 6.9 MPa
Rock cohesion ... 5.5 MPa
Rock friction angle ... 25 deg
Coal elastic modulus ... 3447 MPa
Coal shear modulus ... 1379 MPa

Coal strength:
0 m - 3.0 m ... 24.5 MPa peak; 8.5 MPa residual
3.0 m - 6.1 m ... 66.8 MPa peak; 33.4 MPa residual
6.1 m - 9.1 m ... 108.2 MPa peak; 72.5 MPa residual
9.1 m - 12.2 m ... 149.6 MPa peak; 119.7 MPa residual
12.2 m - 15.2 m ... 191.0 MPa peak; 171.9 MPa residual
15.2 m - 18.3 m... 232.4 MPa peak; 232.4 MPa residual
> 18.3 m ... elastic

Backfill Properties:
*(Gob)* a ... 0.69 MPa
b ... 0.50 of seam height

material input data used in the models. Rock properties were estimated through the weighted average techniques from borehole logs in the vicinity and published rock strength data. Coal properties were based on an in-situ strength of 6.7 MPa provided by the mine and the strain-softening parameters were calculated using the methodology described previously. Gob caving was simulated using the Salamon backfill with constants of a = 0.69 MPa and b = 0.50.

Maximum Stress, maximum Convergence and minimum Failure Index values were determined from the three models for 37 locations (entries and crosscuts) corresponding to the observed areas. A portion of that data (along with observed conditions) is listed in Table 5. A series of multiple linear regression analyses were made to relate the Deterioration Indices observed to the BESOL data and resulted in the equations also listed in Table 5. The R-squared values for the PDI (0.79) and the RDI (0.80) were very good, but marginal for the FDI (0.60). It should be noted that the characterization of floor conditions was not a primary concern during the investigation, but sketchy data acquired was used to illustrate the process. The BESOL output was then input into the regression equations to predict (back-calculate) Deterioration Indices for the observed locations, and values describing entry conditions are also listed in Table 5. Most of the predicted PDI and RDI levels match the observed data fairly well, and the trend of higher Deterioration Indices in areas of more severe conditions was evident, even with the FDI.

Table 5. Mine A: Partial BESOL-Deterioration Index Listing and Regression Equation.

| Location | Entry | Max. Stress (MPa) | Max. Conv. (m) | Min. F.I. | Observed | | | Calculated | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | PDI | RDI | FDI | PDI | RDI | FDI |
| Face Area | 1 | 27.6 | .034 | 1.04 | 1.5 | 1.5 | 0.0 | 1.5 | 1.2 | 0.2 |
| | 2 | 46.9 | .059 | 1.09 | 2.0 | 1.8 | 0.3 | 2.5 | 2.4 | 1.2 |
| | 3 | 55.8 | .077 | 0.96 | 3.5 | 3.0 | 1.0 | 3.0 | 2.9 | 1.7 |
| | 4 | 60.7 | .088 | 0.89 | 4.0 | 4.2 | 2.5 | 3.3 | 3.3 | 2.0 |
| | 5 | 60.7 | .094 | 0.87 | 4.0 | 3.5 | 4.0 | 3.3 | 3.4 | 2.1 |
| 1 X-Cut Outby | 1 | 21.4 | .025 | 1.11 | 1.2 | 1.5 | 0.0 | 1.1 | 0.9 | 0.0 |
| | 2 | 37.2 | .049 | 1.16 | 1.5 | 1.5 | 0.0 | 2.0 | 1.9 | 0.8 |
| | 3 | 48.3 | .063 | 1.11 | 2.0 | 2.0 | 0.5 | 2.6 | 2.5 | 1.3 |
| | 4 | 51.7 | .070 | 1.02 | 3.0 | 3.0 | 1.0 | 2.8 | 2.7 | 1.5 |
| | 5 | 51.7 | .068 | 0.94 | 3.0 | 3.0 | 3.0 | 2.7 | 2.6 | 1.4 |
| 3 X-Cuts Outby | 1 | 18.7 | .019 | 1.25 | 1.0 | 0.5 | 0.0 | 1.0 | 0.7 | 0.0 |
| | 2 | 26.9 | .027 | 0.93 | 1.5 | 0.8 | 0.0 | 1.3 | 1.1 | 0.0 |
| | 3 | 41.4 | .046 | 1.16 | 1.5 | 1.5 | 0.2 | 2.2 | 2.0 | 0.9 |
| | 4 | 46.3 | .055 | 1.13 | 2.0 | 2.0 | 1.0 | 2.5 | 2.4 | 1.2 |
| | 5 | 50.3 | .062 | 1.21 | 3.0 | 2.5 | 2.0 | 2.7 | 2.6 | 1.4 |
| 3-Right | 2 | 15.4 | .018 | 1.53 | 1.0 | 0.5 | 0.0 | 1.0 | 0.7 | 0.0 |
| | 4 | 17.7 | .021 | 1.41 | 1.4 | 1.0 | 0.0 | 1.1 | 0.8 | 0.0 |
| | 5 | 19.4 | .022 | 1.45 | 1.5 | 1.4 | 0.1 | 1.2 | 0.9 | 0.0 |
| 1-Right | 2 | 10.5 | .012 | 2.13 | 1.0 | 0.2 | 0.0 | 1.0 | 0.6 | 0.0 |
| | 4 | 11.7 | .014 | 1.91 | 1.0 | 1.0 | 0.0 | 1.0 | 0.6 | 0.0 |
| | 5 | 12.3 | .014 | 2.00 | 1.0 | 1.0 | 0.0 | 1.0 | 0.7 | 0.0 |

*PDI = 0.0389•STR + 10.696•CONV + 0.380•F.I. - 0.384* $r^2$ = 0.79
*RDI = 0.0381•STR + 15.105•CONV + 0.309•F.I. - 0.644* $r^2$ = 0.80
*FDI = 0.0247•STR + 19.993•CONV + 0.600•F.I. - 1.824* $r^2$ = 0.60

Figure 7 presents a composite of maximum pillar stress and convergence levels predicted by the BESOL model of the roof fall site. Note the correlation of BESOL stress and convergence with the degree of deterioration observed underground (Figure 6). The zone of high convergence (> 0.076 m) and stress (> 65.5 MPa) encompasses the area of deteriorating conditions at the pillar line, including the roof fall. Lower stress and convergence levels correspond to

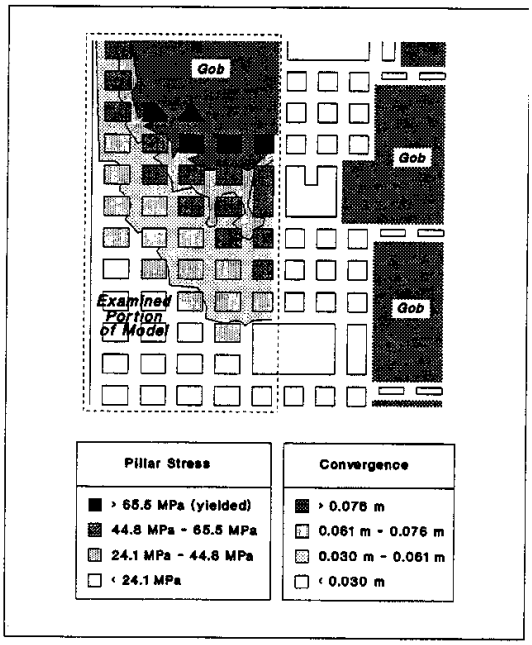


Figure 7. Mine A: BESOL Output for Pillaring Section - Roof Fall Area.

zones of lesser deterioration and the more severe conditions predicted on the right side of the mains (indicating the influence of the adjacent gob) also match the conditions observed underground. These correlations, coupled with the good fit of the regression analysis (Deterioration Indices) confirmed the accuracy of the model (and properties used) to simulate conditions at the mine. Confidence was further enhanced by an evaluation of the BESOL model with a face position several crosscuts inby the fall. The results showed significantly lower stress and convergence levels in the face area that correlated well with the improved mining conditions actually encountered.

It was concluded that the roof fall (and deteriorating conditions) encountered resulted from a combination of stresses from the active and adjacent gobs overriding the pillar line (yielding) and focusing outby the face. The small pillar size employed (9.1 m x 12.2 m) on the Mains, the lack of protection provided by the combination of chain and barrier pillars from the adjacent gob, and the depth of cover (over 245 m) contributed to the problems encountered.

Several additional models were created to evaluate the performance of various pillar sizes at different mining depths that would be encountered. Figure 8 illustrates the pillaring plan to be implemented using a 61 m barrier between adjacent panels that would be roomed and retreated along with the panel being extracted. Stresses and convergences were examined at four entry locations near the face (during retreat of the second panel), as illustrated in Figure 9. Threshold levels delineating expected conditions (from the 1 Left models) were established as follows:

**Severe Conditions:**

Stress > 55.2 MPa; Convergence > 0.076 m
PDI > 3.5; RDI > 3.5; FDI > 3.5

**Borderline Conditions:**

Stress > 44.8 MPa; Convergence > 0.055 m
PDI > 2.5; RDI > 2.5; FDI > 2.5

**Desirable Conditions:**

Stress < 44.8 MPa; Convergence < 0.055 m
PDI < 2.5; RDI < 2.5; FDI < 2.5

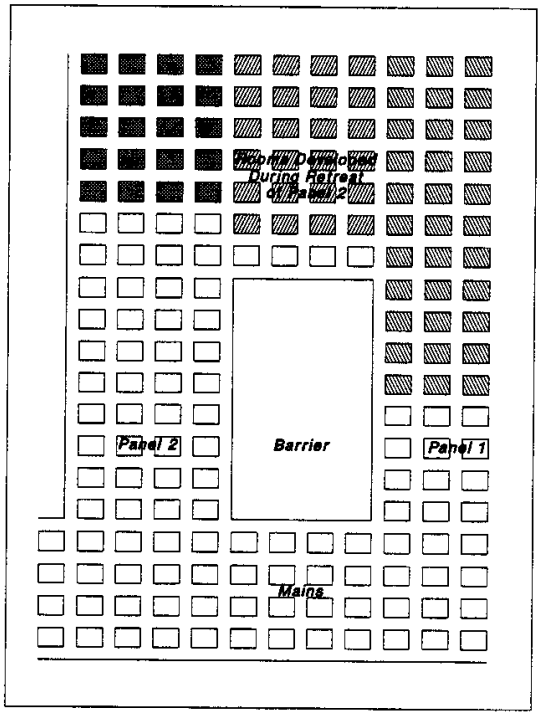


Figure 8. Mine A: Proposed Full Pillaring Plan.

It was predetermined that good (desirable) mining conditions should exist at locations 3 and 4 (Figure 9) since no supplemental supports (posts) would be installed in those areas. Borderline conditions could be tolerated at locations 1 and 2 (posts are set in this area), but the occurrence of severe conditions should be avoided or at least limited to location 1.

Table 6 presents the BESOL and predicted Deterioration Index data for each of the four locations for a number of scenarios. Values indicating severe conditions are shown in bold face type, those suggesting borderline conditions are presented in italics and those listed in standard type correspond to desirable mining conditions. The analysis indicated that the use of 12.2 m x 9.1 m. pillars would result in good conditions through a depth of 120 m and that

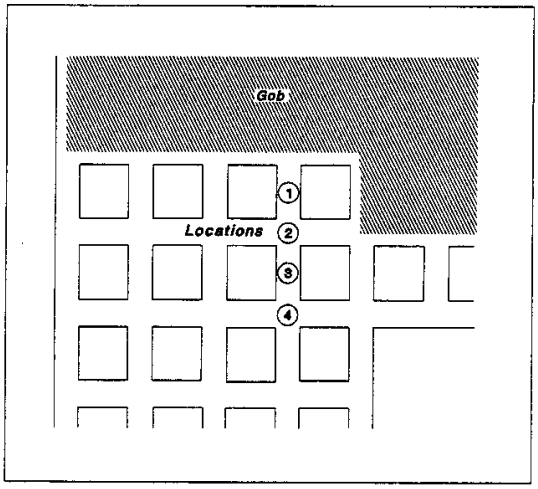


Figure 9. Mine A: Full Pillaring Analysis Locations.

12.2 m x 12.2 m pillars would be effective up to 185 m of cover. Pillars 15.2 m x 15.2 m in size would be needed for deeper cover areas although severe conditions could still occur at locations 1 and 2 as the depth approaches 275 m.

Table 6. Mine A: Full Pillaring BESOL Output and Predicted Deterioration Indices.

| Pillar Size (mxm) | Depth (m) | Location | Max. Stress (MPa) | Max. Conv. (m) | PDI | RDI | FDI |
|---|---|---|---|---|---|---|---|
| 15.2x15.2 | 275 | 1 | **57.2** | **0.089** | *3.0* | *3.1* | 1.7 |
| | | 2 | **56.5** | *0.076* | *3.1* | *3.1* | 1.9 |
| | | 3 | 40.7 | *0.056* | 2.1 | 2.0 | 0.8 |
| | | 4 | 38.6 | 0.049 | 2.2 | 2.0 | 1.0 |
| 12.2x12.2 | 275 | 1 | **66.8** | **0.117** | **3.8** | **4.0** | *2.7* |
| | | 2 | **66.8** | **0.105** | **3.8** | **3.9** | *2.6* |
| | | 3 | **60.0** | *0.075* | *3.0* | *3.0* | 1.6 |
| | | 4 | **57.2** | *0.070* | *3.1* | *3.0* | 1.7 |
| 12.2x12.2 | 245 | 1 | **66.8** | **0.093** | **3.5** | **3.6** | 2.2 |
| | | 2 | **66.8** | **0.082** | **3.6** | **3.5** | 2.2 |
| | | 3 | *46.9* | *0.060* | 2.4 | 2.3 | 1.0 |
| | | 4 | *45.5* | 0.055 | *2.5* | 2.4 | 1.3 |
| 12.2x12.2 | 185 | 1 | *50.3* | *0.062* | *2.6* | *2.5* | 1.2 |
| | | 2 | *49.3* | 0.052 | *2.7* | *2.5* | 1.4 |
| | | 3 | 24.1 | 0.029 | 1.2 | 1.0 | 0.0 |
| | | 4 | 23.4 | 0.027 | 1.3 | 1.0 | 0.1 |
| 12.2x9.1 | 120 | 1 | 30.3 | 0.035 | 1.5 | 1.3 | 0.2 |
| | | 2 | 29.0 | 0.030 | 1.4 | 1.2 | 0.1 |
| | | 3 | 18.3 | 0.019 | 1.0 | 0.7 | 0.0 |
| | | 4 | 16.0 | 0.016 | 1.1 | 0.8 | 0.0 |

**Case Study: Mine B**

Mine B is located in Southwest Pennsylvania and is predominantly a longwall operation. An investigation of conditions at the mine was prompted by the occurrence of a number of roof falls in tailgate entries, generally in intersections, just outby the longwall face. Figure 10 illustrates a typical longwall panel and the general location of tailgate problems. The mine is extracting the Pittsburgh Seam (1.7 m thick) and average mining height is about 2.1 m. Four entry gate roads are developed 4.7 m wide by miners with integral bolters. Entry centers are 15.2 m for the outside "yield" pillars and 45.7 m for the large abutment pillar. Crosscuts are turned at 60-degree angles on 42.7 m

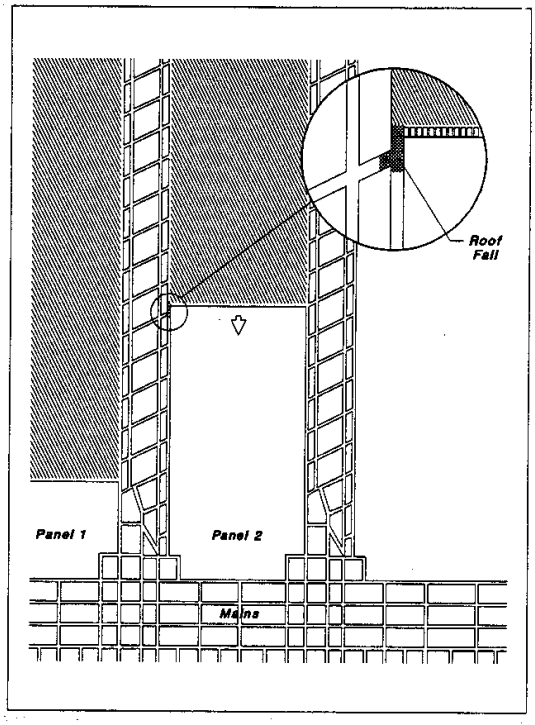


Figure 10. Mine B: Typical Longwall Panel and Tailgate Roof Fall.

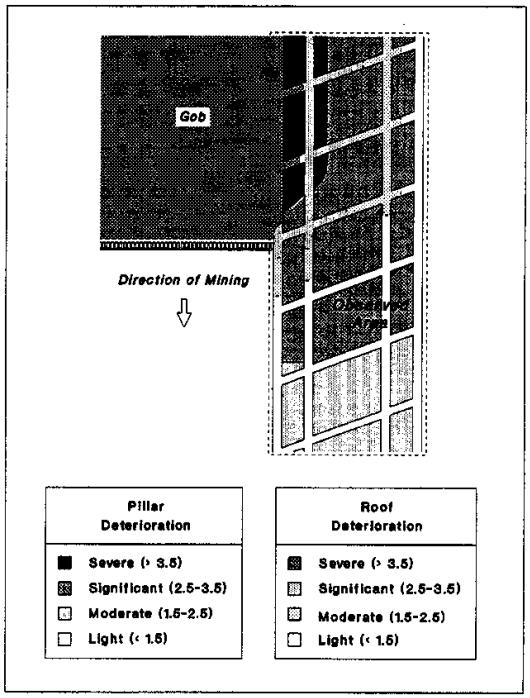


Figure 11. Mine B: Observations on Longwall Headgate at a Depth of 290 m.

to 48.8 m centers. Cover over the mine varies from less than 150 m in stream valleys to over 300 m under the ridges. The immediate roof consists of a dark shale which varies from 0.3 m to 3.0 m in thickness. The Pittsburgh sandstone overlies the shale and also varies in thickness from 1.5 m to 20 m. The remainder of the overburden is comprised of sequences of limestone, shales, sandy shales, coal and massive sandstones. Roof support in the longwall gate roads consists of channels secured to the roof with three 2.4 m mechanically anchored, resin-assisted bolts on 1.2 m centers. Each intersection is supplemented with 9 to 12 super bolts, 3.7 m in length. The tailgate of each panel contains two rows of cribs (0.76 m apart spaced at 1.2 m intervals) throughout, and two extra cribs across each intersection.

Visual observations were made of a number of areas throughout the mine and representative deterioration indices were assigned to each location. Conditions were examined on development sections, main entries, and both head- and tailgate entries of active longwall panels under depths of cover ranging from 150 m to 290 m. Only pillar and roof deterioration were noted as floor stability was not an issue. In general, more severe conditions were observed in areas of deeper cover on development sections, and, as expected, near the face in both the head- and tailgate entries. Significant cutters, broken roof and heavy pillar sloughing were noted in the more extreme areas while flaking, light cutting and moderate pillar sloughing were evident in most other areas. Figure 11 presents a composite Deterioration Index drawing of conditions observed on the headgate of one active longwall panel with about 290 m of cover. Caving occurred just behind the shields in the No. 1 entry and propagated through the crosscuts to the No. 2 entry rib line. Continually improving conditions were noted ahead and to the solid side of the gob.

A total of nine BESOL models were created to simulate mining in the areas observed underground. The models included development sections and active longwall panels under various depths of cover. Table 7 summarizes the stress, geometric and material data used in the analyses. As in the previous case study, rock properties were estimated from borehole logs and published strength data. Coal strength data was based on an in-situ strength of 5.5 MPa obtained from several literature sources. Gob caving was again simulated using the Salamon backfill with constants a = 0.69 MPa and b = 0.50.

Table 8 lists the maximum stress, maximum convergence and minimum Failure Index data for a portion of the observed areas modeled. The regression equations relating the observed PDI and RDI values (also listed) to the BESOL output and correlation coefficients are also shown. Again, the R-squared values (0.69 and 0.70) are reasonable. The calculated PDI and RDI values compare well to the observed data, and a trend of Higher Deterioration Indices in more severe ground conditions is apparent. Figure 12 presents a composite drawing of the BESOL output (Stress and Convergence) for one longwall headgate observed underground (Figure 11). Comparing the two illustrations shows a definite correlation. Severe conditions (caving)

Table 7. Mine B: BESOL Input Data.

**General Input Data**

Seam Height ... 1.7 m
Mining Height ... 2.1 m
Mining Depth ... 150 m - 290 m
Entry Width ... 4.6 m
Horizontal stress ... 1/2 vertical
Seam element size ... 2.3 m

**Material Properties**

Rock elastic modulus ... 13583 MPa - 15996 MPa
Rock poissons ratio ... 0.21
Rock tensile strength ... 6.9 MPa
Rock cohesion ... 11.2 MPa
Rock friction angle ... 26 deg
Coal elastic modulus ... 2413 MPa
Coal shear modulus ... 965 MPa

Coal strength:
0 m - 2.3 m ... 10.3 MPa peak; 3.4 MPa residual
2.3 m - 4.6 m ... 23.3 MPa peak; 11.6 MPa residual
4.6 m - 6.9 m ... 36.2 MPa peak; 23.5 MPa residual
6.9 m - 9.1 m ... 49.1 MPa peak; 36.8 MPa residual
9.1 m - 11.4 m ... 62.0 MPa peak; 54.6 MPa residual
11.4 m - 13.7 m ... 74.9 MPa peak; 74.9 MPa residual
› 13.7 m ... elastic

Backfill Properties:
(Gob) a ... 0.69 MPa
b ... 0.50 of seam height

Table 8. Mine B: Partial BESOL-Deterioration Index Listing and Regression Equations.

| Location | Entry | Depth (m) | Max. Stress (MPa) | Max. Conv. (m) | Min. F.I. | Observed | | Calculated | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | PDI | RDI | PDI | RDI |
| Dev 1 | Outside | 168 | 6.6 | .005 | 2.66 | 1.6 | 1.2 | 1.9 | 1.5 |
| Dev 1 | Inside | 168 | 7.1 | .006 | 2.50 | 1.2 | 1.2 | 2.0 | 1.6 |
| Dev 2 | Outside | 230 | 9.5 | .007 | 1.81 | 2.6 | 1.8 | 2.6 | 1.9 |
| Dev 2 | Inside | 230 | 9.9 | .008 | 1.92 | 2.4 | 1.9 | 2.6 | 1.9 |
| Dev 3 | Outside | 230 | 9.5 | .007 | 1.81 | 2.6 | 1.5 | 2.6 | 1.9 |
| Dev 3 | Inside | 230 | 9.9 | .008 | 1.92 | 2.6 | 1.5 | 2.6 | 1.9 |
| Dev 4 | Outside | 230 | 9.3 | .008 | 1.76 | 2.0 | 2.5 | 2.6 | 2.0 |
| Dev 4 | Inside | 230 | 9.3 | .007 | 2.01 | 2.0 | 1.7 | 2.5 | 1.8 |
| Dev 5 | Outside | 245 | 10.2 | .009 | 1.72 | 2.9 | 2.3 | 2.7 | 2.0 |
| Dev 5 | Inside | 245 | 10.1 | .009 | 1.65 | 3.0 | 2.6 | 2.7 | 2.1 |
| Dev 6 | Outside | 290 | 11.2 | .011 | 1.51 | 3.0 | 1.8 | 2.9 | 2.2 |
| Dev 6 | Inside | 290 | 11.8 | .013 | 1.55 | 3.0 | 1.9 | 2.9 | 2.3 |
| Dev 7 | Outside | 290 | 11.2 | .011 | 1.51 | 3.2 | 2.6 | 2.9 | 2.2 |
| Dev 7 | Inside | 290 | 11.8 | .013 | 1.55 | 3.1 | 2.8 | 2.9 | 2.3 |
| Head 1 | 2-Face | 290 | 17.9 | .020 | 1.29 | 3.0 | 2.0 | 3.5 | 2.7 |
| Head 1 | 3-Face | 290 | 15.2 | .017 | 1.50 | 3.0 | 1.8 | 3.2 | 2.5 |
| Head 1 | 4-Face | 290 | 14.6 | .015 | 1.50 | 2.8 | 2.8 | 3.1 | 2.4 |
| Head 1 | 2-Inby | 290 | 23.1 | .029 | 1.24 | 3.5 | 2.9 | 3.9 | 3.1 |
| Head 2 | 1-Face | 290 | 19.3 | .029 | 0.97 | 3.5 | 2.8 | 3.7 | 3.3 |
| Head 2 | 2-Face | 290 | 17.9 | .020 | 1.29 | 2.9 | 2.8 | 3.5 | 2.7 |
| Head 2 | 3-Face | 290 | 15.2 | .017 | 1.50 | 2.9 | 2.0 | 3.2 | 2.5 |
| Head 2 | 4-Face | 290 | 14.6 | .015 | 1.50 | 3.0 | 1.8 | 3.1 | 2.4 |
| Head 2 | 2-Inby | 290 | 23.1 | .029 | 1.24 | 4.5 | 3.5 | 3.9 | 3.1 |

*PDI = 0.0828•STR - 8.238•CONV - 0.581•F.I. + 2.915* $r^2$= 0.69
*RDI = -0.0409•STR + 68.041•CONV - 0.386•F.I. + 2.508* $r^2$= 0.70

correspond to zones of high stress (> 27.6 MPa) and convergence (> 0.0.43 m) while moderate to significant deterioration was observed where stress exceeded 13.8 MPa and convergence was greater than 0.015 m. Examination of the tailgate side of the model showed that a 0.034 m convergence contour encompassed the potential roof fall zone that extended into the first outby crosscut.

It was concluded that the BESOL model provided a reasonable simulation of mine conditions, and the following threshold levels describing expected conditions were established:

**Severe Conditions:**

Stress > 27.6 MPa; Convergence > 0.043 m
PDI > 4.0; RDI > 4.0

**Borderline Conditions:**

Stress > 20.7 MPa; Convergence > 0.034 m
PDI > 3.5; RDI > 3.5

**Moderate Conditions:**

Stress > 13.8 MPa; Convergence > 0.015 m
PDI > 2.5; RDI > 2.5

**Good Conditions:**

Stress < 13.8 MPa; Convergence < 0.015 m
PDI < 2.5; RDI < 2.5

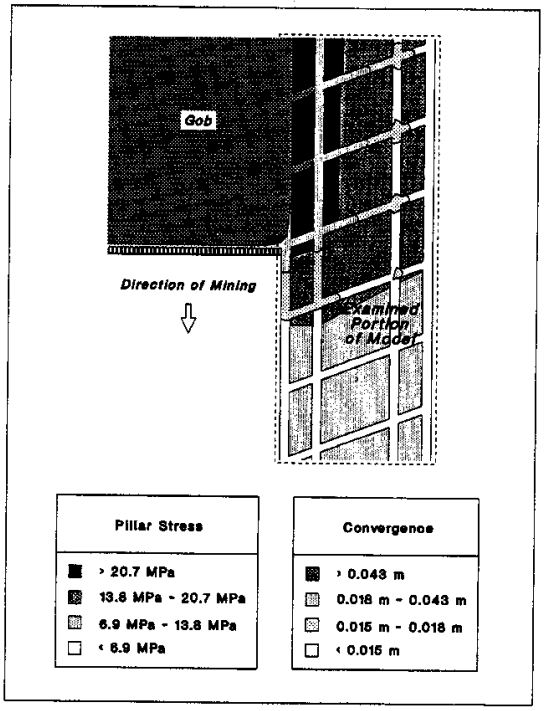


Figure 12. Mine B: BESOL Output for Longwall Headgate at Depth of 290 m.

Ten additional models were created to evaluate the effect of gate road configurations on head- and tailgate stability at a depth of 260 m. A variety of pillar sizes (ranging from 2.1 m to 50 m) and crosscut orientations (60 degrees and 90 degrees) were analyzed just outby the longwall face as shown in Figure 13. Table 9 summarizes the results at a position about 3 m outby the face.

In general, it was concluded that the current gate road configuration (15.2 m - 45.7 m - 15.2 m centers; 60-degree

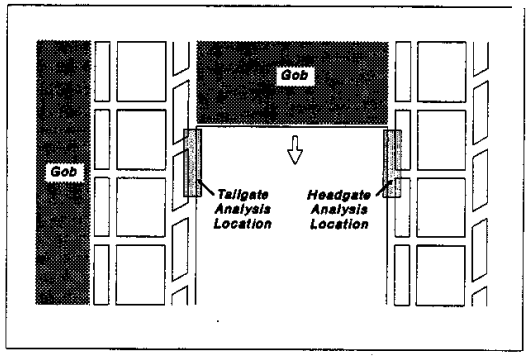


Figure 13. Mine B: Gateroad Analysis Locations.

Table 9. Mine B: Gateroad BESOL Output and Predicted Deterioration Indices at a Depth of 260 m.

| Gateroad Configuration Data | | | | | | Location | Max. Stress (MPa) | Max. Conv. (m) | PDI | RDI |
|---|---|---|---|---|---|---|---|---|---|---|
| Pillar Width (m) | | | X-cut Orientation (deg) | | | | | | | |
| Head | Abut | Tail | Head | Abut | Tail | | | | | |
| 11.2 | 41.1 | 11.2 | 60 | 60 | 60 | Head | 21.2 | .023 | 3.9 | 2.8 |
| 11.2 | 41.1 | 11.2 | 90 | 90 | 90 | Head | 20.6 | .023 | 3.8 | 2.8 |
| 11.2 | 41.1 | 11.2 | 90 | 90 | 60 | Head | 20.7 | .023 | 3.9 | 2.8 |
| 13.7 | 36.6 | 13.7 | 90 | 90 | 60 | Head | 20.3 | .022 | 3.8 | 2.8 |
| 11.2 | 36.6 | 15.8 | 90 | 90 | 60 | Head | 20.7 | .023 | 3.9 | 2.8 |
| 11.2 | 45.7 | 6.7 | 90 | 90 | 60 | Head | 20.6 | .023 | 3.8 | 2.8 |
| 11.2 | 47.9 | 4.6 | 90 | 90 | 60 | Head | 20.7 | .023 | 3.9 | 2.8 |
| 11.2 | 50.3 | 2.1 | 90 | 90 | 60 | Head | 20.6 | .023 | 3.8 | 2.8 |
| 6.7 | 50.3 | 6.7 | 90 | 90 | 60 | Head | 22.5 | .025 | 4.0 | 2.9 |
| 25.0 | 27.4 | 11.2 | 90 | 90 | 60 | Head | 19.8 | .021 | 3.8 | 2.7 |
| 11.2 | 41.1 | 11.2 | 60 | 60 | 60 | Tail | 23.4 | .034 | 4.0 | 3.5 |
| 11.2 | 41.1 | 11.2 | 90 | 90 | 90 | Tail | 23.2 | .030 | 4.0 | 3.2 |
| 11.2 | 41.1 | 11.2 | 90 | 90 | 60 | Tail | 23.2 | .032 | 4.0 | 3.4 |
| 13.7 | 36.6 | 13.7 | 90 | 90 | 60 | Tail | 23.3 | .030 | 4.0 | 3.3 |
| 11.2 | 36.6 | 15.8 | 90 | 90 | 60 | Tail | 22.8 | .030 | 4.0 | 3.3 |
| 11.2 | 45.7 | 6.7 | 90 | 90 | 60 | Tail | 23.2 | .036 | 4.0 | 3.6 |
| 11.2 | 47.9 | 4.6 | 90 | 90 | 60 | Tail | 23.2 | .038 | 3.9 | 3.6 |
| 11.2 | 50.3 | 2.1 | 90 | 90 | 60 | Tail | 23.2 | .036 | 3.9 | 3.6 |
| 6.7 | 50.3 | 6.7 | 90 | 90 | 60 | Tail | 23.2 | .037 | 4.0 | 3.7 |
| 25.0 | 27.4 | 11.2 | 90 | 90 | 60 | Tail | 23.3 | .031 | 4.0 | 3.3 |

crosscuts) provided a reasonable level of stability. Increasing the size of the head- and tailgate pillars, however, would result in a slight improvement in conditions as stress, convergence and PDI on the headgate would be lowered and both the RDI and convergence on the tailgate would be reduced to the moderate range. Additional benefits could be attained, especially on the tailgate side, by turning crosscuts at 90-degree angles rather than 60 degrees.

The evaluation also showed that the 10.7 m wide "yield" pillars were acting as stable or conventional pillars, and not yielding until they were well into the gob. The models suggested that a pillar roughly 2.3 m in size would be required to achieve stress relief at the tailgate corner. The mine subsequently increased head- and tailgate pillars to 13.7 m in width (with a 36.6 m wide abutment pillar) and turned the headgate and abutment pillar crosscuts on 90-degree angles. Because of operational constraints associated with the full-face miner, it was not possible to change the 60-degree tailgate crosscut orientation. It was, therefore, recommended that additional supports be strategically located in the tailgate intersection crosscuts to better establish an "artificial rib" and limit the resultant exposed roof span.

## Conclusion

Boundary Element modeling has proven to be an effective tool that can be used by mining engineers to resolve complex ground control problems. The techniques set forth in this paper describing coal, rock and gob behavior have been effectively used to evaluate a variety of mining scenarios. While they are supported by a number of in situ measurements and have resulted in near duplication of underground conditions in many instances, they provide only a first estimate of parameters that must be validated. Successful numerical simulation requires a substantial effort including the observation of conditions in many areas and the often repetitive process of calibrating model parameters. The use of techniques such as the Deterioration Index - Regression method has greatly facilitated the linking of observed and simulated mine conditions. It can not be over-emphasized, however, that in order to be of any value, a numerical model must be validated and provide a realistic representation of the underground environment for which it is applied.

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