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NADCON

THE APPLICATION OF MINIMUM-CURVATURE-DERIVED SURFACES IN THE TRANSFORMATION OF POSITIONAL DATA FROM THE NORTH AMERICAN DATUM OF 1927 TO THE NORTH AMERICAN DATUM OF 1983

Warren T. Dewhurst

Rockville, MD
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LCDR Warren T. Dewhurst, NOAA
National Geodetic Survey
Charting and Geodetic Services
National Ocean Service, NOAA
Rockville, MD 20852

ABSTRACT. The application of minimum curvature is described as it applies to the transformation of positional data between various geodetic datums. Specifically, the author has implemented this approach and technique in a computer program, known as NADCON (an acronym standing for North American Datum CONversion). NADCON, discussed in detail within this publication, is designed to transform or convert data between the North American Datum of 1927 (NAD 27) and the North American Datum of 1983 (NAD 83). It should be noted, however, that the method is equally applicable to other datums as well. All that is required are data with two different and distinct coordinate values representing the exact same point on the surface of the Earth. The actual differences between coordinate values are modeled; no knowledge of datum origins or ellipsoidal parameters is required. The application of minimum curvature within this context is new and somewhat innovative. Readers requiring more detailed knowledge are urged to consult the references.

This approach, as implemented within NADCON, has resulted in a method that is simple, accurate, cost-effective, and rapid. NADCON allows for transformations within all states and territories with great confidence. The approach introduces no more than approximately 15 cm (1σ) of uncertainty within the conterminous United States (CONUS), 4 cm (1σ) within Puerto Rico, and 50 cm (1σ) within Alaska. Hawaii, although modeled to 15 cm (1σ) appears to have large datum inconsistencies which require attention prior to the application of any transformation technique. The accuracy of the transformations represents at least an order of magnitude improvement upon the traditional methods of Molodensky and the regression equations as implemented by the Defense Mapping Agency (DMA), designed to be used to transform data from NAD 27 to the World Geodetic Datum of 1984 (WGS 84), a datum which is almost identical in ellipsoidal parameters and definition to NAD 83 (Defense Mapping Agency, 1987). NADCON is sufficiently accurate for mapping at a scale of 1:200 or smaller, ideal for Geographic Information Systems/Land Information Systems (GIS/LIS) and low-order land surveys.

BACKGROUND

The North American Datum of 1983 (NAD 83) represents the single most accurate and comprehensive geodetic survey datum in the history of the United States. NAD 83, historically the third official U.S. datum, supersedes the North American Datum of 1927 (NAD 27).

The datum redefinition effort, international in scope, has spanned more than a decade in time at a cost of more than $35 million. This effort, consisting of the accumulation, validation, automation, and adjustment of horizontal survey information, involved 1,785,772 geodetic observations connecting 266,436 control stations within the United States, Canada, Mexico, and Central America. Greenland, Hawaii, and the Caribbean islands were connected independently by Doppler observations and the application of Very Long Baseline Interferometry (VLBI).

The Geodetic Survey of Canada and the Danish Geodetic Institute (representing Greenland) participated in the validation and adjustment of data for their respective territories. Information for Central America was collected by the Inter American Geodetic Survey and compiled by the Defense Mapping Agency (DMA). The National Geodetic Survey was responsible for the overall project and the publication of the results within the United States.

NAD 83 provides a consistent datum for the country at a higher level of accuracy. In brief, NAD 83 supersedes any application of NAD 27.

What were the problems with NAD 27? What makes this new datum preferable to use? These questions are often asked, because the "old" datum has proven useful for so long.

There are several fundamental reasons why the geodetic reference system for the United States required reconsideration. Increasing demands by the surveying community, the introduction of highly accurate electronic measurement systems, and the advent of satellite tracking systems such as Doppler and the Global Positioning System (GPS) all contributed to the identification of weaknesses in NAD 27. Discrepancies between existing control and newly established surveys predicated the establishment of an entirely new datum rather than a "repair" to NAD 27. The arguments articulating the necessity for a new datum are well documented by Whitten and Burroughs (1969), Whitten (1971), and the National Academy of Sciences (1971). In summary, NAD 27 suffers from: (1) an outmoded and obsolete mathematical representation of the Earth, (2) inconsistencies arising from partial adjustments of data on a regional basis, and (3) limitations due to outdated survey instrumentation. These inconsistencies needed to be reconciled in order to provide a consistent datum from coast to coast and between neighboring nations within North America (Schwarz 1989).

It is important to remember that users of geodetic data and, hence, those affected by changes in geodetic datum can be roughly grouped into three distinct categories: primary users---geodesists and land surveyors who employ the coordinate information directly; secondary users---those who employ the work of primary users in some way, adding value in the form of cartography and digital interpretation; and tertiary users---those who employ the work of secondary users in order to gain knowledge and insight. A pilot or navigator would be an example of a tertiary user. Typically, knowledge of surveying and geodetic datums decreases with each category. A tertiary user may have absolutely no concept of geodesy. However, it may be critical for such a user to recognize a datum inconsistency and be able to obtain coordinates in either datum with confidence and little fanfare. Tertiary users of coordinate data greatly outnumber primary and secondary users.

Complete Datums

A complete horizontal datum consists of: (1) all of the parameters necessary to define a particular coordinate system, and (2) a set of control points whose geometric relationships are known, either through measurement or calculation. Published coordinate values, derived from above, serve as the basis and standard for subsequent surveys. NAD 27 as well as NAD 83 are both complete datums. Permanent survey monuments, and their respective coordinate values, serve as reference points and are the most evident and perhaps important aspect of the datum.

The shifts or offsets between NAD 27 and NAD 83 arise from a difference in the assumed coordinate systems as well as with the difference in approach taken in the calculation of coordinate values. Figure 1 shows the magnitude of the shifts between these datums within the conterminous United States.

The largest discrepancies can be attributed to the choices in reference ellipsoids (of revolution) for each datum. Reference ellipsoids or spheroids, a geometric approximation for the Earth, are necessary for the definition of coordinate systems. NAD 27 depends upon an early approximation, known as the Clarke Spheroid of 1866, while NAD 83 relies upon the more exacting Geodetic Reference System of 1980 (GRS 80). Datum differences attributed to differences in ellipsoid can be in excess of 100 m in amplitude. The Clarke Spheroid of 1866 was designed to optimize or fit the shape of the conterminous United States while GRS 80 is global in extent. GRS 80 was made possible due to advances in satellite tracking technology and modeling. The primary advantage of GRS 80 is that it facilitates the computation of correct geometric relationships on a global as well as a continental scale. GRS 80 is a geocentric ellipsoid, indirectly employing the Earth's center of mass in the specifications for the coordinate system; the center of the reference ellipsoid is defined as the Earth's center of mass. The Clarke Spheroid of 1866 employed a specific coordinate pair, as well as a geometric relationship (orientation), in order to define the survey datum. The coordinate pair for a station known as MEADES RANCH in Kansas served as the origin, rather than the Earth's center of mass.

In addition to discrepancies caused by differences in the definition of coordinate systems, small differences or local distortions (on the order of 10 m) arise due to differences in adjustment and survey methodologies. Coordinate values referenced to NAD 27 were obtained in layers, with accurate geodetic networks serving as rigid framework for less accurate observations.

NAD 27 was performed prior to the advent of digital computers and all of the computations and data assimilation were done manually. A large-scale, totally comprehensive adjustment of all horizontal control data within the United States represented an impossible task.

In contrast, NAD 83 represents the simultaneous consideration of all geodetic survey information; a layered computational approach was not utilized. This ensured that observations connecting stations were all related, regardless of survey date, instrumentation, or relative accuracy. Appropriate weighing and mathematical modeling guaranteed that the "mixed" observations were properly considered. Optimal, in a least squares sense, coordinates resulted for the NAD 83 adjustment. This adjustment process resulted in consistency between political boundaries as well as within surveys of differing origin.

The digital computer, as well as advances in higher mathematics, permitted the simultaneous adjustment of all observations and the resolution of all unknowns for NAD 83.

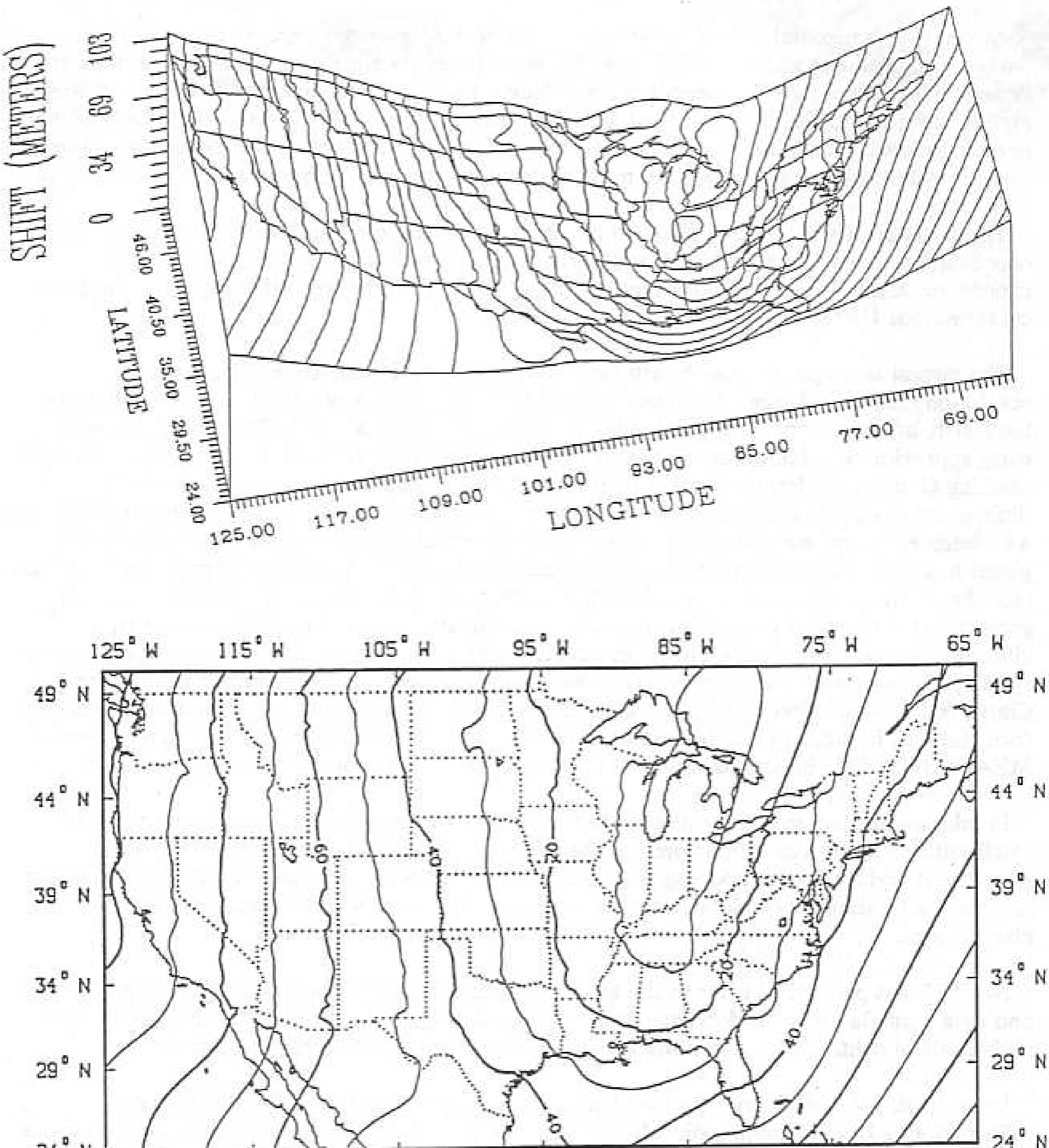

Figure 1.--Three dimensional perspective and contour map of the conterminous United States showing the magnitude (meters) of total shift between NAD 27 and NAD 83.

Datum Transformations Before NADCON

Several different methods for transforming coordinate data are widely accepted in the geodetic and surveying communities, and were available prior to the development of NADCON. All of these rely upon a closed form expression relating the conformal mapping of coordinate systems from one datum to another. These approaches are thus all analytical and exact.

Unfortunately, the differences between NAD 83 and NAD 27, as mentioned above, consist of more than a difference in coordinate systems. Local distortions, implicit within NAD 27, are evident and require correction.

Several approaches are often employed for the transformation of data from NAD 27 to NAD 83. All rely upon knowing the exact differences at a number of existing control points.

A transformation can be accomplished in curvilinear (geodetic) or in rectangular space. For a geodetic coordinate this becomes:

$$\phi_{NAD\,83} = \phi_{NAD\,27} + \delta\phi$$

and

$$\lambda_{NAD\,83} = \lambda_{NAD\,27} + \delta\lambda \quad . \tag{1}$$

From the above it is clear that this procedure is only possible if the datum shifts, $\delta\phi$ and $\delta\lambda$, are known or estimated.

DMA has prepared tables and articulated methodologies for the transformation of data from many worldwide local datums into the World Geodetic System of 1984, a new global and geocentric datum which is practically identical to NAD 83 for the United States. Average shift values for various locations have been computed and tabulated (Defense Mapping Agency 1987). The shifts are based solely on a limited number of Doppler stations and hence do not adequately model any global and local distortion. Use of average shifts for the United States would most likely yield inadequate accuracy, introducing tens of meters of uncertainty.

Another approach, the Molodensky or abridged Molodensky formulas, provides a more accurate methodology for obtaining shift values between local and geocentric datums (Defense Mapping Agency 1987). These formulas rely upon more accurate parameter estimates, based upon additional Doppler information. For the United States, 405 Doppler stations were used to obtain transformation parameters used in the formulas for obtaining shift values. Scale, geometric-ellipsoidal center shifts, and orientation are all accounted for within the Molodensky formulation. Even though this method is superior to the direct application of average shifts, it provides a transformation that is reliably accurate only to approximately 5-10 m for the United States.

In an effort to simplify transformation between datums and increase overall accuracy, DMA also provides another solution to the problem. A two-dimensional polynomial was fit to the actual (observed) datum shifts between NAD 27 and WGS 84. This approach, utilizing the theory of least squares (e.g., multiple regression), achieves more accurate results than a Molodensky formula, but most probably insufficient accuracy for land survey and large-scale mapping purposes. Again, a total of only 405 Doppler observations was available for analysis within the conterminous United States, and the polynomial is only of order nine. In addition, Doppler positions are only accurate to about 1 m (1σ). Any transformation approach which only utilizes Doppler information cannot be better than 1 m in accuracy. However, this method often provides results with an accuracy of better than 10 m.

To satisfy the more demanding applications within the Federal surveying and mapping communities, NGS developed a least squares approach (simplicity or affine transformation) that relies upon actual and observed coordinates expressed in both NAD 27 and NAD 83. Since these coordinates are the same as used in the actual definition of NAD 83, and are known to millimeters, a much more accurate transformation is possible. A computer program, known as "LEFTI," implements the approach (Vincenty 1980). In areas of good geodetic coverage, accuracies of 1 m or better are possible. The approach, however, requires geodetic expertise to use and access to valid geodetic data. In addition, application of different geodetic control data within the program will yield different results, depending on the accuracy of the control data and their spatial distribution relative to the unknown points. This method is therefore suitable for experts, those that can select the correct data to use as control. In addition, the availability of good control data surrounding the unknown point, or the geometry of the problem, greatly impacts the results. Knowledge of both is paramount. The differences between LEFTI and NADCON results, when both are used properly, are small, often less than 10 cm.

LEFTI is well founded and appropriate, especially amongst well informed primary users, but unfortunately, these well-informed and sophisticated users represent a minority of those affected by the datum change. Major objections occur when addressing the needs of secondary and tertiary users.

Several private concerns also have developed methodologies for the transformation of data within local areas. These independently derived methods often optimize the transformations within a particular area, such as the Gulf of Mexico, while sacrificing transformation accuracy elsewhere in the United States. They are often the result of a limited number of actual control points, perhaps even outside of the National Geodetic Reference System (NGRS). The methods are often either complex or overly simplistic, attending to the perceived requirements of the organization responsible for their development. These methods range from applying average shifts within regional zones, with obvious discontinuities at borders, to locally fitting very high-order polynomials to coordinate data. Other approaches require the computation of Molodensky transformation parameters (dx, dy, dz) for local regions and rely upon the estimation of geoid heights based upon the Clarke 1866 spheroid and GRS 80. Almost certainly these methods require expertise to use and knowledge of limitations.

In brief, several different methodologies have been developed, but none satisfies the majority of users. Table 1 provides a summary of the options, followed by a comparison of a new method.

NADCON

The author recognized that the majority of users of coordinate data are not expert geodesists and that a transformation methodology should yield consistent results. Therefore an intuitive method was developed (NADCON), utilizing a homogeneous subset of first-, second-, and third-order horizontal control data resulting directly from the readjustment of NAD 83. They represent the best available (e.g., most accurate) data associated with the establishment of NAD 83.

NADCON was designed to satisfy the majority of users, providing a uniform methodology for the Nation and minimizing the technical confusion concerning the application of NAD 83. In

Table 1.--Comparison of various transformation methodologies

Methods	Originator	Advantages	Disadvantages	Approximate accuracy (m)	Field use (Yes/No)
Molodensky Abridged Molodensky	DMA	Defined worldwide	General Doppler-derived	~5 - 10	Yes
Regression analysis	DMA	Defined worldwide	Inaccurate Cumbersome Local Dependency	~3 - 5	Yes
LEFTI	NGS	Documented	External data required Geometry dependent Awkward Expert required	1 - 5	No
NADCON	NGS	Fast Accurate Continuous Standardized Single source Consistent	Interpolation and extrapolation	0.15 - 0.5	Yes
Independently derived	varies	Tailored for user	Not standardized Expert may be required Discontinuous	Varies	Perhaps

particular, the method was designed for: (1) simplicity, (2) accuracy (1 m or better was the target accuracy), (3) completeness, and (4) availability (low cost). The overall goal was, again, to stabilize the technicalities of the transformation between datums and facilitate the application of NAD 83 within the United States.

In essence, a gridded data set of standard datum shifts has been prepared and a simple interpolation routine provides estimates of values at non-nodal points. The preparation of the gridded values was the difficult aspect. The actual application of the processed data is accomplished by the user in a simple application program. Both are available from NGS.

In summary, NADCON is a two-step process: (1) the development of gridded data sets--essentially the $\delta\phi$ and $\delta\lambda$ of eq. 1, and (2) the estimation to non-nodal points using interpolations based on the grids. The difficult aspect of data selection and the computation of the grids has been done by NGS, and is of little concern to most users.

Interpolation in General

Two approaches are available for the interpolation of discrete data. The first relies upon the definition of a continuous function in two space variables, and the second relies upon the estimation of representative data at nodal points, either on a regular rectangular grid or an irregular polygonal pattern. A development of a continuous function can naturally lead to a grid of estimates, as noted by Briggs (1974), but the numerical estimates on a grid cannot lead, in itself, to the definition of a continuous function.

Examples of the first approach would include regression analysis, either linear or multivariate, least squares collocation (Moritz 1973), and multiquadrics (Hardy 1978). All of these rely upon the fitting of data to a mathematical model which explicitly defines a function. The fitting process yields parameters to the model which allow for the prediction of values at desired locations.

This approach is well accepted within geodesy and discussions can be found elsewhere. It must be remembered that the estimates from the fitting are only as good as the model. A model consisting of a deterministic function with well understood systematic (modeled) noise or even random (white) noise can usually be dealt with. Unmodeled systematic noise and anisotropy among the data can be troublesome. Assumptions in the modeling are usually simplistic, precluding the analysis of highly variable data, as might be encountered with geodetic data gathered by differing agencies, using differing procedures, with differing instrumentation over various epochs. Even with the limitations, fitting usually produces reasonable results, provided the data are consistent and the model is appropriate.

The estimation of nodal values on the other hand does not necessarily require that a mathematical model or function be defined. The only requirement is that the estimates at the nodes approach or duplicate the original data, as the original data approach the location of a nodal point. This has several advantages, not the least being that a complete knowledge of the systematic and random errors in the data is unnecessary as well as the exact form of a mathematical function. In addition, the fitting of a large number of observations to a fairly complicated model usually requires significant computer resources.

The definition of a deterministic model for an ellipsoidal transformation is well understood and not debatable provided sufficient parameters are used. Typically three, four, or seven parameters are employed. Implicit with such a transformation is an assumption of comformality, the preservation of angular magnitude and orientation. Anisotropic variations of scale and orientation at data points prove to be problematic. Therefore, transformation parameters for one location may not be appropriate for another. In addition, the distance between zones of appropriateness will vary. The transformation parameters for one location will simply be unreliable for another. Therefore, the first and most common approach, that of fitting data to obtain a parametric function, is complicated at best and inappropriate at worst.

The fitting of polynomials, which may have no particular physical significance other than approaching the "shape" of the data, does have some attraction. First, the parameters can be easily computed, with the matrix inversion dependent solely upon the order of the polynomial. The lower the order, the less complicated the computation. In addition, polynomials can most often be found which follow the shape of real, continuously varying data, provided one adds enough terms. One of the difficulties, however, is with the establishment of exactly how many terms are sufficient. For example, a straight line can be modeled with a high-order polynomial, but not as well as a first-order polynomial; linear regression would be simpler and superior. The nonlinearity of the spatial variability representative of differences between NAD 27 and NAD 83 provides an additional challenge to polynomial modeling. Location-dependent coefficients may have to be computed, perhaps causing discontinuities between zones of grossly dissimilar parameter values. Certainly one can fit a polynomial to shift data; the question becomes one of sufficiency of order, zonal descriptions, and edge blending. Desired accuracy as well as region size (e.g., only a county or perhaps the entire country) determines how much these complicating factors need to be addressed.

The other approach, that taken with NADCON, relies upon the direct estimation of grid values rather than the determination of polynomial coefficients or the reliance upon some mathematical model describing the mapping between datums.

Modeling of NADCON Grids with Minimum Curvature

To reiterate, NADCON is a two-step process: (1) the development of gridded data sets--essentially the $\delta\phi$ and $\delta\lambda$ of eq. (1), and (2) the estimation to non-nodal points.

The gridded data sets are prepared using a technique known as "minimum curvature" (Briggs, 1974; Swain, 1976; Webring, 1981). This approach mathematically minimizes the total curvature, or rate of bending, associated with a smooth surface describing the shift values between datums.

A total of two gridded data sets is required for the complete computation of a transformation: one for latitude shifts and another for longitude shifts. Thus, two mathematical surfaces must be prepared for each major region of the country.

The regions identified during this development, along with their geographic limits are given in table 2 and figure 2.

Minimum curvature has its origin within mechanical engineering, geophysics, and the mathematics of finite differences. The differential equations pertaining to the deformation or bending of plates form the basis for the method. For completeness, a development of the mathematics is presented. Additional discussion may be found in the above mentioned references, particularly Briggs (1974). The FORTRAN code utilized within NADCON has its origins with that of Webring (1981).

Minimum total curvature provides an approach for the interpolation of irregularly spaced data either on a profile (e.g., one dimension) or a surface (e.g., two dimensions). The dimensionality is of little consequence; multidimensional developments of minimum curvature have been done in tensor notation, notably by Cloutier (1983). Surface or bivariate analysis is sufficient for a discussion of datum transformations.

Let us review exactly why the two datums result in different coordinate values. In essence there are two sources for the differences, the first being the conformal mapping between differing ellipsoids and the second a result of varying technologies, not particularly well understood but completely recognized as existing. This second source is commonly known as distortion, as mentioned above. The first source results in differences which are smoothly varying over the Earth, the curvature of the differences being continuous. The second may result in highly location-dependent differences, but usually confined within regions and of lower amplitude than the first. Smoothness may or may not be evident with differences resulting from the second source. In other words, surveyors usually confine projects to small areas, such as neighborhoods, counties, and municipalities. Quite possibly stations of differing origin but physically located next to each other may describe entirely different datum differences, apart from the well understood conformal mapping.

Global smoothness then is important; local smoothness is as well, but to a lesser extent provided data fully describing offsets from a global fit are available. The density or distribution of the data, as well as the variability, determines the relative importance of local smoothness to a solution.

Table 2.--NADCON regions, geographic limits and gridding particulars

Area	Minimum Latitude (deg.)	Minimum Longitude (w) (deg.)	Maximum Latitude (deg.)	Maximum Longitude (w) (deg.)	Number rows	Number columns	Grid increment (arc minute)
CONUS	24	66	50	126	105	241	15.0
Alaska	50	128	74	191	193	505	7.5
Hawaii	18	154	23	161	201	281	1.5
Puerto Rico and the Virgin Islands	17	64	19	68	41	81	3.0
St. George Island	56	169	57	171	61	121	1.0
St. Laurence Island	62	168	64	172	41	81	3.0
St. Paul Island	57	169	58	171	21	41	3.0

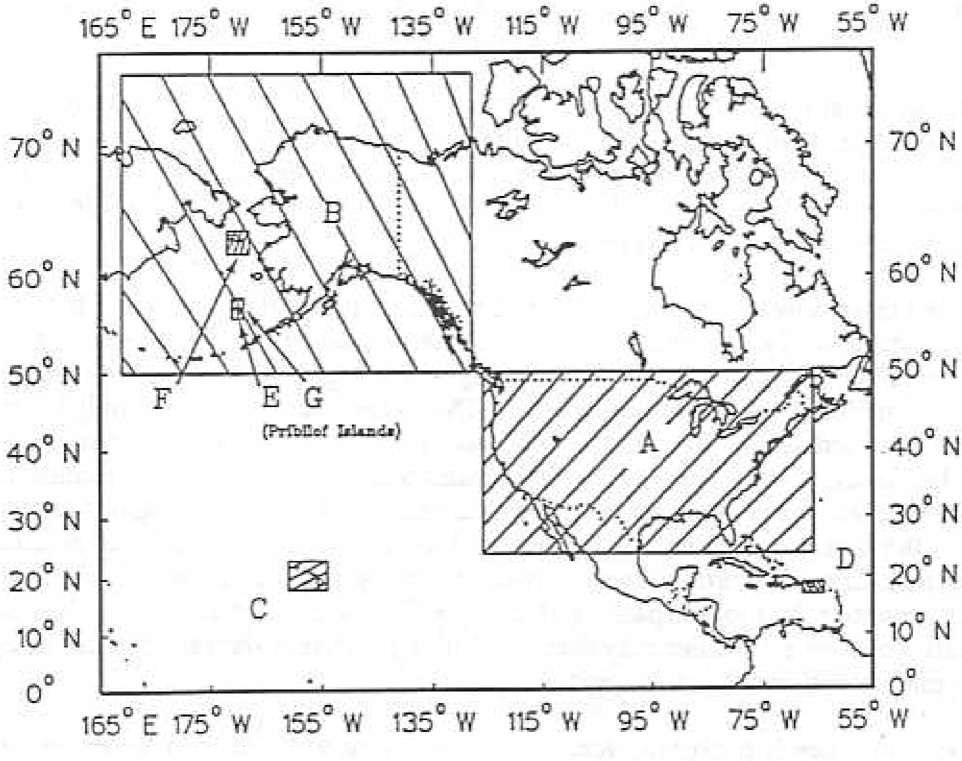

Figure 2.--Regions of coverage for NADCON. Each grid region is shown as a rectangle with cross-hatching: A) conterminous United States, B) Alaska, C) Hawaii, D) Puerto Rico and the Virgin Islands, E) St. George Island, F) St. Laurence Island, and G) St. Paul Island.

Cubic splines are polynomials of third order, having one highly desirable feature: they represent the smoothest fit of a continuous function through data points. For this reason, splines provide an apparent advantage, global smoothness. Piecewise continuous splines provide an even more convincing argument, allowing for global and local smoothness. Best of all, however, would be the preservation of global smoothness, allowance for local smoothness, and consideration of anisotropic (location dependent) variation. Minimization of curvature, through the establishment of grid estimates, provides this approach. The direct establishment of a grid of estimates provides a ready means to address location dependency.

It is instructive to compare a surface of gridded datum shift values to plate theory in mechanical engineering. Consider a large thin sheet (thickness much less than either lateral dimension), with point forces acting perpendicular to the sheet. (See fig. 3.) Shear and tensional/compressional forces are not present. Bending or deformation occurs, with the magnitude of deformation at each point source dependent or constrained by the force applied at the point. Deformation away from the point sources would be dependent upon close forces, with smooth bending between points evident unless the forces exceed some threshold and actually "crimp" or puncture the plate. The datum values at station locations would be equivalent to the point forces and the smooth bending being representative of the variation of shift values dependent upon location. Smoothness of both the thin sheet and the surface describing the shift values is highly desirable, provided discontinuities or punctures are to be avoided.

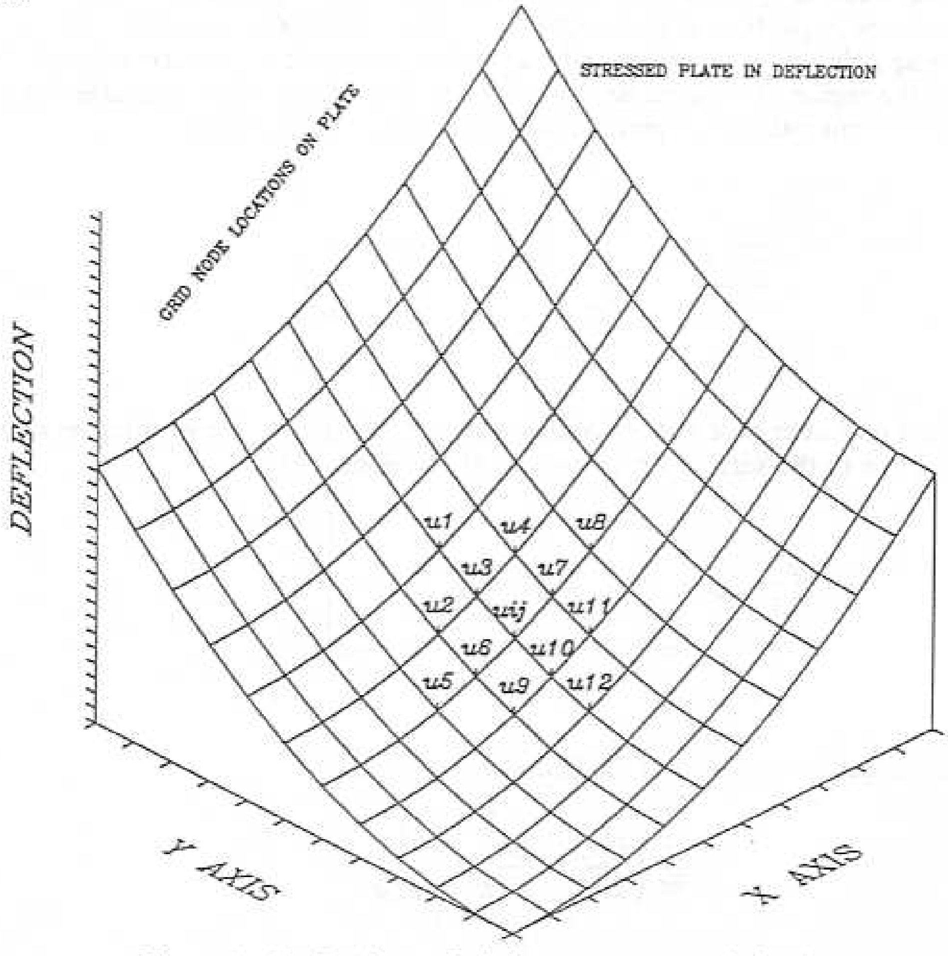

Figure 3.--Deflections of minimum-curvature-derived surface with node locations as represented in eq. (7).

The biharmonic partial differential equation describing the deformation of the plate is given by

$$\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{P}{D}$$

or

$$D \nabla^4 w = P$$

where D is a scale factor dependent upon the material, known as the flexural rigidity, P the force, and w the normal displacement of the sheet.

Note that a solution to this differential equation is a third-order polynomial in two space variables. Continuity is of course a condition of solution, requiring that the deformation at point forces equals that resulting from the force at the point. Mathematically,

$$u(x_m, y_m) = w_m$$

where $u(x,y)$ is the displacement at point m.

Boundary conditions must be physically meaningful. For a plate, one would not expect the slope of deformation to change outside the region of influence due to point forces, provided that gravitational forces are neglected. It also seems logical that datum shift values would vary continuously among points of observation and that variation would tend toward a global influence outside the region of observation, with changes in tendency (slope) outside this region unacceptable. This is equivalent to a plate or surface with free edges. Thus,

$$\left.\frac{\partial^2 u}{\partial x^2} = 0 \right|_{edge} \quad and \quad \left.\frac{\partial^2 u}{\partial y^2} = 0 \right|_{edge} . \qquad (2)$$

In addition, since no forces exist outside of this region, by definition, the bending moment about a tangential line to the surface would be zero at the edges. Thus,

$$\left.\frac{\partial^3 u}{\partial x^3} = 0 \right|_{edge} \quad and \quad \left.\frac{\partial^3 u}{\partial y^3} = 0 \right|_{edge} .$$

These two latter requirements lead to the boundary condition

$$\left.\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0 \right|_{edge} .$$

As noted by Briggs (1974), the biharmonic equation can also be derived from the principle of minimum curvature, and with the free-edge or Kirchoff conditions can also be solved numerically using difference equations. Briggs also notes that these difference equations can be formed either through the application of Taylor's theorem or directly through the principle of minimum curvature. Different approaches simplify the derivation depending upon the location of the nodal points under examination.

Consider curvature, C_{ij}, at a point (x_i,y_i) on a grid of unit spacing as

$$C_{i,j} = \frac{\partial^2 u_{i,j}}{\partial x^2} + \frac{\partial^2 u_{i,j}}{\partial y^2} = u_{i+1,j} + u_{i-1,j} + u_{i,j} + u_{i,j-1} - 4u_{i,j} \quad . \tag{3}$$

The discrete total squared curvature can then be defined as

$$C = \sum_{i=1}^{I} \sum_{j=1}^{J} (C_{i,j})^2 \quad .$$

Note that the curvature, C, is a function of the values on the grid, u_{ij} , and its neighboring points. The accuracy of the estimate of total curvature, C, is dependent upon the total number of neighbors one is able or willing to consider. Quite often, depending upon the density and distribution of data, one may only have to sum over a small area surrounding the data, allowing values in sparse regions to be fixed by linear extrapolation or some other technique. Note that a minimum of three points is necessary to estimate curvature.

Minimization of the discrete total curvature, eq. 3, differentiating with respect to each grid point and setting the result to zero, yields

$$4\ C_{i,j} = C_{i+1,j} + C_{i-1,j} + C_{i,j+1} + C_{i,j-1} \quad . \tag{4}$$

This difference equation of curvature can be used, through substitution of eq. (3), to solve for the estimates u_{ij}. In general, this leads to a new difference equation in terms of the grid estimates, u_{ij},

$$\begin{aligned} & u_{i+2,j} + u_{i,j+2} + u_{i-2,j} + u_{i,j-2} \\ & + 2(u_{i+1,j+1} + u_{i-1,j+1} + u_{i+1,j-1} + u_{i-1,j-1}) \\ & - 8(u_{i+1,j} + u_{i-1,j} + u_{i,j-1} + u_{i,j+1}) + 20u_{i,j} = 0 \quad . \end{aligned} \tag{5}$$

Modification of the above for various locations, for example along the edges of the grid, is necessary due to numerical differentiation, the free-edge boundary conditions, and the finite dimensions of the surface.

Briggs (1974) provides a summary of the requisite difference equations. For completeness, they appear below. Note that there are exactly as many of these equations to solve as there are nodal or grid points.

Normal or general situation
Away from edges and observations the difference equation is:

$$u_{i+2,j} + u_{i,j+2} + u_{i-2,j} + u_{i,j-2} + 2(u_{i+1,j+1} + u_{i-1,j+1} + u_{i+1,j-1} + u_{i-1,j-1})$$
$$- 8(u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}) + 20u_{i,j} = 0.$$

Edge
For the edge $j=1$ and away from corners:

$$u_{i-2,j} + u_{i+2,j} + u_{i,j+2} + u_{i-1,j+1} + u_{i+1,j+1} - 4(u_{i-1,j} + u_{i,j+1} + u_{i+1,j}) + 7u_{i,j} = 0.$$

One row from edge
For the row $j=2$, and away from corners, use

$$u_{i-2,j} + u_{i+2,j} + u_{i,j+2} + 2(u_{i-1,j+1} + u_{i+1,j+1}) + u_{i-1,j-1}$$
$$+ u_{i+1,j-1} - 8(u_{i-1,j} + u_{i,j+1} + u_{i+1,j}) - 4u_{i,j-1} + 19u_{i,j} = 0.$$

The corners
For the corner $i=1, j=1$, use

$$2u_{i,j} + u_{i,j+2} + u_{i+2,j} - 2(u_{i,j+1} + u_{i+1,j}) = 0.$$

Next to corner
For a grid point next to the corner and lying on a diagonal $i=2, j=2$, use

$$u_{i,j+2} + u_{i+2,j} + u_{i-1,j+1} + u_{i+1,j-1} + 2u_{i+1,j+1}$$
$$- 8\ (u_{i,j+1} + u_{i+1,j}) - 4\ (u_{i,j-1} + u_{i-1,j}) + 18u_{i,j} = 0.$$

Edges next to corner
For the grid point next to the corner point, which lies on the edge, $i=2, j=1$, use

$$u_{i,j+2} + u_{i+1,j+1} + u_{i-1,j+1} + u_{i+2,j} - 2u_{i-1,j} - 4(u_{i+1,j} + u_{i,j+1}) + 6u_{i,j} = 0.$$

The original data, the datum shift values in this case, do not often coincide with the location of grid nodes. Curvature at a grid point in which original data are nearby can be defined as

$$C_{i,j} = \sum_{k=1}^{4} b_k u_k - u_{i,j} \sum_{k=1}^{5} b_k + b_5 w_n \ . \tag{6}$$

This expression can be used in eq. (4) from above to yield yet another difference equation in lieu of eq. (5). This can then be thought of as a special case, where data exist within a grid cell. w in eq. (6) represents the actual data values and b_k are internal weights applied to the surrounding grid points. (See Briggs 1974.) Equation (6) is the result of considering the data point as part of an irregular grid, together with the vertices of the regular grid surrounding the point. Two limitations need to be noted; the matrix yielding the weights, b_k, becomes singular as data approach the grid location at u_{ij}, and the data must be close enough to the desired node so the approximations leading to eq. (6) remain valid.

Equation (6) assumes that only one data point is within a given cell. This too is a limitation. Aliasing can occur when the grid spacing is larger than the wavelengths present in the data. Care must be taken in choosing the grid interval. In addition, a distance weighing function is used to combine multiple data points within a grid cell. The single value for a pseudo-observation to be used in eq. (6) is thus obtained.

A threshold approach to data selection was used by Webring (1981) as well as by this author. Briefly, all nodes in which data points are located within a certain radius are set to the exact value of the data; data outside a certain radius are ignored and the general curvature equation is used. For the purpose of this study the separation thresholds are 0.05 and 0.75 of the grid spacing. A threshold of 0.05 times the grid spacing guarantees that data located nearby will have the most influence on an estimate and that data farther away will not be as heavy an influence. This is also in agreement with the "boundary" condition, that grid nodes which coincide exactly with data points attempt to assume the value of the data points.

Let us now examine some of the practical aspects of the above discussion. Consider a matrix of grid locations shown in figure 3. Note that an estimate at an unknown grid location, u_{ij}, appearing in the center and away from the edges, can be defined by eq. (5) as

$$-\frac{1}{20}[u_1+u_5+u_8+u_{12}+2(u_2+u_4+u_9+u_{11})-8(u_3+u_6+u_7+u_{10})]=u_{ij}, \qquad (7)$$

where the subscripts are the same as shown in figure 3.

A similar relationship can be found for each point within this matrix grid. This set of equations can be solved, either through linear algebra or through iteration. Iteration accommodates a large amount of data without potentially unstable matrix inversion (e.g., singularities). However, iteration also requires that initial values of u_{ij} be provided. Theoretically, any initial values could be used (including zero). Practically, values which are closest to the asymptotic solution minimize the total number of iterations necessary for convergence. In addition, the method used to initialize the grid should be independent and should not introduce false trends or long-wavelength information.

These difference equations are solved iteratively, starting with a coarse grid representing the long-wavelength trend, and dividing the dimensions of the area by two until the desired size and grid spacing are achieved. The regional grid is determined by linear interpolation, row by row, at an interval of four times that of the final grid.

Webring (1981) suggests constraining areas within the final grid, which are devoid of data, to the values of this regional grid. This approach can save considerable computer time and effort and "coaches" the final grid in the direction of the regional trend. However, this was not done in this study. The difference equations were employed throughout the grid, even in areas of sparse data coverage. This was done for several reasons. It was believed that the NAD 27 distortions closest to an area of sparse coverage should influence the shift estimates within the area, since surveyors tend to use the closest existing control stations, and computer time was of no concern, with the gridding of the entire conterminous United States accomplished in less than 30 minutes.

Smoothness is assured by this method. For any data set (a minimum of four values are required) and a given grid interval, this method will determine the smoothest surface. However, as mentioned above, the grid interval is of great importance. Too coarse an interval will alias the data and too fine an interval will not improve the regional trend resulting from clustered or linearly dependent sampling. In fact, too fine an interval may yield a false picture of the regional trend.

In summary, minimum curvature provides the modeling technique within the NADCON method. The modeling is analogous to predicting thin-plate deflections, and relies on a biharmonic differential equation describing curvature. This equation is solved using finite differences; a total of seven difference equations is required. The difference equations are solved iteratively, using progressively finer grid intervals. Initialization of the grid is done through linear interpolation. Distance weighing is used when a grid cell contains more than one data point. Convergence is rapid, and grid estimates tend toward the values of the original data as these data approach a grid node. Any original data within a threshold (0.05 * grid spacing) surrounding a node determine the value at the node. Table 2 provides the gridding particulars for each region.

Interpolation of Grid

The data grids require interpolation to be useful. For this investigation, a locally fit polynomial is used in the interpolation, equivalent to a bilinear interpolation. Other methods may work; this method is simple and sufficiently accurate.

The polynomial surface, fit to the four surrounding nodal points, is defined as

$$z = a + bx + cy + dxy$$

where z is the estimate at the unknown point, x and y are positional indices, and a, b, c, and d are coefficients of the polynomial. This equation is solved using index location based on the row and column organization of the grid. The grids are organized from minimum latitude to maximum latitude (e.g., rows) to minimum longitude to maximum longitude (e.g., columns). In the above equation , x and y are defined as

$$x = [(x_{pt} - x_{min})/dx + 1] - j_{sw}$$

and

$$y = [(y_{pt} - y_{min})/dy + 1] - i_{sw}$$

where x_{pt} and y_{pt} represent the coordinates of the unknown point, x_{min}, y_{min} represent the minimum coordinates for the overall grid, dx, dy are the grid intervals in each direction, and j_{sw}, i_{sw} are the indices of the lower left (e.g., southwest) corner of the cell in which the unknown point resides.

The coefficients a, b, c, and d are all functions of the shift values of the surrounding nodal points. Clockwise from the southwest corner of the cell, these values can be defined as t_1, t_2, t_3, and t_4. Within this scheme, the coefficients become

$$
\begin{aligned}
a &= t_1 \\
b &= t_3 - t_1 \\
c &= t_2 - t_1 \\
d &= t_4 - t_3 - t_2 + t_1 \, .
\end{aligned}
$$

This interpolation is extremely fast, requiring few operations. In addition, given a grid of points sufficiently dense (e.g., dx and dy small) then this technique will work as well as more complicated approaches.

The NADCON program that is available from NGS provides for the transparent application of the interpolation method outlined above on all of the gridded data sets listed in table 2 and shown in figure 2. The program user merely has to ensure that the proper gridded data set resides in the same directory or storage area as the program. Latitude and longitudes provided by the user will be converted between datums via the program. NADCON allows for the conversion of individual points interactively as well as files of points (e.g., batch processing). Several different file formats are allowed. The sorting of data, the selection of applicable cell as well as the selection of the appropriate grid files (e.g., area) are all done automatically. Hence, a user may easily transform multiple points from various distant areas. For example, a user could compile a list or file of several hundred points in Hawaii, Alaska, Puerto Rico, and CONUS and transform all with a single effort. Such a file would not have to be divided based upon regional considerations.

Contour Maps of Shifts

Figures 4 and 5 depict contour maps of shifts resulting from the minimum-curvature modeling for the conterminous United States. These maps are in very good agreement with the predicted shifts of Vincenty (1979). It should be noted that the long-wavelength trends are well preserved and local effects or distortions are also evident. A complete suite of contour maps for all NADCON regions may be found in Dewhurst and Drew (1990a).

Accuracy and Speed

It is difficult to judge the exact accuracy of NADCON without some basis to use as truth. The most reliable geodetic control has been used in the minimum curvature modeling process. All of the legitimate NGS first- and second-order data have been used. Occasionally, third-order data provided the only means for computing estimates. However, third-order control was used sparingly, and only when necessary.

Ideally, the best and most general test of validity would be a comparison of NADCON estimates with independent truth data. Since independent data are scarce, overall accuracy can be defined as an extension of precision or the ability of the estimation process to replicate the original data used in the modeling.

A comparison can be made to other geodetic or survey data of a lesser accuracy, such as third-order data not used in the modeling. This comparison does result in an independent verification. However, this comparison is somewhat poor, since the basis of comparison involves these same data previously rejected during the modeling phase.

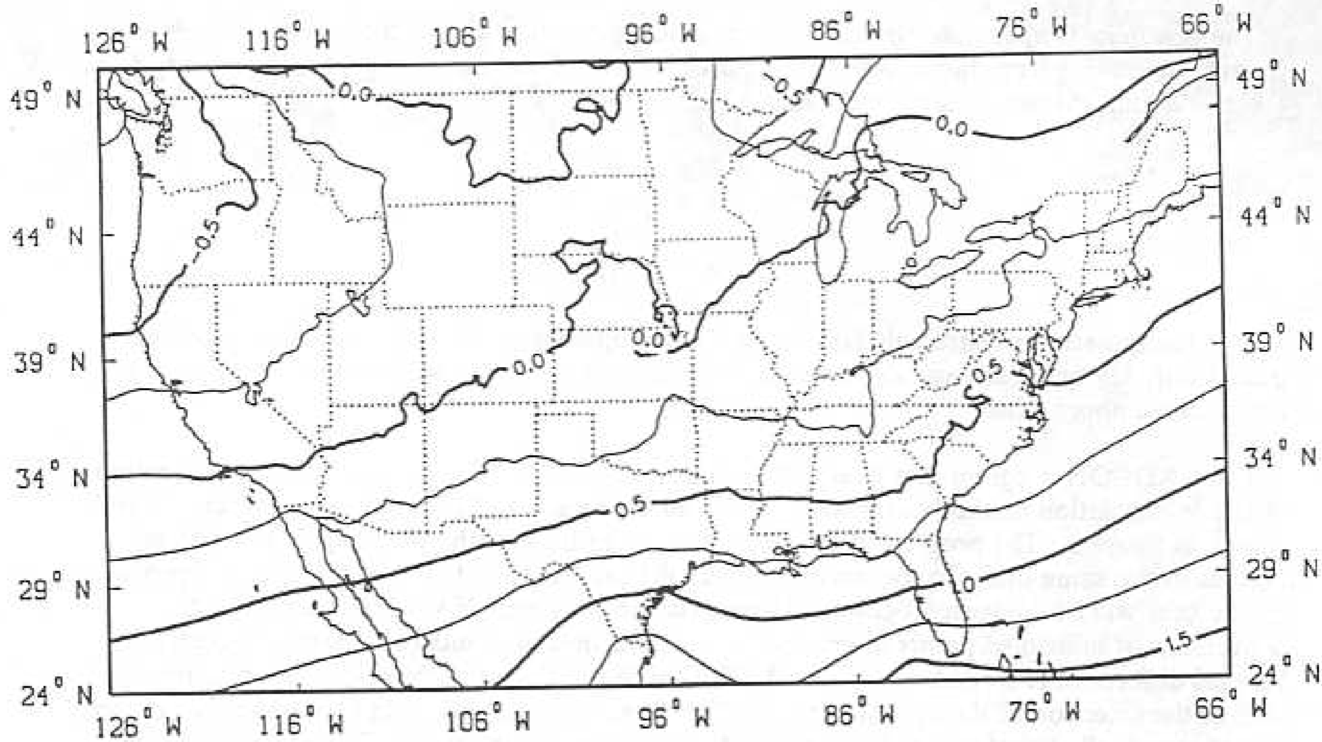

Figure 4a.--Latitude shifts in the conterminous United States in arc seconds (NAD 83 minus NAD 27).

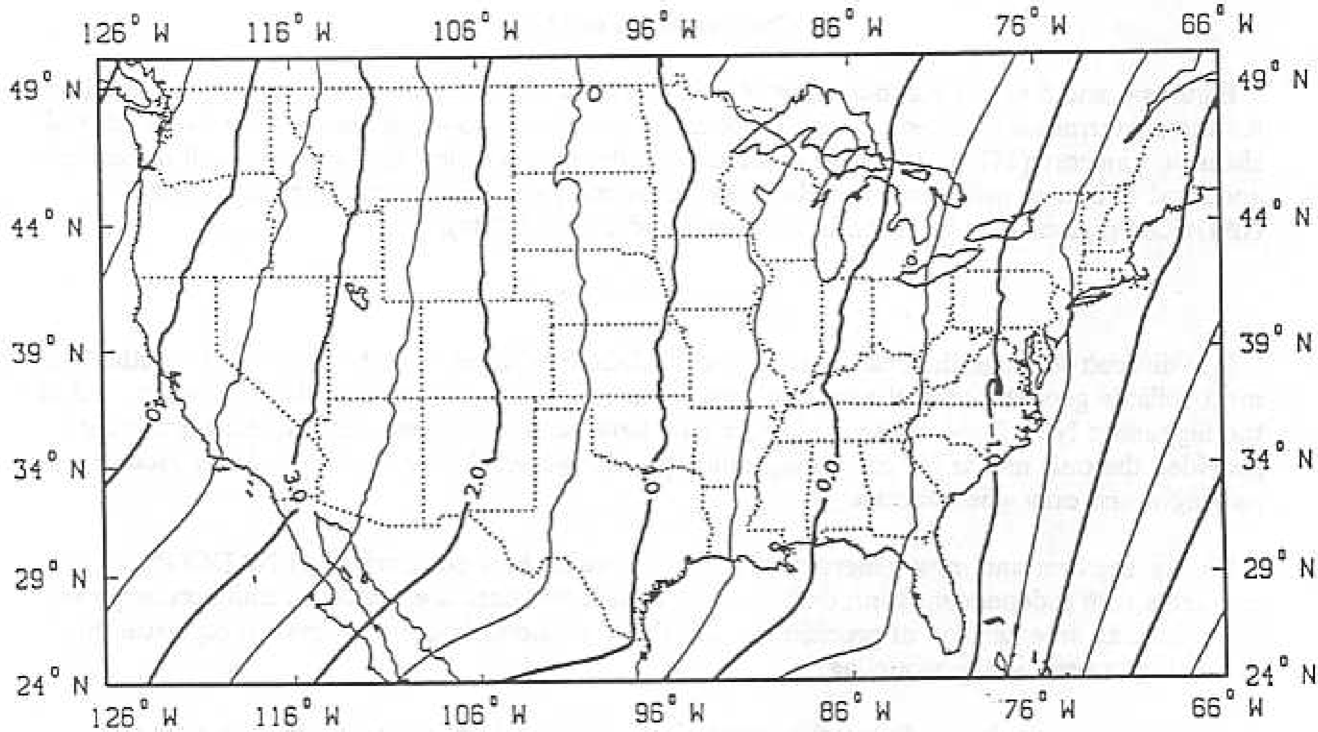

Figure 4b.--Longitude shifts in the conterminous United States in arc seconds (NAD 83 minus NAD 27).

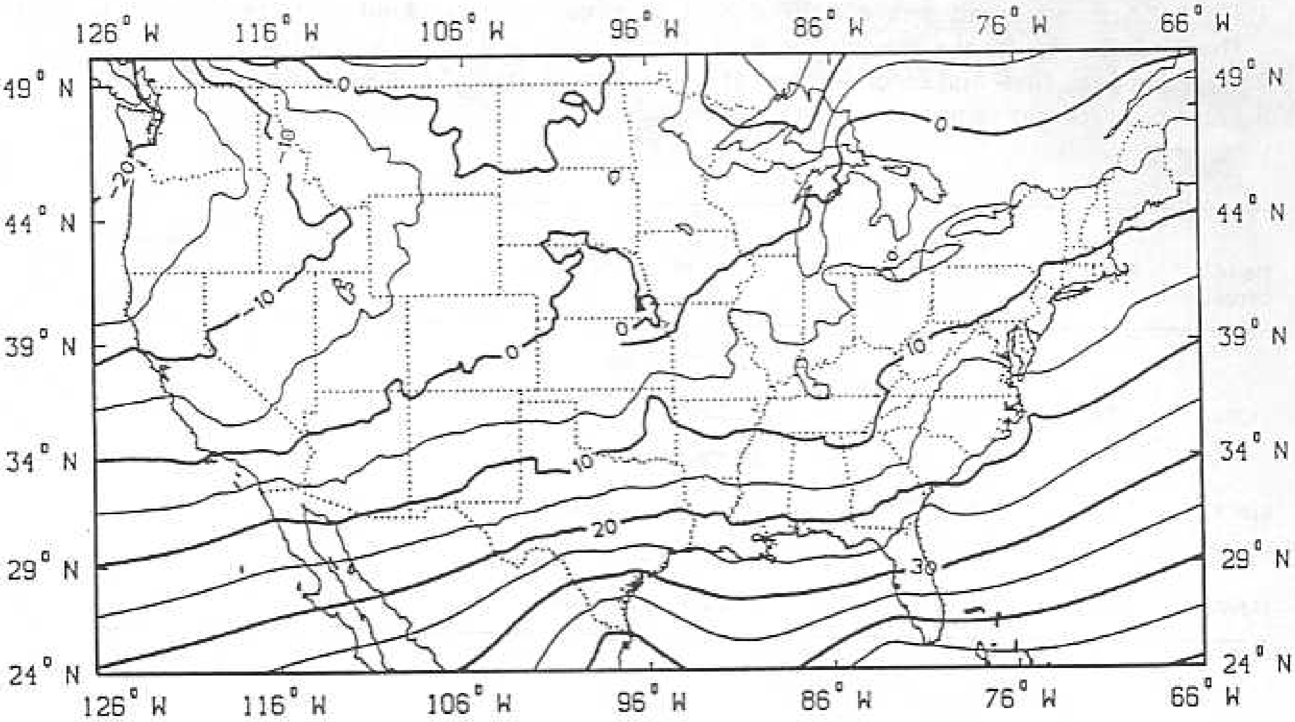

Figure 5a.--Latitude shifts in the conterminous United States in meters (NAD 83 minus NAD 27).

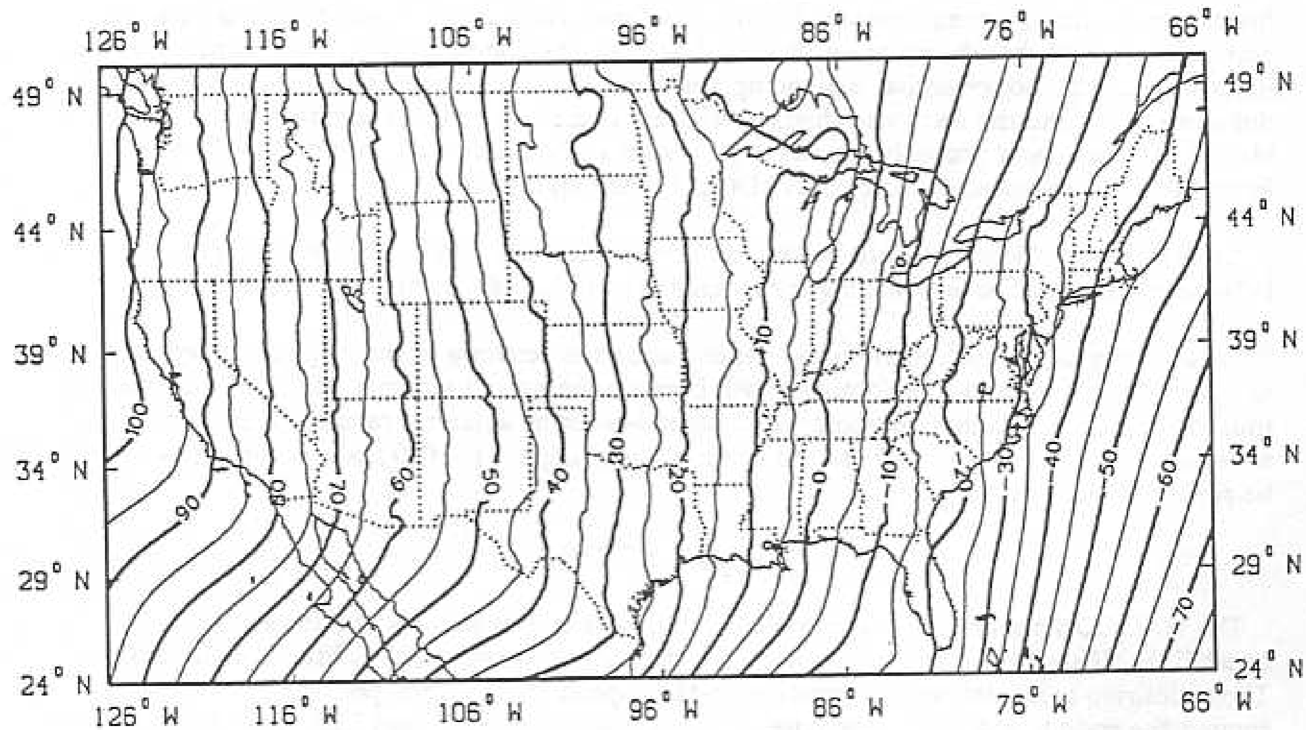

Figure 5b.--Longitude shifts in the conterminous United States in meters (NAD 83 minus NAD 27).

Table 3.-- Accuracy results with NADCON within the conterminous United States (CONUS). These results represent a comparison against original input data to the minimum-curvature modeling (e.g., first- and second-order) as well as against unmodeled data (e.g., third-order). Third-order results represent a worst case situation.

			Latitude		Longitude	
No. of comparisons	No. of outliers	Estimates within 1 m (%)	Mean residual (m)	Standard deviation (m)	Mean residual (m)	Standard deviation (m)
			FIRST ORDER			
33,280	90	99.7	0.002	0.145	0.001	0.131
			SECOND ORDER			
81,803	240	99.7	-0.001	0.162	0.002	0.149
			THIRD ORDER			
42,070	1,765	96	0.009	0.454	0.010	0.446

Table 3 summarizes the tests described above. Additional summaries can be found in Dewhurst and Drew (1990a) and appendix A. This table reveals that NADCON can replicate the original first- and second-order data to approximately 15 cm within the conterminous United States. In addition, a comparison with third-order data reveals that transformations with an accuracy of better than 50 cm are probable. Note that the comparison with third-order may be independent but also somewhat misleading; third- and lower-order geodetic data are often of dubious quality and the accuracies highly variable. These independent comparisons must be viewed cautiously and probably represent a worst-case situation. Such information provides insight into the expected accuracy of NADCON based upon location.

Table 4 provides summary shift values and statistics for each NADCON region. This information is useful in determining the variability of datum shifts within an area.

The accuracy of a transformation will depend upon the accuracy of the original survey data and the control used; no transformation will improve the overall accuracy of the coordinates. A third-order position cannot be improved to second-order by a datum transformation. In addition, coordinate values that are the result of digitizing a 1:100,000 scale manuscript cannot be rendered more accurate.

APPLICATION

The NADCON application program has been designed to accommodate users of all capabilities. Therefore, the user interface is straightforward and requires little explanation. Those desiring more detailed information on the application program, perhaps in order to convert the code to a different computer, are referred to Dewhurst and Drew (1990b), a companion document specifically for this purpose.

Table 4.--Shift statistics for each NADCON region. The range gives an indication of variability. Areas where little difference is reported in the minimum and maximum values would indicate that a fairly uniform shift may be suitable, such as in Puerto Rico and the Virgin Islands. The average values would represent appropriate shifts (see eq. 1 in text) and the standard deviation would be an indication of confidence (67% level) for these average values.

SHIFT STATISTICS
(NAD 83 - NAD 27)

	Latitude (arc seconds)	Longitude (arc seconds)	Latitude (meters)	Longitude (meters)
CONUS				
Range				
Minimum	-0.814	-3.773	-25.152	-106.632
Maximum	1.658	5.149	51.023	105.417
Average	0.317	0.873	9.746	20.690
Standard deviation	0.509	1.977	15.691	48.372
ALASKA				
Range				
Minimum	-6.014	5.368	-185.828	77.597
Maximum	0.595	12.700	18.432	206.190
Average	-2.600	9.033	-80.473	126.416
Standard deviation	1.489	1.775	46.043	23.871
HAWAII				
Range				
Minimum	-20.251	-10.261	-622.975	-294.455
Maximum	-3.523	-9.725	-108.310	-284.085
Average	-11.617	-9.993	-357.282	-289.432
Standard deviation	2.899	0.114	89.223	2.622
PUERTO RICO & VIRGIN ISLANDS				
Range				
Minimum	-7.279	-1.547	-223.801	-45.477
Maximum	-6.990	-1.305	-214.887	-38.181
Average	-7.128	-1.417	-219.158	-41.694
Standard deviation	0.079	0.066	2.428	1.951
ST. GEORGE ISLAND				
Range				
Minimum	3.028	7.889	93.664	136.726
Maximum	4.190	9.694	129.581	163.640
Average	3.160	8.789	111.664	150.300
Standard deviation	0.251	0.395	7.754	5.720
ST. LAURENCE ISLAND				
Range				
Minimum	-2.090	-3.551	-64.704	-51.679
Maximum	-1.537	-2.723	-47.584	-37.009
Average	-1.778	-3.020	-55.035	-42.553
Standard deviation	0.136	0.184	4.207	3.395

Table 4.--Continued

SHIFT STATISTICS
(NAD 83 - NAD 27)

ST. PAUL ISLAND	Latitude (arc seconds)	Longitude (arc seconds)	Latitude (meters)	Longitude (meters)
Range				
Minimum	2.122	8.092	65.650	136.605
Maximum	2.762	8.650	85.453	142.080
Average	2.454	8.436	75.918	140.482
Standard deviation	0.189	0.141	5.839	1.327

In general, NADCON is appropriate for all applications where coordinate data require transformation between NAD 27 and NAD 83. Users requiring additional accuracy, perhaps for geodetic surveying, should always reacquire data or readjust original observations. However, NADCON has been successfully used to transform coordinates arising from GPS surveys in NAD 83 to NAD 27 for subsequent classical surveys within a NAD 27 framework at the third-order level (P. Crabtree, Airport Surveys Section, NOS, personal communication).

NADCON has proven to be extremely useful for obtaining NAD 27 coordinates derived from GPS surveys. These surveys are adjusted in a NAD 83 framework and coordinates are subsequently transformed into NAD 27 for publication. The results have proven more reliable than attempting an adjustment of GPS vectors in NAD 27. Most probably this is a result of the superior accuracies of GPS compared to the distortions of NAD 27. GPS vectors often exceed the accuracies of even first-order control stations within NAD 27. This technique has become routine at NGS.

Cartography provides an extremely appropriate application for NADCON. NADCON results can be readily used for the conversion of maps at a variety of scales. Figure 6 shows the relative accuracies required at various scales to meet National Mapping Accuracy Standards (U. S. Geological Survey/National Oceanic and Atmospheric Administration, 1978) within the Federal Government. It is clear from this figure that NADCON transformations meet these standards at scales as large as approximately 1:200. This provides the GIS/LIS community with a digital transformation method necessary to avoid datum inconsistencies. Datum transformations must be sufficiently accurate to avoid falsely distorting the overall depiction of a display, diagram, or map. In general, GIS/LIS systems allow for zooms, pans, and rubber-sheeting at various scales, often on a continuous basis. NADCON would permit a user to closely examine a small area within a GIS/LIS, even when data from differing sources and datum are present.

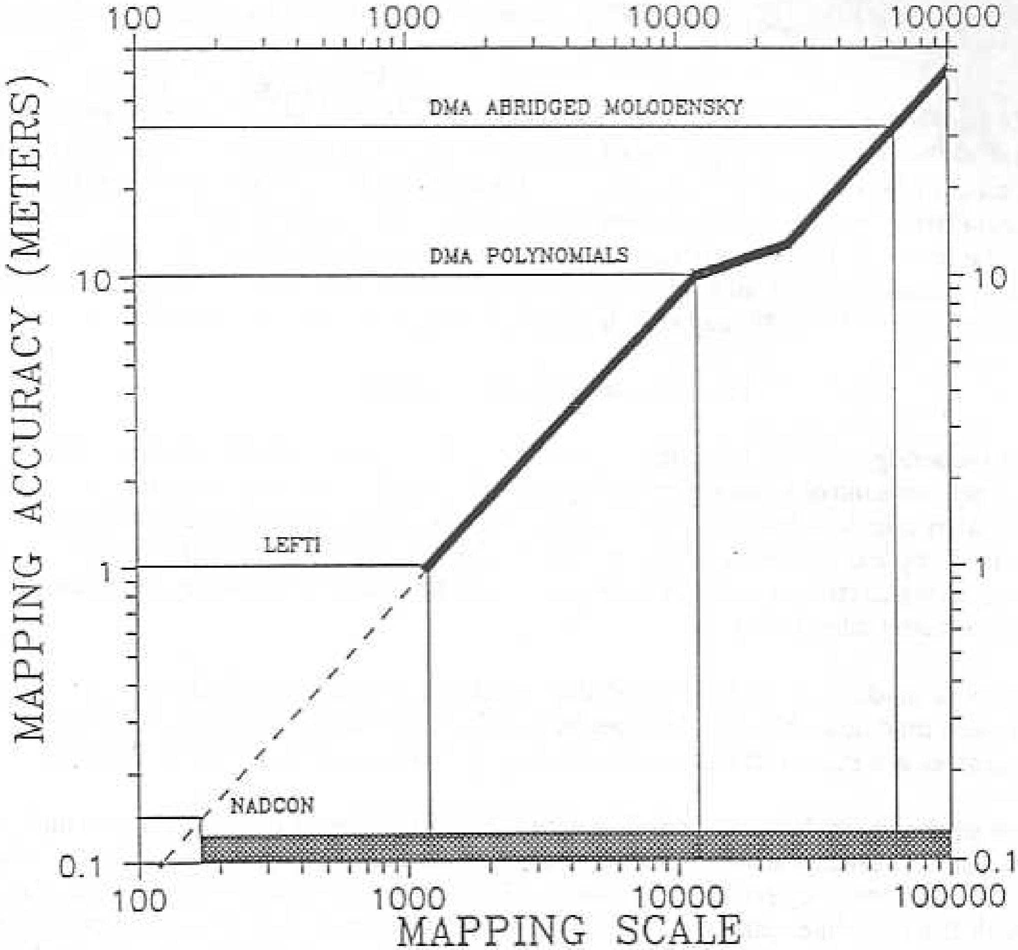

Figure 6.--Graph of National Map Accuracy Standards as established by the U.S. Geological Survey and the National Ocean Service, NOAA (1978). The graph compares various transformation methods, two from the Defense Mapping Agency (1987) and two from the National Geodetic Survey. X-axis, mapping scales are expressed as the denominator of a scale fraction (e.g., a value of "10000" indicates a mapping scale of 1:10,000). The shaded area indicates an approximate continuum of scales at which transformations performed with NADCON would meet the accuracy standards.

Again, NADCON will not, nor will any procedure, improve inferior data. Users must always be aware of the limitations inherent with the original accuracies. A detailed examination of data obtained from digitizing a 1:100,000 scale manuscript will often reveal a jagged line and the individual points digitized rather than a better view of a map. At the very least, however, NADCON will not further complicate the situation by falsely distorting the relationship of lines and points among themselves.

GIS/LIS systems require and depend upon large quantities of coordinate data. Hundreds of thousands, and often millions, of coordinates are necessary to depict an area. For example, the Tiger files available from the Census Bureau incorporate more than 311 Mbytes of data for the State of Colorado alone. Such volume is typical. Any transformation procedure must be fast enough to accommodate the transformation of a complete data base in a reasonable length of time.

NADCON has been successfully employed to transform a data base containing more than 150,000 coordinates--a subset of Tiger data for the Denver, Colorado, region--in less than 30 minutes on an IBM 80386-based microcomputer. Depending upon the size of a GIS/LIS coordinate data base, the conversion process could perhaps be accomplished within a few hours. Remember, the conversion process occurs only once; repeated computations are unnecessary. Additional transformations are only necessary when different NAD 27 data become available. Thus, the conversion to NAD 83 may only involve a few hours of computational time.

Land Versus Offshore Estimates

NADCON modeling employed the data within the National Geodetic Reference System. These data form the control upon which surveying and mapping in this country are based. The coordinates within this data base are, for the most part, on stable ground and in all cases monumented. Temporary stations or way points are not considered part of the NGRS. Data close to shore, along coastlines, were included, provided they met the accuracy requirements of the other surrounding inland stations.

No attempt was made to artificially "pad" the raw shift data with pseudo-data. Only official NGRS data were modeled. NADCON is perfectly suited to obtaining estimates on land. Offshore estimates are more difficult to predict, largely due to the lack of data offshore.

Predictions offshore are, however, possible with NADCON, due to the modeling technique. These estimates will reflect the overall trend of the differences between the two datums, with generally decreasing accuracy farther from shore. The offshore estimates will yield results consistent with the shoreline stations, but with more emphasis placed on long-wavelength trends. In general, the scale of nautical charts and other Federal offshore mapping products tends to decrease with distance offshore. Scales of 1:80,000 are not uncommon. In fact, such small scale maps may not require conversion to NAD 83 at all, since the map accuracy at that scale is worse than the magnitude of the datum shift. Although testing results offshore are scarce, NADCON results compare favorably at the 1- to 3-meter level in the Gulf of Mexico, compared to data of the Shell Oil Company (Henk Krijnen, private communication) and other private concerns offshore (L. Harold Spradley, National Ocean Industries Association, private communication). It should be noted that these comparisons involve unverified and independent data, separate from the NGRS. They were not used in the modeling. Estimates in Alaska may be somewhat worse, due to lack of adequate control, but still should be sufficiently accurate for Federal land management and cartographic purposes.

DISCUSSION

As previously addressed, NAD 27 and NAD 83 do not exactly map into one another. NADCON can be criticized for being an interpolation technique rather than a more traditional geodetic computation. This is true, but no "traditional" geodetic transformation technique can adequately accommodate NAD 27 distortions.

Minimization of total curvature guarantees smoothness and "gap filling" in areas of sparse data, without the specific knowledge of covariance functions required by collocation.

The estimation of shift data in regions of sparse coverage, such as far out at sea, is a difficult problem. No technique is sufficiently capable of providing estimates where no original data exist. Some information must be available for any technique to work.

NADCON provides transformations in most areas with great reliability. The quality of these transformations is directly related to the quality and coverage of the first- and second-order geodetic control incorporated within the NAD 83 adjustment. Users should expect the accuracy of a transformation to decline in inverse proportion to the distance away from good control. For example, the transformation of coordinates that are tens to hundreds of miles at sea can be expected to be no better than several meters, or perhaps in a worst case situation several tens of meters, while the transformation of data within the conterminous U.S. land mass can be expected to be much better than 1 m. NADCON results meet the requirements of the National Mapping Accuracy Standards (U. S. Geological Survey/National Oceanic and Atmospheric Administration 1978).

SUMMARY

The author has designed a transformation technique that facilitates the conversion of quantities of coordinate data from the North American Datum of 1927 to the North American Datum of 1983. This approach relies upon modeling first- and second-order geodetic data, originally involved with the creation of NAD 83, and interpolating shift values as correctors to desired points. The modeling method employs the minimization of total curvature associated with surfaces defining the differences between datums.

The accuracy of the datum transformations is a function of original data quality as well as proximity of existing control to new or desired points. In general, transformations can be expected at the 15-centimeter level for land mass of the conterminous United States and several meters far offshore. These accuracies are believed to be sufficient for most of the Nation's land surveying and mapping requirements. In all, NADCON results meet National Mapping Accuracy Standards and should be appropriate for a vast number of applications.

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APPENDIX A.--COMPARISON RESULTS FOR EACH OF THE NADCON REGIONS.

Each table represents a comparison of NADCON results against the original data used in the modeling for that area. A total of seven regions is represented.

Table A1.--Comparison results for the conterminous United States (CONUS)

Number of comparisons: 115296

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-8.774	3.388	-8.101	4.172	0.000	10.200
Range of differences (seconds)	-0.285	0.109	-0.381	0.165	0.000	0.381
Mean differences (meters)	-0.001		0.002		0.136	
Std. dev. of mean differences	0.159		0.148		0.170	
Mean differences (seconds)	0.000		0.000		0.005	
. Std. dev. of mean differences	0.005		0.006		0.006	

Table A2.--Comparison results for Alaska

Number of comparisons: 16284

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-6.905	8.646	-12.215	6.339	0.000	12.290
Range of differences (seconds)	-0.223	0.279	-1.131	0.416	0.000	1.132
Mean differences (meters)	0.003		-0.003		0.299	
Std. dev. of mean differences	0.479		0.463		0.596	
Mean differences (seconds)	0.000		0.000		0.014	
Std. dev. of mean differences	0.015		0.030		0.031	

Table A3.--Comparison results for Hawaii

Number of comparisons: 1257

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-0.989	0.974	-0.930	0.980	0.001	1.175
Range of differences (seconds)	-0.032	0.032	-0.032	0.034	0.000	0.040
Mean differences (meters)	0.047		-0.006		0.152	
Std. dev. of mean differences	0.215		0.147		0.217	
Mean differences (seconds)	0.002		0.000		0.005	
Std. dev. of mean differences	0.007		0.005		0.007	

Table A4.--Comparison for Puerto Rico and the Virgin Islands

Number of comparisons: 873

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-0.222	0.158	-0.241	0.266	.000	.297
Range of differences (seconds)	-0.007	0.005	-0.008	0.009	.000	.010
Mean differences (meters)	0.000		-0.002		0.040	
Std. dev. of mean differences	0.041		0.045		0.046	
Mean differences (seconds)	0.000		0.000		0.001	
Std. dev. of mean differences	0.001		0.002		0.002	

Table A5.--Comparison results for St. George Island

Number of comparisons: 23

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-0.030	0.088	-0.028	0.040	.002	.088
Range of differences (seconds)	-0.001	0.003	-0.002	0.002	.000	.003
Mean differences (meters)	0.003		0.001		0.027	
Std. dev. of mean differences	0.028		0.018		0.019	
Mean differences (seconds)	0.000		0.000		0.001	
Std. dev. of mean differences	0.001		0.001		0.001	

Table A6.--Comparison results for St. Laurence Island

Number of comparisons: 146

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-0.094	0.127	-0.081	0.091	0.001	0.127
Range of differences (seconds)	-0.003	0.004	-0.006	0.007	0.000	0.007
Mean differences (meters)	0.001		-0.001		0.022	
Std. dev. of mean differences	0.025		0.017		0.022	
Mean differences (seconds)	.000		.000		.001	
Std. dev. of mean differences	.001		.001		.001	

Table A7.--Comparison results for St. Paul Island

Number of comparisons: 41

	Latitude		Longitude		Magnitude	
	MIN	MAX	MIN	MAX	MIN	MAX
Range of differences (meters)	-0.131	1.007	-0.263	0.280	0.013	1.031
Range of differences (seconds)	-0.004	0.032	-0.016	.016	.000	0.035
Mean differences (meters)	0.041		0.004		0.125	
Std. dev. of mean differences	0.174		0.101		0.162	
Mean differences (seconds)	0.001		0.000		0.006	
Std. dev. of mean differences	0.005		0.006		0.006	